

Distance Measurement and Wave Dispersion in a Liouville-String Approach to Quantum Gravity

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Abstract

Within a Liouville approach to non-critical string theory, we discuss space-time foam effects on the propagation of low-energy particles. We find an induced frequency-dependent dispersion in the propagation of a wave packet, and observe that this would affect the outcome of measurements involving low-energy particles as probes. In particular, the maximum possible order of magnitude of the space-time foam effects would give rise to an error in the measurement of distance comparable to that independently obtained in some recent heuristic quantum-gravity analyses. We also briefly compare these error estimates with the precision of astrophysical measurements.

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1 Introduction

Two of the most important problems of Modern Physics are the lack of a consistent quantum theory of gravity, and a satisfactory description of the quantum measurement process. As regards the first of these problems, so far there is no mathematically consistent local field theory of gravity which is compatible with quantum mechanics. Although the classical limit of gravity, General Relativity, has been verified experimentally with good accuracy, profound difficulties arise when one seeks to combine it with the quantum features of our world. These difficulties have many aspects. The absence of a (renormalizable) path integral for quantum geometries in four space-time dimensions is a very basic formal issue [1]. Puzzles and inconsistencies arise even in certain weak-coupling limits of general relativity with quantum mechanics. One such example is the semiclassical quantization of macroscopic black holes: when one quantizes the fields of low-energy point particles in such backgrounds, there appears Hawking radiation due to quantum evaporation of the black hole [2]. The latter results in entropy production and therefore the apparent evolution of pure quantum-mechanical states to mixed ones. As the macroscopic black hole evaporates, the energy and entropy of the radiation field increases. Obviously, such a situation *cannot* be described by treating the quantum field theory subsystem as *closed*. Indeed, the general view is that there is information loss across the event horizon surrounding the black hole.

This apparent difficulty also presents another problem associated with the ill-defined nature of the quantum-gravitational path-integral measure. If *microscopic* Planck-scale fluctuations of the black hole type exist, which is to be expected since they exist classically, the possibility arises then that the associated Hawking radiation will affect the purity of quantum-mechanical states interacting with this quantum-gravitational space-time foam [3, 4]. At present there is no known consistent way of formulating scattering-matrix elements for the propagation of quantum matter incorporating recoil effects in black hole backgrounds. And the black hole is but one example of a topologically non-trivial gravitational fluctuation: there are likely to be others that could cause similar or worse problems.

The measurement problem is one of the great mysteries of quantum theory itself, even if gravitational interactions are ignored. Quantum mechanics is a non-deterministic theory, whereas the physical laws governing the macroscopic limit of the observable world have deterministic formulations. Many physicists have attempted to shed some light on this question by exploring various mechanisms that could lead to the transition from the microscopic quantum world to the macroscopic classical world. The most influential idea in the field is that of the *decoherence* of a quantum system in interaction with a macroscopic measuring apparatus [5, 6]. The idea is that, during a measurement process, the quantum system is an *open* system, and as

such it should be described by a density matrix. After some time t_D , characteristic of the system, the off-diagonal elements of its density matrix decay *exponentially* in time. The time t_D is inversely proportional to the number of ‘atoms’ of the measuring device [6]. Hence, the collapse time for macroscopic systems is diminished significantly, and the transition to classicality is achieved.

Until recently, there were objections [5] to this approach to classicality based on the argument that the decoherence (or density matrix) approach refers to an ensemble of theories, and hence does not describe correctly the behaviour of a *single* quantum mechanical system, which should in principle be described by a state vector. As a counterargument, we would emphasize that the above approach - which is more general than the state vector approach - is the correct one, since the interaction with the measuring device opens up the microscopic system. This means that one must average over possible histories in the path integral, and hence over theories [7]. The state-vector picture is inadequate in such a case, whilst the density-matrix picture is an appropriate description.

A mathematically satisfactory approach which formally reconciles the two pictures has been developed recently in ref. [8]. It was shown that, for quantum mechanical systems in interaction with an environment whose details are irrelevant to a major extent, a state-vector picture can be written down, but the time evolution is *not* of Schrödinger type. Instead, it resembles *stochastic* evolution of the Ito type [8], known previously for open statistical systems. The importance of this formulation is that it leads, under some rather generic assumptions, to *localization* of the quantum system in one of its states [8]. This localization occurs *independently* of the presence of a measuring apparatus, and may be viewed as a state-vector representation of the environment-induced decoherence previously described in the density matrix approach [4, 6]. According to the theorem of ref. [8], the passage to classicality - in the above sense - will occur for *all* non-isolated quantum systems, i.e., for all realistic physical systems in our world. However, this approach does not deal with the accuracy (uncertainty) in the measurement. It is important to note, as we shall discuss later on, that localization in this state-vector approach does *not necessarily* imply that the uncertainty in, say, length measurements, will be reduced as a result of decoherence. On the contrary, the opposite seems true. Indeed, it has been known for some time that decoherence effects may lead to uncertainties in length measurements that depend on the size of the distances measured [9].

It seems natural, in the light of the above discussion, to expect that quantum gravity, in solving the apparent incompatibility between quantum mechanics and classical gravity, leads to a mechanism for decoherence [6, 4, 10]. This can be realized by thinking of quantum gravity as providing a space-time ‘medium’ through which low-energy quantum-mechanical particles propagate [11]. The suggestion that quantum gravity

induces decoherence finds encouragement in some analyses of thought experiments, in which the only assumption was that quantum gravity should reproduce smoothly quantum mechanics and classical gravity in their respective limits of validity. These analyses indicate [9, 12, 13, 14] that a length L can only be defined operationally in quantum gravity up to an uncertainty $\min\{\delta L\}$ that is L -dependent, with the largest contribution to this uncertainty [13, 14] growing with L like \sqrt{L} . Moreover, it is known [12, 9] that such an L -dependence in $\min\{\delta L\}$ can be associated with decoherence effects.

In recent years, a candidate quantum-gravity model leading naturally to decoherence has been provided by a non-critical formulation of string theory, exemplified by a treatment of quantum black holes [11]. At present, string is the only apparently consistent quantum theory incorporating gravity along with the rest of the fundamental interactions. It should be noted that non-critical strings constitute a more general framework than critical ones, and that their precise quantum structure is still not known. The approach of ref. [11] includes a treatment of the nature of time in such non-critical strings. It has been argued [11] that time may be identified with a renormalization-group scale on the world sheet of the first-quantized version of the string. As such, the passing of time is associated with entropy production, reflecting information loss across event horizons, microscopic as well as macroscopic.

In this picture, space-time foam is a medium of global, non-propagating gravitational stringy degrees of freedom, which interact with propagating low-energy particles via stringy gauge symmetries. In the specific example of two-dimensional black hole studied in ref. [11], these global Planckian modes fail to decouple from the propagating low-energy modes in the presence of microscopic black-hole space-time backgrounds. Quantum coherence can be preserved only when the infinite set of such modes are taken into account. This can be done in the two-dimensional quantum field theory on the world sheet, but local space-time scattering experiments cannot detect these non-propagating gravitational modes. Thus, the low-energy world is effectively an *open* system, and decoherence arises in a stochastic way for specifically *stringy* reasons [11]. This is the first explicit realization of the idea that the peculiar features of quantum gravity may provide a ‘medium’ for the propagation of ordinary matter. Attempts have been made to use these two-dimensional stringy black-hole models as building blocks in the construction of realistic four-dimensional quantum-gravity backgrounds in string theory [11, 15]. This is still an open issue, though, and should not be considered as complete at this stage. For our purposes, however, we shall view the two-dimensional non-critical string [11] as a concrete example of a stochastic formulation of quantum gravity.

In this paper we discuss issues related to uncertainties in measurement in the presence of quantum gravity entanglement of the stochastic type mentioned above.

We shall derive bounds on the precision with which lengths can be measured, that are in agreement with the general arguments concerning quantum-gravity-induced decoherence which were mentioned above. The structure of the article is as follows: in section 2 we review the results of refs. [13, 14] on the growth of the measurement error $\min\{\delta L\}$ with L like \sqrt{L} . Then, in section 3 we review the type of vacuum entanglement present in the non-critical string theory of ref. [11], discuss from various points of view its maximum possible order of magnitude, and motivate the ensuing modification of the state-vector formalism of conventional quantum mechanics. As we discuss in more detail in section 4, this suggests that low-mass particle waves may experience dispersion as they propagate through the space-time foam. If this has the maximal order of magnitude discussed earlier, we show in section 5 that it leads naturally to $\min\{\delta L\} \sim \sqrt{L}$ as in the previous approach to combining quantum mechanics and classical gravity.

This striking observation adds support to both approaches. It is important for the generic approach of ref. [14], because it provides a candidate microscopic theory which leads to bounds on measurement errors of similar order of magnitude to those derived previously on the basis of heuristic considerations on the low-energy limit of quantum gravity. For the string framework [11], it is important because the derivation of the measurement bounds relies, as we shall see, on an as yet unverified assumption on the *maximal* order of magnitude of quantum-gravity effects. The compatibility of the bounds derived within the two different approaches constitutes an additional argument in favour of the correctness of these order-of-magnitude estimates for low-energy physics. Section 6 contains a summary of our results, presents some conclusions, and compares the distance uncertainty derived here with various astrophysical limits.

2 Bounds on the Possible Accuracy of Distance Measurements from Heuristic Considerations in Quantum Gravity

Difficulties in introducing local observables in quantum (and even classical) gravity originate from the fact that general coordinate invariance washes away the previous specification of individual points of the space-time manifold using a coordinate system. This specification of individual points can be regained by labelling them using particles of matter¹, *i.e.*, introducing a material reference system (MRS), but then one must take into account the dynamics of this matter and its back-reaction on the space-time manifold [16].

¹This is an idea which has been discussed extensively in the quantum gravity literature. Refs. [16, 17] recently reviewed this subject, presenting various viewpoints.

The problem of measuring distances in such a conceptual framework was discussed in ref. [14], and bounds were derived on the accuracy with which distances can be measured in an abstract MRS composed of identical bodies, defining the space points, with some internal physical variables, defining the time instants [16]. The measurement procedure considered in ref. [14] (see also ref. [13]) involves the exchange of a “light signal”, *i.e.*, a signal propagating at the speed of light and so composed of massless particles, between two of the bodies composing the MRS. Using 40-year-old purely quantum-mechanical arguments [18], it was shown that the uncertainty in measuring the distance between two such MRS bodies must satisfy the following inequality:

$$\delta L \geq \sqrt{\frac{L}{M}}, \quad (1)$$

where M is the common mass of the identical bodies composing the MRS, and we use natural units such that $\hbar=c=1$. The condition (1) follows from the fact that, as their mass is increased, the bodies of the MRS behave more and more classically with respect to the other systems (clocks, detectors, etc.) participating in the measurement process. The classical laboratory limit is reached [14], in complete consistency with quantum mechanics², in the $M \rightarrow \infty$ limit, in which case it is possible to have $\delta L \sim 0$.

However, as discussed in ref. [13, 14], the ideal scenario $\delta L \sim 0$ cannot be realized in presence of gravitational interactions, since the required large values of the mass M necessarily lead to great distortions of the geometry. Well before reaching the $M \rightarrow \infty$ limit, the measurement procedure can no longer be accomplished in the way envisaged in conventional quantum mechanics. In particular, the minimal requirement[14] that a horizon should not form around the bodies constituting the MRS leads to the condition

$$M \leq \frac{s}{L_P^2}, \quad (2)$$

where s is the common size of the bodies. Eqs. (1) and (2) combine to yield the following measurability bound:

$$\delta L \geq \sqrt{\frac{LL_P^2}{s}}. \quad (3)$$

In particular, since $L_P \sim 10^{-33}$ cm is expected to be the minimum length attainable in a single-scale theory of quantum gravity such as may be provided by string theory,

²Conventional quantum mechanics is a theory that predicts the results of measurements of (quantum) observables by some classical (infinitely-massive) apparatus. In this non-gravitational quantum-mechanical framework there are no measurability bounds: any observable can be measured with perfect accuracy, at the price of renouncing all information on its conjugate observable. The effects we discuss in this paper go beyond this traditional uncertainty principle.

it is interesting to consider the special case $s \sim L_P$. In this case, eq. (3) reduces to

$$\delta L \geq \sqrt{LL_P} . \quad (4)$$

As we shall see shortly, a bound of the same order may be obtained from entirely different considerations within certain stochastic approaches to quantum gravity.

3 Space-Time Foam Effects on the Propagation of Low-Energy Particles

In this section we shall take a different approach, discussing the propagation of low-energy particles through space-time foam in the light of an analysis of possible coherence loss in quantum gravity, which is supported by one formulation of non-critical string theory [11].

As we mentioned briefly in the Introduction, in this approach we identify time with a Liouville field which serves as a (local) renormalization-group scale for non-critical strings, leading to a stochastic evolution in time which causes initially-pure quantum-mechanical states to become mixed, with accompanying entropy production [11]. A concrete example of such evolution at a microscopic level is provided by two-dimensional target-space string theory in black-hole backgrounds [19, 11]. In this theory, the only propagating degrees of freedom, to be associated with observables in local scattering experiments, are massless scalar fields. However, due to the persistence of infinite-dimensional stringy W_∞ gauge symmetries in target space, there are non-trivial couplings between the propagating degrees of freedom and the ‘environment’ of global, delocalized stringy states, which may be interpreted as solitons and figure among the higher-level string states [11]. These couplings disappear in flat space-times, and appear only in the presence of microscopic fluctuations in the space-time background, represented in this case by black holes. This phenomenon constitutes an explicit realization of the idea that quantum gravity generates a space-time foam ‘medium’, through which low-energy matter propagates with non-trivial couplings. It is suggested that there are observable consequences of this evolution, which follow from the fact that low-energy dynamics will be inherently *open*, due to its couplings to quantum-gravitational degrees of freedom, which find an explicit model representation. In this approach, time itself is a measure of information loss/entropy production due to the interaction with this microscopic environment. As a result, an open-system modification of conventional quantum-mechanical time evolution, with genuine *CPT* violation [20], will characterize the temporal evolution of the low-energy (observable) world. The stochastic nature of the quantized renormalization-group flow on the world sheet yields stochastic time evolution according to this picture.

We provide below a brief review of the corresponding modifications of low-energy quantum mechanics believed to be induced by such an approach to quantum gravity, specifically in the context of the non-critical string analysis of ref. [11]. Our intention here is only to recapitulate briefly the basic results of the approach for the benefit of the non-workers in the field, and to estimate the maximum possible order of magnitude of deviations from conventional quantum mechanics. Further details can be found in ref. [11].

The evolution of pure states into mixed ones necessitates the introduction of density matrices $\rho(t)$ for the description of low-energy dynamics [4]. The modified form of temporal evolution may be deduced [11] from the renormalization-group invariance of physical quantities in non-critical string theory, and may be written as a modified quantum Liouville equation for $\rho(t)$ [11, 21]:

$$\begin{aligned}
Q \frac{\partial}{\partial t} \rho &\equiv Q \dot{\rho} = i[\rho, H] + \delta \mathbb{H} \rho & ; & \quad \delta \mathbb{H} \equiv i Q \dot{g}^i G_{ij} [g^j,] \\
Q &= \sqrt{\frac{|25 - c[g, t]|}{3}} & & (5)
\end{aligned}$$

where the g^i are fields representing string couplings, G_{ij} is a metric on the space of these couplings, and $c[g]$ is the running central charge of the non-critical string. It is important to stress that the first-quantized approach to non-critical strings entails quantization of the background fields g^i , as a consequence of resummation over higher-genus world-sheet topologies in the Polyakov path integral over string histories. The quantities $Q \dot{g}^i \equiv \beta^i$ are the world-sheet renormalization-group β -function coefficients in a formulation of non-critical string theory in terms of general two-dimensional field theories (σ models) on the world sheet. In critical string theory, the coefficients β^i vanish, as a result of world-sheet conformal invariance. In the presence of fluctuations in the space-time background, such as quantum black holes, however, apparent conformal invariance is lost for a low-energy observer who observes only propagating fields. From the explicit example of two-dimensional (spherically-symmetric four-dimensional) stringy black holes, we know that conformal invariance is guaranteed only if the infinite set of global string modes is turned on as backgrounds. The latter cannot be observed by localized scattering experiments. When these are not taken into account, the result is that the effective β^i do not vanish, inducing Liouville field- (scale-) dependence of the renormalized coupling/fields g^i , as described in ref. [11]. In a quantum theory of non-critical strings, such apparent violations of world-sheet conformal invariance are allowed, in the sense that one can dress up the theory with Liouville scale factors so as to restore conformal symmetry. It is this dressing that leads to the flow of time in the interpretation of ref. [11, 21], and it is the semi-group nature of the renormalization group that causes this time flow to have an arrow, accompanied by coherence loss and entropy gain.

At present we are not in a position to calculate reliably the order of magnitude of any such deviation from conformal invariance. However, various different methods point towards similar estimates, as we now discuss.

- In the *non-critical-string* framework [11], within which the mechanism for the ‘opening’ phenomenon is traceable directly to Planckian physics, we expect that the string σ -model coordinates g^i obey renormalization-group equations of the general form

$$Q\dot{g}^i = \beta^i M_P \quad : \quad |\beta^i| = \mathcal{O}\left(\frac{E^2}{M_P^2}\right) \quad (6)$$

where the dot denotes differentiation with respect to the target time, measured in string ($L_P = M_P^{-1}$) units, and E is a typical energy scale in the low-energy observable matter system. Since the metric G_{ij} and the couplings g^i are themselves dimensionless numbers of order unity, we expect that

$$|\delta\mathbb{H}| = \mathcal{O}\left(\frac{E^2}{M_P}\right) \quad (7)$$

in general. This is a *maximal* estimate for such effects. Formally, (6) stems from a (target-space) derivative expansion of the closed-string β^i , of the generic form $\sum_{n=1}^{\infty} c_n (\alpha' \partial_{X^M}^2)^n$, where we keep only the first term. However, the coefficient c_1 is not at present known for any physical system. There are expected to be system-dependent numerical factors that depend on the underlying string model, $|\delta\mathbb{H}|$ might be suppressed by further (E/M_P) -dependent factors, and it could even vanish for some systems. Nevertheless, equation (7) gives us a natural order-of-magnitude estimate for the decoherence effects due to stringy quantum gravity in a generic physical system.

- More heuristic *quantum gravity* considerations yield a similar order-of-magnitude estimate. The term proportional to $\delta\mathbb{H}$ in (5) can be interpreted as a non-canonical contribution to the rate of change of the density matrix ρ due, in some sense, to ‘scattering’ off evanescent black holes [4], which one might be able to estimate by the generic formula

$$\delta(Q\rho) \simeq \sigma \mathcal{N} \rho \quad (8)$$

where σ is the ‘scattering cross section’, and \mathcal{N} is the number density of microscopic ‘target’ black holes. Within this heuristic approach, it is natural to expect that σ would be proportional to the square of some ‘scattering amplitude’ obtainable from some effective particle/black hole interaction. The latter could be expected to contain a factor $\mathcal{O}(1/M_P^2)$, as for example in a four-fermion interaction of the generic form [4, 6]

$$\mathcal{L} = \mathcal{O}\left(\frac{1}{M_P^2}\right) \bar{\psi}\psi\bar{\psi}\psi a_{BH} + h.c. \quad (9)$$

where the ψ are fermionic fields, a_i is a black hole annihilation operator, and the corresponding creation operator appears in the Hermitian conjugate term in (9). Squaring the generic ‘scattering amplitude’ generated by the interaction (9) yields a factor $\mathcal{O}(1/M_P^4)$ in the cross section σ in (8). We would expect that the black hole density

$$\mathcal{N} = \mathcal{O}(M_P^3) \tag{10}$$

corresponding to $\mathcal{O}(1)$ microscopic Planck-mass black hole per Planck volume in space, in much the same way as one expects one QCD instanton per unit volume $\simeq \Lambda_{QCD}^3$ in the strong-interaction vacuum. We know of no evidence from generic quantum-gravity considerations, or more specifically from string theory, which would argue for any parametric suppression of the generic estimate (10), e.g., of the form $e^{-1/\alpha}$ or $e^{-1/\sqrt{\alpha}}$. Combining (9) and (10), therefore, we arrive [6] at the estimate

$$\delta(Q\dot{\rho}) = \mathcal{O}\left(\frac{E^2}{M_P}\right) \tag{11}$$

where the energy factors in the numerator follow from naive dimensional analysis.

- The above estimates receive some support from a *semi-classical* black-hole calculation, in which a scalar field with gauge interactions is quantized in a four-dimensional black-hole background. Calculating the dependence of the density matrix for the external scalar field on the radius of the black-hole horizon, and recalling that the latter evolves with time as a result of Hawking radiation, we have arrived [22] at the same order-of-magnitude estimate for the ‘open’ term in the modified quantum Liouville equation (5).

The next step is to explore the consequences of this estimate for the propagation of what would normally be a coherent quantum-mechanical wave. Although we have used above the density-matrix formalism, which is in many applications the most appropriate for a description of mixed states, it is also possible to cast the problem of environmental entanglement in a state-vector formalism, and this turns out to be instructive for a first discussion of wave propagation. We follow the approach due to Gisin and Percival [8], which involves stochasticity. As we argued above, stochasticity is an essential feature of the identification of target time with a (stochastic) renormalization group scale variable on the world sheet.

The Lindblad formalism [23, 4] admits a state-vector picture involving a modified Schrödinger equation [8]. If $|\Psi\rangle$ is the state vector, environmental entanglement

may be represented as a *stochastic* differential Ito process for $|\Psi\rangle$ [8]:

$$Q|d\Psi\rangle = -\frac{i}{\hbar}H|\Psi\rangle dt + \sum_m (\langle B_m^\dagger \rangle_\Psi B_m - \frac{1}{2}B_m^\dagger B_m - \frac{1}{2}\langle B_m^\dagger \rangle_\Psi \langle B_m \rangle_\Psi) |\Psi\rangle dt + \sum_m (B_m - \langle B_m \rangle_\Psi) |\Psi\rangle d\xi_m \quad (12)$$

where H is the Hamiltonian of the system, and B, B^\dagger are ‘environment’ operators. In our case, these may be defined as appropriate ‘squared roots’ of the various partitions of the operator $\beta^j G_{ij} \dots g^i$ [11], and the $d\xi_m$ are complex differential random variables, associated with stochastic white-noise Wiener or Brownian processes, which represent environmental effects averaged out by a low-energy local observer. In our quantum-gravity/string-theory picture, such effects are due to global Planckian modes [11] of the string which cannot be detected by local scattering experiments. As such, these effects are therefore inherently stringy/quantum gravitational.

It is clear from (12) that among the effects of the environmental/quantum-gravity entanglement represented by the operators B are extra, frequency-dependent phase shifts in the Schrödinger wavefunction for the modes g^i , defined by $\langle g^i | \Psi \rangle = \Psi_{g^i}$, as well as attenuation effects on the amplitudes of the waves. All such effects are expected to be of the generic order of magnitude $O[B^\dagger B] = O[\beta^i \dots G_{ij} g^j]$. The frequency-dependent attenuation effects can be shown to lead to decoherence, in the sense of an exponential (with time) vanishing of the off-diagonal elements of the density matrix [6] in (5). In the case of interest here, the precise order of such effects can be found in principle by solving (12) in the case of massless modes propagating in a stringy quantum gravity medium, as appropriate for the *gedanken* experiments of section 2.

4 Wave Dispersion in Non-Critical String Backgrounds

A more detailed discussion of the possible dispersive effects of space-time foam on wave propagation requires a deeper discussion of the non-critical string approach [11] than we have provided so far in this paper. We will now recall some relevant features of this formalism, and then explore their specific consequences for wave dispersion.

An appropriate quantum formalism for discussing non-critical string is that of generalized σ models on a fixed lowest-genus topology of the world sheet, with dressing by a Liouville field ϕ , which in the interpretation of ref. [24, 11, 21] plays the rôle

of target time. The relevant formalism is that of non-conformal deformations V_i of σ -model actions $S[g]$:

$$S[g] = S[g^*] + \int d^2z \sqrt{\gamma} g^i V_i \quad (13)$$

where the $\{g^i\}$ denote massless background fields in the string target space, $\{g^*\}$ represents an equilibrium conformal solution around which we perturb, and γ is the world-sheet metric. In the approach of ref. [11], for generic curved world sheets the renormalization scale may be taken as a local function on the Riemann surface, which then is quantized to incorporate fluctuating topologies. This necessitates the introduction of the Liouville mode ϕ , which in this approach plays the rôle of a covariant quantum renormalization scale [11, 21]: $\gamma = e^\phi \gamma^*$, where γ^* is a reference world-sheet metric. The renormalization-group picture we adopt is that of Wilson, according to which non-trivial scaling follows from integrating out degrees of freedom in an effective theory consisting of the massless modes $\{g^i\}$ only.

Coupling the theory (13) to two-dimensional quantum gravity restores conformal invariance at the quantum level, by making the gravitationally-dressed operators $[V_i]_\phi$ *exactly* marginal, *i.e.*, ensuring the absence of any covariant dependence on a world-sheet scale. The final result for the gravitationally-dressed matter theory is then

$$S_{L-m} = S[g^*] + \frac{1}{4\pi\alpha'} \int d^2z \{ \partial_\alpha \phi \partial^\alpha \phi - QR^{(2)} + \lambda^i(\phi) V_i \} \quad (14)$$

where α' is the inverse of the string tension. To order $O(g^2)$, the gravitationally-dressed couplings $\lambda(\phi)$ are given by:

$$\lambda^i(\phi) = g^i e^{\alpha_i \phi} + \frac{\pi}{Q \pm 2\alpha_i} c_{jk}^i g^j g^k \phi e^{\alpha_i \phi} + \dots \quad (15)$$

with

$$\alpha_i^2 + \alpha_i Q = \text{sgn}(25 - c)(h_i - 2) \quad (16)$$

where the $h_i - 2$ are the scaling dimensions of the operators V_i before Liouville dressing. The operator product expansion coefficients c_{jk}^i are defined as usual by coincident limits of the products of pairs of vertex operators V_i :

$$\lim_{\sigma \rightarrow 0} V_j(\sigma) V_k(0) \simeq c_{jk}^i V_i\left(\frac{\sigma}{2}\right) + \dots \quad (17)$$

where the completeness of the set $\{V_i\}$ is assumed.

From the quadratic equation (16) for α_i , only the solution

$$\alpha_i = -\frac{Q}{2} + \sqrt{\frac{Q^2}{4} - (h_i - 2)} \quad (18)$$

for $c \geq 25$ is kept, due to the Liouville boundary conditions. It is worth noticing that the case $c \geq 25$ automatically implies a Minkowski signature for the Liouville mode ϕ , enabling it to be interpreted as target time [11, 24]. Indeed, equation (5) was derived [11] by identifying the physical Minkowski time t with $-i\phi$, an identification which has been supported by model black-hole calculations.

The structure of eq. (15) suggests that the effects of the non-criticality are quite complicated in general. However, the form of the Liouville time dependence implies that one of the physical effects of the non-criticality is a modification of the time-dependence of the λ^i , which may be described to order $O(g^2)$ by the following approximation to (15):

$$\lambda^i(t) \sim g^i e^{i(\alpha_i + \Delta\alpha_i)t} \quad (19)$$

where the $\Delta\alpha_i$ depend on the c_{jk}^i , which encode the interactions with quantum-gravity fluctuations in the space-time background.

We are now ready to discuss the issue of wave dispersion. From a target-space point of view, the g^i may be viewed as the Fourier transforms, *i.e.*, the polarization tensors, of target-space background fields, and the vertex operators are wave operators³. For instance, for a deformation corresponding to a scalar mode $T(X)$, one can write

$$\int d^2z \sqrt{\gamma} g^i V_i \equiv \int d^2z \sqrt{\gamma} \int d^Dk e^{ik_M X^M(z, \bar{z})} \tilde{T}(k) \quad (20)$$

where the summation over i includes target-space integration over k , on a D -dimensional Euclidean space. For massless string modes, as opposed to higher modes, $h_i - 2 = k^M k_M$. For our purposes, it suffices to consider the case of an almost flat space time with small quantum-gravity corrections coming from the interactions of massless low-energy modes with the environment of Planckian string states, implying that the string is close to a fixed point for which $Q^2 = c[g^*] - 25 = 0$. This is the case in certain cosmological backgrounds of interest to us [25]. In such a case, we see from (18) that $\alpha_i \sim |\vec{k}|$.

³Here the concepts of Fourier transforms and plane waves should be understood as being appropriately generalized to curved target spaces, with the appropriate geodesic distances taken into account. For our purposes below, the details of this will not be relevant. We shall work with macroscopically-flat space-times, where the quantum-gravity structure appears through quantum fluctuations of the vacuum, leading simply to non-criticality of the string, in the sense of non-vanishing β^i functions.

Our basic working hypothesis, which has been confirmed explicitly in the two-dimensional string black hole example [11], is that the matter deformations in such a black-hole background are not exactly marginal unless the couplings to non-propagating Planckian global modes are included in the analysis. This implies that the set of the operator product expansion coefficients c_{jk}^i in (15) includes non-zero couplings between massless and Planckian modes in the presence of non-trivial metric fluctuations of black-hole type, and more generally in the presence of structures with a space-time boundary. These couplings arise in string theory from the infinite set of stringy gauge W_∞ symmetries mentioned in the previous section. They imply that the massless modes constitute an open system, and that they lose information to these non-propagating higher-level modes which are not detected in local scattering experiments, inducing apparent decoherence [11].

The order of magnitude of such couplings is not known at present, and precise calculations would require a full second-quantized string theory. However, in the same spirit of the *maximal* estimate (7), which was also adopted in ref. [11, 20], one may assume that

$$\Delta\alpha_i \sim \eta \frac{|\vec{k}|^2}{M_P} \quad (21)$$

with η a dimensionless quantity, which parametrizes our present ignorance of the quantum structure of space time.

For the purposes of the present work, we restrict ourselves to non-critical strings on fixed-genus world sheets, in which case η in (21) is real. This should be viewed as describing only part of the quantum-gravity entanglement, namely that associated with the presence of global string modes [11]. The full string problem involves a summation over genera, which in turn implies *complex* η 's, arising from the appearance of imaginary parts in Liouville-string correlation functions on resummed world sheets, as a result of instabilities pertaining to microscopic black-hole decay in the quantum-gravity space-time foam [11]. Such imaginary parts in η will produce frequency-dependent attenuation effects in the amplitudes of quantum-mechanical waves for low-energy string modes. This type of attenuation effect leads to decoherence of the type appearing in the density-matrix approach to measurement theory [6, 11]. For our purpose of deriving bounds on the possible accuracy of distance measurements, however, such attenuation effects need not be taken into account, and the simple entanglement formula (21), with real η , will be sufficient.

The value of η depends in general on the type of massless field considered. In particular, in the case of photons η is further constrained by target-space gauge invariance, which restricts the structure of the relevant σ -model β function. With

this caveat in mind, (21) represents a maximal estimate of $\Delta\alpha_i$, compatible with the generic structure of perturbations of the σ -model β functions for closed strings.

The equations (15), (20) and (21) indicate that the dressed σ -model deformation (14) corresponds to waves of the form⁴

$$e^{i|\vec{k}|t+i\vec{k}\cdot\vec{X}+i\eta\frac{|\vec{k}|^2}{M_P}t} \quad (22)$$

The corresponding modified dispersion relation

$$E \sim |\vec{k}| + \eta|\vec{k}|^2/M_P, \quad (23)$$

which was dictated by the overall conformal invariance of the dressed theory, implies that massless particles propagate in the quantum-gravity ‘medium’ with a velocity that is effectively energy-dependent: $v \sim 1 + \eta E/M_P$.

5 Distance Measurement Errors within the Non-Critical String Approach

We now use the result (22) to derive bounds on the precision with which macroscopic distances can be measured, from the quantum-gravity point of view suggested by our non-critical approach to string theory. For comparison with the previous estimate of section 2, we consider the use of massless probes propagating through space-time foam, modelling the latter as a stochastic environment. We consider only contributions to the distance uncertainty that are associated with the emission and propagation of such probes. All other possible contributions to the uncertainty, associated, for example, with the finite extent of the ‘clock’ used to measure time delays, will be ignored.

We start by considering the contribution to the distance uncertainty associated with the vacuum entanglement of the probe during its motion between the bodies whose distance is to be measured. The fact that (22) describes particle propagation with an energy-dependent speed $v(E)$, leads to a (positive or negative, depending on the sign of η) time shift, with respect to conventional relativistic $v = 1$ propagation. After a time t , this shift amounts to

$$\Delta t \sim \eta L_P E t . \quad (24)$$

⁴Note that eq. (22) is consistent with target-space gauge invariance, since it may describe one of the polarization states of the photon, and the transversality condition on the photon polarization tensor, $k^\mu A_\mu = 0$, can be maintained.

From the point of view of an observer attempting to use the time of travel T of one such probe to measure a length L , this correction can be taken systematically into account. However, an uncertainty in the relation between T and L is introduced by the quantum uncertainty in the energy of the probe. Specifically, the time of travel of a wave packet of energy spread δE exchanged between two bodies whose distance L is being measured, has an uncertainty

$$\delta L_1 \sim \eta L_P \delta E T \sim \eta L_P \delta E L , \quad (25)$$

where we have used on the right-hand side the fact that the time of travel is, for a typical experimental set up, of order L . We also keep track of the fundamental uncertainty for a low-energy observer, in quantum gravity [26] and string theories [27], due to the natural cut-off in lengths set by the Planck or string length:

$$\delta L_2 \sim L_P . \quad (26)$$

In the measurement analysis, these contributions of quantum-gravitational origin must be combined with the other, well known, purely quantum-mechanical contributions to the uncertainty. In particular, it follows from the time-energy uncertainty principle that

$$\delta E \delta T^{emission} \sim 1 \quad (27)$$

where $\delta T^{emission}$ is the uncertainty in the time of emission of the probe. Since this obviously contributes to the uncertainty in the time of travel measured in the experiment, from (27) one finds the contribution

$$\delta L_3 \sim \frac{1}{\delta E} \quad (28)$$

to the uncertainty in the measurement of L .

From (25), (26), and (28) one finds a total uncertainty in the length measurement that may be written as

$$\delta L_{total} \geq \eta L_P L \delta E + \frac{1}{\delta E} + L_P \quad (29)$$

The observer can minimize the uncertainty (29) only by tuning δE , *i.e.*, by preparing a suitable wave packet. It is, then, straightforward to see that the following bound holds for the error in the measurement of L :

$$\min\{\delta L\} \sim \sqrt{\eta L L_P} + L_P . \quad (30)$$

in the context of the non-critical string approach to quantum gravity outlined in the previous section.

If, possibly as a result of string-theoretical cancellations as yet unknown, the actual wave dispersion due to space-time foam turned out to be weaker than the maximal order-of-magnitude estimate (7), a correspondingly weaker dependence on L of $\min\{\delta L\}$ would be found. For example, as the reader can easily verify, if (7) is replaced by $|\delta\mathbb{H}| = \mathcal{O}(\frac{E^n}{M_P^{n-1}})$, one finds that $\min\{\delta L\} \sim L^{1/n} L_P^{1-1/n}$.

6 Summary and Conclusions

We have derived bounds on the precision of macroscopic distance measurements which follow from a stochastic approach to quantum gravity, motivated by the formulation of non-critical string theory in ref. [11]. Within this approach, distance uncertainties arise from the observation that propagating particles are subject to dispersion by the foamy ‘medium’ of microscopic fluctuations in space time, causing them to acquire an energy- and time- dependent phase shift. This is a result of their entanglement with the quantum-gravitational ‘environment’, and such dispersion would lead to an L -dependent limit on the accuracy with which a macroscopic length L could be measured.

We have reviewed and developed previous arguments suggesting a specific maximal estimate (7) for the order of magnitude of the quantum-gravitational entanglement, which leads to a limit on the accuracy of distance measurement that is proportional to \sqrt{L} . Interestingly, this is of the same order as the bound predicted by a previous independent analysis combining quantum mechanics and classical general relativity [13, 14], which we briefly reviewed in Sec.2. We find this similarity remarkable, because the two physical mechanisms involved in deriving the \sqrt{L} -dependent bound are *a priori* different and independent. This agreement could even be construed as providing additional support for the assumption (7).

It is important to note that the type of vacuum entanglement effects appearing in (7) are within the range allowed by the present experimental accuracy. For instance, for lengths of the order of the size of the present Universe, 10^{10} light years, the formula (30) predicts a distance uncertainty of only $\eta^{\frac{1}{2}} 10^{-3}$ cm. Even the most accurate astrophysical experimental data appear to constrain only marginally the value of our phenomenological parameter η . Pulsars provide an excellent long-distance laboratory, because of their excellent timing. However, even for 1 GeV signals, at the upper end of the energies of observed pulsar signals, and for a pulsar distance of the order of 10^4 light years, which would correspond to a pulsars on the other side of our galaxy, the energy-dependent time delay described in (24) is only of order $\eta 10^{-8}$ s. Similarly, (24) predicts that the neutrinos from supernova 1987a, which had energies of order 10 to 100 MeV and came from a distance of about $1.6 10^5$ light years, might have

experienced a time dispersion of only about $\eta 10^{-7} s$ as a result of their entanglement with the quantum-gravity medium/vacuum.

This brief discussion indicates that a value $\eta \sim 1$, corresponding to the maximal estimate (7), may not be in contradiction with any present data. However, a more systematic investigation of terrestrial and astrophysical data is needed to set an accurate experimental bound on η . To the extent that the maximal estimate (7) cannot be excluded, experimental searches for quantum-gravity effects in particle and other laboratory physics motivated by this estimate should not be discouraged. We would cite in particular the searches for open quantum-mechanical *CPT* violation in neutral kaons[20], which could even be on the verge of experimental observation, if (7) holds, and also possible searches in macroscopic quantum-mechanical systems such as SQUIDS [6, 28].

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References

- [1] *Conceptual Problems of Quantum Gravity, Proc. Osgood Hill Conference*, North Andover, USA, May 15-18, 1988, A. Ashtekar and J. Stachel, eds. (Birkhaeuser, 1991).
- [2] S. Hawking, *Comm. Math. Phys.* 43 (1975), 199.
- [3] S. Hawking, *Comm. Math. Phys.* 87 (1982), 395.
- [4] J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, *Nucl. Phys.* B241 (1984), 381.
- [5] W.H. Zurek, *Phys. Rev. D* 26 (1982), 1862; *Physics Today* 44 (October 1991), 33.
- [6] J. Ellis, S. Mohanty and D.V. Nanopoulos, *Phys. Lett.* B221 (1989), 113; *ibid.* B235 (1990), 305.
- [7] M. Gell-Mann and J. B. Hartle, *Int. Conf. on H.E. Phys.* (World Scientific, Singapore, 1990), 1303; *Strong Decoherence*, preprint USCBTH-95-28 (1995).
- [8] N. Gisin, *Helv. Phys. Acta* , Vol. 62 (1989), 363; N. Gisin and I. Percival, *J. Phys.* A26 (1993), 2233.
- [9] F. Karolyhazy, *Il Nuovo Cimento* A42 (1966), 390.
- [10] R. Penrose, *The Emperor's New Mind* (Oxford Univ. Press 1989).
- [11] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, *Phys. Lett.* B293 (1992), 37; *Mod. Phys. Lett.* A10 (1995), 425, and hep-th/9305117; *Lectures presented at the Erice Summer School, 31st Course: From Supersymmetry to the Origin of Space-Time*, Ettore Majorana Centre, Erice, July 4-12 1993, published in the *Proc. Subnuclear Series* Vol. 31, p.1 (World Scientific, Singapore, 1994), hep-th/9403133; For a pedagogical review of this approach see: D.V. Nanopoulos, *Riv. Nuov. Cim.* Vol. 17, No. 10 (1994), 1.
- [12] Y.J. Ng and H. Van Dam, *Mod. Phys. Lett.* A9 (1994), 335; *Mod. Phys. Lett.* A10 (1995), 2801.
- [13] G. Amelino-Camelia, *Mod. Phys. Lett.* A9 (1994), 3415.
- [14] G. Amelino-Camelia, gr-qc/9603013, *Mod. Phys. Lett.* A in press.
- [15] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, CERN-TH/96-81, CTP-TAMU-11/96, ACT-04/96, OUTP-96-15-P, hep-th/9605046.
- [16] C. Rovelli, *Class. Quantum Grav.* 8 (1991), 297; *ibid.* 8 (1991), 317.

- [17] J.D. Brown and D. Marolf, *Phys. Rev. D* 53 (1996), 1835.
- [18] E.P. Wigner, *Rev. Mod. Phys.* 29 (1957), 255;
H. Salecker and E.P. Wigner, *Phys. Rev.* 109 (1958), 571.
- [19] E. Witten, *Phys. Rev. D* 44 (1991), 344.
- [20] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, *Phys. Lett. B* 293 (1992), 142,
and *Int. J. Mod. Phys. A* 11 (1996), 1489;
J. Ellis, J. Lopez, N.E. Mavromatos and D.V. Nanopoulos, *Phys. Rev. D* 53
(1996), 3846;
CPLEAR Collaboration, R. Adler et al., and J. Ellis, J.L. Lopez, N.E. Mavro-
matos and D.V. Nanopoulos, *Phys. Lett. B* 364 (1995), 239.
- [21] I. Kogan, preprint UBCTP-91-13 (1991); *Proc. Particles and Fields 91*, Vancou-
ver 18-21 April 1991 (eds. D. Axen, D. Bryman and M. Comyn, World Scientific,
1992); see also *Phys. Lett. B* 265 (1991), 269.
- [22] J. Ellis, N.E. Mavromatos, D.V. Nanopoulos and E. Winstanley, preprint CERN-
TH/95-336, CTP-TAMU-04/95, ACT-02/95, OUTP-96-05P, gr-qc/9602011.
- [23] G. Lindblad, *Comm. Math. Phys.* 48 (1976), 119.
- [24] I. Antoniadis, C. Bachas, J. Ellis, and D.V. Nanopoulos *Phys. Lett. B* 211 (1988),
393; *Nucl. Phys. B* 328 (1989), 117; *Phys. Lett. B* 257 (1991), 278.
- [25] C. Schmidhuber and A.A. Tseytlin, *Nucl. Phys. B* 426 (1994), 187.
- [26] T. Padmanabhan, *Class. Quantum Grav.* 4 (1987), L107;
M. Maggiore, *Phys. Lett. B* 304 (1993), 65;
S. Doplicher, K. Fredenhagen, and J.E. Roberts, *Phys. Lett. B* 331 (1994), 39;
L.J. Garay, *Int. J. Mod. Phys. A* 10 (1995), 145;
A. Kempf, G. Mangano, and R.B. Mann, *Phys. Rev. D* 52 (1995), 1108.
- [27] D. Amati, M. Ciafaloni, G. Veneziano, *Phys. Lett. B* 216 (1989), 41;
K. Konishi, G. Paffuti, P. Provero, *Phys. Lett. B* 234 (1990), 276.
- [28] A.J. Leggett, *Prog. Theor. Phys.* 69 (1980), 80;
C.D. Tesche, *Phys. Rev. Lett.* 64 (1990), 2358.