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The Chromomagnetic Dipole Operator and the B Semileptonic Branching Ratio

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ABSTRACT

We consider the possibility of having a large branching ratio for the decay $b \to sg$ coming from an enhanced Wilson coefficient of the chromomagnetic dipole operator. We show that values of $BR(b \to sg)$ up to $\sim 10\%$ or more are compatible with the constraints coming from the CLEO experimental results on $BR(B \to X_s\gamma)$ and $BR(B \to X_s\phi)$. Such large values can reconcile the predictions of both the semileptonic branching ratio and the charm counting with the present experimental results. We also discuss a supersymmetric model with gluino-mediated flavour violations, which can account for such large values of $BR(b \to sg)$.

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1 Introduction

The world average of all measurements of the B-meson semileptonic branching ratio is [1]

$$BR_{SL}^{exp} \equiv BR(B \to X e \bar{\nu}_e) = (10.4 \pm 0.4)\% \ . \tag{1}$$

This result is considerably smaller than the theoretical prediction in the parton model, where $BR_{SL} \sim$ 13–15% [2]. Furthermore $1/m_Q^2$ non-perturbative corrections cannot reduce the predicted BR_{SL} below 12.5% [3]. However it has recently been found that charm mass corrections to $\Gamma(b \to c\bar{c}s)$ are large and can further reduce the theoretical prediction for BR_{SL} [4]:

$$BR_{SL} = \begin{cases} (12.0 \pm 0.7 \pm 0.5^{+0.9}_{-1.2} \pm 0.2)\% & \text{on-shell scheme} \\ (11.3 \pm 0.6 \pm 0.7^{+0.9}_{-1.7} \pm 0.2)\% & \overline{\text{MS}} \text{ scheme} \end{cases} \tag{2}$$

Here the errors correspond to the uncertainties on m_b, $\alpha_s(M_Z)$, the renormalization scale μ and the other parameters present in the calculation. It may now seem that the discrepancy in the B semileptonic branching ratio has essentially disappeared. However, as we lower the theoretical prediction for BR_{SL} by increasing $\Gamma(b \to c\bar{c}s)$, we simultaneously increase the prediction for n_c, the average number of charm hadrons produced per B decay. Indeed the theoretical prediction for n_c obtained from eq. (2) is

$$n_c = \begin{cases} 1.24 \pm 0.05 \pm 0.01 & \text{on-shell scheme} \\ 1.30 \pm 0.03 \pm 0.03 \pm 0.01 & \overline{\text{MS}} \text{ scheme} \end{cases} \tag{3}$$

where the first error corresponds to the quark mass uncertainties, the second to the α_s variation in $\overline{\text{MS}}$ and the last to all the other uncertainties. This prediction is larger than the present world average of the n_c measurements [1]

$$n_c^{exp} = 1.117 \pm 0.046 \ . \tag{4}$$

The real problem is therefore to understand the presence of a low BR_{SL} together with a low value of n_c. The theoretical difficulty is summarized in fig. 1.

It could well be that this is just an experimental problem; measurements of n_c rely on model assumptions about charm hadron production which have been questioned in ref. [5]. Recently it has been proposed [6] that "spectator effects" can be at the origin of the problem. Although these effects cannot be fully computed without a knowledge of the hadronic parameters, potentially they can bring BR_{SL} close to the experimental value. They also result in a simultaneous, although modest, decrease of n_c [6]. On the other hand, if "spectator effects" are responsible for the observed low value of the ratio $\tau(\Lambda_b)/\tau(B_d) = 0.76 \pm 0.05$, they then tend to increase the prediction for BR_{SL}, making the problem even more pressing. Recently it has also been suggested [7] that the data indicate the presence of $1/m_Q$ corrections in non-leptonic decays, which are not present in HQET. This has been attributed to a possible failure of the operator product expansion near the physical cut and a corresponding violation of the local quark–hadron duality. In particular, using the phenomenological recipe of replacing the quark mass by the decaying hadron mass in the m^5 factor in front of all non-leptonic widths, BR_{SL}, n_c, and $\tau(\Lambda_b)/\tau(B_d)$ can be reconciled with their measured values [7].

In this paper we discuss the effect of new physics on BR_{SL} and n_c. The possibility that large contributions to $\Gamma(b \to sg)$ can eliminate the disagreement between experiments and Standard Model predictions was first proposed in ref. [8], and then studied in much further detail in ref. [9]. Our goal here is to refine previous analyses and consider new constraints.

In sect. 2 we perform a model-independent analysis of $\Gamma(b \to sg)$ and the possible constraints coming from $\Gamma(B \to X_s\gamma)$ and $\Gamma(B \to X_s\phi)$, extending the results of ref. [9]. In particular we show that $\Gamma(B \to X_s\phi)$, in spite of its sensitivity to penguin operators, provides a poor probe of New Physics effects. In sect. 3 we discuss a supersymmetric model with gluino-mediated flavour violations, first suggested by Kagan [9], which can explain a large enhancement of $\Gamma(b \to sg)$.

2 Model-Independent Analysis

The effective Hamiltonian relevant for $\Delta B = 1$ decays is given by [10]

$$\begin{aligned} H_{eff}^{\Delta B=1} &= \frac{G_F}{\sqrt{2}}\Big[V_{cb}^* V_{cs}\left(C_1(\mu)Q_1^c(\mu) + C_2(\mu)Q_2^c(\mu)\right) + \\ &\quad V_{ub}^* V_{us}\left(C_1(\mu)Q_1^u(\mu) + C_2(\mu)Q_2^u(\mu)\right) - V_{tb}^* V_{ts}\sum_{i=3}^{12} C_i(\mu)Q_i(\mu)\Big], \end{aligned} \tag{5}$$

where V_{ij} are the CKM matrix elements and $C_i(\mu)$ are the Wilson coefficients evaluated at the scale μ of order m_b. The dimension-six local operator basis Q_i is given by

$$\begin{aligned} Q_1^q &= (\bar{s}_\alpha q_\beta)_{V-A}(\bar{q}_\beta b_\alpha)_{V-A} \\ Q_2^q &= (\bar{s}_\alpha q_\alpha)_{V-A}(\bar{q}_\beta b_\beta)_{V-A} \end{aligned}$$

Parameter	Value
$\lvert V_{ts}^* V_{tb}\rvert^2/\lvert V_{cb}\rvert^2$	0.95
$\alpha_s(M_Z)$	0.117
m_t (GeV)	174
m_b (GeV)	5
m_s (MeV)	500
m_c/m_b	0.316
M_ϕ (GeV)	1.019
g_ϕ^2 (GeV^4)	0.0586

Table 1: *Central values of the input parameters used in the analysis*

$$\begin{aligned}
Q_{3,5} &= (\bar{s}_\alpha b_\alpha)_{V-A}\sum_q(\bar{q}_\beta q_\beta)_{V\mp A} \\
Q_{4,6} &= (\bar{s}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V\mp A} \\
Q_{7,9} &= \frac{3}{2}(\bar{s}_\alpha b_\alpha)_{V-A}\sum_q e_q(\bar{q}_\beta q_\beta)_{V\pm A} \\
Q_{8,10} &= \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V\pm A} \\
Q_{11} &= \frac{g_s}{16\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu}_{V+A} t^A_{\alpha\beta} b_\beta G^A_{\mu\nu} \\
Q_{12} &= \frac{e}{16\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu}_{V+A} b_\alpha F_{\mu\nu}.
\end{aligned} \tag{6}$$

Here $(\bar{q}_1 q_2)(\bar{q}_3 q_4)$ denotes current–current products, $V \pm A$ indicates the chiral structure and α, β are colour indices. Moreover, g_s (e) is the strong (electromagnetic) coupling constant, $G^A_{\mu\nu}$ ($F_{\mu\nu}$) is the gluon (photon) field strength, and the t^A are the $SU(N)$ colour matrices normalized so that $Tr(t^A t^B) = \delta^{AB}/2$.

This Hamiltonian is known at the next-to-leading order (NLO), as far as one separately considers the current–current, gluon and photon penguin operators Q_1-Q_{10} on the one hand [11] and the magnetic-type operators Q_{11}-Q_{12} on the other hand [12]. Unfortunately the mixing between these two classes of operators is at present known only at the leading order (LO) [13].

The branching ratio of the decay $b \to sg$ is given by [14]

$$BR(b \to sg) = BR_{SL}^{exp}\frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2}\frac{2\alpha_s(\mu)}{\pi g(m_c/m_b)}|C_{11}^{eff}(\mu)|^2\,, \tag{7}$$

where BR_{SL}^{exp} is the measured semileptonic branching ratio, the phase-space factor $g(z)$ is given in the Appendix, $\mu = O(m_b)$ and C_{11}^{eff} is the renormalization-scheme invariant coefficient[1] introduced in ref. [15]. With the input values given in table 1, the LO Standard Model prediction is

$$BR^{SM}(b \to sg) = (2.3 \pm 0.6) \times 10^{-3} \; , \tag{8}$$

where the error is mainly due to the variation of μ between $m_b/2$ and $2m_b$ and to the uncertainty on $\alpha_s(M_Z)$. Inclusion of the known part of the NLO corrections reduces the central value and the μ dependence, but introduces a significant scheme dependence [14].

In the effective Hamiltonian approach, physics beyond the weak scale affects only the initial conditions of the Wilson coefficients, namely $C_i(M_W)$. Therefore these coefficients can be used to parametrize new physics effects without referring to a specific model. In the case at hand, we assume that the initial condition of the Wilson coefficient $C_{11}(M_W)$ is an independent variable and define

$$r_g = \frac{C_{11}(M_W)}{C_{11}^{SM}(M_W)} \; , \tag{9}$$

where

$$\begin{aligned} C_{11}^{SM}(M_W) &= g_1(m_t^2/m_W^2) \; , \\ g_1(x) &= \frac{x(-x^2+5x+2)}{4(x-1)^3} - \frac{3x^2 \log x}{2(x-1)^4} \; . \end{aligned} \tag{10}$$

In terms of r_g, the branching ratio of $b \to sg$ is given by

$$BR(b \to sg) = 2.73 \times 10^{-2}(0.15 + 0.14 \; r_g)^2 \; , \tag{11}$$

assuming central values for all the relevant parameters.

An enhanced value of $BR(b \to sg)$ affects the relation between n_c and BR_{SL}. We find

$$n_c = 2 - (2 + R_\tau + R_{ud} + 2R_{\not{c}})BR_{SL} - 2BR(b \to sg) \; , \tag{12}$$

where [16]

$$\begin{aligned} R_\tau &= \frac{\Gamma(b \to c\tau\nu)}{\Gamma(b \to ce\nu)} = 0.25 \\ R_{ud} &= \frac{\Gamma(b \to c\bar{u}d')}{\Gamma(b \to ce\nu)} = 4.0 \pm 0.4 \\ R_{\not{c}} &= \frac{\Gamma(b \to \text{no charm})}{\Gamma(b \to ce\nu)} = 0.25 \pm 0.10. \end{aligned} \tag{13}$$

[1]Using calculations in the 't Hooft–Veltman scheme, as we do, $C_{11,12}^{eff} = C_{11,12}^{HV}$. Therefore we denote them $C_{11,12}$ in the following.

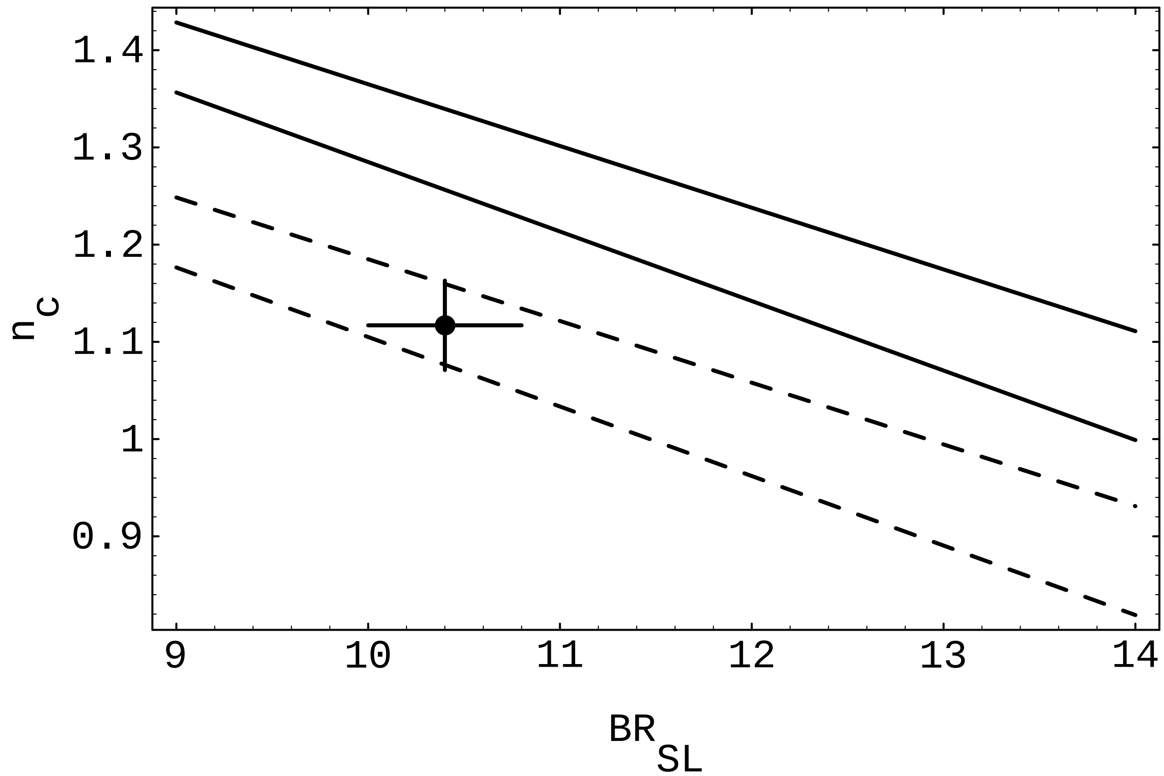

Figure 1: *Correlation between the semileptonic branching ratio BR_{SL} and the number of charms per B decay n_c. The solid band is the Standard Model prediction, including the theoretical uncertainties, while the dashed one is obtained assuming a $BR(b \to sg) = 9\%$. The experimental data point is also shown.*

Here $\Gamma(b \to \text{no charm})$ is the sum of all B decay widths into charmless final states different from sg. Notice that in eq. (12) we have eliminated the dependence on $\Gamma(b \to c\bar{c}s)$, which is the main source of theoretical uncertainty. If we require that the experimental central values of BR_{SL} and n_c, eqs. (1) and (4), lie within the theoretical uncertainty, we need $7\% < BR(b \to sg) < 11\%$. This corresponds to $10.2 < r_g < 13.1$ or $-12.3 < r_g < -15.2$. For instance the effect of $BR(b \to sg) = 9\%$ in eq. (12) is shown in fig. 1 (see the dashed band).

We now want to perform a model-independent analysis of the constraints on r_g. The first constraint comes from the observation of the inclusive decay $B \to X_s\gamma$. The LO expression for $BR(B \to X_s\gamma)$ is [14, 15]

$$BR(B \to X_s\gamma) = BR_{SL}^{exp} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{3\alpha_e}{2\pi g(m_c/m_b)} |C_{12}^{eff}(\mu)|^2. \tag{14}$$

It is well known that this branching ratio has very large QCD corrections. It also has a significant theoretical uncertainty, mainly due to the scale dependence in the Wilson coefficient, to be taken into account. Also in this case, the known NLO terms decrease the central value and the μ dependence, but introduce a sizeable scheme dependence, to be cancelled by the unknown terms.

The dependence on r_g in eq. (14) comes from the operator mixing in the QCD renormalization group equations. Analogously to eq. (9), we define

$$r_\gamma = \frac{C_{12}(M_W)}{C_{12}^{SM}(M_W)} \; , \tag{15}$$

where

$$\begin{aligned} C_{12}^{SM}(M_W) &= f_1(m_t^2/m_W^2) \; , \\ f_1(x) &= \frac{x(-8x^2-5x+7)}{12(x-1)^3} + \frac{x^2(3x-2)\log x}{2(x-1)^4} \; . \end{aligned} \tag{16}$$

We can now rewrite eq. (14) in terms of r_g and r_γ as

$$BR(B \to X_s\gamma) = 7.10 \times 10^{-4}(0.017\, r_g + 0.273\, r_\gamma + 0.313)^2, \tag{17}$$

where the theoretical error is not shown. This formula gives a model-independent parametrization of $BR(B \to X_s\gamma)$. Figure 2 shows the ranges of r_g and r_γ allowed by the measurement $BR(B \to X_s\gamma) = (2.32 \pm 0.67) \times 10^{-4}$ [17]. The existence of two different bands corresponds to the overall sign ambiguity of $C_{12}^{eff}(\mu)$ in eq. (14). The size of the bands accounts for both the experimental error at the 1σ level and a (linearly added) theoretical error of 30% [15]. Figure 2 shows that very large enhancements of $C_{11}(M_W)$ are allowed, since they can be compensated by appropriate, and relatively small, changes of $C_{12}(M_W)$. A large values of $C_{11}(M_W)$ can result in a modification of the photon energy spectrum in $B \to X_s\gamma$ and could be tested in the future if the threshold in the observed E_γ is appropriately lowered[2] [18]. The solution of the BR_{SL}–n_c problem in terms of $\Gamma(b \to sg)$ singles out the four disconnected regions in the r_g-r_γ plane shown in fig. 2.

The extraction of analogous constraints from the measured upper limit [19]

$$BR(B \to X_s\phi) < (1.1 - 2.2) \times 10^{-4} \qquad (90\% \text{ CL}) \tag{18}$$

is more involved. Available theoretical estimates [20, 21] rely on some assumptions which are questionable. In particular, hadronic matrix elements are calculated by combining perturbation

[2] We thank A. Ali for pointing out this possible experimental test.

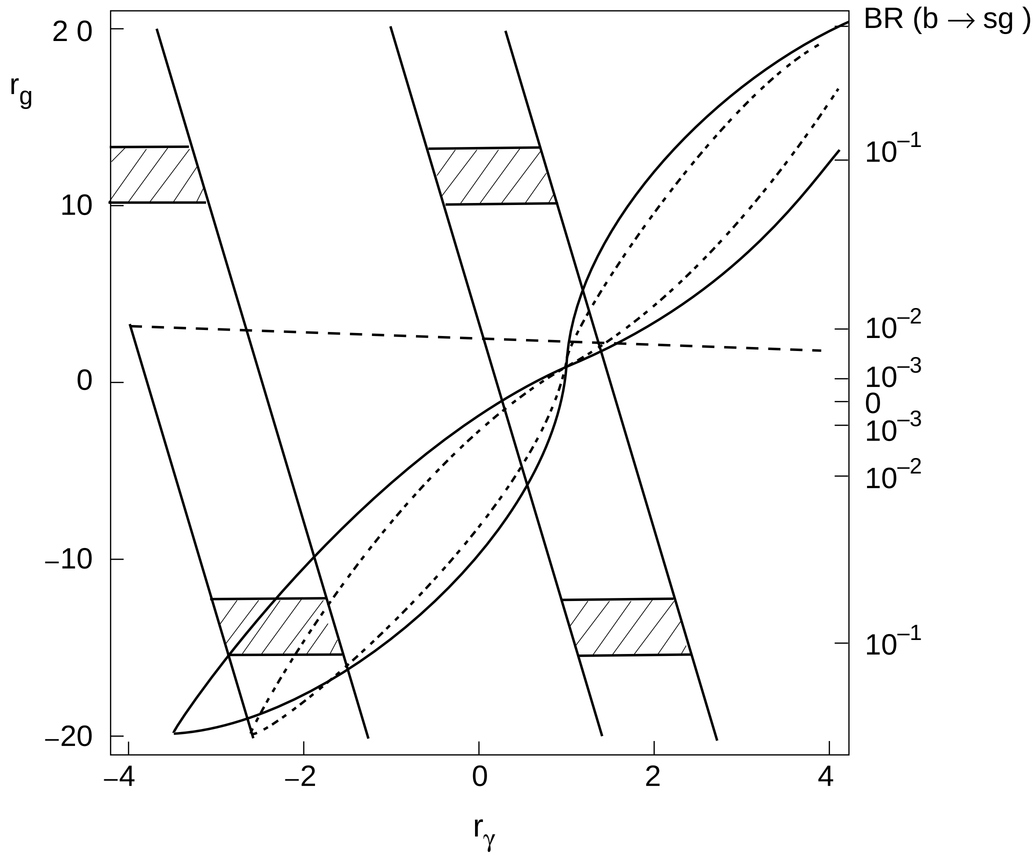

Figure 2: *Correlation between the ratios r_g and r_γ, defined in eqs. (9) and (15). The scale on the right shows the corresponding values of $BR(b \to sg)$. The regions outside the two oblique bands are excluded by the measured $BR(B \to X_s\gamma)$. The region above the dashed line is excluded by the upper limit on the $BR(B \to X_s\phi)$, assuming $\xi = 1/3$. However, the theoretical error is large and not completely under control in this case, so that the limit is plotted only for illustrative purpose. Finally the two oblated regions delimit the ranges allowed in the supersymmetric model with LL (dotted line) and LR (solid line) flavour mixing obtained for $m_{\tilde{g}} > 200$ GeV and $\tilde{m} > 100$ GeV.*

theory with the factorization method. Moreover they made a strong assumption on the quark momenta distribution inside the ϕ meson ($p_s = p_{\bar{s}} = p_\phi/2$). These hypotheses result in a pure two-body decay $b \to s\phi$, while for example string fragmentation models indicate high multiplicity final states for the $b \to sg$ decay [22]. The same assumptions are also used to extract the experimental limit from the measurements. Thus, constraints coming from this limit have large theoretical uncertainties and should be taken with caution.

Nonetheless, in the following, we calculate the constraints in the r_g-r_γ plane, coming from the limit on $BR(B \to X_s\phi)$, under the same assumptions made in refs. [20, 21], which allow a straightforward calculation of the branching ratio. In terms of r_g-r_γ, the following expression is obtained:

$$\begin{aligned}BR(B \to X_s\phi) &= a_1|C_\phi(m_b)|^2 + a_2\alpha_s(m_b)Re(C_\phi(m_b))r_g + a_3\alpha_e Re(C_\phi(m_b))r_\gamma \\ &\quad + a_4\alpha_s(m_b)^2 r_g^2 + a_5\alpha_e^2 r_\gamma^2 + a_6\alpha_s(m_b)\alpha_e r_g r_\gamma, \end{aligned} \tag{19}$$

where the coefficients a_i are functions of the masses m_b, M_ϕ, m_s. They are explicitly given in the Appendix.

Beside C_{11} and C_{12}, eq. (19) depends on the coefficient C_ϕ. It appears when the matrix elements of the relevant penguin and electropenguin operators are evaluated using the factorization method and it is given by [20]

$$\begin{aligned}C_\phi(\mu) &= C_3^{eff}(\mu) + C_4^{eff}(\mu) + C_5^{eff}(\mu) + \xi\left(C_3^{eff}(\mu) + C_4^{eff}(\mu) + C_6^{eff}(\mu)\right) \\ &\quad -\frac{1}{2}\left[C_7^{eff}(\mu) + C_9^{eff}(\mu) + C_{10}^{eff}(\mu) + \xi\left(C_8^{eff}(\mu) + C_9^{eff}(\mu) + C_{10}^{eff}(\mu)\right)\right], \end{aligned} \tag{20}$$

where $\xi = 1/N$. Here N is the number of colours, taken as a free parameter to account for possible deviations from the factorization result. Since the magnetic operators contribute to eq. (19) through $O(\alpha_s)$ $(O(\alpha_e))$ matrix elements, it is mandatory to calculate C_ϕ at NLO. The effective coefficients[3] $C_3^{eff} - C_{10}^{eff}$ are defined so that they include the NLO contributions coming from the matrix elements at $O(\alpha_s)$ $(O(\alpha_e))$, calculated in perturbation theory [20, 23]. With such a definition, $C_3^{eff} - C_{10}^{eff}$ become complex functions. We note that the calculations of these coefficients in refs. [20, 23] are not complete. A full NLO perturbative calculation of the matrix elements would allow for a complete cancellation (at the considered order) of the scheme dependence of the NLO Wilson coefficients that appear in C_ϕ. In our case, we use NLO coefficients $C_1 - C_{10}$ calculated in the 't Hooft–Veltman scheme, including only the largest contribution to the NLO matrix elements coming from Q_2, as in ref. [20]. We have estimated the residual scheme dependence ($\sim 15\%$), by comparing our effective coefficients to those given in ref. [20]. We consider it negligible, compared with the other uncertainties present in the calculation.

The presence of C_ϕ in eq. (19) could make our analysis much more involved, since this coefficient depends on several initial conditions other than $C_{11}(M_W)$ and $C_{12}(M_W)$. However

[3]Notice that the denomination "effective coefficient" refers to completely different definitions in the case of $C_3 - C_{10}$ and $C_{11} - C_{12}$.

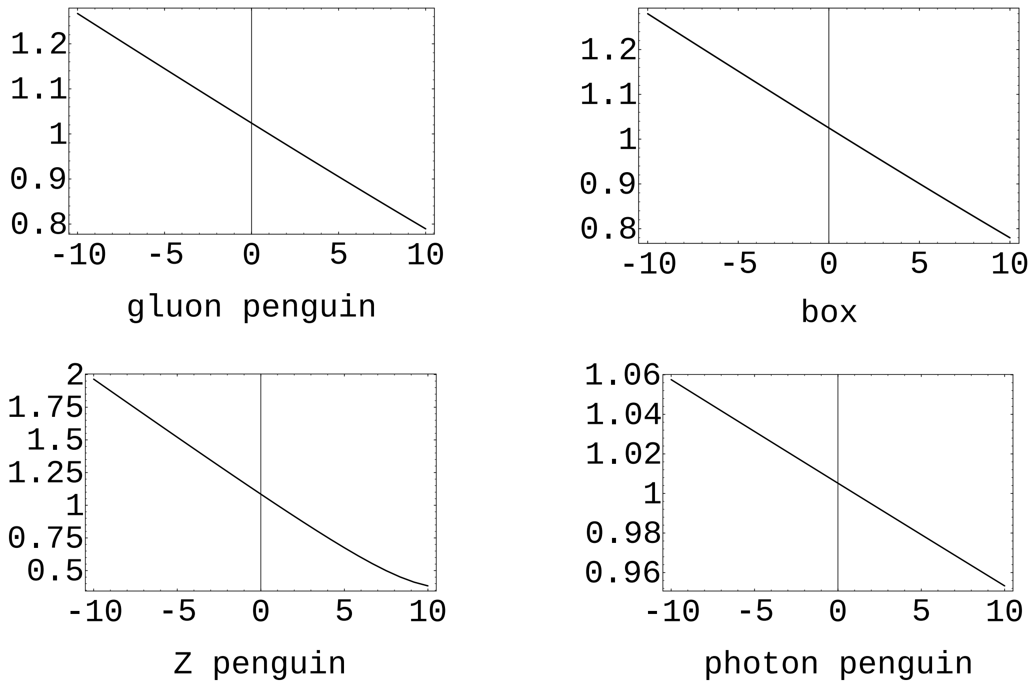

Figure 3: *The coefficient $|C_\phi|$ as a function of $C_3(M_W)$-$C_{10}(M_W)$, normalized to the Standard Model. The eight initial conditions are parametrized in terms of the four contributions coming from the box and the g, Z, γ penguin diagrams.*

C_ϕ is quite insensitive to the initial conditions of the penguin and electropenguin operators $Q_3 - Q_{10}$. In fig. 3 we show $|C_\phi|$ as a function of these initial conditions, parametrized in terms of the contributions of the gluon, Z, γ penguin and the box diagrams, normalized to the Standard Model. The reason of this weak dependence is twofold. On the one hand, the largest terms in C_ϕ come from the gluon penguin operators $Q_3 - Q_6$, which are insensitive to their initial conditions because the large mixing with $Q_1 - Q_2$ dominates their renormalization group evolution down to $\mu = O(m_b)$. On the other hand, the effective coefficients C_i^{eff} get a large NLO contribution from the $O(\alpha_s)$ matrix elements, which do not depend on the initial conditions of the Wilson coefficients. This implies that $BR(B \to X_s\phi)$ in general gives a poor probe of new physics effects, contrary to what is often stated in the literature.

Therefore, it is reasonable to assume C_ϕ as a constant in the analysis, so that the $BR(B \to X_s\phi)$ could be used to put constraints in the $r_g - r_\gamma$ plane. Unfortunately this constraint suffers from large theoretical uncertainties. For example the predicted branching ratio changes by a factor of 2 as we vary ξ between 1/2 and 1/3, which is a popular way to account for the uncertainty of the factorization method [20]. Moreover the assumption on the decay kinematics is not under control and the related uncertainty is not quantifiable. Just for illustrative purposes we show in fig. 2 the constraint coming from eq. (19), assuming $\xi = 1/3$. For larger values of ξ, the bound on r_g from $B \to X_s \phi$ becomes more stringent, disfavouring even further the solutions of the BR_{SL}–n_c problems corresponding to positive r_g. However if we assign an overall uncertainty of a factor of 3 in the prediction of the branching ratio, any significant bound on r_g disappears.

An enhanced $b \to sg$ decay rate also affects the exclusive decays $B \to K\pi$ [21]. We estimate $BR(B^- \to \bar{K}^0 \pi^-) \simeq 10^{-5}(1 + 0.1\ r_g)^2$, which corresponds to an effect of order 1 for $|r_g| \sim 10$. Again the theoretical uncertainty of this prediction is large and not under control, so we prefer not to show this constraint in fig. 2.

We have seen how present constraints allow large enhancements of $BR(b \to sg)$. Measurements of $BR(B \to X_s \gamma)$ require, however, a precise correlation between r_g and r_γ. The main difficulty to solve the BR_{SL}–n_c problem in terms of new physics is to explain this correlation. We now turn to a discussion of models in which this is possible.

3 An Illustrative Model

The suggestion that anomalously large $BR(b \to sg)$ can explain a reduction of BR_{SL} was first made in ref. [8], where it was assumed that the new effective bsg interaction is mediated by virtual charged Higgs boson exchange. This possibility is now ruled out by a combination of the constraints from $B \to X\tau\bar{\nu}$, $B \to X_s\gamma$, and $Z \to b\bar{b}$. The main difficulty of models where the bsg interaction arises from charged-Higgs exchange, shared by most other models with weakly-interacting new particles, is that generically $r_g \sim r_\gamma$. Constraints from $BR(B \to X_s\gamma)$ then allow only small enhancements of r_g.

The possibility that flavour-changing quark–squark–gluino interactions can generate large coefficients for the chromomagnetic operator was first suggested in ref. [9]. Now the loop generating O_{11} has a large Casimir factor, which is not present in the loop generating O_{12}, and one obtains $r_g/r_\gamma \sim$ 5–7. This allows a significant enhancement of $b \to sg$, especially if we consider solutions with negative r_γ.

We have in mind the case in which the squark mass matrix is not diagonal in the quark mass eigenbasis. This situation is generic in supersymmetric models derived from supergravity with non-minimal Kähler metric. For simplicity we consider separately two possibilities. In the first case, we assume that flavour non-diagonal entries of the squark mass matrix appear only in the left sector and there is no left–right squark mixing. The new contributions to the Wilson coefficients C_{11} and C_{12}, evaluated at the scale of supersymmetric particle masses, are [24]

$$C_{11} = \frac{Z}{m_{\tilde{g}}^2} \sum_{i=1}^{3} U_{ib}^* U_{is} g_2(m_{\tilde{g}}^2/\tilde{m}_i^2) \tag{21}$$

$$g_2(x) = \frac{x(-11x^2+40x+19)}{36(x-1)^3} + \frac{x^2(x-9)\log x}{6(x-1)^4} \tag{22}$$

$$C_{12} = \frac{Z}{m_{\tilde{g}}^2} \sum_{i=1}^{3} U_{ib}^* U_{is} f_2(m_{\tilde{g}}^2/\tilde{m}_i^2) \tag{23}$$

$$f_2(x) = \frac{2x(-2x^2-5x+1)}{27(x-1)^3} + \frac{4x^3\log x}{9(x-1)^4} \tag{24}$$

$$Z \equiv \frac{\sqrt{2}\pi\alpha_s}{V_{tb}^* V_{ts} G_F} \tag{25}$$

Here $m_{\tilde{g}}$ is the gluino mass and $\tilde{m}_i^2$ are the eigenvalues of the down-squark squared-mass matrix, which is diagonalized by the 3×3 unitary matrix U. If $\tilde{m}_i$ are nearly degenerate, eqs. (21) and (23) can be Taylor-expanded around the common squark mass $\tilde{m}$:

$$C_{11} = Z\frac{\delta\tilde{m}_{b_L s_L}^2}{\tilde{m}^4} g_3(m_{\tilde{g}}^2/\tilde{m}^2) \tag{26}$$

$$g_3(x) = \frac{(x^2+172x+19)}{36(x-1)^4} + \frac{x(x^2-15x-18)\log x}{6(x-1)^5} \tag{27}$$

$$C_{12} = Z\frac{\delta\tilde{m}_{b_L s_L}^2}{\tilde{m}^4} f_3(m_{\tilde{g}}^2/\tilde{m}^2) \tag{28}$$

$$f_3(x) = \frac{2(-17x^2-8x+1)}{27(x-1)^4} + \frac{4x^2(x+3)\log x}{9(x-1)^5} \tag{29}$$

Here $\delta\tilde{m}_{b_L s_L}^2 \equiv \sum U_{ib}^* U_{is} \tilde{m}_i^2$ corresponds to the flavour non-diagonal mass insertion.

In the second case we consider, the down-squark mass matrices for the left and right sectors are both diagonal, but there are flavour non-diagonal left–right (LR) mixing terms. Since these terms are not generated by supersymmetry breaking, in the limit of vanishing Yukawa coupling constants, we will assume here that they are proportional to the corresponding Yukawa coupling. In this case, the mass-insertion approximation is always adequate and the new contributions to the Wilson coefficients C_{11} and C_{12} are

$$C_{11} = Z \frac{\delta \tilde{m}^2_{b_R s_L}}{m_b \tilde{m}^3} g_4(m^2_{\tilde{g}}/\tilde{m}^2) \tag{30}$$

$$g_4(x) = \sqrt{x} \left[-\frac{2(x+11)}{3(x-1)^3} + \frac{(-x^2+16x+9)\log x}{3(x-1)^4} \right] \tag{31}$$

$$C_{12} = Z \frac{\delta \tilde{m}^2_{b_R s_L}}{m_b \tilde{m}^3} f_4(m^2_{\tilde{g}}/\tilde{m}^2) \tag{32}$$

$$f_4(x) = \sqrt{x} \left[\frac{4(5x+1)}{9(x-1)^3} - \frac{8x(x+2)\log x}{9(x-1)^4} \right]. \tag{33}$$

The two cases here considered are the simplest because they generate only operators with the same chiral structure as in the Standard Model. They also describe the generic features of more complicated squark mass matrices.

By varying the relevant supersymmetric parameters under the requirement that all squark masses are larger than 100 GeV and $m_{\tilde{g}} > 200$ GeV, we find that the new interactions can generate values of r_g and r_γ within the regions illustrated in fig. 2. The prediction has a strong correlation in the r_g–r_γ plane because the ratio r_g/r_γ depends on the supersymmetric parameters only weakly, through the loop functions. Thus, in spite of its generic features, the model can make a rather precise prediction of this ratio. It is interesting that it is indeed possible to reach a region in which $b \to sg$ has the correct enhancement to solve the BR_{SL}–n_c puzzle. The solution always corresponds to the case in which the signs of both C_{11} and C_{12} are opposite to the Standard Model results. In the case of left–left (LL) mixings this region is achieved when the mixing between the s and b squark is maximal[4] and when the lightest squark mass[5] is around 100 GeV and $m_{\tilde{g}}$ around 200 GeV. This means that supersymmetry could be soon discovered at the Tevatron, but no light squark has to be expected at LEP2. Similar conclusions apply to the case of LR mixing. However if the LR mixing were not proportional to m_b, the same effects could be obtained for much heavier squarks and gluinos, although the existence of colour-breaking minima could impose significant constraints on the parameter

[4]Indeed the mixing is so large that the mass-insertion approximation is not valid. The region in fig. 2 was obtained by using the complete expressions in eqs. (21) and (23).

[5]Here we refer to the mass of the lightest squark, which can be considerably lighter than the other squarks, because of the large mixing. Tevatron bounds on squark masses [25] do not directly apply here, since they are obtained under the assumption of three families of degenerate squarks.

space. Finally notice that, in the case of a very light gluino ($m_{\tilde{g}} \sim 1$ GeV) the ratio r_g/r_γ can become much larger than that shown in fig. 2, especially for LR mixings. In this case it is possible to have r_g as small as -10 for positive values of r_γ, and almost reach the shaded region in fig. 2 corresponding to positive r_γ and negative r_g.

Given a definite model of flavour violation, we can consider more tests on its consistency than in the case of the model-independent analysis of sec. 2. We now assume that gluino-mediated flavour violations occur only between the second and third generation of the down quark–squark sector and consider different processes with $|\Delta B| \neq 0$, which can be of experimental interest.

The $|\Delta B| = 2$ transitions will affect the B_s–$\bar{B}_s$ mixing. For LL mass insertions, the new contribution to Δm_{B_s} is [24]

$$\Delta m_{B_s} = \frac{4}{3} B_{B_s} f_{B_s}^2 m_{B_s} \frac{\alpha_s^2}{\tilde{m}^2} \left(\frac{\delta \tilde{m}^2_{b_L s_L}}{\tilde{m}^2} \right)^2 G(m_{\tilde{g}}^2/\tilde{m}^2) \tag{34}$$

$$G(x) = \frac{1}{216} \left[\frac{(-2x^3 + 27x^2 + 144x + 11)}{(x-1)^4} - \frac{2x(51x+39)\log x}{(x-1)^5} \right] . \tag{35}$$

This contribution is much larger than the Standard Model one, making the discovery of B_s–$\bar{B}_s$ mixing even more arduous. However, for $x \simeq 2.4$, the function $G(x)$ has a zero and therefore, for particular values of the squark and gluino masses, we can reduce the total contribution to Δm_{B_s}. Nevertheless, the generic prediction of the model is that B_s–$\bar{B}_s$ mixing will not be discovered soon.

The $|\Delta B| = 1$ transitions can induce the flavour-changing decay $Z \to \bar{b}s$. For LL transitions [26]

$$BR(Z \to \bar{b}s) = \frac{\alpha \alpha_s^2 m_Z}{18\pi^2 \sin^2\theta_W \cos^2\theta_W} \left(1 - \frac{2}{3}\sin^2\theta_W \right)^2 \left| U_{ib}^* U_{is} F(\tilde{m}_i^2, m_{\tilde{g}}^2, m_Z^2) \right|^2 \tag{36}$$

$$F(\tilde{m}_i^2, m_{\tilde{g}}^2, m_Z^2) = \int_0^1 dx \int_0^{1-x} dy \log \left[\frac{\tilde{m}_i^2(x+y) + m_{\tilde{g}}^2(1-x-y) - m_Z^2 xy}{\tilde{m}_i^2(1-x) + m_{\tilde{g}}^2 x} \right] . \tag{37}$$

Assuming the validity of the mass insertion approximation, eq. (36) can be simply expressed in terms of r_g. For $\tilde{m}^2 = m_{\tilde{g}}^2 > m_Z^2$, we find

$$BR(Z \to \bar{b}s) = 5 \times 10^{-8}\ r_g^2 . \tag{38}$$

Unfortunately, observation of this decay mode represents a real experimental challenge. LEP1 can at best reach a sensitivity on $BR(Z \to \bar{b}s)$ of about 10^{-3} [27].

Finally we consider FCNC decay processes of B mesons, such as $B \to X_s\nu\bar{\nu}$ and $B \to X_s\ell^+\ell^-$. It is known [24] that gluino-mediated interactions do not affect the Z penguin operator. Indeed, the effective b-s-Z vertex turns out to be proportional to the b-s-γ vertex. QED gauge invariance then implies that the Z penguin is suppressed by a factor q^2/m_Z^2, where q^2 is the momentum transfer. Because of this suppression we do not expect new contributions to $B \to X_s\nu\bar{\nu}$. On the other hand, the process $B \to X_s\ell^+\ell^-$ receives new contributions from the gluino-mediated γ penguin operator and from the electromagnetic dipole operator, which has here a sign opposite to the Standard Model result. This change of sign implies important modifications both in the rate and the lepton asymmetries of $B \to X_s\ell^+\ell^-$ [28]. The new contribution to the γ penguin is however rather modest. We find that LL transitions give

$$C_e = \frac{Z}{m_{\tilde{g}}^2}\sum_{i=1}^{3} U_{ib}^* U_{is} f(m_{\tilde{g}}^2/\tilde{m}_i^2) \tag{39}$$

$$f(x) = \frac{2x(11x^2-7x+2)}{81(x-1)^3} - \frac{12x^4\log x}{81(x-1)^4}\,, \tag{40}$$

where C_e is the Wilson coefficient at the weak scale of the operator

$$O_e = \frac{\alpha_e}{2\pi}(\bar{s}_\alpha b_\alpha)_{V-A}(\bar{e}e)_V\,. \tag{41}$$

Using the mass-insertion approximation and taking for simplicity $m_{\tilde{g}}^2/\tilde{m}_i^2 = 1$, eq. (39) can be rewritten as

$$\frac{C_e}{C_e^{SM}} \simeq 5\times 10^{-2}\; r_g\,. \tag{42}$$

4 Conclusions

In conclusion we have discussed how large $BR(b \to sg)$ can reconcile the predictions for BR_{SL} and n_c with present experimental measurements. Constraints from $BR(B \to X_s\gamma)$ require a precise correlation between new physics effects in the coefficients of the chromomagnetic and electromagnetic dipole operators. On the other hand $BR(B \to X_s\phi)$ does not set any significant constraints on the parameter space, mainly because of the large theoretical uncertainties involved in the calculation. We have also shown that $BR(B \to X_s\phi)$ does not provide a good probe of new physics effects because the relevant coefficient $C_\phi(m_b)$ is rather insensitive to the initial conditions at the weak scale of the gluon–penguin operators.

Finally we have discussed how a supersymmetric model with gluino-mediated flavour violations can account for a large value of $BR(b \to sg)$, consistently with all other constraints from $\Delta B = 1$ and $\Delta B = 2$ FCNC processes. The theory has a rather precise correlation between the predictions for r_g and r_γ, which is fairly independent of the specific model assumptions. Although the enhancement of $BR(b \to sg)$ is achieved only for particular values of the parameter space and it is evidently not a general consequence of the model, it is interesting to know that supersymmetry can potentially predict $BR(b \to sg) \simeq 5$–10%, and solve the BR_{SL}–n_c puzzle. Future experiments will certainly be able to test this scenario.

Note added

After completion of this work, we received a note by A. Kagan who informed us that their new results on $B \to X_s \phi$ agree with those presented in this paper [29].

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Appendix

In this appendix we report on the result of our calculation of $BR(b \to s\phi)$. As in refs. [20, 21], we assume a pure two-body decay and use the factorization method to estimate the relevant matrix elements. We have computed both the contributions of Q_{11} and Q_{12}. The amplitude we obtain is

$$\begin{aligned} A(b \to s\phi) \;=\; & -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* i g_\phi \epsilon_\mu \left\{ 2C_\phi(m_b) \bar{s}\gamma^\mu L b + \frac{\alpha_s(m_b) m_b}{8\pi k^2} \frac{N^2-1}{2N^2} C_{11}(m_b) \left[m_b \bar{s}\gamma^\mu L b \right.\right. \\ & + \left(6 + 4\frac{m_s^2}{M_\phi^2}\right) p_b^\mu \bar{s} R b - m_s \left(1 - 2\frac{m_b^2 + 2M_\phi^2 - m_s^2}{M_\phi^2}\right) \bar{s}\gamma^\mu R b - 4\frac{m_b m_s}{M_\phi^2} p_b^\mu \bar{s} L b \Big] \\ & + \frac{\alpha_e m_b}{8\pi k^2} C_{12}(m_b) \left[9 m_b \bar{s}\gamma^\mu L b - \left(10 - 4\frac{m_s^2}{M_\phi^2}\right) p_b^\mu \bar{s} R b - m_s \left(1 \right.\right. \end{aligned}$$

$$-2\frac{m_b^2+2M_\phi^2-m_s^2}{M_\phi^2}\Bigg)\,\bar{s}\gamma^\mu Rb-4\frac{m_bm_s}{M_\phi^2}p_b^\mu\bar{s}Lb\Bigg]\Bigg\}, \tag{43}$$

where $k^2=(m_b^2+m_s^2-M_\phi^2/2)/2$, $C_\phi(m_b)$ has been introduced in eq. (20) and g_ϕ is defined by $\langle\phi|\bar{s}\gamma^\mu s|0\rangle=ig_\phi\epsilon^\mu$. This result does not fully agree with those of refs. [20, 21][6].

From eq. (43), we obtain the branching ratio

$$\begin{aligned}
BR(b &\to s\phi)=\frac{\Gamma(b\to s\phi)}{\Gamma(b\to ce\nu)}BR_{SL}^{exp}=\frac{|V_{tb}V_{ts}^*|^2}{|V_{cb}|^2}\frac{6\pi^2g_\phi^2BR_{SL}^{exp}}{g(m_c/m_b)\Omega(m_c/m_b,m_b)m_b^4}\lambda(m_b^2,M_\phi^2,m_s^2)^{1/2}\\
&\Bigg[|C_\phi(m_b)|^2\left(2+2\frac{m_b^2}{M_\phi^2}-4\frac{M_\phi^2}{m_b^2}+2\frac{m_s^4}{m_b^2M_\phi^2}+2\frac{m_s^2}{m_b^2}-4\frac{m_s^2}{M_\phi^2}\right)\\
&+\frac{\alpha_s(m_b)}{8\pi}\frac{2\mathrm{Re}\left(C_\phi(m_b)\right)C_{11}(m_b)}{k^2}\left(\frac{N^2-1}{2N^2}\right)\Bigg(-5m_b^2-20\frac{m_b^2m_s^2}{M_\phi^2}+4\frac{m_b^4}{M_\phi^2}+M_\phi^2\\
&+16\frac{m_s^4}{M_\phi^2}-23m_s^2\Bigg)+\frac{\alpha_e}{8\pi}\frac{2\mathrm{Re}\left(C_\phi(m_b)\right)C_{12}(m_b)}{k^2}\Bigg(19m_b^2-20\frac{m_b^2m_s^2}{M_\phi^2}+4\frac{m_b^4}{M_\phi^2}\\
&-23M_\phi^2+16\frac{m_s^4}{M_\phi^2}+m_s^2\Bigg)+\left(\frac{\alpha_s(m_b)C_{11}(m_b)}{8\pi k^2}\right)^2\left(\frac{N^2-1}{2N^2}\right)^2\Bigg(\frac{31}{2}m_b^2M_\phi^2\\
&-4\frac{m_b^2m_s^6}{M_\phi^4}+16\frac{m_b^2m_s^4}{M_\phi^2}-54m_b^2m_s^2-19m_b^4-4\frac{m_b^4m_s^4}{M_\phi^4}+4\frac{m_b^6m_s^2}{M_\phi^4}\\
&+8\frac{m_b^6}{M_\phi^2}+\frac{15}{2}M_\phi^2m_s^2-\frac{9}{2}M_\phi^4+4\frac{m_s^8}{M_\phi^4}-8\frac{m_s^6}{M_\phi^2}+m_s^4\Bigg)+\left(\frac{\alpha_eC_{12}(m_b)}{8\pi k^2}\right)^2\Bigg(-\frac{177}{2}m_b^2M_\phi^2\\
&-4\frac{m_b^2m_s^6}{M_\phi^4}+112\frac{m_b^2m_s^4}{M_\phi^2}-62m_b^2m_s^2+93m_b^4-4\frac{m_b^4m_s^4}{M_\phi^4}-96\frac{m_b^4m_s^2}{M_\phi^2}+4\frac{m_b^6m_s^2}{M_\phi^4}\\
&+8\frac{m_b^6}{M_\phi^2}+\frac{47}{2}M_\phi^2m_s^2-\frac{25}{2}M_\phi^4+4\frac{m_s^8}{M_\phi^4}-8\frac{m_s^6}{M_\phi^2}-7m_s^4\Bigg)\\
&+\frac{\alpha_s(m_b)C_{11}(m_b)}{8\pi k^2}\frac{\alpha_eC_{12}(m_b)}{8\pi k^2}\left(\frac{N^2-1}{2N^2}\right)\Bigg(-41m_b^2M_\phi^2-8\frac{m_b^2m_s^6}{M_\phi^4}+192\frac{m_b^2m_s^4}{M_\phi^2}\\
&-212m_b^2m_s^2+10m_b^4-8\frac{m_b^4m_s^4}{M_\phi^4}-128\frac{m_b^4m_s^2}{M_\phi^2}+8\frac{m_b^6m_s^2}{M_\phi^4}+16\frac{m_b^6}{M_\phi^2}-65M_\phi^2m_s^2\\
&+15M_\phi^4+8\frac{m_s^8}{M_\phi^4}-48\frac{m_s^6}{M_\phi^2}+90m_s^4\Bigg)\Bigg],
\end{aligned} \tag{44}$$

where N is the number of colours, $\lambda(m_1,m_2,m_3)=(1-m_2/m_1-m_3/m_1)^2-4m_2m_3/m_1^2$, $g(z)=1-8z^2+8z^6-z^8-24z^4\ln(z)$ is the phase-space correction and $\Omega(z,\mu)\simeq 1-\frac{2\alpha_s(\mu)}{3\pi}\left[\left(\pi^2-\frac{31}{4}\right)(1-z)^2+\frac{3}{2}\right]$ is the QCD correction to the semileptonic decay rate.

The cofficients a_i of eq. (19) can be immediately obtained from eq. (44).

[6] Numerically our branching ratio is $\sim 25\%$ smaller, including the effect of the residual scheme dependence.

References

[1] T.E. Browder, preprint UH-511-893-95, to appear in the *Proceedings of the Int. Europhysics Conf. on High Energy Physics*, Brussels, Belgium, July 1995 [hep-ex/9602009];
T. Skwarnicki, preprint HEPSY-95-05, to appear in the *Proceedings of the 17th Int. Conf. on Lepton-Photon Interactions*, Beijing, China, August 1995 [hep-ph/9512395].

[2] G. Altarelli and S. Petrarca, *Phys. Lett.* **B261** (1991) 303.

[3] I. Bigi, B. Blok, M.A. Shifman and A. Vainshtein, *Phys. Lett.* **B323** (1994) 408.

[4] E. Bagan, P. Ball, V.M. Braun and P. Gosdzinsky, *Nucl. Phys.* **B432** (1994) 3; *Phys. Lett.* **B342** (1995) 362; Erratum [hep-ph/9602364];
see also M. Neubert, preprint CERN-TH/95-307, to appear in the *Proceedings of the 17th Int. Conf. on Lepton-Photon Interactions*, Beijing, China, August 1995 [hep-ph/9511409].

[5] I. Dunietz, P.S. Cooper, A.F. Falk and M.B. Wise, *Phys. Rev. Lett.* **73** (1994) 1075.

[6] M. Neubert and C.T. Sachrajda, preprint CERN-TH/96-19 [hep-ph/9603202].

[7] G. Altarelli, G. Martinelli, S. Petrarca and F. Rapuano, preprint CERN-TH/96-73 [hep-ph/9604202].

[8] B. Grzadkowski and W.-S. Hou, *Phys. Lett.* **B272** (1991) 383.

[9] A.L. Kagan, *Phys. Rev.* **D51** (1995) 6196.

[10] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Nucl. Phys.* **B370** (1992) 69 and **375**(1992) 501.

[11] A.J. Buras, M. Jamin, M.E. Lautenbacher, *Nucl. Phys.* **B400** (1993) 37 and 75;
M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *Nucl. Phys.* **B415** (1994) 403.

[12] M. Misiak and M. Münz, *Phys. Lett.* **B344** (1995) 308.

[13] M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini, *Phys. Lett.* **B316** (1993) 127;
M. Ciuchini, E. Franco, L. Reina and L. Silvestrini, *Nucl. Phys.* **B421** (1994) 41;

G. Cella, G. Curci, G. Ricciardi and A. Viceré, *Nucl. Phys.* **B431** (1994) 417;
M. Misiak, *Nucl. Phys.* **B439** (1995) 461.

[14] M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini, *Phys. Lett.* **B334** (1994) 137.

[15] A.J. Buras, M. Misiak, M. Münz and S. Pokorski, *Nucl. Phys.* **B424** (1994) 374.

[16] G. Buchalla, I. Dunietz and H. Yamamoto, *Phys. Lett.* **B364** (1995) 188.

[17] M.S. Alam et al. (CLEO Collaboration), *Phys. Rev. Lett.* **74** (1995) 2885.

[18] A. Ali and C. Greub, work in progress.

[19] K.W. Edwards et al. (CLEO Collaboration), preprint CLEO-CONF-95-8, EPS0162 (1995).

[20] N.G. Deshpande and X.-G. He, *Phys. Lett.* **B336** (1994) 471.

[21] N.G. Deshpande, X.-G. He and J. Trampetić, preprint OITS-582 [hep-ph/9509346].

[22] J. Swain, preprint NUB-3101 [hep-ph/9501415].

[23] G. Kramer, W.F. Palmer and H. Simma, *Nucl. Phys.* **B428** (1994) 77.

[24] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, *Nucl. Phys.* **B353** (1991) 591.

[25] J. Hauser (CDF Collaboration), *Proceedings of the 10th Topical Workshop on Proton-Antiproton Collider Physics*, Batavia, IL, May 1995, edited by R. Raja and J. Yoh (AIP Conference Proceedings, 357);
S. Abachi et al. (D0 Collaboration), preprint FERMILAB-CONF-95/193-E, to appear in the *Proceedings of the Int. Europhysics Conf. on High Energy Physics*, Brussels, Belgium, July 1995.

[26] G. Gamberini and G. Ridolfi, *Nucl. Phys.* **B287** (1987) 304.

[27] Y. Khokhlov, private communication.

[28] A. Ali, G.F. Giudice and T. Mannel, *Z. Phys.* **C67** (1995) 417.

[29] A. Kagan and A. Perez, work in progress.