

# Z' PHYSICS

Contribution to the workshop on Physics at LEP 2

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# 1 Introduction

Direct production of new particles (with special emphasis on Higgs and supersymmetric partners) and possible indirect effects due to deviations from the predictions of the Standard Model (hereafter denoted as SM), in particular in the presence of anomalous triple gauge couplings, will be soon thoroughly searched for at LEP2 in the forthcoming few years. In both cases, typical and sometimes spectacular experimental signatures would exist, allowing to draw unambiguous conclusions if a certain type of signal were discovered.

At LEP2, one extra Z (to be called  $Z'$  from now on) would not be directly produced, owing to the already existing mass limits from Tevatron. Its indirect effects on several observables might be, though, sizeable, since it would enter the theoretical expressions at tree level. In this sense,  $Z'$  effects at LEP2 would be of similar type to those coming from anomalous triple gauge couplings (hereafter denoted as AGC), although the responsible mechanism would be of totally different physical origin. This peculiar feature of a  $Z'$  at LEP2 has substantially oriented the line of research of our working group. In fact, we have tried in this report to answer two complementary questions.

The first one was the question "which information on a  $Z'$  can one derive if no indirect signal of any type is seen at LEP2?". To answer this point leads to the derivation, to a certain conventionally chosen confidence limit, of (negative) bounds on the  $Z'$  mass  $M_{Z'}$ . This has been done for a number of "canonical"  $Z'$  models, and the resulting bounds (that are typically in the TeV range) will be shown in section 2 together with those for more general  $Z'$ 's that might not be detected at an hadronic collider.

The second question is: "if a signal of indirect type were seen at LEP2, would it be possible to decide whether it may come from a  $Z'$  or, typically, from a model with anomalous triple gauge couplings?". The answer to this question, which is essentially provided from measurements in the final leptonic channel, is given in section 3. In section 4, the role of the final WW channel, that might a priori not be negligible for a  $Z'$  of most general type, has also been investigated and shown to be irrelevant at LEP2. Section 5 is devoted to a short final discussion, that will conclude our work.

## 2 Derivation of bounds.

Theoretical motivations for the existence of a  $Z'$  have already been given by several authors, and excellent reviews are available [1], where the most studied models are listed and summarized. In the following, we will limit ourselves in defining as "canonical" cases those of a  $Z'$  from either  $E_6$  [2] or Left-Right symmetric models [3] type. For sake of completeness we shall also consider the often quoted case of a "Sequential" Standard Model  $Z'$  [4] (hereafter denoted as SSM), whose couplings to fermions are the same of those of the SM  $Z^0$ . For these models, derivations of bounds for  $Z'$  parameters ( $Z - Z'$  mixing angle and  $M_{Z'}$ ) have been obtained from present

data[5],[6] and calculations of discovery limits for  $M_{Z'}$  performed for future colliders[7],[8], including also a discussion of  $Z'$  model identification. Therefore, the first question that we shall answer in our report will be that of how do the LEP2 indirect mass limits compare to the direct ones achievable at Tevatron now and in a not too far future (i.e. assuming an integrated luminosity of  $1fb^{-1}$ ). In fact a motivation of our work was also that of deriving limits on a  $Z'$  whose couplings to fermions are completely free, including cases that would not be detectable by any present or future hadronic collider (for example for negligibly small  $Z'$  quark couplings). For this purpose, the final leptonic channel at LEP2 provides all the necessary experimental information, and we shall consequently begin our analysis with the detailed examination of the role of this channel.

To fix our normalization and conventions, we shall write the expression of the invariant amplitude for the process  $e^+e^- \rightarrow l^+l^-$  (where  $l$  is a generic charged lepton) in the Born approximation and in the presence of a  $Z'$ . Denoting  $q^2$  as the squared center of mass energy this amplitude reads in our notations:

$$A_{ll'}^{(0)}(q^2) = A_{ll'}^{(0)\gamma,Z}(q^2) + A_{ll'}^{(0)Z'}(q^2) \quad (1)$$

where:

$$A_{ll'}^{(0)\gamma}(q^2) = i \frac{e_0^2}{q^2} \bar{v}_l \gamma_\mu u_l \bar{u}_{l'} \gamma^\mu v_{l'} \quad (2)$$

$$A_{ll'}^{(0)Z}(q^2) = \frac{i}{q^2 - M_Z^2} \frac{g_0^2}{4c_0^2} \bar{v}_l \gamma_\mu (g_{Vl}^{(0)} - \gamma^5 g_{Al}^{(0)}) u_l \bar{u}_{l'} \gamma_\mu (g_{Vl'}^{(0)} - \gamma^5 g_{Al'}^{(0)}) v_{l'} \quad (3)$$

and(note the particular normalization):

$$A_{ll'}^{(0)Z'}(q^2) = \frac{i}{q^2 - M_{Z'}^2} \frac{g_0^2}{4c_0^2} \bar{v}_l \gamma_\mu (g'_{Vl}{}^{(0)} - \gamma^5 g'_{Al}{}^{(0)}) u_l \bar{u}_{l'} \gamma_\mu (g'_{Vl'}{}^{(0)} - \gamma^5 g'_{Al'}{}^{(0)}) v_{l'} \quad (4)$$

In the previous equations  $e_0 = g_0 s_0$ ,  $c_0^2 = 1 - s_0^2$ ,  $g_{Al}^{(0)} = I_{3L} = -\frac{1}{2}$  and  $g_{Vl}^{(0)} = -\frac{1}{2} + 2s_0^2$ . Following the usual approach we shall treat the SM sector at one loop and the  $Z'$  contribution at tree level. The  $Z'$  width will be assumed "sufficiently" small with respect to  $M_{Z'}$  to be safely neglected in the  $Z'$  propagator. Moreover the  $Z - Z'$  mixing angle will be ignored since the limits for this quantity provided by LEP1 data from the final leptonic channel are enough constraining to rule out the possibility of any observable effect in the final leptonic channel at LEP2 (this has been shown in a previous paper [6] for the "canonical" cases and for a general  $Z'$  case in a more recent preprint [8]). If we stick ourselves to the final charged leptonic states, we must therefore only deal with two "effective" parameters, that might be chosen as the adimensional quantities  $g'_{Vl} \sqrt{\frac{q^2}{M_{Z'}^2 - q^2}}$  and  $g'_{Al} \sqrt{\frac{q^2}{M_{Z'}^2 - q^2}}$  (this would be somehow reminiscent of notations that are common for models with AGC, with  $M_{Z'}$  playing the role of the scale of new physics). In practice, for the specific purpose of the derivation of bounds, a convenient choice was that of

the following rescaled couplings [9]:

$$v_l^N = g'_{Vl} \sqrt{\frac{q^2}{M_{Z'}^2 - q^2}} \sqrt{\frac{\alpha}{16c_W^2 s_W^4}} \quad (5)$$

and

$$a_l^N = -g'_{Al} \sqrt{\frac{q^2}{M_{Z'}^2 - q^2}} \sqrt{\frac{\alpha}{16c_W^2 s_W^4}} \quad (6)$$

with  $\alpha = \frac{1}{137}$  and  $s_W^2 = 1 - c_W^2 = 0.231$ .

As previously stressed our first task has been that of the derivation of constraints for the two previous rescaled couplings from the non observation of effects in the leptonic channel. We considered as observables at LEP2 the leptonic cross section  $\sigma_l$  and the forward-backward asymmetry  $A_{FB}^l$ , obtained from measurements of  $\mu$  and  $\tau$  final states, and also the final  $\tau$  polarization  $A_\tau$  (we have not yet included the electron channel because a full assessment of the corresponding experimental precision is more complicated at this stage). Three different energy-integrated luminosity configurations were considered, i.e.  $\sqrt{q^2} = 140$  GeV and  $\int Ldt = 5pb^{-1}$ , 175(500) and 192(300). Table 1 gives the SM predictions for the three leptonic quantities together with the expected experimental accuracies in the three cases. For each energy the first block of three lines contains convoluted quantities, whereas the second does not.

A short technical discussion about the way we have calculated the effects of QED emission is now appropriate. In fact two main approaches exist that use either complete Feynmann diagrams evaluation to compute photonic emission from external legs [10] or the so called QED structure function formalism [11], based on the analogy with QCD factorization and on the use of the Lipatov-Altarelli- Parisi evolution equation [12]. In the calculation of the limits on rescaled parameters performed in this section we have used the code ZEFIT [13] together with ZFITTER [14]. These programs utilize the first approach [10]. More precisely ZFITTER has all SM corrections and all possibilities to apply kinematical cuts. The code ZEFIT contains the additional  $Z'$  contributions including the full first order QED correction with soft photon exponentiation. The first version of this combination, applied to LEP1 data, which was restricted to definite  $Z'$  models, has been adapted for the model independent analysis that we are now performing at LEP2.

In practice the largest contribution is due to initial state radiation. The corresponding expressions for the cross section and forward-backward asymmetry read:

$$\sigma_T = \int_0^\Delta dk \sigma_T^0(s') R_T^e(k) \quad (7)$$

$$A_{FB} = \frac{1}{\sigma_T} \int_0^\Delta dk \sigma_{FB}^0(s') R_{FB}^e(k) \quad (8)$$

The reduced energy  $s'$  reads  $s' = q^2(1 - k)$  and  $\Delta = \frac{E_\gamma}{E_{beam}}$ . To first order in  $\alpha$ , improved by soft photon exponentiation, the two functions  $R_T^e(k)$  and  $R_{FB}^e(k)$  are given by the following

expressions:

$$R_{T,FB}^e(k) = (1 + S_e)\beta_e k^{\beta_e - 1} + H_{T,FB}^e(k) \quad (9)$$

where

$$\beta_e = 2\frac{\alpha}{\pi}e_e^2 \ln\left(\frac{q^2}{m_e^2} - 1\right), \quad (10)$$

the soft radiation part reads:

$$S_e = \frac{\alpha}{\pi}e_e^2 \left( \frac{\pi^2}{3} - \frac{1}{2} + \frac{3}{2} \ln\left(\frac{q^2}{m_e^2} - 1\right) \right) \quad (11)$$

and the hard radiation parts:

$$H_T(k) = \frac{\alpha}{\pi}e_e^2 \left\{ \frac{1 + (1-k)^2}{k} \ln\left(\frac{q^2}{m_e^2} - 1\right) \right\} - \frac{\beta_e}{k} \quad (12)$$

$$H_{FB}(k) = \frac{\alpha}{\pi}e_e^2 \left( \frac{1 + (1-k)^2}{k} \frac{1-k}{(1-\frac{k}{2})^2} (\ln\frac{q^2}{m_e^2} - 1 - \ln\frac{1-k}{(1-\frac{k}{2})^2}) \right) - \frac{\beta_e}{k} \quad (13)$$

The value of  $\Delta$  is chosen by requiring that the invariant mass of the final fermion pair  $M_{ll}^2 = (1 - \Delta)q^2$  is "sufficiently" greater than  $M_Z$ , to exclude the radiative return to the  $Z$  peak. This has very important implications for searches of  $Z'$  effects, since it is well known that the radiative tail can enhance the SM cross section by a factor 2 – 3, then completely diluting the small  $Z'$  effects, as fully discussed in a previous paper [15]. Results shown in table 1 are obtained for an invariant mass of the final fermion pair  $M_{ll}$  greater than 120 GeV.

One clearly sees from inspection of Table 1 that the most promising of the three energy-luminosity combinations for what concerns the relative size of the error is 175 GeV and  $500pb^{-1}$ .

From now on we shall therefore concentrate on this configuration and evaluate the bounds on  $Z'$  rescaled parameters that would follow from the non observation of any virtual effect. With this purpose we have made full use of the code ZEFIT and chosen  $\Delta = .7$ , although smaller values like for instance the one used in table 1 would lead to practically the same conclusions.

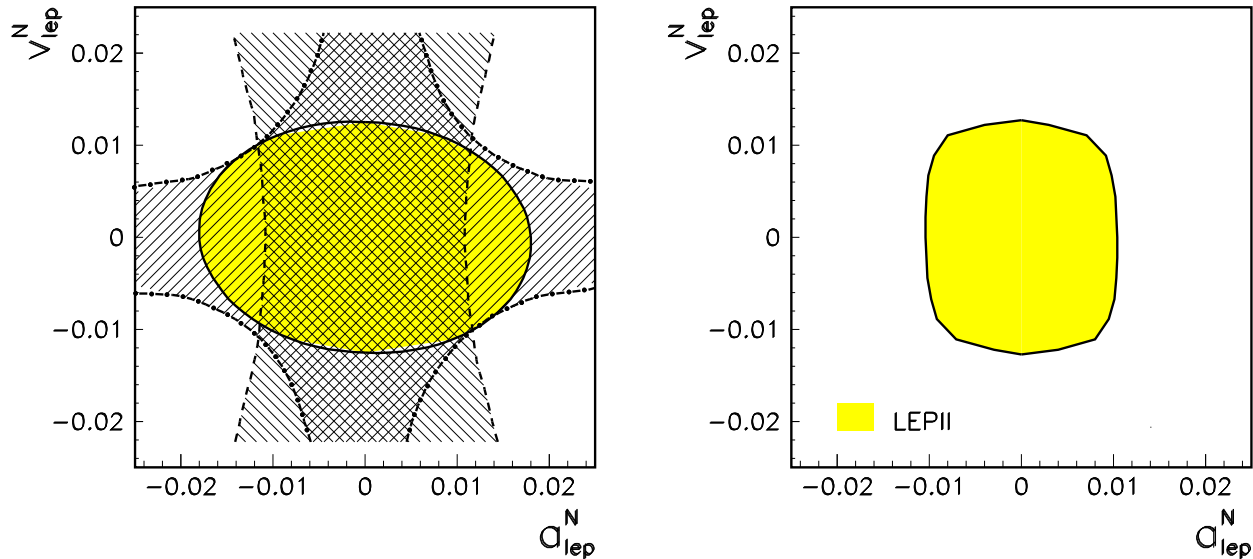
To obtain exclusion limits, we calculate the SM predictions of all observables  $O_i(SM)$  and compare them with the prediction  $O_i(SM, v_i^N, a_i^N)$  from a theory including a  $Z'$ . In our fits we use the errors  $\Delta O_i$  calculated using the same assumptions as in Table 1 and define:

$$\chi^2 = \sum_{O_i} \left( \frac{O_i(SM) - O_i(SM, Z')}{\Delta O_i} \right)^2. \quad (14)$$

$\chi^2 < \chi_{min}^2 + 5.99$  corresponds to 95% confidence level for one sided exclusion bounds for two parameters.

	$E_{cm}$	Lumi	Acc	$\sigma(pb)$	$\Delta\sigma_{stat}$	$\Delta\sigma_{syst}$	$\Delta\sigma$	Error	$A_{FB}$	$(\Delta A_{FB})$	$A_\tau$	$(\Delta A_\tau)$
$\mu$	140.	5.	.90	6.43	1.20	.05	1.20	18.6%	.684	.136		
$\ell$	140.	5.	.90	6.51	0.85	.04	0.85	13.1%	.684	.095		
$\tau$	140.	5.									-.104	0.61
$\mu$	140.	5.	.90	6.98	1.25	.05	1.25	17.9%	.666	.133		
$\ell$	140.	5.	.90	6.98	0.88	.04	0.88	12.6%	.666	.094		
$\tau$	140.	5.									-.102	0.59
$\mu$	175.	500.	.90	4.13	0.10	.03	0.10	2.4%	.602	.019		
$\ell$	175.	500.	.90	4.15	0.07	.03	0.07	1.7%	.602	.013		
$\tau$	175.	500.									-.082	0.08
$\mu$	175.	500.	.90	4.01	0.09	.03	0.10	2.5%	.586	.01		
$\ell$	175.	500.	.90	4.01	0.07	.02	0.07	1.8%	.586	.01		
$\tau$	175.	500.									-.079	0.08
$\mu$	192.	300.	.90	3.47	0.11	.02	0.12	3.3%	.579	.02		
$\ell$	192.	300.	.90	3.49	0.08	.02	0.08	2.4%	.579	.01		
$\tau$	192.	300.									-.085	0.11
$\mu$	192.	300.	.90	3.28	0.11	.02	0.11	3.4%	.565	.02		
$\ell$	192.	300.	.90	3.28	0.08	.02	0.08	2.5%	.565	.02		
$\tau$	192.	300.									-.081	0.11

Table 1: SM predictions for leptonic observables including experimental accuracies. As a simple simulation of the detector acceptance, we impose that the angle between the outgoing leptons and the beam axis is larger than  $20^\circ$ , leading to an acceptance of about 0.9. The first line gives the muon cross section and the forward-backward asymmetry and errors. The second line gives the averaged  $\mu$  and  $\tau$  cross section and asymmetry (and errors) whereas the third line contains the  $\tau$  polarization (obtained by using only the  $\rho$  and  $\pi$  channels and assuming an average sensitivity of .5). Concerning systematics we assumed .5% relative error for  $\mu$  and  $\tau$  selections and also for luminosity. For all asymmetries we did not consider any systematic error. All quoted errors refer to a single LEP experiment. Taking into account the type of systematic errors and the relative size of systematics vs statistics, it is a good approximation to divide the error by 2 to estimate the combined error of the four experiments.



**Fig. 1** The areas in the  $(a_\ell^N, v_\ell^N)$  plane excluded with 95% CL at LEP 2 by different observables i.e.  $\sigma_\ell$  (ellipse) and  $A_{FB}^l$  (crossed lines). The remaining contours, that do not improve the limits, would correspond to a measurement of  $A_\tau$  with an accuracy twice better than the realistic one quoted in Table 1.

**Fig. 2** The area in the  $(a_i^N, v_i^N)$  plane excluded with 95% confidence at LEP 2 by the combination of all the leptonic observables.

Figures 1 and 2 give our model independent constraints to the rescaled leptonic  $Z'$  couplings including all radiative corrections, i.e. the QED radiation and the electroweak corrections, that have a very small influence on the results. In figure 1 the constraint from each observable is shown separately. The combined exclusion region is depicted in the next figure 2 and a few comments on the previous figure are now appropriate. It represents in fact the most general type of constraints that can be derived on a  $Z'$  from the absence of signals in the leptonic channel at LEP2, under the assumption that the  $Z'$  couples to charged leptons in a universal way. In particular, from this figure one might derive bounds on the parameters of  $Z'$  that were only coupled to leptons and would therefore escape detection at any hadronic machine. For such models, the limit on  $M_{Z'}$  would be then derivable to a very good (and conservative) approximation, for given  $Z'$  couplings, by the simple expression (derived from eq. (5) and eq. (6)):

$$\frac{M_{Z'}^2}{q^2} \geq \frac{1}{r^2} \frac{g'_{Vl}{}^2 + g'_{Al}{}^2}{\frac{\alpha}{16c_w^2 s_w^4}} \quad (15)$$

where  $r$  is the distance from the origin in the  $(v_i^N, a_i^N)$  plane of the intersection between the boundary region of figure 2 and the straight line:

$$\frac{g'_{Vl}}{g'_{Al}} = -\frac{v_i^N}{a_i^N} = k \quad (16)$$

where  $k$  is fixed by the considered model and  $r$  will vary between .01 and .015. (Numerically  $\frac{\alpha}{16c_w^2 s_w^4} \sim 81$ .)

	$E_{cm}$	Lumi	Acc	$\sigma(pb)$	$\Delta\sigma_{stat}$	$\Delta\sigma_{syst}$	$\Delta\sigma$	Error	$A_{FB}$	$(\Delta A_{FB})$	$R$	Error
$b$	140.	5.	0.25	10.46	2.89	.31	2.91	27.8%	.499	.379		
$h$	140.	5.	1.00	59.03	3.44	.66	3.50	5.9%			9.181	14.3%
$R_h$	140.	5.									0.177	28.4%
$R_b$	140.	5.										
$b$	140.	5.	0.25	10.62	2.92	.32	2.93	27.6%	.509	.373		
$h$	140.	5.	1.00	60.49	3.48	.68	3.54	5.9%			8.666	13.9%
$R_h$	140.	5.									0.176	28.2%
$R_b$	140.	5.										
$b$	175.	500.	0.25	5.15	0.20	.15	.26	5.0%	.543	.052		
$h$	175.	500.	1.00	31.24	0.25	.35	.43	1.4%			7.572	2.1%
$R_h$	175.	500.									0.165	5.1%
$R_b$	175.	500.										
$b$	175.	500.	.25	4.69	0.19	.14	0.24	5.1%	.562	.054		
$h$	175.	500.	1.00	28.90	0.24	.32	0.40	1.4%			7.215	2.1%
$R_h$	175.	500.									0.162	5.3%
$R_b$	175.	500.										
$b$	192.	300.	0.25	4.08	0.23	.12	.26	6.5%	.554	.079		
$h$	192.	300.	1.00	25.22	0.29	.28	.40	1.6%			7.265	2.8%
$R_h$	192.	300.									0.162	6.6%
$R_b$	192.	300.										
$b$	192.	300.	.25	3.62	0.22	.11	0.25	6.8%	.577	.078		
$h$	192.	300.	1.00	22.79	0.28	.25	0.38	1.6%			6.942	2.9%
$R_h$	192.	300.									0.159	6.9%
$R_b$	192.	300.										

Table 2: SM predictions for hadronic observables including experimental accuracies. The first line gives the  $b$  quark cross section and forward-backward asymmetry and the corresponding experimental errors. The second line gives the total hadronic cross section (and errors) whereas the third line contains the ratio  $R_h = \frac{\sigma_h}{\sigma_t}$  and the fourth line the ratio  $R_b = \frac{\sigma_b}{\sigma_h}$ . Concerning systematics we assumed 1% relative error for hadron selection and 3% relative error for  $b$  quark selection. We did not consider any systematic error for all asymmetries. Concerning the  $b$  quark cross section we assume a tagging efficiency of 25% (vertex tag) and 10% (lepton tag) for the asymmetry. The first block of numbers (upper four lines) refer to convoluted quantities where proper cuts have been applied, whereas the lower four lines contain deconvoluted quantities.



In a less special situation, the  $Z'$  couplings to quarks will not be vanishing. In these cases, to derive meaningful bounds, the full information coming from the final hadronic channel should be also exploited. At LEP2, we assumed the availability of three different measurements, i.e. those of the total hadronic cross section  $\sigma_h$  and those of the cross section and forward-backward asymmetry for  $b\bar{b}$  production,  $\sigma_b$  and  $A_{FB}^b$ . In table 2 we give the related expected experimental accuracies, for the three energy-luminosity configurations already investigated for the final leptonic channel in Table 1, and under the same general assumptions listed in the discussion preceding the presentation of this table.

From the combination of the leptonic and hadronic channels, a fully general investigation of the six rescaled  $Z'$  couplings (there would be four extra rescaled couplings for "up" and "down" type quarks) might be, in principle, carried through if at least four hadronic independent observables were measured at LEP2. This could be obtained if one more hadronic asymmetry were measured. In practice, though, the utility of such an approach is somehow obscured by practical considerations (like the realistic achievable experimental accuracy). For these reasons, we have therefore decided to make full use of the hadronic observables to derive limits on  $M_{Z'}$  only for a number of "canonical" models where the  $Z'$  couplings to fermions are constrained. As relevant examples to be investigated, we shall discuss  $E_6$  models [2] and Left-Right symmetric models [3], for which the  $Z'$  current can be decomposed as:

$$J_{Z'}^\mu = J_\chi^\mu \cos \beta + J_\psi^\mu \sin \beta \quad (17)$$

or

$$J_{Z'}^\mu = J_{LR}^\mu \alpha_{LR} - J_{B-L}^\mu \frac{1}{2\alpha_{LR}} \quad (18)$$

Table 3a

f	$\frac{g'_{Vf}}{s_w}$	$\frac{g'_{Af}}{s_w}$
l	$\frac{2}{\sqrt{6}} \cos \beta$	$\frac{1}{\sqrt{6}} \cos \beta + \frac{\sqrt{10}}{6} \sin \beta$
u	0	$-\frac{1}{\sqrt{6}} \cos \beta + \frac{\sqrt{10}}{6} \sin \beta$
d	$-\frac{2}{\sqrt{6}} \cos \beta$	$\frac{1}{\sqrt{6}} \cos \beta + \frac{\sqrt{10}}{6} \sin \beta$

Table 3b

f	$\frac{g'_{Vf}}{s_w}$	$\frac{g'_{Af}}{s_w}$
l	$\frac{1}{\alpha_{LR}} - \frac{\alpha_{LR}}{2}$	$\frac{\alpha_{LR}}{2}$
u	$-\frac{1}{3\alpha_{LR}} + \frac{\alpha_{LR}}{2}$	$-\frac{\alpha_{LR}}{2}$
d	$-\frac{1}{3\alpha_{LR}} - \frac{\alpha_{LR}}{2}$	$\frac{\alpha_{LR}}{2}$

Table 3: Couplings of ordinary fermions (f=l,u,d) to  $Z'$  boson a) from  $E_6$  models as a function of the parameter  $\cos \beta$  b) from Left-Right models as a function of the parameter  $\alpha_{LR}$ .

In table 3 we have given the  $Z'$  couplings to l, u and d fermions for the two models. Some specific relevant cases in the  $E_6$  sector are the so called  $\chi$  model (corresponding to  $\cos\beta = 1$ ),  $\psi$  model ( $\cos\beta = 0$ ) and  $\eta$  model ( $\arctan\beta = -\sqrt{\frac{5}{3}}$ ). Special cases for Left-Right symmetric models are obtained for  $\alpha_{LR} = \sqrt{\frac{2}{3}}$  (this case reproduces the  $\chi$  model) and  $\alpha_{LR} = \sqrt{2}$  (the so called manifestly L-R symmetric model). Finally we also chose the  $Z'$  of the Sequential Standard Model (which has the same fermionic couplings as those of the SM Z) as an additional benchmark.

Table 4 shows the CL bounds on  $M_{Z'}$  obtainable from the non observation of any effect at LEP2 in the configuration: 175 GeV,  $500pb^{-1}$ . In fact one can easily show that for virtual  $Z'$  searches this configuration is the best of the three that we have considered for LEP2, since the simple scaling law for the achievable limit  $(M_{Z'})_{max} \sim (q^2 \int L)^{\frac{1}{4}}$  applies. The different lines show the influence of the hadronic observables. As one sees, this is indeed relevant for the SSM  $Z'$ . In the other cases it improves the bounds derived from purely leptonic observables by an amount of less than (typically) a relative 10%.

	$\chi$	$\psi$	$\eta$	LR	SSM
$\sigma_\ell, A_{FB}^\ell$	870	640	525	838	1238
$\sigma_\ell, A_{FB}^\ell, \sigma_{had}$	900	642	550	854	1530
$\sigma_\ell, A_{FB}^\ell, R_b, A_{FB}^b, \sigma_{had}$	930	666	560	880	1580

Table 4: Maximal  $Z'$  masses  $M'_{Z'}$  excluded by leptonic and hadronic observables.  $\chi^2 < \chi^2_{min} + 2.7$  (95% CL, one sided limits).

The more general analysis of the two models of extra gauge, that corresponds to values of  $\cos\beta$  ranging from  $-1$  to  $+1$  (positive  $\sin\beta$ ) and  $\alpha_{LR}$  ranging from  $\sqrt{\frac{2}{3}}$  to  $\sqrt{2}$ , has been summarized in figures 3 and 4 (full lines). One sees that the best  $M_{Z'}$  limits correspond to models where  $\cos\beta \sim 1$  and where  $\alpha_{LR}$  is at the boundary of its allowed interval, for which the bounds derivable at LEP2 would be about 1 TeV.

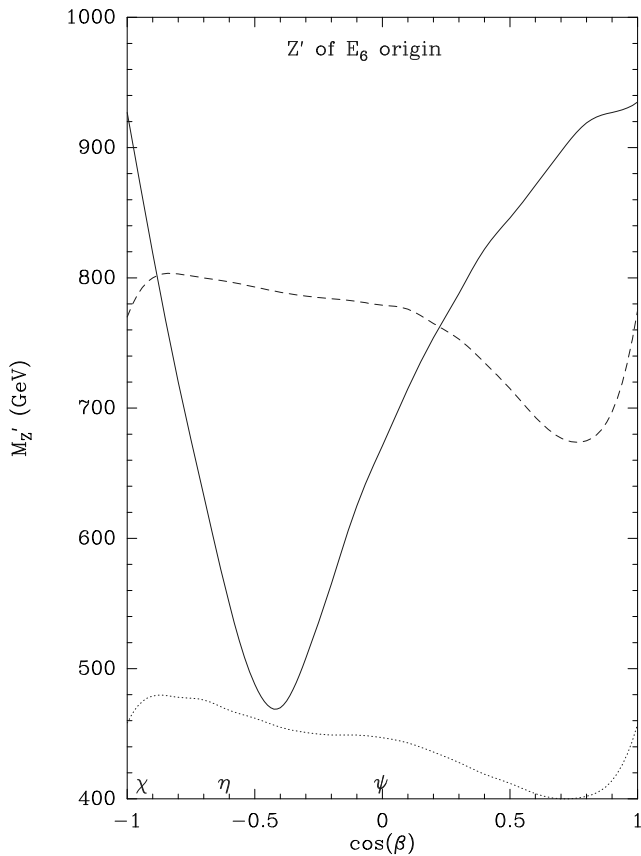


Fig. 3

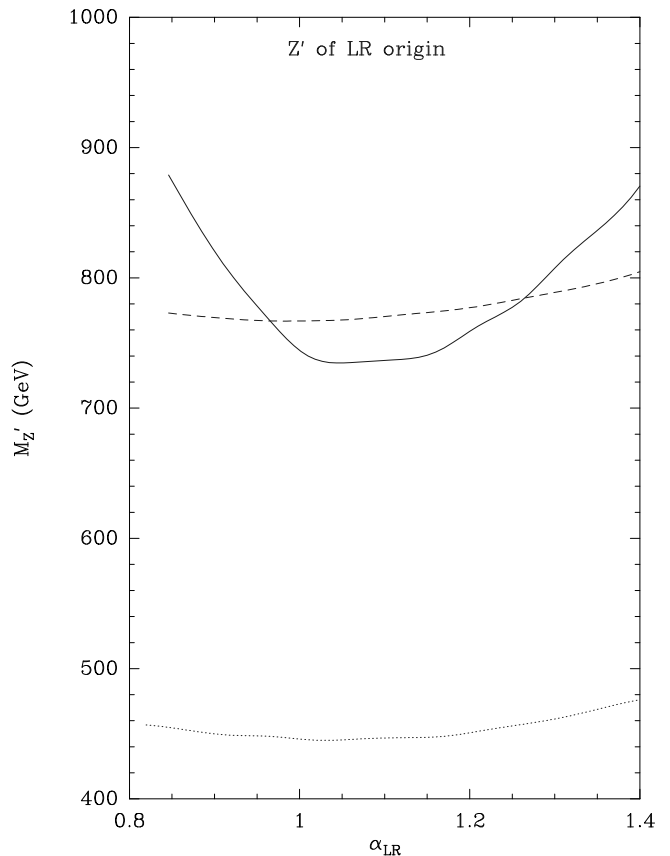


Fig. 4

Maximal  $Z'$  masses excluded at LEP2 (full curve) and at Tevatron with a luminosity of  $1fb^{-1}$  (dashed curve) or  $20pb^{-1}$  (dotted curve) for  $E_6$  models (Fig. 3) and for Left-Right models (Fig. 4).

The values that we have derived should be compared with those already available and with those reachable in a not too far future at Tevatron. To fix the scales for the comparison, we have considered the limits that would correspond to an energy of 1.8 TeV with an integrated luminosity of  $1fb^{-1}$  and drawn on the same figures 3 and 4 (dashed lines) the expected Tevatron limits, that would "compete" with the LEP2 results (the present Tevatron limits for  $20pb^{-1}$  correspond to the dotted curves). The limits correspond to 95% CL bounds on  $M_{Z'}$ , based on 10 events in the  $e^+e^- + \mu^+\mu^-$  channel, assuming that  $Z'$  can only decay in the three conventional fermion families. The values that we plot are in agreement with those quoted in a recent report [16]. One sees from figures 3 and 4 that for the  $E_6$  models the LEP2 limits are in a sense complementary to those of Tevatron in the future configuration, providing better or worse indications depending on which range is chosen for  $\cos\beta$ . LEP2 is better if  $\cos\beta$  lies in the vicinity of  $-1$  and  $\cos\beta \geq 0.4$ . On the contrary, for Left-Right symmetric models LEP2 appears to do much better, except for  $\alpha_{LR}$  ranging between 1 and 1.2 where Tevatron could provide limits a bit higher; concerning the SSM  $Z'$ , LEP2 does systematically better since it can reach 1.5 TeV whereas the future Tevatron limit is around 900 GeV. Note that, should other exotic or supersymmetric channel be open for  $Z'$  decay, the Tevatron limits might decrease by as much as 30%, depending on the considered model [16]. We conclude therefore that, until

the Tevatron luminosity will reach values around  $10fb^{-1}$ , the canonical LEP2 bounds will be, least to say, strongly competitive.

Our determination of bounds is at this point finished for what concerns the final fermionic channel. In the next section we shall try to derive some model-independent criterion to identify  $Z'$  signals at LEP2.

### 3 Search for signals: the leptonic channel.

In this section, we shall assume that some virtual signal has been seen at LEP2 in the leptonic channel. In this case, we shall show that it would be possible to conclude whether this signal is due to a  $Z'$  or not.

This can be easily understood if one compares the  $Z'$  effect to a description that includes the SM effects at one loop, and we shall briefly summarize the main points. For what concerns the treatment of the SM sector, a prescription has been very recently given [17], that corresponds to a "Z-peak subtracted" representation of four fermion processes, in which a modified Born approximation and "subtracted" one loop corrections are used. These corrections, that are "generalized" self-energies, i.e. gauge-invariant combinations of self-energies, vertices and boxes, have been called in [17] (to which we refer for notations and conventions)  $\tilde{\Delta}\alpha(q^2)$ ,  $R(q^2)$  and  $V(q^2)$  respectively. As shown in ref [17], they turn out to be particularly useful whenever effects of new physics must be calculated. In particular, the effect of a general  $Z'$  would appear in this approach as a particular modification of purely "box" type to the SM values of  $\tilde{\Delta}\alpha(q^2)$ ,  $R(q^2)$  and  $V(q^2)$  given by the following prescriptions:

$$\tilde{\Delta}\alpha^{(Z')}(q^2) = -\frac{q^2}{M_{Z'}^2 - q^2} \frac{1}{4c_1^2 s_1^2} g_{Vl}^2 (\xi_{Vl} - \xi_{Al})^2 \quad (19)$$

$$R^{(Z')}(q^2) = \left(\frac{q^2 - M_Z^2}{M_{Z'}^2 - q^2}\right) \xi_{Al}^2 \quad (20)$$

$$V^{(Z')}(q^2) = -\left(\frac{q^2 - M_Z^2}{M_{Z'}^2 - q^2}\right) \frac{g_{Vl}}{2c_1 s_1} \xi_{Al} (\xi_{Vl} - \xi_{Al}) \quad (21)$$

where we have used the definitions:

$$\xi_{Vl} = \frac{g'_{Vl}}{g_{Vl}} \quad (22)$$

$$\xi_{Al} = \frac{g'_{Al}}{g_{Al}} \quad (23)$$

with  $g_{Vl} = \frac{1}{2}(1 - 4s_1^2)$ ;  $g_{Al} = -\frac{1}{2}$  and  $c_1^2 s_1^2 = \frac{\pi\alpha(0)}{\sqrt{2}G_\mu M_Z^2}$ . To understand the philosophy of our approach it is convenient to write the expressions at one-loop of the three independent leptonic

observables that will be measured at LEP2, i.e. the leptonic cross section, the forward-backward asymmetry and the final  $\tau$  polarization. Leaving aside specific QED corrections extensively discussed in the previous section, these expressions read:

$$\begin{aligned} \sigma_l(q^2) = & \sigma_l^{Born}(q^2) \left\{ 1 + \frac{2}{\kappa^2(q^2 - M_Z^2)^2 + q^4} [\kappa^2(q^2 - M_Z^2)^2 \tilde{\Delta}\alpha(q^2) \right. \\ & \left. - q^4(R(q^2) + \frac{1}{2}V(q^2))] \right\} \end{aligned} \quad (24)$$

$$\begin{aligned} A_{FB}^l(q^2) = & A_{FB}^{l,Born}(q^2) \left\{ 1 + \frac{q^4 - \kappa^2(q^2 - M_Z^2)^2}{\kappa^2(q^2 - M_Z^2)^2 + q^4} [\tilde{\Delta}\alpha(q^2) + R(q^2)] \right. \\ & \left. + \frac{q^4}{\kappa^2(q^2 - M_Z^2)^2 + q^4} V(q^2) \right\} \end{aligned} \quad (25)$$

$$\begin{aligned} A_\tau(q^2) = & A_\tau^{Born}(q^2) \left\{ 1 + \left[ \frac{\kappa(q^2 - M_Z^2)}{\kappa(q^2 - M_Z^2) + q^2} - \frac{2\kappa^2(q^2 - M_Z^2)^2}{\kappa^2(q^2 - M_Z^2)^2 + q^4} \right] [\tilde{\Delta}\alpha(q^2) \right. \\ & \left. + R(q^2)] - \frac{4c_1 s_1}{v_1} V(q^2) \right\} \end{aligned} \quad (26)$$

where  $\kappa$  is a numerical constant ( $\kappa^2 = (\frac{\alpha}{3\Gamma_l M_Z})^2 \simeq 7$ ) and we refer to [17] for a more detailed derivation of the previous formulae.

A comparison of eqs. (24-26) with eqs. (19-21) shows that, in the three leptonic observables, only two effective parameters, that could be taken for instance as  $\xi_{Vl} \frac{M_Z}{\sqrt{M_{Z'}^2 - q^2}}$  and  $(\xi_{Vl} - \xi_{Al}) \frac{M_Z}{\sqrt{M_{Z'}^2 - q^2}}$  (to have dimensionless quantities, other similar definitions would do equally well), enter. This leads to the conclusion that it must be possible to find a relationship between the relative  $Z'$  shifts  $\frac{\delta\sigma_l}{\sigma_l}$ ,  $\frac{\delta A_{FB}^l}{A_{FB}^l}$  and  $\frac{\delta A_\tau}{A_\tau}$  (defining, for each observable  $O_i = O_i^{SM} + \delta O_i^{Z'}$ ) that is completely independent of the values of these effective parameters. This will correspond to a region in the 3-d space of the previous shifts that will be fully characteristic of a model with the most general type of  $Z'$  that we have considered. We shall call this region "Z' reservation".

To draw this reservation would be rather easy if one relied on a calculation in which the  $Z'$  effects are treated in first approximation, i.e. only retaining the leading effects, and not taking into account the QED radiation. After a rather straightforward calculation one would then be led to the following approximate expressions that we only give for indicative purposes:

$$\left( \frac{\delta A_\tau}{A_\tau} \right)^2 \simeq f_1 f_3 \frac{8c_1^2 s_1^2}{v_1^2} \frac{\delta\sigma_l}{\sigma_l} \left( \frac{\delta A_{FB}^l}{A_{FB}^l} + \frac{1}{2} f_2 \frac{\delta\sigma_l}{\sigma_l} \right) \quad (27)$$

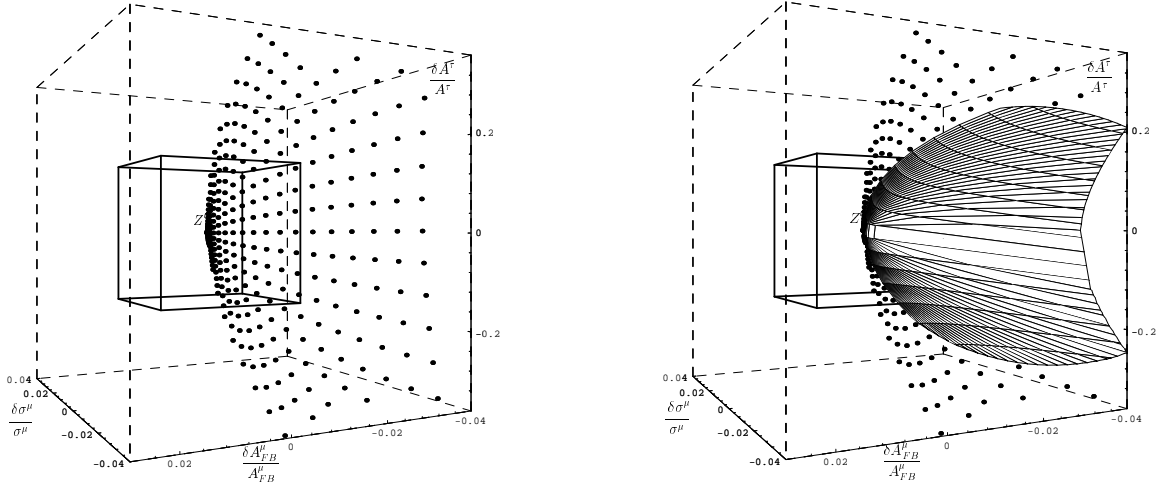
where the  $f_i$ 's are numerical constants, whose expressions can be found in [17].

Eq. (27) is an approximate one. A more realistic description can only be obtained if the potentially dangerous QED effects are fully accounted for. In order to accomplish this task, the QED structure function formalism [11] has been employed as a reliable tool for the treatment of large undetected initial-state photonic radiation. Using the structure function method amounts to writing, in analogy with QCD factorization, the QED corrected cross section as a convolution of the form:

$$\sigma(q^2) = \int dx_1 dx_2 D(x_1, q^2) D(x_2, q^2) \sigma_0((1 - x_1 x_2) q^2) (1 + \delta_{fs}) \Theta(cuts) \quad (28)$$

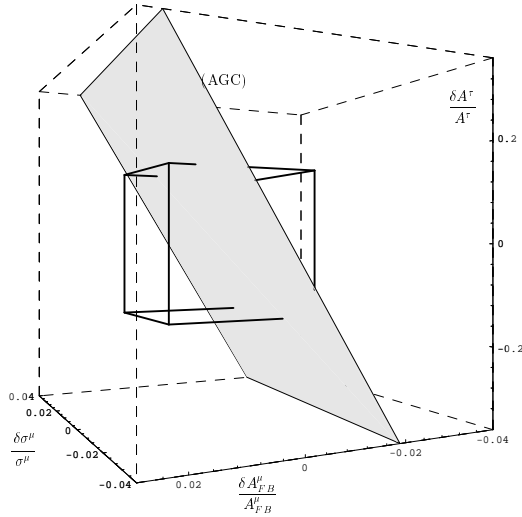
where  $\sigma_0$  is the lowest order kernel cross section, taken at the energy scale reduced by photon emission,  $D(x, q^2)$  is the electron (positron) structure function,  $\delta_{fs}$  is the correction factor taking care of QED final-state radiation and  $\Theta(cuts)$  represents the rejection algorithm to implement possible experimental cuts. Its expression, obtained by solving the Lipatov-Altarelli-Parisi evolution equation in the non-singlet approximation, can be found in [12] together with a complete discussion of the method. In order to proceed with the numerical simulation of the  $Z'$  effects under realistic experimental conditions, the master formula eq. (28) has been implemented in a Monte Carlo event generator which has been first checked against currently used LEP1 software [18], found to be in very good agreement and then used to produce our numerical results. The  $Z'$  contribution has been included in the kernel cross section  $\sigma_0$  computing now the s-channel Feynman diagrams associated to the production of a leptonic pair in  $e^+e^-$  annihilation mediated by the exchange of a photon, a SM Z and an additional  $Z'$  boson. In the calculation, which has been carried out within the helicity amplitude formalism for massless fermions and with the help of the program for algebraic manipulations SCHOONSCHIP [19], the  $Z'$  propagator has been included in the zero-width approximation. Moreover, the bulk of non QED corrections has been included in the form of the Improved Born Approximation, choosing  $\bar{\alpha}(s)$ ,  $M_Z$ ,  $G_F$  and  $\Gamma_Z$  as input parameters. The values used for the numerical simulation are [20]:  $M_Z = 91.1887$  GeV,  $\Gamma_Z = 2.4979$  GeV. The center of mass energy has been fixed to  $\sqrt{q^2} = 175$  GeV and the cut  $x_1 x_2 > 0.35$  (that would correspond to the choice  $\Delta = 0.65$  in the notations of the previous section) has been imposed in order to remove the events due to Z radiative return and hence disentangle the interesting virtual  $Z'$  effects. These have been investigated allowing the previously defined ratios  $\xi_{Vl}$  and  $\xi_{Al}$  to vary within the ranges  $-2 \leq \xi_{Al} \leq 2$  and  $-10 \leq \xi_{Vl} \leq 10$ . Higher values might be also taken into account; the reason why we chose the previous ranges was that, to our knowledge, they already include all the most known models.

The results of our calculation are shown in figure 5 [21]. One sees that the characteristic features of a general  $Z'$  effect are the fact that the shifts in the leptonic cross section are essentially negative. This can be qualitatively predicted from the Born approximation formula eq. (24) because the dominant photon exchange contribution to  $\sigma_l$  is clearly negative since  $\tilde{\Delta}^{(Z')} \alpha(q^2)$  is negative. Away from  $\xi_{Al} \simeq 0$  the forward-backward asymmetry will be also negative, as easily inferred from eq. (25).



**Fig. 5**  $\frac{\delta A^\tau}{A^\tau}$  versus  $\frac{\delta\sigma^\mu}{\sigma^\mu}$  and  $\frac{\delta A_{FB}^\mu}{A_{FB}^\mu}$ . The central "dead" area where a signal would not be distinguishable corresponds to an assumed (relative) experimental error of 1.5% for  $\sigma_\mu$  and to 1% (absolute) errors on the two asymmetries. The region that remains outside the dead area represents the  $Z'$  reservation at LEP2, to which the effect of the most general  $Z'$  must belong.

**Fig. 6** The same as Fig. 5, comparing the realistic results obtained via Monte Carlo simulation with the approximate ones according to Born approximation.



**Fig. 7** The region corresponding to Anomalous Gauge Couplings according to a Born approximation.

One might be interested in knowing how different the realistic figure 5 is from the approximate Born one, corresponding to the simplest version given in eq. (27). This can be seen in figure 6 where we have drawn the allowed regions, the points corresponding to the realistic situation, already shown in figure 5. The region inside the parallelepiped, where a signal would not be detectable, corresponds to an assumed relative experimental error of 1.7% for  $\sigma_l$  and to an absolute error of 1% for  $A_{FB}^l$ . For the  $\tau$  asymmetry an absolute error of 2% has been assumed, that is extremely optimistic. The domain that remain outside this area represents the  $Z'$  reservation at LEP2, to which the effect of the most general  $Z'$  must belong. One sees that the simplest Born calculation is, qualitatively, a reasonable approximation to a realistic estimate, which could be very useful if one first wanted to look for sizeable effects.

The next relevant question that should be now answered is whether the correspondence between  $Z'$  and reservation is of the one to one type, which would lead to a unique identification of the effect. We have tried to answer this question for one specific and relevant case, that of virtual effects produced by anomalous gauge couplings. In particular, we have considered the case of the most general dimension 6  $SU(2) \otimes U(1)$  invariant effective lagrangian recently proposed [22]. This has been fully discussed in a separate paper [23], where the previously mentioned "Z-peak subtracted" approach has been used. The resulting AGC reservation in the  $(\sigma_l, A_{FB}^l, A_\tau)$  has been calculated for simplicity in the Born approximation, as suggested by the previous remarks. This AGC area is plotted in figure 7. As one sees, the two domains do not overlap in the meaningful region. Although we cannot prove this property in general, we can at least conclude that, should a clear virtual effect show up at LEP2, it would be possible to decide unambiguously to which among two well known proposed models it does belong.

The results that we have shown so far have been obtained by exploiting the information provided by the final fermionic channel. We shall devote the next section 4 to a brief discussion of the WW channel at LEP2.

## 4 Search for effects in the WW channel.

The virtual effects of a  $Z'$  in W pair production from  $e^+e^-$  annihilation can be described, at tree level, by adding to the photon, SM Z and neutrino exchanges the diagram with an additional  $Z'$  boson exchange. The overall effect in the scattering amplitude reads:

$$A_{lW}^{(0)}(q^2) = A_{lW}^{(0)(\gamma, Z, \nu)}(q^2) + A_{lW}^{(0)Z'}(q^2) \quad (29)$$

where we assume universal couplings. Separate expressions can be easily derived for eq. (29). We shall only give here the relevant  $Z'$  contribution:

$$A_{lW}^{(0)Z'}(q^2) = \frac{i}{q^2 - M_{Z'}^2} \frac{g_0}{2c_0} \bar{v}_l \gamma_\mu (g'_{Vl} - \gamma^5 g'_{Al}) u_l e_0 g_{ZW} P^{\alpha\beta\mu} \epsilon_\alpha^*(p_1) \epsilon_\beta^*(p_2) \quad (30)$$

where:

$$P_{\alpha\beta\mu} = g_{\mu\beta}(p_1 + 2p_2)_\alpha + g_{\beta\alpha}(p_1 - p_2)_\mu - g_{\mu\alpha}(2p_1 + p_2)_\beta \quad (31)$$



and  $p_{1,2}$  are the four momenta of the outgoing W bosons. In the expression eq. (30) we have assumed that the  $Z'WW$  vertex has the usual Yang-Mills form. We do not consider here the possibility of anomalous magnetic or quadrupole type of couplings. An analysis with anomalous  $ZWW$  and  $Z'WW$  couplings is possible along the lines of [24] but is beyond the scope of this report. Our analysis will be nevertheless rather general as the trilinear  $Z'WW$  coupling will be treated as a free parameter, not necessarily proportional to the  $Z - Z'$  mixing angle as for example it would appear in a "conventional"  $E_6$  picture.

For the purposes of this working group, it will be particularly convenient to describe the virtual  $Z'$  effect as an "effective" modification of  $Z$  and  $\gamma$  couplings to fermions and W pairs. As one can easily derive, this corresponds to the use of the following modified trilinear couplings that fully describe the effect in the final process  $e^+e^- \rightarrow W^+W^-$ :

$$g_{\gamma WW}^* = g_{\gamma WW} + g_{Z'WW} \frac{q^2}{M_{Z'}^2 - q^2} g_{Vl} (\xi_{Vl} - \xi_{Al}) \quad (32)$$

$$g_{ZWW}^* = g_{ZWW} - g_{Z'WW} \frac{q^2 - M_Z^2}{M_{Z'}^2 - q^2} \xi_{Al} \quad (33)$$

In the previous equations, the same definitions as in eq. (22) and in eq. (23) have been used. In the following we shall use the results obtained on  $\xi_{Vl}$  and  $\xi_{Al}$  in the previous section. Our normalisation for trilinear couplings is such that:  $g_{\gamma WW} = 1$  and  $g_{ZWW} = \frac{c_0}{s_0}$ .

Adopting the notations that are available in the recent literature [25], we find for the  $Z'$  effect:

$$\delta_{\gamma}^{(Z')} = g_{\gamma WW}^* - 1 = g_{Z'WW} \frac{q^2}{M_{Z'}^2 - q^2} g_{Vl} (\xi_{Vl} - \xi_{Al}) \quad (34)$$

$$\delta_Z^{(Z')} = g_{ZWW}^* - \cot \Theta_W = -g_{Z'WW} \frac{q^2 - M_Z^2}{M_{Z'}^2 - q^2} \xi_{Al} \quad (35)$$

From eq. (34) and eq. (35) one can derive the following constraint:

$$\frac{\delta_{\gamma}^{(Z')}}{\delta_Z^{(Z')}} = \frac{-q^2}{q^2 - M_Z^2} \left( \frac{\xi_{Vl} - \xi_{Al}}{\xi_{Al}} \right) g_{Vl} \quad (36)$$

We then notice that the virtual effect of a general  $Z'$  in the WW channel is, at first sight, quite similar to that of a possible model with anomalous gauge couplings, that would also produce shifts  $\delta_{\gamma}$ ,  $\delta_Z$  both in the  $\gamma WW$  and in the  $ZWW$  couplings. But the  $Z'$  shifts satisfy in fact the constraint given in eq. (36), that corresponds to a certain line in the  $(\delta_{\gamma}, \delta_Z)$  plane whose angular coefficient is fixed by the model i.e. by the values of  $\xi_{Al}$  and  $(\xi_{Vl} - \xi_{Al})$ .

We shall now introduce the following ansatz concerning the theoretical expression of  $g_{Z'WW}$ , that we shall write as:

$$g_{Z'WW} = \left( c \frac{M_Z^2}{M_{Z'}^2} \right) \cot \Theta_W \quad (37)$$

The constant  $c$  would be of the order of one for the "conventional" models where the  $Z'$  couples to  $W$  only via the  $Z - Z'$  mixing (essentially contained in the bracket of eq. (37)). But for a general model,  $c$  could be larger, as one can see for some special cases of composite models[26] or when the  $Z'$  originates from a strong coupling regime[27]. In fact, a stringent bound on  $c$  comes from the request that the partial  $Z'$  width into  $WW$  has to be "small" compared to the  $Z'$  mass. Imposing the reasonable limit:

$$\Gamma_{Z'WW} \leq \frac{1}{10} M_{Z'} \quad (38)$$

leads to the condition:

$$c \leq 10 \quad (39)$$

that we consider a rather "extreme" choice.

We shall now discuss the observability limits on  $\delta_\gamma$  and  $\delta_Z$ . According to [25], six equidistant bins in the cosine of the production angle are chosen for the generation of data, such that each bin contains a reasonable number of events ( $\geq 4$ ). A binned maximum likelihood method has been used. The result for one parameter fit  $\delta_Z$  is:  $-0.2 \leq \delta_Z \leq 0.25$  for the configuration  $\sqrt{q^2} = 175$  GeV and  $\int L dt = 500 pb^{-1}$  and similarly for  $\delta_\gamma$ . We have then considered a number of possible illustrative situations, as extensively discussed in [28] and found that even in correspondence to the available present CDF limits and for the optimistic choice  $c = 10$ , one would get an effect of about 1%, i.e. well below the expected LEP2 observability limit.

In conclusion, a  $Z'$  of even pathologically small mass, for extreme values of its assumed couplings, would be unable to produce observable effects in the  $WW$  channel at LEP2. Therefore in the derivation of bounds or searches for visible effects, the final fermionic channel provides all the relevant information.

## 5 Concluding remarks.

We have tried in this report to be as concise and essential as possible, partially owing to the lack of space. In this spirit, we feel that a proper conclusion to our work might be that of stressing that LEP2, under realistic experimental conditions and in a rather near future, will be able to perform a clean and competitive, in some respects quite unique, search for effects of

a  $Z'$  whose mass is not above the TeV boundary. For  $M_{Z'}$  values beyond this limit, only more energetic machines will be able to continue this task.

## Acknowledgements.

This work has been partially supported by EC contract CHRX-CT94-0579.

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