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## Supergravity Domain Walls

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## Abstract

We review the status of domain walls in  $N = 1$  supergravity theories for both the *vacuum* domain walls as well as *dilatonic* domain walls. We concentrate on a systematic analysis of the nature of the space-time in such domain wall backgrounds and the special rôle that supersymmetry is playing in determining the nature of such configurations. Isotropic vacuum domain walls that can exist between isolated minima of a  $N = 1$  supergravity matter potential fall into three classes: (i) extreme walls which are static planar walls between supersymmetric minima, (ii) non-extreme walls which are expanding bubbles with two centres and (iii) ultra-extreme walls which are bubbles of false vacuum decay. Dilatonic walls arise in  $N = 1$  supergravity with the linear supermultiplet, an additional scalar field—the dilaton—has no perturbative self-interaction, which however, couples to the matter potential responsible for the formation of the wall. The dilaton drastically changes the global space-time properties of the wall. For the extreme ones the space-time structure depends on the strength of the dilaton coupling, while for non- and ultra-extreme solutions one always encounters naked singularities (in the absence of non-perturbative corrections to the dilaton potential). Non-perturbative effects may modify the dilaton coupling so that it has a discrete non-compact symmetry ( $S$ -duality). In this case the non- and ultra-extreme solutions can reduce to the singularity-free vacuum domain wall solutions. We also summarize domain wall configurations within effective theory of  $N = 1$  superstring vacua, with or without inclusion of non-perturbative string effects, and also provide a comparison with other topological defects of perturbative string vacua.

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# 1 Introduction

Topological defects can occur in a physical system when the vacuum manifold of the system possesses a non-trivial topology. Domain walls correspond to a specific type of topological defects that can occur when the vacuum manifold consists of (energetically degenerate) disconnected components. Among the topological defects, domain walls are the most extended ones, and thus may have the most “disruptive” implications for the nature of the non-trivial ground state of the physical system.

In fundamental theories of elementary particles the spontaneous (gauge) symmetry breaking via the Higgs mechanism [106, 82, 125, 124, 111, 126] plays a central rôle. Such theories in general possess a degenerate vacuum manifold with a non-trivial topology, as specified by the potential of the scalar (Higgs) field(s). Therefore it is plausible that topological defects could be important in basic theory, in particular in its application to cosmology.

In particular, domain wall solutions exist in theories where the scalar field potential has isolated minima. The walls are surfaces interpolating between separate minima of the scalar potential with different vacuum expectation values of the scalar field. In this case the scalar field changes with spatial position and settles in one minimum at one spatial infinity while in the other direction it settles in another disconnected minimum. The interpolating region of rapid change of the scalar field corresponds to the domain wall. In the thin wall approximation the variation of the scalar field energy density is localized at the domain wall surface, and is replaced by the delta function. In the case when all the matter fields are constant on each side of the wall, i.e. they are settled at the minimum of the potential, the domain walls are referred to as vacuum domain walls.

In the early universe such a domain structure can form by the Kibble [144, 227] mechanism whereby different regions of a hot universe cool into different isolated minima of the matter potential. Domain walls [227] can also form as the boundary of a (true) vacuum bubble created by the quantum tunneling process of false vacuum decay [50]. Additionally, the universe could be born through a quantum tunneling process from nothing [223, 224, 120, 226, 169, 95] into different domains with walls in between.

## 1.1 Classes of domain walls

Because of its extended nature, the space–time around the domain wall is drastically affected. Tension reduces the gravitational mass, and in the case of a domain wall where tension is equal to the energy-density and where there are two spatial directions contributing with tension and only one time-direction contributing with an energy-density, the total gravitational mass is negative. The nature of space–time in the presence of domain walls is a central topic of this review. Let us first briefly summarize some earlier developments in this direction.

The first solution of Einstein’s field equations for the gravitational field produced by a thin planar domain wall was found by Vilenkin [221] in the linear approximation for the gravitational field. In this solution, the vacuum energy or the cosmological constant vanishes on both sides of the wall. Since the linear approximation for the gravitational field breaks down at large distances, this solution could not say anything about the global structure of the gravitational field, but as one should expect, test-particles near the wall were found to be gravitationally repelled by the wall. However, the physical meaning of this approximate solution remained obscure since it does not correspond to *any* exact static solution of Einstein’s field equations: in fact, no such solution exists [73]. Instead, the corresponding exact (thin wall) solution [225] has a *time-dependent* metric; it is the (2+1)-dimensional de Sitter space on the wall’s world volume and Minkowski space–time elsewhere. A coordinate transformation revealed that Vilenkin’s [225] exact thin “planar wall” solution is a segment of an accelerated sphere [136, 135] which comes in from infinity, turns around, and heads back out to infinity.

The gravitational effects of spherically symmetric thin vacuum bubbles had earlier been studied in connection with vacuum decay [52]. Using Israel’s method [137] of singular layers Berezin, Kuzmin and Tkachev [21, 23] studied a spherically symmetric domain wall separating regions of true or false vacua with arbitrary non-negative energy densities (and also allowing for a non-vanishing Schwarzschild mass parameter). The vacuum decay bubbles were distinguished from those originating as results of phase transitions by their different surface energy density.

Domain walls between Minkowski, de Sitter, Schwarzschild, and Schwarzschild–de Sitter spaces are discussed in Refs. [136, 154, 198, 30, 10, 24, 90].

Another class of domain walls arise in theories where certain scalar fields (dilaton) do not possess isolated minima of the matter potential, but can couple to the matter potential responsible for the formation of the wall. In this case the dilaton can vary with the spatial distance from the wall, thus forming a new type of walls: dilatonic walls. Particular examples are dilatonic walls in the Brans–Dicke theory and certain effective theories describing perturbative string vacua. As specific examples, a static planar domain wall in general relativity coupled to a conformally coupled massless scalar field [110] and a solution for general Brans–Dicke coupling  $\omega$  [153] have been found.

Most of the analysis of the domain wall solutions has been done within the thin wall approximation. However, also *thick* domain wall solutions have been addressed [242, 241, 104, 177, 9, 8]. The interest in such solutions was boosted by the suggestion that late-time phase transitions could produce soft topological defects, such as very light domain walls [127] and even more when it was suggested that the Great Attractor could be such a domain wall [207].

## 1.2 Walls in $N = 1$ supergravity

Spontaneously broken  $N = 1$  supergravity theory coupled to Yang–Mills fields provides one of a very few viable theories, which can explain low energy phe-

phenomena. Such theories are therefore a subject of intense research of their phenomenological [156] as well as certain cosmological implications. The problem of non-renormalizability of supergravity theory has been removed by realizing that supergravity is an effective low-energy theory of superstring theory, which is believed to be a *finite* theory of gravity and gauge interactions.

It is therefore important to address different aspects of  $N = 1$  supergravity theory, and in particular those that arise as an effective theory of four dimensional superstring vacua. In this paper we shall review the status of domain wall configurations in a specific theory of elementary particles, i.e. in a theory of gravity and gauge interactions, that possesses (spontaneously broken)  $N = 1$  supersymmetry.

The first type of  $N = 1$  supergravity walls are vacuum domain walls between vacua with non-positive cosmological constants (Minkowski and anti-de Sitter vacua). They can be classified [63, 62] according to the values of their energy densities  $\sigma$ . In particular, for a special value of the domain wall energy density there exist [172] static, reflection symmetric, planar domain wall with anti-de Sitter space-time on both sides. They turn out to correspond to a special case of a *static* planar *extreme* domain walls with a *supersymmetric embedding* in  $N = 1$  supergravity theory [60, 59, 57] interpolating between supersymmetric minima of the scalar potential. They have (in the thin wall approximation) a fixed energy density  $\sigma = \sigma_{\text{ext}}$ , which is specified by the values of the cosmological constant on each side of the wall. The non-extreme (isotropic) walls have  $\sigma > \sigma_{\text{ext}}$  and correspond to accelerating two-centered bubbles, while ultra-extreme wall solutions have  $\sigma < \sigma_{\text{ext}}$  and correspond to the false vacuum decay bubbles [63, 62]. Non- and ultra-extreme walls do not have supersymmetric embeddings, with the extreme solution (with supersymmetric embedding) providing a dividing line between the two classes of them.

Another class, dilatonic domain walls, arise within  $N = 1$  supergravity coupled to the linear supermultiplet. For special couplings of the dilaton field describe certain effective theories of perturbative string vacua. A specific example, a static planar domain wall in general relativity coupled to a conformally coupled massless scalar field [110], corresponds to a special case of a supersymmetric dilatonic domain walls [54] of  $N = 1$  supergravity coupled to the linear supermultiplet with a particular value for the dilaton coupling. Classification of dilatonic domain walls according to the value of its energy density has been given in Ref. [65].

Here we shall review the status of domain wall configurations within a general class of  $N = 1$  supergravity theories, and within effective theories of superstring vacua as a special example. The emphasis will be on a systematic analysis of the nature of the space-time in such domain wall backgrounds and the special rôle that supersymmetry is playing in determining the nature of such configurations. We will summarize the analysis for both the *vacuum* domain walls as well as *dilatonic* domain walls. In addition, the analysis will incorporate the non-supersymmetric generalizations of these domain wall backgrounds, and will include the majority of the results for the domain wall examples discussed in the literature.

The review is organized in the following way: In Section 2 we present the structure of the Lagrangian of  $N = 1$  supergravity coupled to Yang–Mills and matter fields focusing on the bosonic part of the action responsible for formation of defects. Section 3 gives an overview of the physics of topological defects as a general background to the subject of this review. Then in Section 4 the space–time symmetry assumptions, the metric ansatz, and the thin wall formalism and the general relativistic field equations are presented. In Section 5 we show how these domain walls can be embedded in  $N = 1$  supergravity theory. Section 6 is devoted to a study of vacuum domain walls and their induced space–times. These results are generalized to the dilatonic case in Section 7. In Section 8 we discuss the connection of the supergravity domain walls to other topological defects of four-dimensional supergravity theories. Here we also discuss implications for the domain walls for effective supergravity vacua from superstring theory. Conclusions are given in Section 9.



## 2 Supergravity theory

In this section we shall review the structure of the effective Lagrangian of  $N = 1$  supergravity coupled to the Yang–Mills and matter fields.  $N = 1$  refers to the fact that the Poincaré algebra is extended with a single spinor generator. We shall primarily concentrate on the bosonic part of the Lagrangian, since this is the one responsible for the formation of the topological defects. In addition, since the matter fields which make up the defects are assumed to be neutral under the gauge symmetry, we will be most specific about the part of the Lagrangian that involves the gravitational and gauge neutral matter supermultiplets.

We also spell out the supersymmetry transformations for the gravitational and matter super-fields. Again, we concentrate on supersymmetry transformations of the fermionic fields, since those are the non-trivial supersymmetry transformations that are preserved by the supersymmetric (classical) bosonic domain wall backgrounds.

There is a number of excellent reviews addressing the supergravity theories in general and  $N = 1$  supergravity theory in particular, and in much more details than covered in this chapter. We refer the interested reader to Refs. [85, 218, 182, 92, 240, 239].

Throughout this paper we use units such that  $\kappa \equiv 8\pi G = c = 1$ . Our sign convention for the metric, the Riemann tensor, and the Einstein tensor, is of the type  $(- + +)$  as classified by Misner, Thorne and Wheeler [176]. Also, we use the conventions:  $\gamma^\mu = e^\mu{}_a \gamma^a$  where  $\gamma^a$  are the flat space–time Dirac matrices satisfying  $\{\gamma^a, \gamma^b\} = 2\eta^{ab} I_4$ ,  $\gamma^5 = \gamma^0 \gamma^1 \gamma^2 \gamma^3$ ;  $e^a{}_\mu e^\mu{}_b = \delta^a_b$ ;  $a \in \{0, 1, 2, 3\}$ ;  $\mu \in \{t, x, y, z\}$ .

### 2.1 Field content of $N = 1$ supergravity

$N = 1$  supergravity theory preserves local (space–time) dependent supersymmetric transformations. The field content of  $N = 1$  supergravity theory coupled to the Yang–Mills fields and matter fields consists of the gravitational, the gauge field, and matter supermultiplets. The physical field components of the superfields correspond to dynamical fields, which remain in the Lagrangian after elimination of the auxiliary fields through the equations of motion. The physical particle spectrum of the three types of superfields is the following:

- The *gravitational supermultiplet*  $\Psi_{\mu\nu}$  contains the spin-2 component—the graviton field  $g_{\mu\nu}$ , and spin- $\frac{3}{2}$  component—the gravitino  $\psi_\mu^\alpha$ ,
- The *vector (Yang–Mills) superfields*  $\mathcal{W}_\alpha^{(a)}$ , whose components in the Wess–Zumino gauge are spin-1 gauge field  $A_\mu^{(a)}$  and spin- $\frac{1}{2}$  gaugino field  $\lambda^{(a)}$ ,
- The *chiral superfields*  $\mathcal{T}_j$  whose spin-0 component is a complex scalar field  $T_j$  and spin- $\frac{1}{2}$  component is its supersymmetric partners  $\chi_j$ ,

- The *linear supermultiplet*  $\mathcal{L}$ , which are gauge neutral fields, and contain a spin-0 scalar field  $\phi$ , an anti-symmetric field  $b_{\mu\nu}$ , which is related to a pseudo-scalar field through a duality transformation, and the supersymmetric partner spin- $\frac{1}{2}$  field  $\eta$ .

The linear supermultiplet  $\mathcal{L}$  can be rewritten (on-shell) in terms of a chiral supermultiplet  $\mathcal{S}$ , by performing a duality transformation [86, 28, 185, 3].<sup>1</sup> The chiral multiplet  $\mathcal{S}$  has the property that, due to the left-over Peccei–Quinn-type symmetry, it cannot appear in the superpotential, and thus, its bosonic component cannot have a potential. In the following we shall follow the description in terms of chiral superfields, only.

In superstring theory,<sup>2</sup> some of the chiral supermultiplets do not have any self-interaction, i.e. their superpotential is zero. They are referred to as moduli, since the vacuum expectation values of their scalar components parameterize the compactification space of string theory. In superstring theory the dilaton superfield is the linear supermultiplet whose vacuum expectation value of its scalar component parameterizes the strength of the gauge coupling in string theory.

## 2.2 Bosonic part of the Lagrangian

In the following we shall write down the bosonic part of the  $N = 1$  supergravity Lagrangian which involves the graviton, the Yang–Mills vector field and the scalar components of the chiral supermultiplets.<sup>3</sup>

The  $N = 1$  supergravity Lagrangian is of a constrained form, specified by the gauge function  $f_{ab}$ , the Kähler potential  $K$  and the super-potential  $W$ , which are specific functions of matter fields, i.e. scalar components  $T_j$  of matter chiral-superfields  $\mathcal{T}_j$ . Some of the chiral super-multiplets may consist of moduli. We also allow for the existence of a linear super-multiplet, which in the Kähler superspace formalism can be rewritten as a chiral superfield  $\mathcal{S}$  (dilaton). It has no superpotential and its Kähler potential is of a special kind, which decouples from the one of  $\mathcal{T}_j$ .

The three functions specifying the  $N = 1$  supergravity Lagrangian are specified as:

- The gauge function  $f_{ab}$  is a holomorphic function of the chiral superfields  $\mathcal{T}_j$  and  $\mathcal{S}$ . In particular, in the bosonic Lagrangian it determines the gauge couplings to the field strengths of the Yang–Mills vector fields  $A_\mu^{(a)}$ . At the tree level of the Lagrangian, the gauge function is specified by the (gauge neutral) fields  $\mathcal{S}$ , only, i.e.  $f_{ab} = \delta_{ab}\mathcal{S}$ , however, at the loop-level

<sup>1</sup>We confine ourselves to one linear supermultiplet, only. For a discussion of more than one linear supermultiplet see, e.g. Ref. [27].

<sup>2</sup>For a review of the structure  $N = 1$  effective Lagrangian from superstring theory, see e.g. Refs. [141, 195].

<sup>3</sup>For the full  $N = 1$  supergravity Lagrangian with bosonic as well as fermionic fields see for example Ref. [239], chapters XXI through XXV and appendix G.

the chiral superfields  $\mathcal{T}_j$  can also contribute and thus in general:

$$f_{ab} = f_{ab}(\mathcal{S}, \mathcal{T}_j). \quad (2.1)$$

- The superpotential  $W$  is a holomorphic function of the chiral matter superfields,  $\mathcal{T}_j$ . In particular, in the bosonic part of the Lagrangian  $W$  specifies the potential for the matter fields. The superfield  $\mathcal{S}$  has *no* (perturbative) superpotential<sup>4</sup> ( $W_{\text{dil}}(\mathcal{S}) = 0$ ) thus:

$$W = W_{\text{matt}}(\mathcal{T}_j). \quad (2.2)$$

- The Kähler potential  $K$  is a real function of chiral superfields,  $\mathcal{T}_j$ , and of  $\mathcal{S}$ . In particular, in the bosonic part of the Lagrangian  $K$  specifies the (Kähler) metric of the kinetic energy terms for the scalar components of the matter superfields. At the tree level of the Lagrangian,  $\mathcal{T}_j$ , and  $\mathcal{S}$  do not couple to each other in the Kähler potential,<sup>5</sup> and thus the Kähler potential is of the form:

$$K = K_{\text{dil}}(\mathcal{S}, \mathcal{S}^*) + K_{\text{matt}}(\mathcal{T}_j, \mathcal{T}_j^*). \quad (2.3a)$$

where

$$K_{\text{dil}}(\mathcal{S}, \mathcal{S}^*) = -\alpha \ln(\mathcal{S} + \mathcal{S}^*). \quad (2.3b)$$

In  $N = 1$  supergravity  $\alpha \geq 0$  is a free parameter, while in string theory  $\alpha = 1$ .

Note that  $N = 1$  supergravity Lagrangian possesses the Kähler invariance associated with the transformation:

$$K_{\text{matt}}(\mathcal{T}_j, \mathcal{T}_j^*) \rightarrow K_{\text{matt}}(\mathcal{T}_j, \mathcal{T}_j^*) + W_{\text{matt}}(\mathcal{T}_j) + W_{\text{matt}}(\mathcal{T}_j^*)^*$$

The bosonic part of the Lagrangian is fully determined by the three functions  $f_{ab}$  (2.1),  $W$  (2.2) and  $K$  (2.3). When confined to the lowest derivative terms, it assumes the following form:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}R - \frac{1}{4}\Re(f_{ab})F_{\mu\nu}^{(a)}F^{(b)\mu\nu} + \frac{1}{8}\Im(f_{ab})\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^{(a)}F_{\rho\sigma}^{(b)} \\ & - K_{T_i T_j^*} \mathcal{D}_\mu T_i \mathcal{D}^\mu T_j^* - K_{S S^*} \partial_\mu S \partial^\mu S^* - V \end{aligned} \quad (2.4)$$

where the potential  $V$  is

$$\begin{aligned} V = e^K & \left[ K^{T_i T_j^*} \mathcal{D}_{T_i} W \mathcal{D}_{T_j^*} W^* - \left( 3 - K_S K_{S^*} K^{S S^*} \right) |W|^2 \right] \\ & + \frac{1}{2} D_{(a)} D^{(a)}, \end{aligned} \quad (2.5)$$

<sup>4</sup>However, non-perturbative effects, e.g. gaugino condensation in certain superstring models, can induce non-perturbative superpotential for  $S$ . For the status of such effects see e.g. Ref. [194].

<sup>5</sup>At the loop level, there could be, however, corrections, which induce mixing interactions either in the Kähler potential or in the gauge function. For such effects within the effective  $N = 1$  supergravity from superstring theory, see e.g. Refs. [86, 28, 185, 3]. We chose to insert the loop effects into the gauge function  $f_{ab}$  (2.1).

where  $D^{(a)}$  (“D-term”) is the Killing potential related to the holomorphic Killing vectors  $X^{(a)}$  of the Kähler manifold in the following way:

$$\begin{aligned} K_{T_i T_j^*} X^{j(a)*} &= i \frac{\partial D^{(a)}}{\partial T_i}, \\ K_{T_i T_j^*} X^{i(a)} &= -i \frac{\partial D^{(a)}}{\partial T_j^*}. \end{aligned}$$

The holomorphic Killing vectors generate the representation of the gauge group, i.e.  $[X^{(a)}, X^{(b)}] = -f_c^{ab} X^{(c)}$  etc. where  $f_c^{ab}$  are the structure constants of the gauge group. The D-term contribution to the scalar potential is due to the scalar components of the chiral supermultiplets  $\mathcal{T}_j$ , which transform as gauge non-singlets under the Yang–Mills gauge group.

Above, in Eq. (2.4), the gauge covariant derivative  $\mathcal{D}_\mu$  is defined as  $\mathcal{D}_\mu T_i \equiv \partial_\mu - A_\mu^{(a)} X^{i(a)}$ , and the gauge field strength as  $F_{\mu\nu}^{(a)} \equiv \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} - f_{bc}^a A_\mu^{(b)} A_\nu^{(c)}$ . The scalar components of the chiral superfields  $\mathcal{T}_i$  and  $\mathcal{S}$ , i.e.  $T_i$  and  $S$ , respectively, specify the following quantities in the bosonic Lagrangian (2.4):  $K_{T_i} = \partial_{T_i} K$  and  $K_{T_i T_j^*} = \partial_{T_i} \partial_{T_j^*} K$  is the positive definite Kähler metric, and  $D_{T_i} W = \partial_{T_i} W + K_{T_i} \bar{W}$ . As usual, summation over repeated indices is implied.

### 2.3 Bosonic Lagrangian and topological defects

The above bosonic Lagrangian (2.4) is a starting point for addressing the topological defect in  $N = 1$  Yang–Mills supergravity theory, in particular, (charged) domain walls, (charged) strings, monopoles and (charged) black holes. In general, the existence of  $S$  (gauge singlet without a perturbative potential) allows for existence of a new types of topological defects, where in the presence of the curved space–time and the non-trivial gauge fields  $A_\mu^{(a)}$  as well as the scalar fields  $T_i$ ,  $S$  vary with the space–time coordinates.

In the following chapters we shall primarily concentrate on the gauge-neutral domain wall configurations. Namely, we shall assume the potential (2.5) has a non-trivial structure, e.g. isolated minima, and that the matter field(s)  $T_i$ —which are gauge singlets, i.e. with flat D-terms—are responsible for the formation of domain walls. Supersymmetric minima of the potential correspond to those with  $D_{T_i} W|_{T_i=T_0} = 0$ .

A particular case which will be addressed in detail is a  $N = 1$  supergravity theory with one matter chiral superfield  $\mathcal{T}$  and a linear supermultiplet, expressed in terms of a chiral supermultiplet  $\mathcal{S}$  with the Kähler potential (2.3) [86, 28, 185, 3]. With the choice of the scalar component of  $\mathcal{S}$  written as  $S = e^{-2\phi/\sqrt{\alpha}} + i\mathcal{A}$ , where  $\mathcal{A}$  is the axion, the potential in Eq. (2.5) is of the form:

$$V = e^{2\sqrt{\alpha}\phi} e^{K_M} \left[ |D_T W|^2 K^{T T^*} - (3 - \alpha) |W|^2 \right]. \quad (2.6)$$

The above potential shall be a starting point for addressing domain wall configurations  $N = 1$  supergravity theory.

We shall also compare the space–time of such domain walls to that of certain charged black holes, which are specified by the space–time metric, the gauge fields  $A_\mu^{(a)}$  and the field  $S$ , all of them respecting the spherical symmetry.

## 2.4 Supersymmetry transformations

The supersymmetry transformations involve the physical components<sup>6</sup> of the gravitational, gauge and chiral superfields. We shall display the relevant supersymmetry transformations of the fermionic fields, since those are the ones that specify the Killing spinor equations, which in turn determine the supersymmetric topological defects. Namely, setting such supersymmetry transformations to zero in the presence of nontrivial bosonic field background defines the non-trivial bosonic field configuration with the minimal energy in its class. The fermionic field supersymmetry transformations with only bosonic fields turned on<sup>7</sup> is of the following form:

$$\delta\psi_\mu = \left[ 2\nabla_\rho + i e^{\frac{K}{2}} (\Re(W) + \gamma^5 \Im(W)) \gamma_\rho - \gamma^5 \Im(K_{T_j} \mathcal{D}_\rho T_j) - \gamma^5 \Im(K_S \mathcal{D}_\rho S) \right] \epsilon, \quad (2.7)$$

$$\delta\lambda^{(a)} = \left[ F_{\mu\nu}^{(a)} \gamma^{\mu\nu} - i D^{(a)} \right] \epsilon, \quad (2.8)$$

$$\delta\chi_j = -\sqrt{2} \left[ e^{\frac{K}{2}} K^{T_i T_j^*} (\Re(D_{T_j} W) + \gamma^5 \Im(D_{T_j} W)) + i (\Re(\mathcal{D}_\mu T_i) + \gamma^5 \Im(\mathcal{D}_\mu T_i)) \gamma^\mu \right] \epsilon \quad (2.9)$$

$$\delta\eta = -\sqrt{2} \left[ e^{\frac{K}{2}} K^{SS^*} (\Re(K_S W) + \gamma^5 \Im(K_S W)) + i (\Re(\mathcal{D}_\mu S) + \gamma^5 \Im(\mathcal{D}_\mu S)) \gamma^\mu \right] \epsilon, \quad (2.10)$$

where  $\epsilon$  is a Majorana spinor, and  $\nabla_\mu \epsilon = (\partial_\mu + \frac{1}{2} \omega^{ab}{}_\mu \sigma_{ab}) \epsilon$  and the Einstein summation convention is implied.

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<sup>6</sup>The auxiliary field components are again eliminated by their equations of motion.

<sup>7</sup>For the full set of supersymmetry transformations for the fermionic as well as bosonic fields see e.g. Ref. [239, chapter 23].

## 3 Topological defects and tunneling bubbles

### 3.1 Overview

#### 3.1.1 Topological defects in physics

Topological defects can be studied in the laboratory. Condensed matter systems provide a wide range of such structures. In the low-temperature regime one has magnetic flux lines of type II superconductors [133], quantized vortex-lines in superfluids [74], and in solid state physics one encounters the dislocation and disclination lines of crystals [149]. When viewed under the polarizing microscope, liquid crystals exhibit a variety of optical textures, each characteristic of defects peculiar to the state of molecular order prevailing in the substance [44]. But probably the most accessible example is the domain structure of ferromagnetic materials. For temperatures  $T$  above the Curie temperature  $T_C$ , all the dipoles are randomly oriented; the ground state is rotationally invariant. For  $T < T_C$ , we have spontaneous magnetization and the dipoles are aligned in some arbitrary direction. The magnetic energy is minimized when the ferromagnet splits into domains with different magnetizations. Domain walls appear at the domain boundaries, and must therefore be present in the equilibrium state. There are also other examples which have defects formed in non-equilibrium states.

Topological defects are related to some form of symmetry breaking which gives rise to a non-trivial set of degenerate ground states. Spontaneous (gauge) symmetry breaking via the Higgs mechanism [106, 82, 125, 124, 111, 126] has come to play a central rôle in modern elementary particle theory. Since such theories generally give rise to a degenerate vacuum manifold with a non-trivial topology, it is plausible that topological defects could be important also in particle physics, in particular in its application to cosmology.

Within field theory, Skyrme [205] found the first three-dimensional defect solution, i.e. the skyrmion, and proposed that such defect states could provide a description of observed particle states, i.e. mesons in nuclear physics. On the other hand, in high energy physics it has by now become a prevailing view that rather than explaining familiar particle excitations, the defect states—as by-products of spontaneous symmetry breaking [179]—provide additional non-perturbative states in the theory, which along with the perturbative spectrum of (particle) excitations, complete the full spectrum of the theory. Such defect states provide diverse and supplementary sectors in fundamental theory with interesting dynamics. Non-perturbative defect states turn out to be particularly important in string theory (for recent reviews see [39, 77].).

In addition, it has been recognized recently that supersymmetric topological defects, play a crucial rôle [134, 244] in establishing non-perturbative dualities in string theory, i.e. establishing the equivalence of certain strongly coupled and weakly coupled string vacua. In addition, the defects provide important clues for uncovering properties of the M-theory, i.e. a ‘mystery’-theory of particles and extended objects, formulated in dimensions  $d \geq 11$  and whose different

limits turn out to correspond to eleven-dimensional supergravity as well as *all* the known string theories.

### 3.1.2 Cosmological implications of topological defects

The possible cosmological significance of topological defects was anticipated by Nambu [178] in 1965. When it was realized in the mid seventies that spontaneously broken symmetries would be restored at high temperatures [147, 148], and that topological defects like domain walls could be formed in the very early universe [237], cosmological consequences of topological defects had to be taken seriously. In addition, the relation between the topology of the vacuum manifold and the different types of defects were pointed out by Coleman [48] and by Kibble [143, 144], thus allowing for existence of not only domain walls, but also strings as topological defects. In somewhat parallel developments Polyakov [190] and 't Hooft [211] demonstrated that monopoles can exist in a non-Abelian gauge theory, containing electromagnetism within a larger compact covering group. These developments provided cornerstones for the study of cosmological implications of topological defects such as domain walls, strings, and monopoles within (grand) unified gauge theories.

Everett [84] and Zel'dovich et al. [248] were the first to quantitatively consider potential consequences of cosmic domain wall structures by discussing wave propagation across their boundaries and their gravitational effects, respectively. Unless the domain walls disappear at a sufficiently early stage, such structures are incompatible with the observed level of cosmic isotropy [248]. The argument is the following: adjoining walls with opposite topological charges will move together and annihilate leaving on the order of one wall per Hubble length  $H_0^{-1}$ . Since the cosmic microwave background is isotropic to an accuracy of about  $10^{-5}$ , the mass of this wall  $M_w \approx \sigma H_0^{-2}$  where  $\sigma$  is the mass per area of the wall, must be small compared with the net mass within the Hubble length  $M_U \approx 1/(GH_0)$ . If the wall is formed by a scalar multiplet  $\phi$  governed by a potential  $V(\phi) = \frac{1}{4}\lambda(\phi^2 - \eta^2)^2$  where  $\lambda$  is a dimensionless coupling constant and the constant  $\eta$  determines the ground state value of  $\phi$ , then the energy per area is  $\sigma \approx \lambda^{1/2}\eta^3$ . The bound on the anisotropy of the thermal cosmic background then translates to

$$\delta \approx \frac{M_w}{M_U} \approx \frac{\lambda^{1/2}\eta^3 G}{H_0} = \frac{\lambda^{1/2}\eta^3 \hbar}{M_{\text{Pl}}^2 H_0} \lesssim 10^{-5}$$

where  $M_{\text{Pl}}$  is the Planck mass. This constraint can be rephrased as

$$\eta \lesssim (M_{\text{Pl}}^2 H_0 10^{-5})^{1/3} \lambda^{-1/6} \approx \lambda^{-1/6} \text{ MeV}.$$

For the range of energies associated with the conventional phase transitions of particle physics, the coupling constant  $\lambda$  would have to exceedingly small: for the GUT scale of  $10^{15}$  GeV it must be least a factor of  $10^{-108}$  less than unity! It is very hard to see how such a small coupling could arise naturally in the theory.

In contrast to cosmic domain walls which had been ruled out by the anisotropy argument, cosmic strings could have played an important rôle in the early universe, e.g. by producing local inhomogeneities [143]. This idea was later taken up by Zel'dovich [246] and Vilenkin [220] who argued that these inhomogeneities could be the cause of galaxy formation. Subsequently, Silk and Vilenkin [203] pointed out that over-dense planar wakes are left behind relativistically moving open strings. This effect is a direct consequence of the conical geometry: matter converge behind the string from both sides and produce an over-dense region. The resulting power spectrum has been computed both for string generated perturbations of cold dark matter [5] and hot dark matter [4]. While the string-induced cold dark matter spectrum is in trouble for lack of power on large scales, a combination of strings and hot dark matter is regarded as a promising alternative [128] to the popular inflationary models.

Magnetic monopoles also provide a serious problem since the concentration of magnetic monopoles left over from a phase transition in the early universe turns out to be unacceptably large [247, 192]. Monopole–anti-monopole annihilation could be enhanced by gravitational clumping but not enough to solve the problem [105]. The only viable way to suppress the monopoles seemed to be to require that the universe supercooled before going through a strongly first-order phase transition and that the monopole mass is much higher than the reheating temperature [192, 115, 81], but this mechanism would require an unnatural fine tuning of the GUT parameters [80]. On the other hand, the corresponding vacuum energy density would lead to an effective cosmological constant [164, 31] which would cause a near exponential expansion of the universe [199]. In fact, this solves the monopole problem, which besides the horizon and flatness problems, was a key motivation for the cosmic inflation paradigm [114, 167, 6] (see Ref. [208] for a recent review of inflation which makes contact with the new observational data). Because of the enormous expansion during the inflationary epoch, the density of primordial monopoles and other dangerous particles are diluted so much that only a few of them exist within the present Hubble horizon.

### 3.2 The kink

In order to prepare for a discussion of domain walls in (super)gravity theory, we shall first present an introduction to the classical finite-energy solutions of field theory: generally called *solitons*. Let us start with a simple example of a Goldstone model [106] described by the Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

where  $\phi$  is a scalar multiplet. Let us specialize to a real scalar field in (1+1)-dimensional space–time and the potential

$$V(\phi) = \frac{\lambda}{4} (\phi^2 - \eta^2)^2; \quad (3.1)$$

$\lambda$  is a dimensionless coupling constant, and  $\eta$  defines the ground state of the theory. This double-well potential is illustrated in Fig. 1.



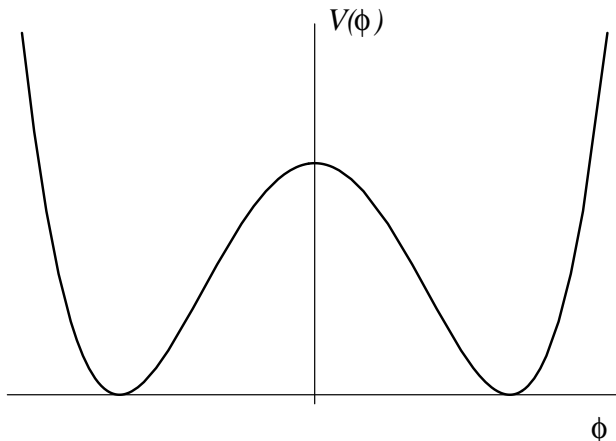


Figure 1: The double-well potential of Eq. (3.1).

The resulting classical field equation

$$\square\phi + \lambda\phi(\phi^2 - \eta^2) = 0$$

has, in addition to the two ground state solutions  $\phi = \pm\eta$ , the exact static solutions

$$\phi_{\pm} = \pm\eta \tanh(\sqrt{\lambda/2}\eta x). \quad (3.2)$$

These are the well-known kink  $\phi_+$  and anti-kink  $\phi_-$  (for the negative sign) solutions. The scalar field interpolates smoothly between the two ground states of the theory. As such it is a lower-dimensional analogue of a domain wall separating two Minkowski vacua. This behaviour is illustrated in Fig. 2.

The total energy of the (anti-)kink is

$$E = \int_{-\infty}^{\infty} \left( \frac{1}{2}\phi'^2 + V(\phi) \right) dx = 2\sqrt{2}\lambda\eta^3/3$$

where a prime stands for a derivative with respect to  $x$ . Thus the kink is a classical finite-energy solution of a field theory and therefore an example of what we call a soliton: a stable particle-like state on a non-linear system.

### 3.2.1 Topological charge

At spatial infinities we have

$$\phi(\infty) - \phi(-\infty) = 2n\eta$$

where  $n = 0$  represents the vacuum,  $n = 1$  the kink and  $n = -1$  the anti-kink. This equation can be rewritten as

$$\int_{-\infty}^{\infty} (\phi') dx = 2n\eta.$$

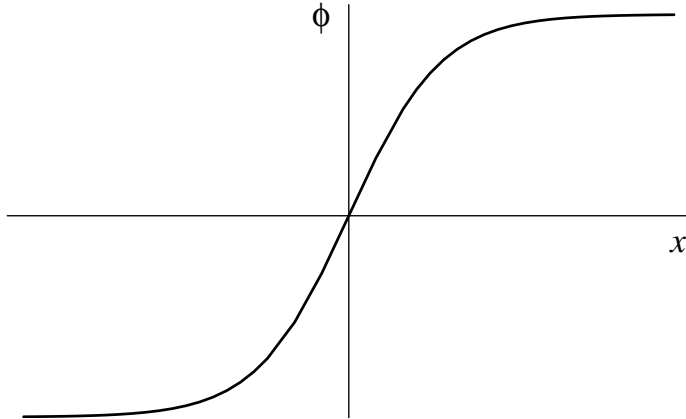


Figure 2: The kink solution  $\phi_+$  of Eq. (3.2).

Let us now define a current by  $j_\mu \equiv \epsilon_{\mu\nu} \partial^\nu \phi$  where  $\epsilon_{\mu\nu}$  is the Lévi-Civita tensor. The antisymmetry of  $\epsilon_{\mu\nu}$  and the commutativity of partial derivatives automatically make this current conserved. The corresponding conserved charge is

$$C = \int_{-\infty}^{\infty} j_0 dx = \int_{-\infty}^{\infty} (\phi') dx = 2n\eta, \quad (3.3)$$

and  $n$  is a conserved quantum number. Hence, there is no transition between kink solutions and ground states, and kinks are stable. The conservation of  $C$  is different in nature from that of conservation of, e.g., electrical charge. The latter is a Noether charge conserved because of a symmetry in the theory, whereas  $C$  is conserved independently of the field equations.

To see how this comes about, let  $\mathcal{M}_0$  be a manifold whose elements are the points in field space which correspond to a ground state, and let  $S$  be the set of points at spatial infinity. A finite-energy solution must have asymptotic field values in  $\mathcal{M}_0$ , i.e.

$$\lim_{x \rightarrow \pm\infty} \phi(x) = \phi \in \mathcal{M}_0.$$

This condition is a mapping  $\Xi : S \rightarrow \mathcal{M}_0$ . In the case of the (1+1)-dimensional  $\lambda\phi^4$  theory, the  $\Xi_0$  of the vacuum state maps the whole of  $S$  either to  $-\eta$  or to  $\eta$ . The mapping  $\Xi_+$  corresponding to a kink  $\phi_+$  maps  $-\infty$  to  $-\eta$  and  $\infty$  to  $\eta$ , and the anti-kink mapping  $\Xi_-$  maps  $-\infty$  to  $\eta$  and  $\infty$  to  $-\eta$ . It is impossible to continuously deform these mappings into one another, i.e. they are topologically distinct. For this reason, a conserved charge such as  $C$  in Eq. (3.3) is called a *topological charge*, and the conservation law a *topological conservation law*.

This simple example has shown how a topological conservation law can divide the set of finite-energy solutions into several different sectors:  $n = 0$  (the vacuum);  $n = 1$  (the kink);  $n = -1$  (the anti-kink); etc. Their existence has

far-reaching implications both at the microscopic and cosmic scales. Clearly, the distinct topological sectors were made possible by the topology of the vacuum manifold  $\mathcal{M}_0$ : With the potential of Eq. (3.1) the vacuum manifold has disconnected components.

### 3.2.2 Higher-dimensional defects

The simple (1+1)-dimensional kink has no direct analogue in higher dimensions. There is a theorem due to Derrick [72] which states that in a scalar theory in with two spatial dimensions or more, the only non-singular time-independent solutions of finite energy are the ground states.

One can evade the dimensional restriction of Derrick's theorem by allowing time-dependence or by adding other fields, e.g. gauge fields. The 't Hooft–Polyakov monopole [211, 190] and the Nielsen–Olesen vortex-line [181] are examples of the latter type. Time-dependent (non-topological or semi-topological) solitons have also been discovered [197, 196, 159, 88, 51] (for reviews of boson stars and non-topological solitons, see Refs. [158, 139]).

## 3.3 Homotopy groups and defect classification

Topological defects can be classified according to the topology of the vacuum manifold. Central to this classification scheme is the concept of *homotopy classes* and *homotopy groups*. It is defined as follows: Let  $\mathcal{X}$  and  $\mathcal{Y}$  be two topological spaces, and let  $\Xi_0(x)$  and  $\Xi_1(x)$  be two continuous mappings from  $\mathcal{X}$  to  $\mathcal{Y}$ . If  $I$  is the unit interval  $I = [0, 1] \subset \mathbb{R}$ , then the two functions  $\Xi_0(x)$  and  $\Xi_1(x)$  are said to be homotopic if and only if there exists a continuous mapping  $F : \mathcal{X} \otimes I \rightarrow \mathcal{Y}$  such that  $F(x, 0) = \Xi_0(x)$  and  $F(x, 1) = \Xi_1(x)$ . The function  $F$  is called the *homotopy*. In other words, two mappings are homotopic if there exists a continuous function which deforms one into the other. In this case the two functions are said to be members of the same *homotopy class*  $[\Xi_0] = [\Xi_1]$ .

Consider now mappings  $\Xi$  from the  $n$ -dimensional sphere  $S^n$  to a vacuum manifold  $\mathcal{M}_0$ . Let us first consider  $n = 1$ . Then each  $\Xi$  represents a closed loop in  $\mathcal{M}_0$  and each loop can be placed in homotopy class  $[\Xi]$ . One can now define a class multiplication by

$$[\Xi_1][\Xi_2] = [\Xi_1 \circ \Xi_2]$$

where loop multiplication is defined as first going through one loop and then the other:

$$\Xi_1 \circ \Xi_2(t) = \begin{cases} \Xi_1(2t), & t \in [0, 1/2] \\ \Xi_2(2t - 1), & t \in [1/2, 1]. \end{cases}$$

The set of homotopy classes is a group under this operation; the identity element consist of all loops which can be contracted to a point and thus to the constant map  $x$ , and the inverse is defined as  $[\Xi]^{-1} \equiv [\Xi^{-1}]$ . Since the base point  $x$  on a connected space is movable through an isomorphism, we can omit it and denote this group  $\pi_1(\mathcal{M})$ . It is called the first homotopy group of the manifold. Similar groups can be defined for all  $n$ -dimensional spheres that can be mapped onto

the manifold. We have seen that  $\pi_1(\mathcal{M})$  is non-trivial if there are loops in the vacuum manifold which cannot be deformed into a point;  $\pi_2(\mathcal{M})$  is non-trivial if there are unshrinkable surfaces, and so on. The case  $n = 0$  corresponds to mappings of a point onto the manifold, and  $\pi_0(\mathcal{M})$  is non-trivial if and only if the manifold has disconnected components as for example in the kink model discussed in Section 3.2.

The topological defects are therefore classified according to the homotopy groups and are shown in Table 1.

Table 1: Classification of topological defects in three space dimensions.

Defect type	Dimension	Homotopy group
Domain walls	2	$\pi_0(\mathcal{M})$
Strings	1	$\pi_1(\mathcal{M})$
Monopoles	0	$\pi_2(\mathcal{M})$
Textures	–	$\pi_3(\mathcal{M})$

In addition, there are hybrid defects such as monopoles connected by strings [157, 222] or walls bounded by strings [229, 145] (see also Refs. [227, 230]).

### 3.4 Formation of topological defects

Besides identifying and explicitly constructing defect solutions, one would like to understand whether or not such structures could form in the early Universe. To this end one needs theories or scenarios for defect formation. In this section we comment on such scenarios.

#### 3.4.1 The Kibble mechanism

In general spontaneously broken symmetries are restored at high temperatures. According to the standard model of the Universe, it began in a very hot state, perhaps with temperatures close to the Planck temperature. It is believed that the effective potential of the Higgs field(s) was different at such high temperatures and that the ground state then was symmetric with respect to the symmetries which we today see as broken symmetries [147, 148, 237].

As the Universe expanded and cooled the effective potential changed and ended up with degenerate ground states with broken symmetries. In different regions of the Universe, the Higgs field(s) could settle down at different parts of the vacuum manifold, and then depending on its topology (as discussed in the previous sections) various types of topological defects can form. This phenomenon is the Kibble mechanism [143] whose realization depends only on the topology of the vacuum manifold and the assumption that the Universe went through a phase transition.

### 3.4.2 Quantum creation

Quantum fields in curved space–time [29] can induce negative energy–momentum densities in the vacuum. This allows particle creation even with a conserved stress energy  $\langle T_{\mu\nu} \rangle$ . Such particle creation also takes place in the de Sitter space–time [96], not only for particles, but also for axionic domain walls [171] and other topological defects [18]. As a result, topological defects with an energy scale not too far above the energy scale of inflation, can still be present after inflation. This could also happen if the defect field  $\phi$  is coupled to the inflation field responsible for inflation [231].

Within the framework of quantum cosmology—where the creation of the whole Universe is envisaged as a quantum tunneling process from “nothing” [223, 224, 226, 95]—the Universe could in principle be born with defects. However, as recently pointed out by Gibbons [94], since their action is infinite, universes with domains of negative cosmological constant separated by super-symmetric domain walls can not be created in this way.

### 3.4.3 False vacuum decay

Another way of forming domain walls is through false vacuum decay [248, 50].<sup>8</sup> In this case the scalar fields have non-degenerate minima, and there is a finite probability of quantum tunneling from a false vacuum (a local minimum) into the true vacuum (the global minimum): Quantum fluctuations may spontaneously generate a “bubble” of the true vacuum.<sup>9</sup> If this bubble has a radius larger than a certain critical size, the bubble will expand rapidly and convert the false vacuum into a true one. Such a true vacuum bubble is separated from the false vacuum by a domain wall. These tunneling bubble walls are *not* topological defects, yet still of fundamental importance in a complete picture addressing domain walls.

## 3.5 Domain walls as a particular type of topological defect

Domain walls provide a special example of topological defects. Therefore the review of the nature of topological defects, their cosmological implications and their dynamical formation, as spelled out in the above subsections, applies to the domain walls as a special case.

In particular, domain walls are topological defects that can occur in the fundamental theory when the vacuum manifold  $\mathcal{M}_0$  has disconnected components, i.e. the homotopy group  $\pi_0(\mathcal{M})$  is non-trivial. An example of such a mapping of a point onto the vacuum manifold  $\mathcal{M}_0$  is the kink model discussed in Section 3.2.

Dynamic formation of domain walls in the early universe may proceed, as discussed in Section 3.4, as:

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<sup>8</sup>Early papers on vacuum decay include Refs. [159, 232, 87, 209, 210, 49, 38, 165, 166].

<sup>9</sup>Gravity makes the concept of degenerate vacua somewhat more complicated. Decay out of a zero-energy minimum is suppressed by gravity [52] and two vacua with zero and negative energy-densities may therefore become degenerate due to gravitational effects.

- a particular case of the Kibble mechanism,
- within the framework of quantum cosmology the domain walls could separate different universes,
- domain walls would provide boundaries of the false vacuum decay bubbles.

Because of their extended structure, the dynamic effects of domain walls, including the gravitational ones, are most drastic. Here we briefly summarize some of their dynamical effects.

As discussed in Section 3.1.2 domain walls that may form via Kibble mechanism, are incompatible with the observed level of cosmic isotropy [248], unless they disappear at a sufficiently early stage. In the case of inflationary universe, the domain walls can effectively disappear by being inflated away [167, 6], if the energy scale of inflation is below the scale of domain wall formation. Another mechanism of getting rid of domain walls may take place, if the walls are bounded by strings. In this case, intersecting walls bounded by strings cut holes in each other, and are also chopped up by intersecting strings [229]. In this way such walls may disappear before they dominate the gravitational field of the universe.

The vacuum decay bubbles are distinguished from those originating as results of phase transitions by their smaller surface energy density. For vacuum decay bubbles, the liberated energy of the false vacuum is converted into kinetic energy of the wall. It could finally be released as thermal energy after bubble collisions.<sup>10</sup> However, in the old inflationary model the phase transition could not be smoothly completed [114, 116, 117]. This is sometimes called the “graceful exit problem”. Moreover, the bubble collisions could not lead to sufficient thermalization of the wall energy [122, 117]. In Ref. [122] it was concluded that bubble collisions could lead to formation of black holes, but a more detailed analysis of the gravitational effects showed that black hole formation is impossible in two-bubble collisions [45]. However, since the domain wall sets up a repulsive gravitational field, one expects black hole pair creation to occur [37].

The graceful exit and thermalization problems were solved by the new inflationary scenario [167, 6] and the chaotic inflationary model [168] in which inflation takes place during a slow-rollover transition. Here the observable universe comes from one inflationary bubble and thermalization is due to dissipation of rapid scalar field oscillations rather than bubble collisions. In these inflationary models the thin wall approximation is inapplicable. However, in the extended inflationary model [152] where gravity is described by an effective Brans–Dicke theory, the phase transition is first-order, and the bubbles can again be treated in the thin wall approximation.

The focus of this Review are the gravitational aspects of domain walls.<sup>11</sup> We shall concentrate on the supersymmetric walls and their non-supersymmetric

<sup>10</sup> The possibility that the vacuum energy is directly released as thermal energy of the medium inside the bubble, a process dubbed as “vacuum burning”, was investigated in Refs. [19, 22].

<sup>11</sup>For more general information about topological defects and their cosmological implications we refer the readers to Ref. [230]; for a recent review on cosmic strings, see Ref. [128]; and

tric generalizations within  $N = 1$  supergravity theory and effective  $N = 1$  supergravity from superstrings, as a particular example. For that purpose, we shall review the space–time symmetry assumptions, the metric ansatz, and the thin wall formalism in Section 4. In Section 5 the embedding of such walls in  $N = 1$  supergravity theory is given, with vacuum domain walls and dilatonic domain walls studied Sections 6 and 7, respectively. Connection of these walls to other topological defects as well as implications for the domain walls within effective supergravity vacua from superstring theory are discussed in Section 8.

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for a review on solitons in superstring theory, see Ref. [77]. Earlier work on domain walls in global supersymmetry is found in Refs. [1, 64].

## 4 Isotropic domain walls

A domain wall is a surface with a timelike velocity vector. As the wall propagates through space–time it defines a set of events that describe the wall’s history. In analogy with a particle’s *world line* and a string’s *world sheet* or *world tube* (for closed strings), the wall’s history shall be referred to as its *world volume*. A priori, the world volume may be spatially closed, semi-closed, or open depending on the topology of the wall.

### 4.1 Domain wall symmetries

An ideal domain wall is an infinitely thin surface in which the tension in all directions equals the energy density. The symmetries of the wall’s energy–momentum tensor imply that it is boost invariant along its spatial directions. This property is characteristic of a vacuum [103]; an unpolarized vacuum has no preferred frame [163, page 367]. Accordingly, a domain wall is said to be vacuum-like. One consequence of this equation of state is that the energy-density is *constant*, even if the wall expands. Covariant energy–momentum conservation dictates that new vacuum energy is created in proportion to the change of volume. This increase of energy in a fixed coordinate-volume (as opposed to a proper volume) is equal to the positive work done by the negative-pressure force on the surroundings, i.e.  $dU = -pdV$ . Exactly the same happens in the de Sitter phase of inflationary cosmologies. Another consequence is that the energy–momentum tensor of a domain wall must be isotropic and homogeneous.

### 4.2 Induced space–time symmetries

Space–time need not have the same symmetries as the energy–momentum tensor. The Kasner cosmology [142] is a very simple example of a symmetric energy–momentum tensor in an anisotropic space–time; here the energy–momentum tensor vanishes identically, but the gravitational field is anisotropic.

Kasner-like solutions for domain walls are also known [215, 138], but here we shall focus on the basic structure of domain wall field configurations without any additional complications coming from anisotropic gravitational background fields. Consequently, we *assume* that the gravitational field has the same high degree of symmetry as the source.

#### 4.2.1 Metric ansatz

It is most convenient to describe the wall system in the comoving coordinates, i.e. in the wall’s rest frame. In this reference frame the wall system is (locally) static, and its stress energy depends only on the spatial distance from the wall surface.

First, we assume that the spatial part of the metric of the wall’s world volume and of the two-dimensional spatial sections “parallel” to the wall are *homogeneous* and *isotropic* in the *comoving frame*. Homogeneity and isotropy



reduce the “parallel” metric to the spatial part of a (2+1)-dimensional [102] Friedmann–Lemaître–Robertson–Walker (FLRW) metric [236, page 412]. In the conventional coordinates the line element has the form

$$(ds_{\parallel})^2 = R^2 [(1 - kr^2)^{-1} dr^2 + r^2 d\phi^2],$$

where the scale factor  $R$  is constant on the surface. This is a surface of constant curvature with Ricci scalar equal to  $2k/R^2$ .

Since any non-zero  $k$  can be absorbed into the scale factor  $R$ , only the sign of  $k$  is important. We can therefore normalize  $k$  to 0, +1, or  $-1$ . This gives three possible wall geometries. If  $k = 0$ , the metric  $(ds_{\parallel})^2$  can be transformed to Cartesian coordinates  $(ds_{\parallel})^2 = R^2(dx^2 + dy^2)$ , and the wall is planar. If  $k = 1$ , then the wall is a bubble in which case one may introduce  $r = \sin \theta$  which gives the line-element  $(ds_{\parallel})^2 = R^2(d\theta^2 + \sin^2 \theta d\phi^2)$ . If  $k = -1$ , then the coordinate transformation  $r = \sinh \varrho$ , with  $\varrho \geq 0$ , brings the line element to  $(ds_{\parallel})^2 = R^2(d\varrho^2 + \sinh^2 \varrho d\phi^2)$ , which corresponds to a Gauss–Bolyai–Lobachevski surface. This non-compact surface cannot be embedded in ordinary 3-dimensional Euclidean space [236, page 5].

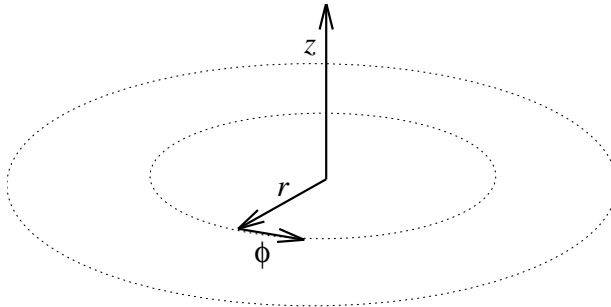


Figure 3: The spatial coordinates in the wall space–time;  $z$  describes the direction orthogonal to the wall surface.

Let us also assume that the two-dimensional space–time sections orthogonal to the wall are *static* as observed in the rest frame of the wall. Hence, if  $z$  denotes a coordinate describing the direction transverse to the wall, and if  $t$  represents the *proper* time as measured by observers at rest on the wall surface, then  $g_{tz} = 0$ , and both  $g_{tt}$  and  $g_{zz}$  depend only on  $z$ . By an appropriate rescaling of the  $z$ -coordinate the orthogonal part of the metric can be written as

$$(ds_{\perp})^2 = e^{2a(z)} (dt^2 - dz^2).$$

With  $ds^2 \equiv (ds_{\perp})^2 - (ds_{\parallel})^2$ , we get

$$ds^2 = e^{2a(z)} (dt^2 - dz^2) - R^2 [(1 - kr^2)^{-1} dr^2 + r^2 d\phi^2],$$

where  $R = R(t, z)$ . Without loss of generality, we can choose the origin of  $z$  at the centre of the wall surface. The range of  $z$  is  $z \in \langle -\infty, \infty \rangle$ , and the range of

the other coordinates is that of a FLRW cosmological model [236, page 412]. In Ref. [62] it was shown that Einstein's equations together with the requirement that each surface of constant  $z$  has a boost invariant extrinsic curvature, imply that  $R(z, t)$  is separable. If we also demand that the world volume has a non-singular and geodesically complete metric, then the space-time metric can be written in the form [62]

$$ds^2 = e^{2a(z)} \{ dt^2 - dz^2 - S^2(t) [(1 - kr^2)^{-1} dr^2 + r^2 d\phi^2] \}$$

$$S(t) = \begin{cases} 1 & k = 0 \\ \cosh \beta t & k = \beta^2 \end{cases} \quad (4.1)$$

where  $\beta$  is a positive real constant. The functional form of  $S(t)$  is determined by the boost-invariance symmetry; the world volume is either a (2+1)-dimensional Minkowski space-time or a (2+1)-dimensional de Sitter space. These two possibilities are characterized by different topologies;  $\mathbb{R}^3$  for the Minkowski case and  $\mathbb{R} \times \mathbb{S}^2$  for the de Sitter case [136, 135, 93, 63]. One could also think of anti-de Sitter space with  $S(t) = \sin \beta t$  as a third possibility, but this option is excluded since this metric would lead to a space-time with singularities periodic in time. Hence with the requirement that the world volume is non-singular, the wall must be either a static plane or an accelerated bubble. Note that by relaxing some of our symmetry assumptions, one can get more general solutions such as for example walls in Schwarzschild, Schwarzschild-de Sitter [30, 24], or Reissner-Nordström backgrounds. In this paper we shall, however, restrict ourselves to domain wall space-times *without* charges or Schwarzschild masses.

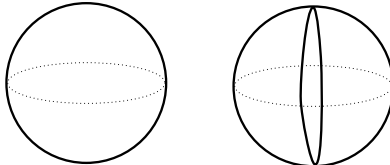


Figure 4: Two-dimensional analogue of the spatial section of the geodesically complete Vilenkin domain wall (domain wall between Minkowski vacua) space-time. The whole spatial geometry is the closed surface (to the left), and the domain string (the analogue of a domain wall in three dimensions) is the dividing circle depicted on the right. Note that angular circles decrease on both sides of the string: both sides are inside the circular domain string.

Very often the line-element of Eq. (4.1) is written on the plane-symmetric [212] de Sitter form with  $S(t) = e^{Ht}$  and  $k = 0$  (see e.g., Refs. [225, 104, 177] and [188, page 282]). It should be observed that the spatially-flat line-element is geodesically incomplete, and even though it is locally plane-symmetric, it describes only a *part* of a spherical bubble, cf. Refs. [136, 135, 93, 63, 150] and [230, page 380]. Nevertheless, recent papers [235, 187] still reveal some confusion about the physical interpretation of these solutions: in support of the view that the wall really is planar, Wang and Letelier [235] argue that the wall

divides space in two and that none of these parts are on the outside of a bubble. This argument makes an unexpressed Euclidean assumption about the spatial topology which imply that only planar walls can divide space in two equal parts. However, if space is closed, such a division can be made with a spherical domain wall (see Fig. 4).

While it is true that the high symmetry of de Sitter space–time allows for different time slicings, these are only different representations of the same geometry. In other words, using a coordinate system covering only part of space–time does not change its physical nature.

### 4.3 The thin wall formalism

Far from a domain wall one expects the gravitational field to be independent of the internal structure of the wall. In particular, the large-scale structure of space–time or its topology should *not* depend on such details. This expectation has been confirmed by studies of “thick” domain walls [241, 104] where the metric approaches that of a thin wall at infinity. Therefore, in order to understand the global properties of domain wall space–times, it is sufficient to study domain wall solutions in the thin wall approximation. In this approximation the wall is regarded as infinitely thin, with a  $\delta$ -function singularity in the energy–momentum tensor and the Einstein tensor. Lichnerowicz’s formalism [162, 213] deals with distribution valued curvature tensors and can be applied to thin domain walls [161]. However, the most used description of domain walls is the Gauss–Codazzi formalism [176, page 514]. It is a method of viewing a four-dimensional space–time as being sliced up into three-dimensional hypersurfaces. The proper junction conditions for such surfaces of discontinuity were worked out by Israel [137]. Later Israel’s formalism has been generalized also to include lightlike surfaces [47, 15] and non-vacuum space–times [20, 155, 173]. The  $\delta$ -function singularities of the singular layers correspond to step-function discontinuities in the first-order derivative of the metric coefficients. The height of the steps determines the components of the energy–momentum tensor of the singular surface layer. More precisely, in Israel’s formalism the energy–momentum of the surface is described by Lanczos’ tensor  $S^i_j$ . This tensor is given by [137]

$$\kappa S^i_j \equiv -[K^i_j]^- + \delta^i_j [K]^- ,$$

where  $K_{ij}$  is the extrinsic curvature of the surface and where  $[\Omega]^- \equiv \Omega^+ - \Omega^-$  signify the discontinuity at the wall. The indices  $\{i\}$  label the components relative to a basis intrinsic to the world volume. The extrinsic curvature describes how the normal vector in the embedding space changes along the world volume. It is given by the covariant derivative of the spacelike unit normal vector. In the comoving frame of the metric ansatz (4.1), the extrinsic curvature is

$$K_{ij} = -\frac{\zeta}{2} e^{-a(z)} g_{ij,z} ,$$

where  $\zeta$  is a sign factor determined by the orientation of the outer normal.

Written in the perfect fluid form, the surface energy–momentum tensor takes the form

$$\kappa S^i_j = \sigma u^i u_j - \tau(\delta^i_j + u^i u_j).$$

Here  $u^i$  is the velocity vector of the surface,  $\sigma$  is its rest mass density (mass per unit area), and  $\tau$  is the wall's tension measured in the comoving frame. For a domain wall the tension equals the energy-density:  $\tau = \sigma$ .

With the metric (4.1) the energy-density of a domain wall placed at  $z = 0$  becomes

$$\sigma = 2\zeta_1 a'|_{0^-} - 2\zeta_2 a'|_{0^+} \quad (4.2)$$

where the  $z$  coordinate has been oriented so that the vacuum of lowest energy will be placed on the  $z < 0$  side and where (without loss of generality) the coordinates have been normalized so that  $a(0) = 0$ . The sign factors  $\zeta_k = \pm 1$  with  $k \in \{1, 2\}$  where the signs are determined by directing the outward normal in the direction of the matter source gradient. We restrict ourselves to the case where the underlying matter source is a kink-like scalar field configuration and then  $\zeta_1 = \zeta_2 = 1$  [62].

Following the above approach for the metric tensor field, one can derive analogous junction conditions for other fields. For a scalar field the result is [65]

$$\phi'|_{0^+} - \phi'|_{0^-} = \int_{0^-}^{0^+} e^{2a} V' dz, \quad (4.3)$$

where  $V(\phi)$  is the effective potential for the scalar field.

### 4.3.1 Lagrangian and field equations

The starting point for the study of the supergravity domain walls is the bosonic part of the action,  $S \equiv \int \mathcal{L} \sqrt{-g} d^4x$ , for the space–time metric  $g_{\mu\nu}$ , the matter field  $\vartheta$  responsible for the formation of the wall and the dilaton field  $\phi$ , which couples to the matter potential. In the Einstein frame the Lagrangian for the Einstein-dilaton-matter system is

$$\mathcal{L} = -\frac{1}{2}R + \partial_\mu \vartheta \partial^\mu \vartheta + \partial_\mu \phi \partial^\mu \phi - V(\phi, \vartheta). \quad (4.4)$$

The potential is of the form:

$$V(\phi, \vartheta) = f(\phi)V_0(\vartheta) + \widehat{V}(\phi). \quad (4.5a)$$

Comparing with Eq. (2.6), the functions in the potential (4.5a) are given in terms of the fields of  $N = 1$  supergravity:

$$f(\phi) = e^{2\sqrt{\alpha}\phi}, \quad (4.5b)$$

$$V_0(T, T^*) = e^{K_M} \left[ |D_T W|^2 K^{TT^*} - (3 - \alpha)|W|^2 \right]. \quad (4.5c)$$

The dilaton couples to the matter potential with the function  $f(\phi)$  which we shall call “the dilaton coupling”. For the sake of generality we include a dilaton self-interaction term  $\hat{V}(\phi)$ . It is not present in the original theory, but is believed to be generated after dynamical supersymmetry breaking. This term is responsible for giving the dilaton a mass. In the thin wall approximation the matter field  $\vartheta$  is frozen outside the wall and its only contribution to the field equations are constant potential terms  $V_0(\vartheta_1)$  and  $V_0(\vartheta_2)$ .

The Einstein tensor for the metric (4.1) is

$$G^z{}_z = 3 \left( \beta^2 - a'^2 \right) e^{-2a}, \quad (4.6a)$$

$$G^i{}_i = \left( \beta^2 - a'^2 - 2a'' \right) e^{-2a}. \quad (4.6b)$$

The corresponding energy-momentum tensor is

$$T^z{}_z = V(\phi) - \phi'^2 e^{-2a}, \quad (4.7a)$$

$$T^i{}_i = V(\phi) + \phi'^2 e^{-2a}, \quad (4.7b)$$

$$(4.7)$$

where  $V(\phi)$  as defined in Eqs. (4.5) is the effective “dilaton potential” on either side of the wall. Einstein’s field equations  $G_{ij} = T_{ij}$  and the second Bianchi identity then lead to

$$e^{2a} V(\phi) + 2\phi'^2 + 3a'' = 0, \quad (4.9)$$

$$-e^{2a} V'(\phi) + 4a'\phi' + 2\phi'' = 0, \quad (4.10)$$

$$3\beta^2 - e^{2a} V(\phi) - 3a'^2 + \phi'^2 = 0. \quad (4.11)$$

Note that the equation of motion for the scalar field is identical to the energy-momentum conservation law (4.10) and consequently there are only two independent field equations. The constraint (4.11) can be used to determine the boundary conditions.

### 4.3.2 Junction conditions as boundary conditions

In the case when the solution for the metric and the dilaton is known on one side of the wall, Israel’s matching condition (4.2) and the dilaton matching condition (4.3) can be used to determine the boundary condition for the metric and the dilaton on the other side of the wall. Thus for walls with Minkowski space-time and a constant dilaton field on one side, these conditions can be used to determine the solution on the other side. For reflection symmetric walls, the boundary conditions are fixed on both sides. Therefore, in both these cases one is able to find numerical solutions to the field equations.

## 5 Supersymmetric embedding

Below we shall derive the Bogomol’nyi bound on the energy density and the Killing spinor equations for supersymmetric (extreme) domain wall configurations. Our calculation will be based on the form of the Lagrangian spelled out in Section 2.

We shall primarily concentrate on supersymmetric configurations, corresponding to extreme walls interpolating between supersymmetric minima of the matter potential.

### 5.1 Bogomol’nyi bound

In order to derive the Killing spinor equations and the Bogomol’nyi bounds for dilatonic domain walls, the technique of the generalized Israel–Nester–Witten form [243, 180], applied to the study of supergravity walls [60, 65], was used. We review the embedding [65]<sup>12</sup> into  $N = 1$  supergravity theory (see Section 2 and specifically, subsection 2.2 for the bosonic part of the Lagrangian). The dilaton field  $S$  and the matter field(s)  $T$ , responsible for the formation of the wall, are assumed to have a general separable Kähler potential (2.3a). Extreme walls with an exponential dilaton coupling [54, 55], as specified by the Kähler potential (2.3b), and ordinary supergravity walls [60] (without dilaton) are thus special examples of such walls. A generalization of the results to more than one “dilaton” field is straightforward, as long as the dilatons have no superpotential and the Kähler potential decouples from the matter fields responsible for wall formation.<sup>13</sup>

Since the extreme domain walls are planar and infinite, we review a derivation of the Bogomol’nyi bound for the energy per unit area of the wall. Note also that a precise definition of the energy density of the wall is possible only in the thin wall approximation, namely, when the “interior” and the “exterior” regions of the wall are clearly separated.

We consider a generalized Nester form [180]:

$$N^{\mu\nu} = \bar{\epsilon} \gamma^{\mu\nu\rho} \widehat{\nabla}_\rho \epsilon \quad (5.1)$$

where  $\epsilon$  is a Majorana spinor. The supercovariant derivative is defined as  $\widehat{\nabla}_\rho \epsilon \equiv \delta_\epsilon \psi_\rho$  and  $\widehat{\nabla}_\rho = 2\nabla_\rho + Q_\rho$ , where  $Q_\rho = i e^{K/2} (\Re(W) + \gamma^5 \Im(W)) \gamma_\rho - \gamma^5 \Im(K_T \partial_\rho T) - \gamma^5 \Im(K_S \partial_\rho S)$  and  $\nabla_\mu \epsilon = (\partial_\mu + \frac{1}{2} \omega^{ab}{}_\mu \sigma_{ab}) \epsilon$ ;  $\psi_\rho$  is the spin  $\frac{3}{2}$  gravitino field. Therefore, the explicit expression for Nester’s form is:

$$N^{\mu\nu} = \bar{\epsilon} \gamma^{\mu\nu\rho} \left[ 2\nabla_\rho + i e^{K/2} (\Re(W) + \gamma^5 \Im(W)) \gamma_\rho - \Im(K_T \partial_\rho T) \gamma^5 - \Im(K_S \partial_\rho S) \gamma^5 \right] \epsilon. \quad (5.2)$$

<sup>12</sup>Analogous procedures were followed in the derivation of the Bogomol’nyi bounds for the mass of the corresponding charged black holes [98, 97, 100, 140, 70].

<sup>13</sup>In superstring theory the additional “dilaton” may be identified with the compactification moduli, if the matter fields responsible for the formation of the wall do not couple to the moduli fields in the Kähler potential and the superpotential. However, matter fields do in general couple to the moduli fields.

Here  $W$  is the superpotential (2.2) and  $K$  is the Kähler potential (2.3). Stokes' theorem ensures the following relationship:

$$\int_{\partial\Sigma} N^{\mu\nu} d\Sigma_{\mu\nu} = 2 \int_{\Sigma} \nabla_{\nu} N^{\mu\nu} d\Sigma_{\mu} \quad (5.3)$$

where  $\Sigma$  is a spacelike hypersurface.

After a lengthy calculation,<sup>14</sup> the volume integral (5.3) yields:

$$\int \left[ \widehat{\nabla}_{\nu} \overline{\epsilon} \gamma^{\mu\nu\rho} \widehat{\nabla}_{\rho} \epsilon + K_{TT^*} \overline{\delta_{\epsilon} \chi} \gamma^{\mu} \delta_{\epsilon} \chi + K_{SS^*} \overline{\delta_{\epsilon} \eta} \gamma^{\mu} \delta_{\epsilon} \eta + (G^{\mu\nu} - T^{\mu\nu}) \overline{\epsilon} \gamma_{\nu} \epsilon \right] d\Sigma_{\mu} \geq 0, \quad (5.4)$$

where  $\delta_{\epsilon} \chi$  and  $\delta_{\epsilon} \eta$  are the supersymmetry transformations of fermionic partners  $\chi$  and  $\eta$  to the matter field  $T$  and the dilaton field  $S$ , respectively;  $T^{\mu\nu}$  is the energy-momentum tensor and  $G^{\mu\nu}$  is the Einstein tensor. The first term in Eq. (5.4) is non-negative, provided the spinor  $\epsilon$  satisfies the (modified) Witten condition, i.e.  $\mathbf{n} \widehat{\nabla} \epsilon = 0$  ( $\mathbf{n}$  is the four-vector normal to  $\Sigma$ ). The Kähler metric coefficients  $K_{TT^*}$  and  $K_{SS^*}$  are positive definite, and thus the second and the third terms in Eq. (5.4) are non-negative as well. The last term in Eq. (5.4) is zero due to Einstein's equations. Thus, the integrand in Eq. (5.4) is always non-negative and it is zero if and only if the supersymmetry transformations (2.7), (2.9), and (2.10) on the gravitino  $\psi_{\rho}$  as well as on  $\chi$  and  $\eta$  vanish, i.e. if the configurations are supersymmetric.

The surface integral of Nester's form in Eq. (5.3) yields the corresponding Bogomol'nyi bound for the energy associated with the configuration. Such a bound can be derived precisely only in the thin wall approximation, because the region inside the wall must be clearly separated from the region outside the wall in order for its energy density to be well defined.

In this case the density of the surface integral of Nester's form in Eq. (5.3) is of the form:

$$\overline{\epsilon}_0 \gamma^0 \epsilon_0 \sigma + \overline{\epsilon}_0 \gamma^{03} e^{K/2} [\Re(W) + \gamma^5 \Im(W)] \epsilon_0 \Big|_{0^-}^{0^+}. \quad (5.5)$$

The spinor  $\epsilon_0$  is defined at the boundaries  $z = 0^+$  and  $z = 0^-$  of the wall. In the first term, we have used the fact that for the thin wall the magnitude of the spinor components does not change. The first term of the surface integral (5.5) of the Nester's form (5.1) can then be identified with the energy density of the wall. The second term corresponds to the topological charge density  $C$  evaluated on both sides of the wall. Positivity of the volume integral (5.4) translates through Eq. (5.3) into the corresponding Bogomol'nyi bound for the energy density of a thin wall:

$$\sigma \geq |C|, \quad (5.6)$$

<sup>14</sup>For details related to the derivation see Ref. [65] and appendices in Ref. [60].

which is saturated if and only if the bosonic background is supersymmetric.

In the following subsection we shall derive the explicit phase factors by which the components of the  $\epsilon_0$  spinor change at the wall boundaries for the case of extreme solutions. These phase factors will in turn allow us to obtain the explicit form of  $\sigma_{\text{ext}} = |C|$ .

Note that the existence of the Bogomol'nyi bound (5.6) crucially depends on the fact that in Eq. (5.4) the spinor  $\epsilon$  satisfies the (modified) Witten condition, i.e.  $\mathbf{n}\widehat{\nabla}\epsilon = 0$  ( $\mathbf{n}$  is the four-vector normal to  $\Sigma$ ). One example, where such a bound (5.6) is satisfied [60], is for domain walls interpolating between the supersymmetric Minkowski vacuum and the anti-de Sitter vacuum, which is either supersymmetric or with spontaneously broken supersymmetry. In general, however, the constraint spinors is not satisfied, and the bound is violated by the existence of the ultra-extreme, false vacuum decay bubbles. Thus, false vacuum decay bubbles are *not* topological defects.

## 5.2 Killing spinor equations

We now write down explicit Killing spinor equations [60, 65], i.e.  $\delta\psi_\mu = \delta\chi = \delta\eta = 0$  (see Eqs. (2.7), (2.9), and (2.10)), which are satisfied by supersymmetric, static configurations. With the metric ansatz (4.1) with  $\beta = 0$ :

$$ds^2 = e^{2a(z)} (dt^2 - dz^2 - dx^2 - dy^2)$$

and  $T(z)$  and  $S(z)$  being functions only of  $z$ , and using the supersymmetry transformations specified in Section 2, the Killing spinor equations are of the form:

$$\delta\psi_x = \left[ -\gamma^1\gamma^3\partial_z a - i\gamma^1 e^{(a+K/2)} (\Re W + \gamma^5\Im W) \right] \epsilon, \quad (5.7a)$$

$$\delta\psi_y = \left[ -\gamma^2\gamma^3\partial_z a - i\gamma^2 e^{(a+K/2)} (\Re W + \gamma^5\Im W) \right] \epsilon, \quad (5.7b)$$

$$\begin{aligned} \delta\psi_z = & \left[ 2\partial_z - i\gamma^3 e^{(a+K/2)} (\Re W + \gamma^5\Im W) \right. \\ & \left. - \gamma^5\Im(K_T\partial_z T + K_S\partial_z S) \right] \epsilon, \end{aligned} \quad (5.7c)$$

$$\delta\psi_t = \left[ \gamma^0\gamma^3\partial_z a + i\gamma^0 e^{(a+K/2)} (\Re W + \gamma^5\Im W) \right] \epsilon, \quad (5.7d)$$

$$\begin{aligned} \delta\chi = & -\sqrt{2} \left[ e^{K/2} K^{TT^*} (\Re(D_T W) + \gamma^5\Im(D_T W)) \right. \\ & \left. + i e^a (\Re(\partial_z T) + \gamma^5(\partial_z T)) \gamma^3 \right] \epsilon, \end{aligned} \quad (5.7e)$$

$$\begin{aligned} \delta\eta = & -\sqrt{2} \left[ e^{K/2} K^{SS^*} (\Re(K_S W) + \gamma^5\Im(K_S W)) \right. \\ & \left. + i e^a (\Re(\partial_z S) + \gamma^5(\partial_z S)) \gamma^3 \right] \epsilon. \end{aligned} \quad (5.7f)$$

We have assumed that the Majorana spinor  $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_2^*, -\epsilon_1^*)$  does not depend on  $x^i \in \{t, x, y\}$ . Note that in Eqs. (5.7) the Kähler potential  $K = K_{\text{dil}}(S, S^*) + K_{\text{matt}}(T, T^*)$  is separable and  $W = W(T)$ , cf. Eq. (2.4). No Killing spinor exist for the anisotropic generalization with  $g_{tt}$ ,  $g_{xx}$ , and  $g_{yy}$  all different [138, 60, 108].



The vanishing of the above expressions yields a set of first-order differential equations<sup>15</sup> (known as the self-dual or Bogomol'nyi equations) for the metric coefficient  $a(z)$ ,  $T(z)$  and  $S(z)$  as well as the constraint on the spinor  $\epsilon$ . The field equations are of the form:

$$0 = \Im(\partial_z T D_T \ln W), \quad (5.8a)$$

$$\partial_z T = \zeta e^{(a+K/2)} |W| K^{TT^*} D_{T^*} \ln W^*, \quad (5.8b)$$

$$\partial_z a = \zeta e^{(a+K/2)} |W|, \quad (5.8c)$$

$$\partial_z S = -\zeta e^{(a+K/2)} |W| K^{SS^*} K_{S^*}. \quad (5.8d)$$

Here  $\zeta = \pm 1$  and it can change sign only when  $W$  crosses zero. There is another constraint on the ‘‘field geodesic’’ motion of the dilaton field, namely  $\Im(K_S \partial_z S) = 0$ . However, by multiplying Eq. (5.8d) by  $K_S$ , this constraint is seen to be automatically satisfied. In this case the right-hand side of the equation is real, since  $K^{SS^*} > 0$  is real and  $K_{S^*} = (K_S)^*$ .

Equations (5.8a) and (5.8b) describe the evolution of the matter field  $T = T(z)$  with  $z$ . The first equation is a ‘‘field geodesic’’ equation, which determines the path of the complex scalar field  $T$  in the complex plane between the two minima  $T_1$  and  $T_2$  of the matter potential. Equation (5.8b) governs the change of the  $T$  field with coordinate  $z$  along this path.

Equations (5.8c) and (5.8d) determine the evolution of the metric coefficient  $a(z)$  and the complex field  $S$ . These two equations imply another interesting relation between the dilaton Kähler potential  $K_{\text{dil}}(S, S^*)$  and  $a(z)$ :

$$2K^{SS^*} |K_S|^2 \partial_z a + \partial_z K_{\text{dil}} = 0.$$

In addition, the Killing spinor equations (5.7) impose a constraint on the phase of the Majorana spinor. Namely, the solution for the Killing spinor component is of the form

$$\epsilon_1 = e^{i\theta} \epsilon_2^* = \mathcal{C} e^{(a+i\theta)/2}, \quad (5.9)$$

where the phase  $\theta(z)$  satisfies

$$\partial_z \theta = -\Im(K_T \partial_z T).$$

The constant  $\mathcal{C}$  can be set to  $\frac{1}{2}$  for the Majorana spinors normalized as  $\epsilon^\dagger \epsilon = 1$ . The constraint (5.9) on the Killing spinor  $\epsilon$  in turn implies that the extreme configurations preserve ‘‘ $N = \frac{1}{2}$ ’’ of the original  $N = 1$  supersymmetry.

The energy density of the wall (in the thin wall approximation) is determined by setting Eq. (5.5) to zero. With the explicit form for the Killing spinor components (5.9), Eq. (5.5) yields:

$$\begin{aligned} \sigma_{\text{ext}} &= |\mathcal{C}| = 2 \left| \left( \zeta e^{K/2} W \right)_{z=0^+} - \left( \zeta e^{K/2} W \right)_{z=0^-} \right| \\ &= 2 e^{K_{\text{dil}}(S_0, S_0^*)/2} \left| \left( \zeta e^{K_{\text{matt}}/2} W \right)_{z=0^+} - \left( \zeta e^{K_{\text{matt}}/2} W \right)_{z=0^-} \right|. \end{aligned} \quad (5.10)$$

<sup>15</sup>Equations (5.7) set to zero can be viewed as ‘‘square roots’’ of the corresponding Einstein and Euler–Lagrange equations; they provide a particular solution of the equations of motion which saturate the Bogomol'nyi bound (5.6).

Here the subscript  $z = 0^\pm$  refers to either side of the wall. Without loss of generality we have normalized  $a(0) = 0$  and set  $S(0) = S_0$ .

**Classification of extreme domain wall solutions:** Solutions to the Bogomol'nyi equations (5.8) fall into three types, depending on whether  $W(T)$  crosses zero or not along the wall trajectory.

**Type I** walls correspond to those where on one side of the wall, say for  $z > 0$ ,  $W(T_1) = 0$ . In this case the energy density of the wall is of the form:  $\sigma_{\text{ext}} = 2 |e^{K/2} W|_{z=0^-}$ , and the side of the wall with  $z > 0$  corresponds to the Minkowski space-time with a constant  $S$ .

**Type II** walls correspond to the walls with  $W(T)$  crossing zero somewhere along the wall trajectory. At  $W = 0$   $\zeta$  changes sign. The energy density of the wall is specified by:  $\sigma_{\text{ext}} = 2 |e^{K/2} W|_{z=0^+} + |e^{K/2} W|_{z=0^-}$ . Reflection symmetric walls fall into this class.

**Type III** walls correspond to the walls where  $W(T) \neq 0$  everywhere in the domain wall background. For this type  $\zeta$  does not change sign. The energy density of such walls is:  $\sigma_{\text{ext}} = 2 ||e^{K/2} W|_{z=0^+} - |e^{K/2} W|_{z=0^-}|$ .

## 6 Vacuum domain walls

If each side of the domain wall corresponds to a vacuum, that is, if on either side all the matter fields have *constant* expectation values, then all the local properties of each side are Lorentz-invariant, and each side is a *vacuum*. Such domain walls shall be called *vacuum domain walls*.

Vacuum domain walls can be classified according to the value of their surface energy density  $\sigma$ , compared to the energy densities of the vacua outside the wall [63, 62]. The three types are: (1) extreme walls with  $\sigma = \sigma_{\text{ext}}$  are planar, static walls. In this case the gravitational mass of the wall is perfectly balanced by that of the exterior vacua; (2) non-extreme walls with  $\sigma = \sigma_{\text{non}} > \sigma_{\text{ext}}$  corresponding to non-static bubbles with two centres and (3) ultra-extreme walls with  $\sigma = \sigma_{\text{ultra}} < \sigma_{\text{ext}}$  representing expanding bubbles of false vacuum decay.

### 6.1 Extreme vacuum walls

The extreme vacuum domain wall solutions were discovered [60, 59] in  $N = 1$  supergravity theory without a dilaton ( $\alpha = 0$ ). The walls represent regions interpolating between isolated supersymmetric vacua of the matter potential. The extreme solutions correspond to static supersymmetric configurations saturating the corresponding Bogomol'nyi bound. The three possible types [59] of these extreme vacuum domain walls are classified according to the nature of the field path in the superpotential. This aspect was discussed in Section 5. Here we shall analyse<sup>16</sup> the space-times induced by these extreme domain walls. We stress that for  $\alpha = 0$  the nature of the potential (2.6) ensures that for supersymmetric minima, i.e. those with  $D_T W = 0$ , the value of the potential  $V \leq 0$ . Thus the extreme walls interpolate between anti-de Sitter vacua or an anti-de Sitter vacuum and a Minkowski vacuum.

#### 6.1.1 The energy density of the extreme walls

In the thin wall approximation, the field equations outside the vacuum domain wall reduce to Einstein's vacuum equations with a non-positive cosmological constant given by the supergravity potential (2.6) with  $\alpha = 0$ . In a supersymmetric vacuum the Kähler covariant derivative of the superpotential vanish  $D_T W = 0$ , and thus the cosmological constant (here parameterized with parameter  $\chi$ ) is

$$\Lambda \equiv V(\phi_0) = -3e^K |W|^2 = -3\chi^2. \quad (6.1)$$

In supergravity without a dilaton, supersymmetric vacua always have a non-positive  $\Lambda$ , but in order to include also the non-supersymmetric case, we let  $\chi$  take a real positive value, if the vacuum energy is negative, and conversely, if the vacuum has a positive energy-density, then  $\chi$  is positive imaginary.

<sup>16</sup>This discussion is based on Refs. [62, 108] where more details can be found. It is included here for completeness.

Hence, the field equations (4.9)–(4.11) reduce to

$$a'^2 = e^{2a}\chi^2,$$

which has the solution

$$a = -\ln(1 \pm \chi z) \tag{6.2}$$

for a real  $\chi$ . The integration constant has been absorbed into a coordinate rescaling so that  $a(0)$  is normalized to unity. To be definite, we place the vacuum with the most negative  $\Lambda$  on the side  $z < 0$ . Then the three possible types of extreme domain walls are characterized by the behaviour of the metric conformal factor (Fig. 5). For a Type I wall it is falling off from the wall on the

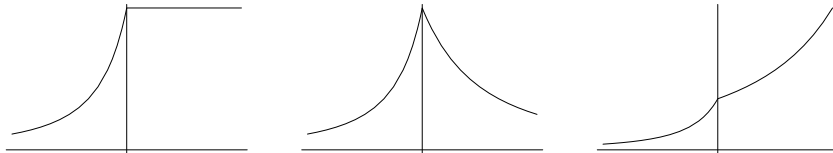


Figure 5: The metric conformal factor as function of the distance from the wall (at the origin) for extreme domain walls of Type I, II, and III, respectively .

anti-de Sitter side and is constant on the Minkowski side. For a Type II wall it is falling off away from the wall on both sides, and for a Type III wall it is increasing on the side with the least negative cosmological constant.

The energy density of an extreme wall is

$$\sigma = 2(\chi_1 \pm \chi_2), \tag{6.3}$$

with the plus sign for Type II walls and the minus sign for Type III. Type I corresponds to  $\chi_2 = 0$ . Linet’s solution [172] is a special case of a reflection symmetric wall of Type II.

One gets the same qualitative results by solving Eqs. (5.8), which in the thin wall approximation reduce to Eq. (5.8c) with all matter fields constant. We would like to emphasize that the self-dual equations (5.8) are numerically tractable without using the thin wall approximation (see Ref. [60, 59] and the examples in Section 8).

### 6.1.2 Gravitational mass for extreme walls

Inertial observers on the Minkowski side of the extreme Type I wall experiences no gravitational effects. One can understand this result by investigating the proper acceleration,<sup>17</sup>  $g^{\hat{\mu}}$ , necessary to be a fiducial observer (an observer at a fixed spatial position). This acceleration is given by [59]

$$g^{\hat{\mu}}(z) = e^{-a} a'(z) \delta^{\hat{\mu}}_{\hat{z}}. \tag{6.4}$$

<sup>17</sup>Hats denote tensor components relative to an orthonormal tetrad frame.

Hence, the metric conformal factor behaves as a gravitational potential, and one can therefore infer the direction of the gravitational acceleration  $\kappa^{\hat{\mu}} = -g^{\hat{\mu}}$  of test particles directly from Fig. 5. For Type I and II walls it is directed *away from* the wall region. Hence, test particles are repelled by the wall which is thus exhibiting “repulsive gravity”.

Clearly when  $a(z)$  is constant as it is on the Minkowski side, the acceleration is zero, and no gravitational effects are felt. On the anti-de Sitter side the fiducial observers have a constant proper acceleration of magnitude  $\chi$  which is *half* the surface mass density of a Type I wall.

For the planar walls it is possible to rewrite Einstein’s field equations as an integral [57, 62] which can be interpreted as a plane-symmetric analogue of Tolman’s [214] gravitational mass. With help of this concept one can understand the “gravitational forces” and their sources. For example, in pure anti-de Sitter space, the effective gravitational mass per volume is positive, thus indicating the attractive nature of the gravitational field produced by a negative vacuum energy. The converse holds for de Sitter space, which is a familiar result from inflationary cosmology where the repulsive nature of a positive vacuum energy drives a near exponential expansion of the universe.

Tolman’s formula for the gravitational mass was originally derived for a static spherically symmetric metric [214]. This result has been adapted [57, 62] to the case of static planar symmetry for which the relevant object is the gravitational mass per area.

In the derivation of Tolman’s mass formula one focuses on the *generalized surface gravity*, that is, the gravitational acceleration as measured with *standard rods* and *coordinate clocks* [109]

$$\kappa^{\hat{i}}(z) \equiv -\sqrt{g_{\hat{t}\hat{t}}g^{\hat{i}\hat{i}}}(z)$$

at any surface. For a metric as given in Eq. (4.1), with  $\beta = 0$ , and with the proper acceleration (6.4), the generalized surface gravity is

$$\kappa^{\hat{i}}(z) = -a'(z)\delta^{\hat{i}}_{\hat{z}}. \quad (6.5)$$

In a spherically symmetric four-dimensional space–time, one relates  $\kappa^r$  to the Tolman mass by defining a mass  $M$  in such a way that the Newtonian force law is reproduced:  $GM \equiv -r^2\kappa^r$ , and in the three-dimensional case  $G_3M_3 \equiv -r\kappa^r$ , where  $G_3$  is the (2+1)-dimensional Newton’s constant [206]. In the plane-symmetric case, considered here, we deal with an essentially two-dimensional problem, and the appropriate Newtonian force law implies  $\Sigma \equiv -2\kappa^z$  where  $\Sigma$  is the *gravitational mass per area* of the plane. The factor of two is included because the gravitational acceleration is half the mass per area in the planar symmetric case (in the reflection symmetric case [225, 7] one finds that the acceleration on both sides is a quarter of the mass density). Hence, Eq. (6.5) leads to

$$\Sigma(z) = 2a'. \quad (6.6)$$

For this equation to make sense in the wall case, we rewrite the right hand side in terms of an integral. Starting from the Einstein tensor (4.6), one finds (in the static case  $\beta = 0$ )

$$G^t_t - G^z_z - G^r_r - G^\phi_\phi = e^{-4a}(2a'e^{2a})'. \quad (6.7)$$

Using  $\sqrt{-g^{(4)}} = e^{4a}$ ,  $\sqrt{g^{(2)}} = e^{2a}$ , Einstein's equations  $G^\mu_\nu = T^\mu_\nu$ , and Eq. (6.7), we can rewrite the right-hand side of Eq. (6.6) as

$$\Sigma(z) = \frac{\int_{-\infty}^z \sqrt{-g^{(4)}} dz' (T^t_t - T^z_z - T^r_r - T^\phi_\phi) \int dx dy}{\sqrt{g^{(2)}}(z) \int dx dy} \quad (6.8)$$

where we used  $a'(-\infty) = 0$ , as is the case for both the asymptotically Minkowski and anti-de Sitter sides of the Type I and II extremal walls. The numerator on the right-hand side of Eq. (6.8) is recognized as the Tolman mass of a static space-time [214], and the denominator is the proper area. Basically, this mass formula expresses the fact that mass and energy are equivalent quantities in relativistic physics: energy in the form of pressure contributes to the gravitational field along with the mass density. On account of this, one can define a gravitational mass density by  $\rho_g \equiv \rho + 3p$  for a perfect fluid in 3+1 dimensions. In the limit where we integrate from  $z = -\infty$  to  $z = \infty$ , we get  $\Sigma(\infty) = 0$ . This means that the total gravitational mass of the Type I and II extreme space-times is zero. It should be noted that the Type III space-time is causally identical to pure anti-de Sitter. Therefore, the effective mass per volume of this system is the relevant object; the effective mass per area—as with pure anti-de Sitter space—is infinite.

In the thin wall approximation, we can distinguish contributions to the Tolman mass per area due to the wall itself and due to the vacuum energy of the adjacent space-time. In this case, a domain wall has an effective gravitational mass per area,  $\Sigma_{\text{wall}} = S^t_t - S^r_r - S^\phi_\phi$ , given by  $\Sigma_{\text{wall}} \equiv \sigma - 2\tau$ . Since the tension,  $\tau$ , is equal to the energy density,  $\sigma$ , for a vacuum domain wall, we find  $\Sigma_{\text{wall}} = -\sigma < 0$ . By use of Eq. (6.3), one finds that  $\sigma = 2\chi$ , which yields

$$\Sigma_{\text{wall}} = -2\chi.$$

This negative gravitational mass per area for the wall, with its *repulsive* gravity, must be compensated by a positive gravitational surface mass density from the anti-de Sitter space-time for there to be no force on the Minkowski side. This is precisely the case as we now show. Again, taking into account the effect of vacuum pressure,  $p_v = -\rho_v$ , the gravitational mass density of anti-de Sitter vacuum is

$$\rho_g = \Lambda - 3\Lambda = 6\chi^2.$$

Integrating out the  $z$ -direction from  $z = -\infty$  to the position of the wall at  $z = 0$  yields the following mass per area for the anti-de Sitter side of the wall:

$$\Sigma_{\text{AdS}} = \lim_{z \rightarrow 0} \left[ \frac{\int_{-\infty}^z (6\chi^2) \sqrt{-g^{(4)}} dz \int dx dy}{\int \sqrt{g^{(2)}} dx dy} \right] = 2\chi.$$

Hence, as seen from the Minkowski side of the domain wall, there are two gravitational surface mass densities on the  $z \leq 0$  side. Firstly, there is a *negative* mass per area coming from the domain wall:  $\Sigma_{\text{wall}} = -2\chi$ . Secondly, there is a *positive* integrated mass per area coming from anti-de Sitter space itself:  $\Sigma_{\text{AdS}} = 2\chi$ , which exactly cancels that of the domain wall.

The analysis used for the extreme Type I wall can also be applied to the extreme Type II wall. When there is a Minkowski metric on one side, the Killing time,  $t$ , corresponds to the proper time of an observer infinitely far away from the wall on this side. In the Type II case, one may use an observer sitting in the center of the wall. Here too, there is a frame where all the connection coefficients vanish, the metric is Minkowskian, and where the proper time of the observer is equal to the Killing time. Thus, in the thin wall approximation, one finds the effective mass per area of the two anti-de Sitter sides to be  $2(\chi_1 + \chi_2)$ . This positive effective mass is exactly cancelled by the negative effective mass of the domain wall separating the two regions of anti-de Sitter space. Likewise, the general expression Eq. (6.8) yields a zero Tolman mass per area for the space-time.

Note that in the above calculations we have integrated along a constant time slice  $-\infty < z < \infty$ . As we shall see below, there is a past and future Cauchy horizon for data placed on such a slice. The above calculation implicitly assumes no contribution to the effective mass arising from the past of the past Cauchy horizon. This is consistent with the extensions of the space-time beyond the Cauchy horizon considered in the next section. It is also consistent with there being a global balance of gravitational “forces”.

### 6.1.3 Global space-times of the extreme walls

The three types of extreme domain walls realized [60, 59] in four-dimensional ( $N = 1$ ) supergravity theory are planar and static. The space-time metric (4.1) induced by these walls have  $\beta \equiv 0$  and is conformally flat with conformal factor  $e^{a(z)}$  becoming unity on the Minkowski side of the Type I wall and falling off as  $(z\chi)^{-2}$  on the anti-de Sitter side, where  $\Lambda = -3\chi^2$ . The Type II conformal factor falls off as  $(z\chi_1)^{-2}$  and  $(z\chi_2)^{-2}$  on the respective sides. For the Type III wall, the conformal factor falls off in the same manner on one side of the wall, but has coordinate singularity at a finite value of  $z$  on the other side. This singularity represents the affine boundary of the space-time. In this section we present geodesically complete extensions of the space-times for the Type I, II, and III extreme domain walls. The global space-times of the Type I wall was first considered in Ref. [57] and the Type II wall in Ref. [93].

For each of the walls, the space-time must be extended across a Cauchy horizon on the anti-de Sitter side [57, 93]. The Cauchy horizons occur on the nulls at  $|z| = \infty$  where  $a(z) = -\infty$ , i.e. where the line element degenerates. Although these nulls are an infinite proper distance away, the geodesic distance is finite. This type of geometry is familiar from the extreme black hole space-times [41, 40, 121, 43]. The comoving coordinates must be extended across the Cauchy horizons on these anti-de Sitter sides. The same need also arises in pure

anti-de Sitter space.

A Cauchy horizon is the boundary of causal evolution. Therefore, one has the possibility of making identifications across the Cauchy horizons which can introduce closed timelike curves (CTCs). The possibility of CTCs is inherited from the anti-de Sitter portion of the space-time. Identifications of this type are especially intriguing for the Type I walls, as CTCs could lead to supersymmetric time-machine [57] for travelers leaving Minkowski space-time by passing across the wall and then re-emerging into the same Minkowski region at an earlier time.

There are three possible extensions across the null Cauchy horizons:

1. One can extend onto a new patch with the scalar field permanently settled into its vacuum, i.e. beyond the Cauchy horizon there is a pure anti-de Sitter vacuum.
2. In the case of the Type II wall, one can shift the old diamond along the null such that the new diamond is oriented just as the old. This extension yields a new wall as well as a jump in the cosmological constant at the Cauchy horizon for non-reflection symmetric walls.
3. The old diamond can be reflected onto the new diamond across the Cauchy horizon. This extension leads to a new wall as well as a smooth matching of the cosmological constant at the horizon.

In the following, we concentrate on geodesic extensions of the third kind. One reason for doing so is that it yields the most interesting causal structure for the resulting space-times. It is for the third approach that the causal structure of the Type I and II space-times exhibit a symmetric lattice structure similar to those first realized by the extensions of Carter for the Kerr and Reissner-Nordström black holes [41, 40]. The extension for the Type I wall realizes the identical causal structure as the extreme Kerr black hole along its symmetry axis [121, 43, 41]. Finally, it is through the infinite lattice for the Type I and II space-times that one eliminates the timelike boundary of the covering space of anti-de Sitter space in exchange for a countably infinite number,  $\aleph_0$ , of isolated vertex points an infinite affine distance away from any interior point (see Figs. 6 and 7). For example, the Cauchy problem for the Type I space-time can be specified by prescribing initial data on one constant time slice in an  $\text{AdS}_4$  region and freely choosing boundary data on past null infinity of the countably infinite number of adjacent Minkowski spaces (see Fig. 6). In contrast, for the anti-de Sitter covering space, the Cauchy problem is defined only after prescribing an infinite amount of boundary data which has to be *self-consistent* with the specified initial data [11, 35].

The three types of extreme space-times, constructed from the third kind of geodesic extension described above, have the conformal diagrams shown in Figs. 6, 7, and 8. In each of the figures, the  $x$ - and  $y$ -coordinates are suppressed; therefore, each point represents an infinite plane. The compact null coordinates  $u'$  and  $v'$  defined by  $\tan(u'/2) = (t - z)\chi$  and  $\tan(v'/2) = (t + z)\chi$  define the



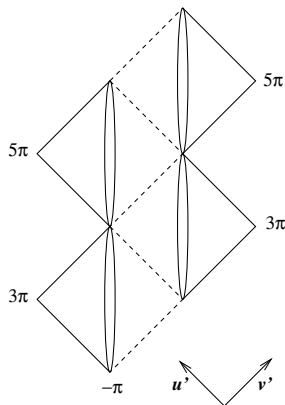


Figure 6: Conformal diagram of the extreme Type I domain wall. The walls are represented by the double timelike arcs splitting the diamonds. The central regions with Cauchy horizons (represented by dashed nulls) consist of anti-de Sitter patches, and the outer regions are semi-infinite Minkowski regions. The vertices are an infinite affine distance away from all other points. The complete extension is an infinite lattice chain which continues forever towards the past and the future.

axes. These coordinates can be extended smoothly across the Cauchy horizons (denoted by the dashed nulls) separating the diamonds on the anti-de Sitter side. This fact is seen explicitly by writing the (1+1)-dimensional line element near the horizon as  $ds^2 = (z\chi)^{-2}(dt^2 - dz^2) = \{\chi \sin[1/2(u' - v')]\}^{-2} du' dv'$  which has a smooth extension across the null specified by  $u' = \pi$  and  $-\pi < v' < \pi$  as well as across all the other Cauchy horizons. Thus, the null  $(u', v')$ -coordinates provide an atlas for describing the global space-time. Note that the full (3+1)-dimensional metric has coordinate singularities in the  $x$ - or  $y$ -directions at the Cauchy horizons.

The extension chosen for the Type I wall [57] in Fig. 6 possesses the same causal structure as the extreme Kerr black hole along its symmetry axis [121, 43, 41]. The extension chosen for the Type II wall in Fig. 7 tiles the whole plane with a lattice of walls. For the Type III wall shown in Fig. 8, the conformal factor diverges at some finite coordinate [59]. This is the edge of the space-time. As a result, the extension of the Type III wall is causally the same as that of pure anti-de Sitter space.

For each of the extensions, the vertices are special points [93] which are an infinite affine distance away from all other points; i.e. they represent an infinite conformal compression. This is analogous to the situation in the extreme Reissner-Nordström and Kerr black holes [121, 43, 41, 40].

The conformal diagram of a space-time containing a Type I or II extreme domain wall centered at  $z = 0$  should be compared to that of pure anti-de Sitter space. The timelike affine infinity of anti-de Sitter space is smoothed out by

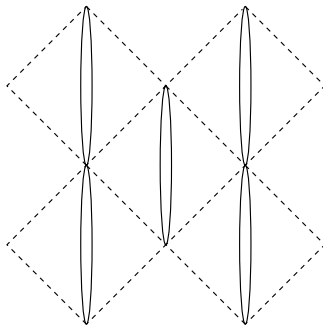


Figure 7: Conformal diagram of the extreme Type II domain wall. Both sides of the walls are anti-de Sitter spaces. The complete extension covers the whole plane.

the wall, which allows for another space-time region across the boundary of pure anti-de Sitter space. In this sense, the wall is located at spatial infinity. It has therefore been speculated [93] that the extreme walls are related to the “*Membrane at the End of the Universe*” in supermembrane theory [25, 26].

## 6.2 Non- and ultra-extreme vacuum walls

As we have seen, the extreme domain walls are characterized by an exact cancellation of the gravitational contributions of the vacua and the walls. By increasing or decreasing the energy density of the wall and thus disturbing this balance, supersymmetry is broken, and the walls become non-static. These solutions are not planar; the walls have spherical topology (if their world volumes are assumed to be geodesically complete). See Section 4 for details about the metric.

By increasing the wall’s energy so that  $\sigma = \sigma_{\text{non}} > \sigma_{\text{ext}}$ , its repulsive gravitational effect increases, and *a priori* one could expect that the total effect of an anti-de Sitter bubble surrounded by such a *non-extreme* domain wall bubble would be an example of a finite body with *negative gravitational mass* which is known to have unusual physical properties [193, 33, 163, Prob. 13.20], but this phenomenon is avoided here; both sides of the wall are on the inside of the wall; it is a two-centered bubble. Hence, one can never be on the outside of the negative mass system. In Ref. [62] a *Positive Mass Conjecture* was formulated saying that there is no singularity free solution of Einstein’s field equations with matter sources (not including the vacuum) obeying the weak energy condition equations for which an exterior observer can see a negative mass object. A global monopole [16, 119, 202] would appear to be a counter example, but in this case the Goldstone fields extend to infinity which means that these object are extended sources and all observers must be inside the system.

Geometrically we define the inside of the spherical domain wall to be the side on which the radius of curvature  $R \equiv e^{a(z)}S(t)/\beta$  (see Eqs. (4.1) for the



Figure 8: Conformal diagram of the extreme Type III domain wall. Both sides of the walls are anti-de Sitter spaces. The outer regions are limited by timelike affine boundaries. The complete extension is an infinite lattice which continues forever towards the future and the past.

definition of  $S(t)$  of concentric shells decreases as one goes away from the wall, and the outside is the side where it increases with distance. As we shall see in the next subsection, this local definition agrees with the global picture.

A domain wall with energy density less than the extreme wall  $\sigma = \sigma_{\text{ultra}} < \sigma_{\text{ext}}$ , is referred to as an ultra-extreme domain wall. As for the non-extreme case, the wall has the topology of a sphere, but now only the side with the smallest vacuum energy is on the inside; this is an ordinary one-centered bubble. It corresponds to a vacuum decay bubble.

When  $\beta \neq 0$  the vacuum Einstein equations reduce to (cf. Eq. (4.11))

$$a'^2 - \beta^2 = e^{2a} \chi^2 \quad (6.9)$$

where  $\chi$  is defined in Eq. (6.1), i.e.  $\chi^2 = -\Lambda/3$  where the cosmological constant  $\Lambda = V(\phi_0)$ .

The solutions are

$$a(z) = \begin{cases} -\ln[\beta \sinh(\beta z - \beta z')/\chi] & \text{for } V(\phi_0) < 0, \\ \pm \beta z & \text{for } V(\phi_0) = 0, \\ -\ln[\beta \cosh(\beta z - \beta z'')/\chi] & \text{for } V(\phi_0) > 0, \end{cases} \quad (6.10)$$

where there are two solutions for each of the integration constants  $z'$  and  $z''$  determined by the normalization condition  $a(0) = 0$ . They are denoted by  $z'_{\pm}$  and  $z''_{\pm}$  and are given by

$$e^{2\beta z'_+} = e^{-2\beta z'_-} = 1 + \frac{2\beta^2}{\chi^2} + \frac{2\beta}{\chi^2} (\chi^2 + \beta^2)^{1/2} \geq 1$$

(with equality in the extreme limit) and

$$e^{2\beta z''_+} = e^{-2\beta z''_-} = -1 - \frac{2\beta^2}{\chi^2} + \frac{2\beta}{\chi^2} (\chi^2 + \beta^2)^{1/2} > 1.$$

From the last equation it is clear that there is no extreme limit  $\beta \rightarrow 0$  in the de Sitter case.

The energy density of a non-extreme domain wall is

$$\sigma_{\text{non}} = 2(\chi_1^2 + \beta^2)^{1/2} + 2(\chi_2^2 + \beta^2)^{1/2}. \quad (6.11)$$

The “planar” reflection symmetric wall discussed by Vilenkin [225] and by Ipers and Sikivie [136] is a reflection symmetric non-extreme wall with vanishing vacuum energy.

An ultra-extreme wall has

$$\sigma_{\text{ultra}} = 2(\chi_1^2 + \beta^2)^{1/2} - 2(\chi_2^2 + \beta^2)^{1/2}, \quad (6.12)$$

where it is understood that the smallest cosmological constant (the most negative one) is on the left side. The ultra-extreme bubbles are the tunneling bubble [21] of false vacuum decay [50, 52]. See Refs. [21, 198, 30, 24] for discussions of false vacuum decay also involving de Sitter space.

The parameter  $\chi$  is real for anti-de Sitter space and imaginary for de Sitter space. Since the energy-density must be real, the cosmological constant is limited by  $\beta^2 \geq \chi^2$  in the de Sitter case [198], again confirming that there is no extreme limit in this case. Yet, the limiting case  $\beta^2 = \chi_{\text{deS}}^2$  is special because for this value of  $\beta$  the gravitational acceleration of fiducial observers vanish near the wall. Hence, just as for the extreme Type I (anti-de Sitter–Minkowski wall), an anti-de Sitter–de Sitter wall with  $\beta^2 = \chi_{\text{deS}}^2$  exactly cancels the gravitational mass of the anti-de Sitter vacuum.

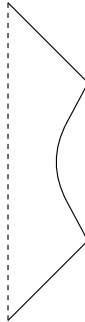


Figure 9: Conformal diagram for the Minkowski side inside a non- or ultra-extreme bubble. This diagram represents the  $(t, r)$ -plane in compactified coordinates. The curved line is the world volume of the wall, which follows radial Rindler motion. The dotted nulls are the Rindler horizons. The stippled line on the left is the center of the bubble.

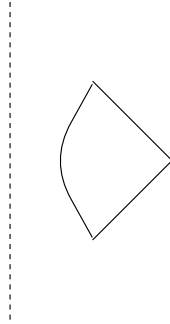


Figure 10: Conformal diagram for the Minkowski side outside an ultra-extreme bubble. This diagram represents the  $(t, r)$ -plane in compactified coordinates. The curved line is the world volume of the wall, which follows radial Rindler motion. The stippled line on the left is the center of the bubble. There are no bubbles with two outsides, so if the interior is a vacuum, it must be filled with anti-de Sitter space.

### 6.2.1 Global structure of non- and ultra-extreme domain wall space-times

For these domain walls, the hyper-space of the world volume is a  $(2 + 1)$ -dimensional de Sitter space. The global structure of the space-time is well known; it can be embedded in a  $(3 + 1)$ -dimensional Minkowski space where it takes the form of a hyperboloid (see, e.g. Ref. [121, 83] for the analogous case of  $(3 + 1)$ -dimensional de Sitter space). Hence, if the inside of the wall is a portion of Minkowski space, it must be the inside of this hyperboloid (Fig. 9). It is then clear that the wall, which is a spacelike section of the hyperboloid, is a sphere with a constant acceleration in the radial direction. In its rest frame one has Rindler horizons at  $|t \pm z| = \infty$ . The extension across these horizons is unique. Conversely, if the Minkowski space is on the outside, it must be the outside the de Sitter hyperboloid (Fig. 10). Timelike observers with insufficient acceleration will eventually be hit by the ultra-extreme wall.

For anti-de Sitter space-time, the Einstein universe coordinates [121] define a frame where the bubbles have kinematic properties analogous to those seen by inertial observers in Minkowski space (see Refs. [62, 108] for details). Non-extreme bubbles accelerate away from observers on both sides, and therefore we get a horizon of the Rindler type also in anti-de Sitter space. Consequently, anti-de Sitter insides of non- or ultra-extreme walls have a similar conformal diagram as the corresponding Minkowski interior, but instead of having a finite diagram one now has Cauchy horizons (Fig. 11) and the possibility to extend to an infinite chain.

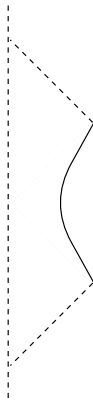


Figure 11: Conformal diagram for the anti-de Sitter side inside a non- or ultra-extreme bubble. The curved line is the world volume of the wall, which follows radial accelerated motion. The straight timelike edge on the right side is the timelike affine boundary of anti-de Sitter space. The stippled line on the left is the center of the bubble.

An ultra-extreme bubble accelerates towards outside observers in anti-de Sitter space, and thus, with exception of the affine boundaries which are null in

the Minkowski case and timelike for anti-de Sitter space, the causal structure (Fig. 12) is similar to that of the Minkowski exterior.

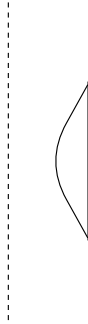


Figure 12: Conformal diagram for the anti-de Sitter side outside an ultra-extreme bubble. This diagram represents the  $(t, r)$ -plane in compactified coordinates. The curved line is the world volume of the wall, which follows radial Rindler motion. The straight timelike edge on the right side is the timelike affine boundary of anti-de Sitter space. The stippled line on the left is the center of the bubble. There are no bubbles with two outsides, so if the interior is a vacuum, it must be filled with anti-de Sitter space with a more negative vacuum energy density.

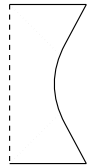


Figure 13: De Sitter side inside a non- or ultra-extreme bubble. The curved line represents the world volume of the wall. The stippled timelike line on the left-hand side is the coordinate singularity  $\psi = 0$  of de Sitter space-time [121, page 127]. Note that in contrast to anti-de Sitter and Minkowski space-time, de Sitter space has a spacelike infinity for timelike and null lines.

The de Sitter inside of a non- or ultra-extreme bubble corresponds to the part of de Sitter space with the metric conformal factor monotonously increasing toward the wall. The wall surface accelerates away from test particles (Fig. 13). In de Sitter space outside an ultra-extreme bubble with  $\chi < \beta^2$ , the wall accelerates toward inertial observers, but only those close to the wall are hit by it (Fig. 14). If  $\chi^2 = \beta^2$ , then the accelerated expansion of the bubble is exactly cancelled by the self-expansion of de Sitter space, and the wall appears to be static (Fig. 15).

To get the complete space-time one must glue the two sides of the wall. In the non-extreme case one must glue two bubble interiors; by gluing together

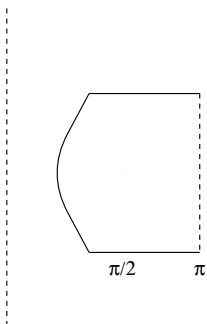


Figure 14: De Sitter space outside an ultra-extreme bubble with  $\chi^2 < \beta^2$ . The curved line is the world volume of the wall. The stippled timelike line on the right-hand side is the coordinate singularity  $\psi = \pi$  of de Sitter space–time [121, page 127]. The stippled line on the left-hand side is the centre of the bubble on the *other* side of the wall.

two copies of the Minkowski interiors of Fig. 9 by identifying points from each of the diagrams along the walls world volume, one gets the maximal extension of Vilenkin’s [225] Minkowski–Minkowski wall. This two-centered space–time is an example of a inhomogeneous closed universe with eternal expansion.

Note again that the domain walls with de Sitter space-times involve vacua where the supersymmetry is (spontaneously) broken. Such vacuum domain walls always fall into a class of non-extreme ( $\mathbb{Z}_2$ -symmetric walls) or ultra-extreme walls (false vacuum decay walls). For more details about vacuum domain walls with de Sitter space-times, we refer the reader to Ref. [24].

### 6.3 Stability

There is a large literature on the instability of Cauchy horizons in Kerr and Reissner–Nordström black holes [204, 113, 112, 174, 183, 129, 189, 184, 34, 175, 245, 151]. In the generic case, such horizons are turned into singularities and one could therefore suspect that the Cauchy horizons of the domain walls have the same kind of instability. This issue has been analyzed by Helliwell and Konkowski [123] who considered the effect of adding infalling and/or outgoing null dust to the space–time. They concluded that—except for one world line where a scalar curvature singularity is formed—the Cauchy horizons of the anti-de Sitter domain wall space–times and vacuum bubbles are stable under such perturbations. Further research on the stability of vacuum domain wall space–times is needed.

The physics of vacuum domain walls also has a bearing upon the stability of supersymmetric vacua. Their stability against decay into other supersymmetric vacua was first shown perturbatively in the gravitational constant expansion [238] and then by a full non-perturbative calculation in Ref. [61]. In addition, supersymmetric Minkowski vacua are known to be absolutely stable [60].



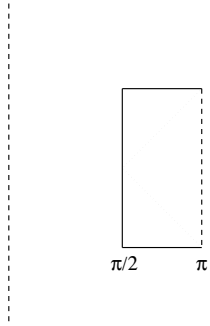


Figure 15: The de Sitter side outside an ultra-extreme bubble with  $\beta^2 = \chi^2$ . The timelike line at  $\psi = \pi/2$  on the left is the world volume of the wall. The stippled timelike line on the right-hand side is the coordinate singularity  $\psi = \pi$  of de Sitter space–time [121, page 127]. The stippled line on the left-hand side is the centre of the bubble on the *other* side of the wall.

## 7 Dilatonic domain walls

The existence of dilatons is a generic feature of unifying theories, including certain classes of supergravity theories, Kaluza–Klein theories, and effective theories from superstrings. The term ‘dilaton’ is here used as a generic name for a scalar field without self-interactions that couples to the matter sources, i.e. the potential of scalar matter fields as well as the kinetic energy of gauge fields. Through this coupling it modulates the overall strength of such interactions (see Section 2 for the  $N = 1$  supergravity case). In the low-energy effective action of superstring theory, the dilaton plays an essential rôle for the “scale-factor duality” [219], which has been taken as an indication of a “dual pre-big-bang” phase as a possible alternative to the initial singularity of the standard cosmological model [91]. The dilaton is believed to play a crucial rôle in dynamical supersymmetry breaking<sup>18</sup> as well. Moreover, in theories with dilaton field(s) topological defects in general, and black holes in particular, have a space–time structure that is drastically changed compared to the non-dilatonic ones. Namely, since the dilaton couples to the matter sources, e.g. the charge of the black hole, or it modulates the strength of the matter interactions, it in turn changes the nature of the space–time. In the past, charged dilatonic black holes have been studied extensively (for review see Ref. [132] and references therein). It is therefore of considerable interest to generalize the vacuum domain wall solutions by including the dilaton, thus addressing the nature of space–time in the domain wall background with a varying dilaton field.

Such configurations may be of specific interest in the study of domain walls in the early universe as they may arise in fundamental theories that include the dilaton, in particular in an effective theory from superstrings. In addition, the nature of ultra-extreme dilatonic domain walls, which describe false vacuum decay, in basic theories that contain one (or more) dilaton fields is of importance.

In the present section we investigate dilatonic domain walls. First, within  $N = 1$  supergravity coupled to a linear supermultiplet, we give the extreme dilatonic solutions to the case with an arbitrary separable dilaton Kähler potential (2.3a). These solutions satisfy Killing spinor equations (5.7) for the static, supersymmetric walls. Since the form of the separable Kähler potential is kept arbitrary, the analysis applies to special cases such as:  $N = 1$  supergravity theory coupled to a linear supermultiplet (see Section 2.2) where the dilaton  $\phi$  corresponds to the scalar component of the linear multiplet and couples with an exponential coupling  $e^{2\sqrt{\alpha}\phi}$  to the potential of the matter scalar fields (see Sections 2.2 and 2.3) as well as examples of the self-dual case, where the Kähler potential has an extremum for a finite dilaton value. We also comment on the effects of a dilaton mass, which in basic theory can be induced as a non-perturbative effect. Such a mass (or any other attractive self-interaction) does not alter the space–time sufficiently to remove the naked singularity.

The first set of extreme dilatonic domain wall solutions [54, 69, 55] is reviewed in Section 7.1. Their space–time structures depend crucially on the

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<sup>18</sup>For a review see, e.g. Ref. [194].

value of the coupling  $\alpha$  of the dilaton to the matter potential (2.6). For  $\alpha \leq 1$  there is a planar null singularity, while for  $\alpha > 1$  the singularity is *naked*.

The major part of this section involves a review of the space–time for non-extreme and ultra-extreme dilatonic domain walls in the thin wall approximation (Section 7.2).

These walls are generalizations of the non- and ultra-extreme vacuum domain walls discussed in Section 6, but now, only numerical solutions have been obtained [65]. For this reason, the analysis can be done only for walls for which the boundary conditions for the dilaton field and the metric can be specified uniquely at the wall surface. Nonetheless, such cases include the physically interesting example of Type I walls, which interpolate between Minkowski space–time with a constant dilaton value and a new type of space–time with varying dilaton, as well as reflection-symmetric (non-extreme) walls.

The nature of the space–time for non- and ultra-extreme dilatonic domain walls, which possesses naked singularities, poses serious constraints on the phenomenological viability of theories with dilaton fields, including a large class of  $N = 1$  supergravity theories as well as the perturbative effective low energy theory from superstrings.

Interestingly, the space–time induced in the dilatonic domain wall backgrounds can also be related to certain cosmological solutions by a complex coordinate transformation where  $z$  is replaced with a cosmic time coordinate and where the potential changes sign (Section 7.3).

## 7.1 Extreme dilatonic walls

In Section 5 we spelled out the formalism for an embedding of extreme domain walls into the corresponding tree level  $N = 1$  supergravity theory as specified in Section 2. Extreme domain walls are static, planar configurations interpolating between *supersymmetric* minima of the corresponding supergravity potential. Such configurations satisfy the Killing spinor equations (5.7) for *any thickness* of the wall, and the energy density of the wall saturates the corresponding Bogomol’nyi bound. In the thin wall approximation the Einstein-dilaton system outside the wall is described by the formalism spelled out in Section 4 with the non-extremality parameter  $\beta = 0$ .

The  $N = 1$  supergravity Lagrangian, described in Section 2.2 contains the (gauge neutral) chiral-superfield  $\mathcal{T}$ , whose scalar component  $T$  is responsible for the formation of the wall. In addition, there is a chiral superfield  $\mathcal{S}$ , which has no superpotential and whose Kähler potential decouples from the one of  $\mathcal{T}$ . In turn, the scalar component  $S$  of the chiral superfield  $\mathcal{S}$  acts as the dilaton field, which couples to and thereby modulates the effect of the matter potential.

The vanishing of the Killing spinor equations (5.7) for the above bosonic field configuration yields first-order differential equations for the metric coefficient  $a(z)$  and the matter fields  $T(z)$  and  $S(z)$  as well as the constraint (5.9) on the spinor  $\epsilon$ , as spelled out in Section 5. In the following we shall concentrate on the explicit form of the extreme solutions for different special cases in the thin

wall approximation.<sup>19</sup>

We shall first summarize<sup>20</sup> the results for extreme walls with a Kähler potential (2.3b) for  $S$ :  $K_{\text{dil}} = -\alpha \ln(S + S^*)$  [55]. Then, we shall study extreme domain walls with self-dual  $K_{\text{dil}}$ , i.e.  $K_S|_{S'} = 0$  for some  $S'$ . An example of the latter class corresponds to a solution of a theory with a strong–weak (dilaton) coupling symmetry, i.e.  $\text{SL}(2, \mathbb{Z})$  invariance of the dilaton coupling. Section 7.1.1 presents the extreme solutions in theories with an exponential dilaton coupling. Their physical properties such as the Hawking temperature associated with the horizons and the gravitational mass of the singularities are also discussed in Section 7.1.2. Section 7.1.3 comments on the self-dual extreme solutions.

### 7.1.1 Extreme solutions for exponential dilaton coupling

Let us first consider the gravitational properties of extreme supersymmetric domain walls with  $K_{\text{dil}} = -\alpha \ln(S + S^*)$ , which has been worked out in Refs. [54, 55]. The energy density of the wall is of the form:

$$\begin{aligned} \sigma_{\text{ext}} &= 2^{1-\frac{\alpha}{2}} e^{\sqrt{\alpha}\phi_0} \left| \left( e^{\frac{K_{\text{matt}}}{2}} W \right)_{z=0^+} \pm \left( e^{\frac{K_{\text{matt}}}{2}} W \right)_{z=0^-} \right| \\ &\equiv 2(\chi_1 \pm \chi_2) \end{aligned} \quad (7.1)$$

where we have chosen the boundary condition for  $a(0) = 0$  and  $\phi(0) = \phi_0$ . The sum corresponds to the Type II wall and the difference to the Type III wall. It is understood that the coordinates are chosen so that  $\chi_2 \leq \chi_1$ . With the choice

$$\chi_{1,2} = 2 \left| e^{K/2} W \right|_{z=0^\mp},$$

the solution of Eqs. (5.8) are satisfied with the following choice for the sign factor  $\zeta$ : on the  $z < 0$  side,  $\zeta = 1$ , while on the side where  $z > 0$ ,  $\zeta = -1$  for Type II walls and  $\zeta = 1$  for Type III walls. Type I corresponds to the case where  $|W|$  is zero on one side of the wall and nonzero on the other.

The Killing spinor equations (5.7)<sup>21</sup> outside the wall region, i.e. when  $\partial_z T \sim 0$ , are the same as those of the second-order equations (4.9)–(4.11) with  $\beta = 0$  and the effective dilaton potential of the type of Eq. (4.5), where

$$f(\phi) = e^{2\sqrt{\alpha}\phi}, \quad V_0(\tau_{1,2}) = -(3 - \alpha)2^{-\alpha} \left( e^{K_{\text{matt}}} |W|^2 \right)_{T_{1,2}}, \quad \widehat{V}(\phi) = 0$$

on either side of the wall.

<sup>19</sup>Explicit numerical solutions of Eqs. (5.8) for a wall of any thickness have the same qualitative features outside the wall region.

<sup>20</sup>Special cases with  $\alpha = 1$ , parameterizing the dilaton coupling in an effective theory from superstring were found in Ref. [54], and  $\alpha = 2, 3, \dots$ , motivated by no-scale supergravity theories, were studied in Ref. [69].

<sup>21</sup>See Refs. [65, 55] for the explicit form of Bogomol'nyi equations (5.8) applied to this particular case.

The value of parameters  $\chi_{1,2}$  in Eq. (7.1) on either side of the wall is related to  $V_0(\tau_{1,2})$  in the following way

$$\begin{aligned}\chi_{1,2} &\equiv 2^{-\frac{\alpha}{2}} e^{\sqrt{\alpha}\phi_0} \left( e^{\frac{K_{\text{matt}}}{2}} |W| \right) |_{T_{1,2}} \\ &= e^{\sqrt{\alpha}\phi_0} \sqrt{-V_0(\tau_{1,2})/(3-\alpha)}.\end{aligned}\quad (7.2)$$

Note that  $\alpha = 3$  corresponds to the point where  $V_0$  changes sign.

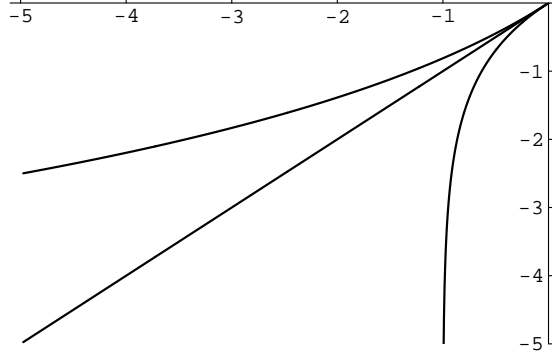


Figure 16: The metric function  $a(z)$  versus  $z$  in units of  $\chi$  for extreme solutions with  $\alpha = 0.5$ ,  $\alpha = 1$ , and  $\alpha = 2$ , respectively. Solutions in models with  $\alpha > 1$  has a naked singularity at a finite value of  $z$ .

The explicit solution on either side of the wall is of the form

$$a = \begin{cases} \chi_1 z, & z < 0 \\ \mp \chi_2 z, & z > 0 \end{cases} \quad (7.3)$$

if  $\alpha = 1$  and

$$a = \begin{cases} (\alpha - 1)^{-1} \ln[1 + \chi_1(\alpha - 1)z], & z < 0 \\ (\alpha - 1)^{-1} \ln[1 \mp \chi_2(\alpha - 1)z], & z > 0 \end{cases} \quad (7.4)$$

if  $\alpha \neq 1$ . The upper and lower signs of the solutions (7.4) correspond to the Type II and Type III solutions, respectively. Type I corresponds to the special case with  $\chi_2 = 0$ , i.e. those are solutions with Minkowski space-time ( $a = 0$ ) and a constant dilaton on the  $z > 0$  side of the wall. The Bogomol'nyi equations also imply that

$$\phi = -\sqrt{\alpha}a$$

everywhere in the domain wall background. Consequently, these solutions are represented by straight lines in the  $(a', \phi')$  phase diagram. For  $\alpha > 1$  the domain walls have a naked (planar) singularity at  $z = -1/[\chi_1(\alpha - 1)]$  and for Type II walls at  $z = 1/[\chi_2(\alpha - 1)]$  as well. For  $\alpha \leq 1$  the singularity becomes null, i.e. it occurs at  $z = -\infty$  and for Type II walls at  $z = \infty$  as well. Note that for

$\alpha < 1$  Type III walls have a coordinate singularity at  $z = 1/[\chi_2(1 - \alpha)]$ . Thus, extreme walls with the “stringy” coupling  $\alpha = 1$  act as an intermediary between the extreme dilatonic walls with naked singularities and those with singularities covered by a horizon. The behaviour of  $a$  for different values of  $\alpha$  is plotted in Fig. 16. The case with  $\alpha = 2$  had earlier been found in the guise of a static plane-symmetric space–time with a conformally coupled scalar field [217, 2]. In Ref. [110] this space–time was shown to be induced by a domain wall.

### 7.1.2 Temperature and gravitational mass per area

Static domain wall configurations with space–time singularities are only possible if there is an exact cancellation of the contributions to the gravitational mass coming from the wall, the dilaton field, and from the singularity.

In the case of an extreme Type I wall, i.e. a static dilatonic wall with a non-zero vacuum energy on one side (say,  $z < 0$  and  $\chi_1 \neq 0$ ) and a Minkowski space on the other ( $z > 0$  and  $\chi_2 = 0$ ), one can employ the concept of gravitational mass per area [62] as discussed in Section 6. It corresponds to a plane-symmetric version of Tolman’s [214] gravitational mass in the spherically symmetric case. The contribution from all sources outside the horizon (or the naked singularity) can be expressed as

$$\Sigma(\infty) = 2a'|_{\infty} - 2a'|_{\text{horizon}}.$$

On the Minkowski side  $a' \equiv da/dz = 0$ , and so  $a'|_{\infty} = 0$ . Hence,  $\Sigma$  is determined by the value of  $a'$  at the horizon:

- For  $\alpha < 1$ , the gravitational mass per area outside the horizon vanishes. Since the total mass vanishes, there is also no mass beyond the horizon. Accordingly, the Hawking temperature associated with this horizon also vanishes [65].
- For  $\alpha = 1$ , the gravitational mass per area from sources outside the horizon is negative:  $\Sigma(\infty) = -2\chi_1$ . In order to have a vanishing total mass, the mass of the singularity must be  $\Sigma_{\text{singularity}} = 2\chi_1$ , but a proper mathematical description would require a distribution-valued metric. We shall not pursue this issue further here, but we note that a source at the singularity of the Schwarzschild metric has recently been identified in terms of a distributional energy-momentum tensor [12]. The Hawking temperature for the extreme Type I wall with  $\alpha = 1$  is finite:  $T = \chi_1/2\pi$  [55].
- For  $\alpha > 1$ , the mass per area outside the singularity is negative and infinite. Accordingly, there must be an infinite positive mass in the singularity, which is then a singularity even in a distributional sense. This naked singularity also has an infinite Hawking temperature.

### 7.1.3 Self-dual dilaton coupling

We now address the case of a self-dual dilaton Kähler potential  $K_{\text{dil}}$ , namely,  $K_{\text{dil}}$  has an extremum ( $K_S|_{S'} \equiv \partial K_{\text{dil}}/\partial S|_{S'} = 0$ ) for some finite  $S = S'$ . In the

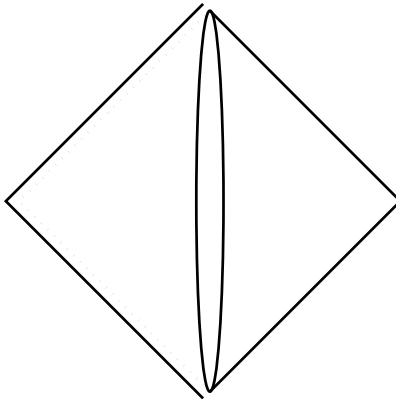


Figure 17: Penrose–Carter diagram in the  $(z, t)$  plane for an extreme Type I dilatonic domain wall with  $0 < \alpha \leq 1$ . The thin arc corresponds to the wall separating the semi-infinite Minkowski space–time and the space–time with a varying dilaton field and the null singularity which coincides with the horizon (the double line).

thin wall approximation, the Bogomol’nyi equations (5.8) are of course satisfied with the choice  $S = S'$  outside the wall region.

In order for such a solution to be stable, one would have to show that as one moves down the wall, the dilaton reaches the point  $S = S'$ , and that from then on it would remain constant, i.e. the point  $S = S'$  is an attractive fixed point. Such solutions would in turn reduce to singularity-free space–times of vacuum domain walls.

However, such a class of extreme solutions is shown [65] to be unstable within  $N = 1$  supergravity theory. In particular, if at  $z \sim 0$ ,  $S(0) = S' + \Delta(0)$ , with  $\Delta(0)$  being an infinitesimal perturbation from the self-dual point,  $S = S'$ , then  $\Delta(z)$  grows indefinitely as  $z \rightarrow -\infty$  and thus the solution with a constant dilaton outside the wall region is not dynamically stable [65].

## 7.2 Non- and ultra-extreme solutions

In this section we shall analyse the non-extreme and ultra-extreme solutions. These are solutions that are not supersymmetric. They correspond to the domain wall backgrounds with moving wall boundaries. Unlike extreme solutions, which have supersymmetric embeddings and where solutions can be given for any thickness of the wall, non- and ultra-extreme solutions have only been obtained [65] in the thin wall approximation, employing the formalism spelled out in Section 4.3.

In the following subsections we review the non-extreme solutions for the exponential dilaton potential, as well as the case with a self-dual dilaton potential. We shall also add a mass term for the dilaton field. Since only numerical solu-

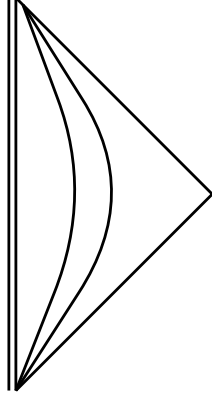


Figure 18: Penrose–Carter diagram in the  $(z, t)$  plane for an extreme Type I dilatonic domain wall with  $\alpha > 1$ . The arcs correspond to the wall separating the semi-infinite Minkowski space–time and the space–time with a varying dilaton field and the naked planar singularity (the double line).

tions for non-extreme walls have been found [65], we confine the analysis to the non-extreme Type I walls (with Minkowski space–time on one side of the wall) and reflection symmetric solutions, for which the boundary conditions on either side of the wall can be specified uniquely. In subsection 7.2.1 the boundary conditions for the non- and ultra-extreme Type I walls are written down explicitly, and the field equations are reduced to a first-order system. The results of the numerical integrations are presented and discussed. Section 7.2.2 contains solutions for the reflection-symmetric cases, and in Section 7.2.3 it is pointed out that the singularity-free self-dual dilatonic domain walls are dynamically unstable. Finally, in Section 7.2.4 the effects of dilaton self-interactions are studied.

### 7.2.1 Walls with Minkowski space–time on one side

We now consider the case where the dilaton potential outside the wall region has the form specified in Eq. (4.5) with  $f = e^{2\sqrt{\alpha}\phi}$  and  $\widehat{V}(\phi) = 0$ . Let the wall be non- or ultra-extreme with a non-vanishing  $V_0$  and a running dilaton on one side ( $z < 0$ ) and a Minkowski space with  $V_0 = 0$  and a constant dilaton on the other ( $z > 0$ ). According to the results of Section 4.3, the boundary conditions are

$$\begin{aligned} \phi'|_{0^-} &= -\frac{1}{2}\sqrt{\alpha}\sigma, & a'|_{0^-} &= \frac{1}{2}\sigma - \beta \\ \phi'|_{0^+} &= 0, & a'|_{0^+} &= -\beta. \end{aligned} \quad (7.5)$$

$\beta < 0$  and  $\beta > 0$  represent an ultra-extreme and a non-extreme wall, respectively. Without loss of generality we have also chosen  $\phi|_0 = 0$  and normalized the metric coefficient  $a|_0 = 0$ . The choice  $\phi|_0 = \phi_0 \neq 0$  would correspond to the



rescaling  $V_0 \rightarrow e^{2\sqrt{\alpha}\phi_0} V_0$ .

At the boundary  $z = 0^-$ , Eq. (4.11) gives

$$V_0 - 3\beta\sigma + (3 - \alpha) \left(\frac{\sigma}{2}\right)^2 = 0. \quad (7.6)$$

For  $\alpha \neq 3$  the above equation reduces to

$$\sigma = \frac{2}{3 - \alpha} \left[ \sqrt{9\beta^2 + (3 - \alpha)^2 \chi_1^2} + 3\beta \right],$$

where we have used the fact that  $\chi_1^2 = -V_0(\tau_1)/(3 - \alpha)$ , as found in Eq. (7.2). In the case  $\alpha = 0$ ,  $\beta \neq 0$ , and  $\chi_1^2 = |V_0|/3$ , one recovers the result from the non- and ultra-extreme anti-de Sitter–Minkowski walls without a dilaton, and if  $\alpha = 0$ ,  $\beta > 0$ , and  $\chi_1 = 0$ , one recovers the dilaton-free non-extreme Minkowski–Minkowski walls [225, 62].

For  $\alpha = 3$ , one finds

$$V_0 = 3\beta\sigma,$$

which indicates that in the non-supersymmetric case with  $\alpha = 3$ , unlike in the supersymmetric case where  $V_0 = 0$ , the potential itself has a non-zero value.

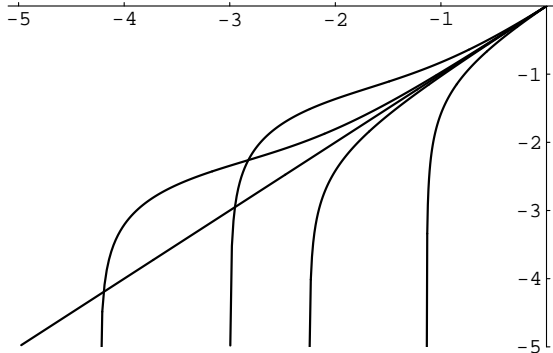


Figure 19: The metric function  $a(z)$  versus  $z$  in units of  $\chi$  with  $\alpha = 1$  for different values of  $\beta$ . Starting from the left at the bottom of the figure where  $a = -5$ , the curves correspond to  $\beta = 0$ ,  $\beta = -0.01$ ,  $\beta = -0.1$ ,  $\beta = 0.01$  and  $\beta = 0.1$ .

The field equations have been integrated numerically (for further details see Ref. [65]). The conformal factor goes to zero faster than in the extreme space-times *both* in the non- and ultra-extreme cases (see Figs. 19 and 20). This is the case for *all* values of  $\alpha$  as long as  $\beta \neq 0$ . As illustrated in Fig. 19, when  $|\beta|$  is increased, the conformal factor decreases even faster. *Such domain walls thus always exhibit naked singularities.*

In order to understand these surprising results, it is instructive to look at the evolution of the dilaton. The extreme solutions with  $0 < \alpha < 3$  are characterized

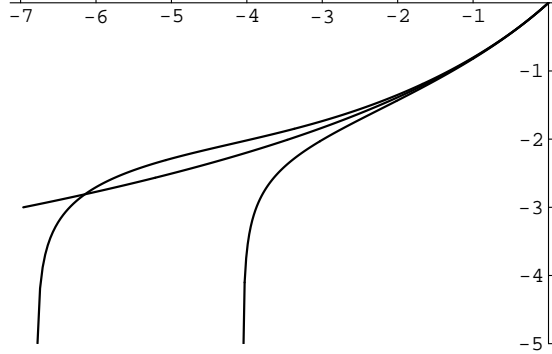


Figure 20: The metric function  $a(z)$  versus  $z$  in units of  $\chi$  with  $\alpha = 1/2$ . The curve starting in the middle corresponds to the extreme solution. The non-extreme case with  $\beta = 0.01$  becomes singular shortly after  $x = -4$ . The third curve corresponds to the ultra-extreme case with  $\beta = -0.01$ . It also ends in a singularity.

by a delicate balancing of the “kinetic” and “potential” energies. As soon as the supersymmetry is broken, i.e.  $\beta \neq 0$ , the dilaton speeds away along its potential; the kinetic energy eventually becomes dominant in both cases and is thus responsible for the appearance of a naked singularity.

### 7.2.2 Reflection-symmetric walls

Now we consider a reflection-symmetric non-extreme wall (a special case of Type II walls) with non-vanishing  $V_0$  and a running dilaton.<sup>22</sup> The potential is taken to be that of Eq. (4.5) with  $f(\phi) = e^{2\sqrt{\alpha}\phi}$  and  $\widehat{V}(\phi) = 0$ . According to Section 4.3, the boundary conditions are

$$\begin{aligned}\phi'|_{0^-} &= -\phi'|_{0^+} = -\frac{1}{4}\sqrt{\alpha}\sigma, \\ a'|_{0^-} &= -a'|_{0^+} = \frac{1}{4}\sigma.\end{aligned}$$

With these boundary conditions, Eq. (4.11) gives

$$-3\beta^2 + V_0 + (3 - \alpha) \left(\frac{\sigma}{4}\right)^2 = 0.$$

If  $\alpha \neq 3$ , we find by use of Eq. (7.2) that

$$\sigma = 4\sqrt{\chi_1^2 + 3(3 - \alpha)^{-1}\beta^2}.$$

For  $\alpha = 0$ , this expression reduces to the one found for reflection-symmetric vacuum domain walls [62]. If  $\alpha = 3$ , then  $\sigma$  remains undetermined from this

<sup>22</sup>Of course, there are no reflection-symmetric ultra-extreme walls.

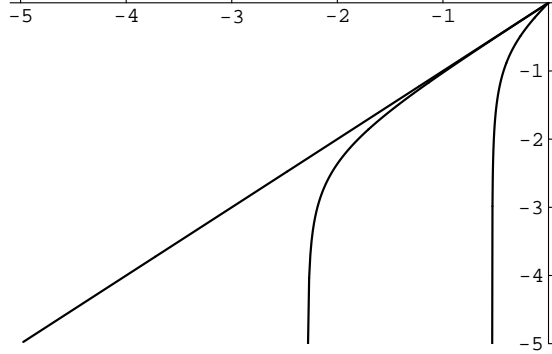


Figure 21: The metric function  $a(z)$  versus  $z$  in units of  $\chi$  for a reflection-symmetric wall with  $\alpha = 1$ . The straight line corresponds to the extreme solution. The other two curves represent non-extreme walls with  $\beta = 0.1$  and  $\beta = 1$ .

expression, but

$$V_0 = 3\beta^2.$$

This result again indicates that in the non-supersymmetric case with  $\alpha = 3$ , the potential itself is modified to be non-zero. Thus, these walls are different from reflection-symmetric domain walls in the background of a Zel'dovich fluid [233] or, which is equivalent, domain walls minimally coupled to a scalar field with no effective potential [234]. Yet, in all these cases one encounters naked singularities. The boundary conditions for reflection-symmetric non-extreme walls are different from those of the wall adjacent to Minkowski space. In this case the singularity is further away from the wall (compare Figs. 19 and 21); however, as for the Type I non-extreme walls, the reflection-symmetric solutions always have naked singularities as well.

### 7.2.3 Self-dual dilaton coupling

We now review the case when the dilaton coupling  $f(\phi)$  is self-dual. Namely, for a finite  $\phi = \phi'$ ,  $f(\phi)$  satisfies:

$$\left. \frac{\partial f}{\partial \phi} \right|_{\phi'} = 0 \quad (7.7)$$

and that  $\widehat{V}(\phi) = 0$  (see Eq. (4.5)).

Note that the equation for the dilaton is of the form (see Eq. (4.10))

$$2\phi'' + 4a'\phi' + e^{2a} \frac{\partial f(\phi)}{\partial \phi} V_0 = 0,$$

with  $\phi' = d\phi/dz$ . With the boundary conditions  $\phi|_0 = \phi_0$  and  $\phi'|_0 = 0$ , the solution of the field equations corresponds to a dilaton frozen at  $\phi = \phi_0$ . The

global space–times of these solutions are then identical to those of the vacuum domain walls [62, 57].

When addressing the stability of the constant dilaton  $\phi(z) = \phi_0$  solutions for extreme, non- and ultra-extreme solutions, one adds to  $\phi(0) = \phi_0$  an infinitesimal virtual displacement  $\delta(0)$ . Supersymmetric embedding of an extreme solution with a constant (complex) dilaton  $S = S'$  is *always* unstable. One can also show an instability of such solutions by solving the corresponding differential equation for  $\delta$ , directly, i.e. without reference to the effective dilaton potential restricted by  $N = 1$  supergravity theory.<sup>23</sup> Thus, the extreme self-dual solutions with a frozen dilaton are always dynamically unstable. The origin of the instability of extreme solutions may be related to an infinite extent of such planar configurations.

On the other hand, the non-extreme and ultra-extreme solutions ( $\beta \neq 0$ ) turn out to be *stable* under this perturbation [65], and may thus be “phenomenologically” stable.

#### 7.2.4 Domain walls with a massive dilaton

Let us now consider the situation where, supersymmetry breaking due to non-perturbative effects, introduces an additional self-interaction term in the dilaton potential. In general such a term is of a complicated form. One might hope that a dilaton self-interaction term, providing a mass for the dilaton, could stabilize the system and keep the dilaton from running away. For the sake of simplicity we consider an additional self-interaction potential  $\widehat{V}$  of the form

$$\widehat{V} = \lambda^2 \chi_1^2 \sinh^2 \omega \phi,$$

where  $\lambda$  and  $\omega$  are real constants. Note that in the cosmological picture, this potential has the opposite sign.

Since  $V(0) = V'(0) = 0$ , the boundary conditions for the equations of motion at  $z = 0^-$  for a wall adjacent to Minkowski space, remain as in Eq. (7.5).

In the non-extreme case adding a mass term forces the dilaton to “run” in a way which causes the appearance of the naked singularity. In the ultra-extreme case with a dilaton self-interaction potential, the dilaton also produces a naked singularity, in general. In the latter case the appearance of the naked singularity can only be avoided by a fine-tuned self-interaction potential whose fine-tuning would have to depend also on  $\beta$ .

The same qualitative features take place in the reflection-symmetric case.

### 7.3 Correspondence with a cosmological model

In this section we point out that the Einstein–dilaton system outside the wall is equivalent to that of an Einstein–dilaton Friedmann–Lemaître–Robertson–Walker (FLRW) cosmology. Formally, one can flip the wall into a spacelike

<sup>23</sup>The extreme solutions with a real dilaton field  $\phi$  can be viewed as corresponding to a special supersymmetric embedding, which renders the imaginary part of the complex field  $S$  constant.

hyperspace by a complex coordinate transformation:

$$z \rightarrow \eta, \quad \cosh \beta t \rightarrow i\beta r. \quad (7.8)$$

If one regards the new coordinates as real, then  $\eta$  becomes the conformal time and  $r$  a spatial coordinate in a metric with opposite signature:  $(-, +, +, +)$ . Changing the sign of the metric implies a change of sign of the curvature scalar,  $R$ , and of all kinetic energy terms in the Lagrangian (2.4). Because the overall sign of the total Lagrangian is arbitrary, one can change back the sign of the metric, if one at the same time changes the sign of the potential  $V(\phi)$ . Thus, the complex coordinate transformation (7.8) *maps the domain wall system onto a cosmological model having a potential with the opposite sign*. As a result, the line-element takes the form

$$ds^2 = e^{2a(\eta)} \left[ d\eta^2 - \frac{dr^2}{1 + \beta^2 r^2} - r^2 d\Omega_2^2 \right].$$

This is a FLRW line-element where the spatial curvature is  $k = -\beta^2$ .

The equivalence of the Einstein-dilaton system outside the wall with the dilaton-FLRW cosmology (by using the coordinate transformations (7.8), as well as identifying  $V(\phi) \rightarrow -V_c(\phi)$  and  $\beta^2 \rightarrow -k$ ) proves useful, because it allows us to carry over results from the corresponding cosmological studies. In the cosmological picture the domain wall is a spacelike hyper-space. It could be interpreted as representing a phase transition taking place simultaneously throughout the whole universe. In our case, the boundary conditions at this hyper-space are fixed by the boundary conditions of the domain wall. In addition, it is useful to compare the Einstein-dilaton system outside the wall with the evolution of corresponding well-known perfect fluid cosmologies and to compute the corresponding effective equation of state for the dilaton. In terms of a perfect fluid description, the energy-momentum tensor is of the form:

$$\begin{aligned} T^\eta_\eta &= V_c(\phi) + \dot{\phi}^2 e^{-2a} \\ T^i_i &= V_c(\phi) - \dot{\phi}^2 e^{-2a}, \end{aligned} \quad (7.9)$$

where  $\eta$  is the conformal time and  $\dot{\phi} = d\phi/d\eta$ . Here the index  $i$  refers to the three spatial coordinates. Note that in the energy-momentum tensor (7.9) the sign of  $V_c(\phi)$  is reversed with respect to the potential of Eq. (4.5). The expressions (7.9) correspond to an energy density

$$\rho \equiv T^\eta_\eta = \dot{\phi}^2 e^{-2a} + V_c(\phi) \quad (7.10)$$

and a pressure

$$p \equiv -T^i_i = \dot{\phi}^2 e^{-2a} - V_c(\phi) \quad (7.11)$$

of a perfect fluid with a four-velocity  $u^\mu = e^{-a} \delta^\mu_\eta$ . It is conventional to parameterize the equation of state of a perfect fluid by the  $\gamma$ -parameter:  $p = (\gamma - 1)\rho$ . The following values of  $\gamma$  are singled out:  $\gamma = 0$  corresponds to the equation of

state of a cosmological constant;  $\gamma = 2/3$  is the equation of state of a cloud of randomly oriented strings;  $\gamma = 1$  represents dust (non-relativistic cloud of particles);  $\gamma = 4/3$  is radiation (ultra-relativistic matter); and  $\gamma = 2$  corresponds to a Zel'dovich fluid (maximally stiff matter). All physical equations of state are confined to the range  $0 \leq \gamma \leq 2$ . This is also the range covered by a minimally coupled scalar field  $\phi$ :

$$\gamma = \frac{2\dot{\phi}^2 e^{-2a}}{\dot{\phi}^2 e^{-2a} + V_c(\phi)}. \quad (7.12)$$

It has  $\gamma = 2$  if the kinetic energy dominates, and  $\gamma = 0$  if the potential energy dominates. Matter satisfying an equation of state with  $\gamma < 1$  has negative pressure. If  $\gamma < 2/3$ , then the repulsive gravitational effect of the negative pressure is greater than the attractive gravitational effect of the energy density. Matter obeying such an equation of state is therefore a source of repulsive gravity. An effective equation of state of this kind is a necessary ingredient in inflationary universe models.

### 7.3.1 Domain wall space-time as a cosmological solution

The equivalence of the domain wall solutions on either side of the wall and a class of cosmological solutions implies that we are able to relate the above solutions to known solutions of inflationary cosmology with exponential potentials [17], which were later generalized to higher-dimensional FLRW cosmologies [36]. Properties of general (extreme and non-extreme) scalar field cosmological models and their corresponding phase diagrams were studied in Ref. [118].<sup>24</sup>

Note that after the substitution  $z \rightarrow \eta$  and  $V_0 \rightarrow -V_{0c}$  the Type II solutions and Type III solutions (on the  $z > 0$  side) correspond to contracting and expanding cosmological solutions, respectively. For cosmological models  $\chi_{1,2}^2 \equiv V_{0c}/(3 - \alpha)$ . The value  $\alpha = 3$  corresponds to the point where  $V_{c0}$  changes sign from positive (for  $\alpha < 3$ ) to negative (for  $\alpha > 3$ ). Since the extreme solutions are characterized by  $\phi = -\sqrt{\alpha}a$  they are represented by straight lines in the  $(\dot{a}, \dot{\phi})$  phase diagram [118].

### 7.3.2 Cosmological horizons and domain wall event horizons

We shall now relate the nature of the cosmological horizons to the event horizons in the domain wall background. If we write the cosmological line element in the standard form

$$ds^2 = d\tau^2 - R^2(\tau) \left( \frac{dr^2}{1 + \beta^2 r^2} + r^2 d\Omega_2^2 \right), \quad (7.13)$$

---

<sup>24</sup>Note that in the cosmological picture, the extreme Type I vacuum domain wall becomes a flat inflationary universe where the inflation (the wall-forming scalar field in the original picture) rolls down the inflation potential with just the right speed so that it stops at a local maximum corresponding to a vanishing cosmological constant. At this point the universe also stops expanding.

then the *convergence* of the integral

$$I = \int_{\tau_0}^{\tau_1} \frac{d\tau}{R(\tau)} = \int_{\eta_0}^{\eta_1} d\eta \quad (7.14)$$

in the limit  $\tau_1 \rightarrow \tau_{\max}$  is a necessary and sufficient condition for the existence of a cosmological event horizon [236]. Note that the complex rotation to the domain wall space–time interchanges a space dimension with the time dimension. Because of this, the sufficient and necessary condition for having an event horizon in the domain wall space–time is that

$$I = \int_{\eta_0}^{\eta_{\max}} d\eta = \eta_{\max} - \eta_0 \quad (7.15)$$

*diverges*. In other words, if there is a singularity at finite  $\eta$ , then this singularity is naked.

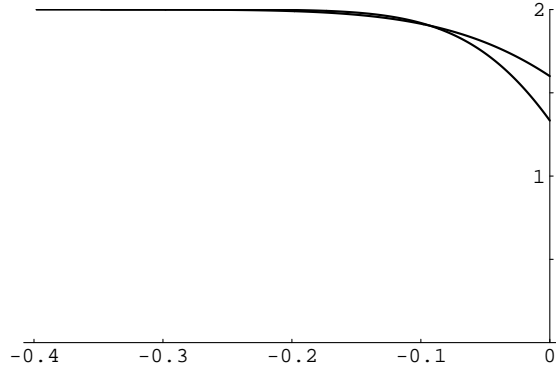


Figure 22: The “equation of state” parameter  $\gamma$  versus  $z$  in units of  $\chi$  for extreme solutions with  $\alpha = 4$  (upper curve) and  $\alpha = 6$ . For  $0 \leq \alpha \leq 3$ , the equations of state are straight lines  $\gamma = 2\alpha/3$ .

### 7.3.3 Equation of state

For  $0 \leq \alpha \leq 3$ , the equation of state is given by

$$\gamma = \frac{2\alpha}{3}. \quad (7.16)$$

The “stringy” value,  $\alpha = 1$ , is therefore the border line between the solutions corresponding to attractive and repulsive equations of state in the cosmological picture. In the domain wall system, this is the dividing line of domain walls with naked singularities ( $\alpha > 1$ ), and domain walls with the singularity hidden behind a horizon ( $\alpha < 1$ ).

For  $\alpha > 3$  the equation of state is time-dependent (see Fig. 22). It approaches  $\gamma = 2$  near the singularity.

## 8 The rôle of supergravity domain walls in basic theory

The purpose of this chapter is two-fold. First, we emphasize a connection of supergravity domain walls to other topological defects of four-dimensional supergravity theories. Specifically, a complementary nature of dilatonic Type I extreme domain walls and certain extreme black holes, some of them appearing as Bogomol'nyi–Prasad–Sommerfield (BPS) saturated states [32, 191, 53] of  $N = 4$  (or  $N = 8$ ) superstring vacua, are summarized. Second, we illuminate how (and when) such domain walls can arise in fundamental theories, like the  $N = 1$  effective supergravity theory of superstring vacua. In particular we shall emphasize different nature of the domain walls when the perturbative as well as non-perturbative effects in superstring theory are included.

### 8.1 Connection to topological defects in superstring theory

The study of topological defects in superstring theory is an important topic extensively discussed in the literature. For recent reviews see, e.g. Refs. [39, 77]. Recently, it has been recognized that supersymmetric solitons, also referred to as BPS-saturated states play a crucial rôle [134, 244] in establishing non-perturbative dualities in string theory. Namely, BPS-saturated states are non-perturbative configurations with the minimal energy in its class, and in the case when the corresponding string vacuum has a large enough supersymmetry, i.e.  $N \geq 2$ , the energy of the BPS-saturated states may not receive quantum (loop) corrections. In this case the expression for the mass of such solitons can be trusted not only in the weak coupling, but also in the strong coupling regime. Since such BPS-saturated states, along with the perturbative string excitations, contribute to the full spectrum of the theory, they provide a nontrivial test to establish equivalence (at the spectrum level) of certain strongly coupled and (dual) weakly coupled string vacua.

The majority of BPS-saturated states in superstring theory, which have been studied in the literature, corresponds to (charged) p-brane configurations in various dimensions ( $d \geq 4$ ) and superstring vacua with supersymmetry  $N \geq 2$ .<sup>25</sup> Such configurations therefore need not bear direct connection to the supergravity walls, which were addressed in previous Sections as nontrivial configurations in four-dimensions within  $N = 1$  supergravity theories. It is however interesting that certain p-brane solutions in  $(p+2)$ -dimensions may still possess similar features as extreme dilatonic domain walls. In Refs. [76, 78] a special example of four-dimensional supersymmetric domain walls of toroidally compactified string theory was found, whose extreme limit [76] corresponds to a specific example of dilatonic domain walls, with the rôle of the cosmological constant being played by the constant gauge field strength.

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<sup>25</sup>For a recent review, see, e.g. Ref. [216] and references therein.



Interestingly, certain p-brane solutions in dimensions higher than four possess [101, 75] the same interesting feature as extreme four-dimensional domain walls; namely, they interpolate between different types of (higher dimensional) supersymmetric vacua.

In the following we shall also see that there is an intriguing complementarity between the space–time structure of certain extreme black holes and certain extreme Type I domain walls as described in the following subsection 8.1.1. Some of these extreme black holes appear as special cases of BPS-saturated black holes of  $N = 4$  (or  $N = 8$ ) superstring vacua as will be shown in the subsequent subsection 8.1.2.

### 8.1.1 Extreme domain wall and black hole complementarity

Interestingly, Type I supersymmetric (extreme) domain walls in the  $(x, y)$  plane of four-dimensional  $N = 1$  supergravity theories with a general dilaton coupling  $\alpha > 0$  (see Sections 2.2 and 2.3) have the same global space–time structure in the  $(t, z)$  slice as of the extreme magnetically charged black holes with the coupling  $1/\alpha$  in the  $(t, r)$  hyperspace [55].

The origin of this complementarity lies in the nature of the  $N = 1$  supergravity Lagrangian (2.4) with one U(1) gauge superfield  $\mathcal{W}$ , a gauge neutral chiral matter superfield  $\mathcal{T}$  (with nonzero superpotential) and one linear supermultiplet rewritten in terms of a gauge neutral chiral superfield  $\mathcal{S}$ . Recall (see Section 2.2), that in this case the theory is specified by the gauge coupling function  $f_{ab} = \delta_{ab}\mathcal{S}$ , superpotential  $W = W_{\text{matt}}(\mathcal{T})$  and a separable Kähler potential  $K = K_{\text{matt}}(\mathcal{T}, \mathcal{T}^*) + K_{\text{dil}}(\mathcal{S}, \mathcal{S}^*)$  where  $K_{\text{dil}}(\mathcal{S}, \mathcal{S}^*) = -\alpha \ln(\mathcal{S}, \mathcal{S}^*)$  and  $S = e^{-2\phi/\sqrt{\alpha}} + iA$  Eqs. (2.1)–(2.3).

The black holes with a general dilaton coupling [99, 130] are spherically symmetric solutions of the theory (2.4) with the matter fields  $T$  turned off, i.e.  $V \equiv 0$ , however, with non-zero U(1) gauge fields  $F_{\mu\nu} \neq 0$ . Note that now the kinetic energy term for the gauge field is of the form:  $\frac{1}{4} e^{-2\phi/\sqrt{\alpha}} F_{\mu\nu} F^{\mu\nu}$ . This particular coupling arises in the  $N = 1$  supergravity Lagrangian with the U(1) Yang–Mills gauge field and a general linear supermultiplet (see Eq. (2.4)). It is, however, not known whether such a term with a general coupling  $\alpha$  of the dilaton to the gauge fields has an embedding into supergravity Lagrangian with  $N \geq 2$ . We shall however see later that for special values of  $\alpha$ , the black hole Lagrangian has an embedding into a consistently truncated  $N = 4$  (as well as  $N = 8$ ) Lagrangian of string vacua. In this case it can be shown that the corresponding extreme black holes are supersymmetric and thus have the minimum energy in their class, i.e. they are BPS-saturated states. We shall defer the discussion of these states to Section 8.1.2.

The (Einstein frame) metric for the extreme magnetically charged solution

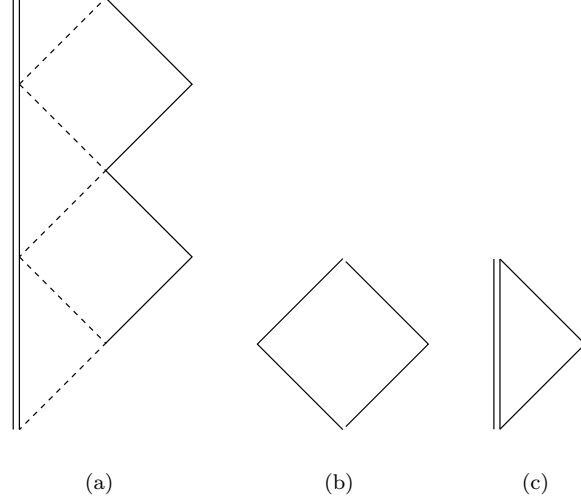


Figure 23: Penrose–Carter diagrams for the extreme magnetically charged black holes with  $\alpha = \infty$ ,  $\infty > \alpha \geq 1$ , and  $1 > \alpha \geq 0$ , respectively. The double lines corresponds to the timelike, null and naked singularity.

with the general dilaton coupling is of the form [99, 89, 130]:<sup>26</sup>

$$ds^2 = \lambda(r)dt^2 - \lambda(r)^{-1}dr^2 - R(r)d\Omega_2^2, \quad (8.1a)$$

with:

$$\lambda(r) = \left(1 - \frac{r_0}{r}\right)^{\frac{2\alpha}{1+\alpha}}, \quad R(r) = r^2 \left(1 - \frac{r_0}{r}\right)^{\frac{2}{1+\alpha}}. \quad (8.1b)$$

The dilaton field  $\phi$  and the magnetic field are of the form:

$$e^{2\phi/\sqrt{\alpha}} = \left(1 - \frac{r_0}{r}\right)^{-\frac{2}{1+\alpha}}, \quad F_{\theta\phi} = P \sin \theta.$$

Here  $P$  is the magnetic charge of the black hole,  $r_0^2 = P^2 \left(\frac{1+\alpha}{\alpha}\right)$  and the mass  $M$  of the black hole is

$$M = |P| \left(\frac{\alpha}{1+\alpha}\right)^{\frac{1}{2}}.$$

The global space–time structure (and the related thermal properties) of the extreme magnetically charged dilatonic black holes bear striking similarities to the one of the domain wall configurations, however, now the role of  $\alpha$  is inverted:

<sup>26</sup>The corresponding electrically charged black holes have the same Einstein frame metric, however, the dilaton solution is related to the corresponding magnetic one by the transformation  $\phi \rightarrow -\phi$ .

- $\alpha = \infty$  corresponds to the case, where the dilaton field does not couple to the gauge fields. The solution therefore corresponds to the extreme Reissner–Nordström black hole, which has a timelike singularity at  $r = 0$  and where  $r = r_0$  corresponds to a Cauchy horizon. Its global space–time structure (see Figure 23a) in the  $(r, t)$  direction is *the same* as the one of the Type I supergravity walls in the  $(z, t)$  direction (cf. Figure 6). In the latter case, however, the timelike singularity is replaced by the wall. The corresponding black hole Hawking temperature vanishes.
- $\infty > \alpha > 1$  corresponds to solutions with the curvature singularity at  $r = r_0$ . Timelike radial geodesics reach  $r = r_0$  in infinite time. Therefore  $r = r_0$  corresponds to a null singularity (see Figure 23b). The temperature vanishes.
- $\alpha = 1$  corresponds to the stringy extreme magnetically charged black hole with a null singularity at  $r = r_0$  (see Figure 23b) however, the temperature is  $T = M/8\pi$ .
- $\alpha < 1$ , corresponds to solutions, where the singularity at  $r = r_0$  is reached by a radial geodesics in a *finite* proper time. Thus, the singularity is *naked* (see Figure 23c), and the temperature  $T$  is infinite.

Thus, the extreme magnetically charged stringy dilatonic black hole with  $\alpha = 1$  and  $T = M/(8\pi)$  serve as a dividing line [99] between extreme charged dilatonic black holes with  $\alpha > 1$  and  $T = 0$  and the naked singularities with  $\alpha < 1$  and  $T = \infty$ .

Therefore the global space–time in the  $(t, z)$  slice for extreme walls with coupling  $\alpha$  is the same as the one in the  $(t, r)$  slice for extreme magnetically charged black holes with coupling  $1/\alpha$  (see Figure 23). Between the two solutions the dilaton coupling  $\alpha$  is inverted, while the rôle of  $W_{\text{matt}}(T)$  on one side of the wall and the magnetic charge  $P$  of the black hole are interchanged.

Near the singularity, the metric (4.1) in the  $(t, z)$  slice of the wall with the coupling  $\alpha$  is the same as the metric (8.1) in the  $(t, r)$  slice of the black hole with the coupling  $1/\alpha$ . Namely, in the region  $r - r_0 \equiv \rho \rightarrow 0^+$ , the coordinates  $(t, \rho)$  of the black hole with the coupling  $\alpha$  and the coordinates  $(t, z)$  of the wall with the coupling  $\tilde{\alpha} \equiv 1/\alpha$  are related in the following way:

$$\frac{\rho}{r_0} = \begin{cases} [1 - \frac{1}{2}(\tilde{\alpha} - 1)\sigma_{\text{ext}}|z|]^{\frac{\tilde{\alpha}+1}{\tilde{\alpha}(\tilde{\alpha}-1)}} & \text{for } \alpha \neq 1 \\ e^{-\sigma_{\text{ext}}|z|} & \text{for } \alpha = 1 \end{cases} \quad (8.2)$$

where  $r_0 = 2/[(1 + \tilde{\alpha})\sigma_{\text{ext}}]$  and  $\tilde{\alpha} \equiv 1/\alpha$ .

Near the singularity the dilaton blows up in both cases, however, unlike the two-dimensional metric slices, the coordinate dependence of the dilaton near the singularity is *different* in either case. This fact is also reflected in the different form of the corresponding two-dimensional effective actions.

The complementarity between the global space–time structure of the extreme dilatonic domain walls with coupling  $\alpha$  and extreme charged dilatonic

black holes with coupling  $1/\alpha$  can be traced back to the nature of the coupling  $e^{2\sqrt{\alpha}\phi}$  of the dilaton to the matter potential (the source for the wall) and the complementary coupling  $e^{-2\phi/\sqrt{\alpha}}$  of the dilaton to the gauge kinetic energy (the source of the charge of the black hole).

The complementarity ( $\alpha \leftrightarrow 1/\alpha$ ) between the extreme wall and extreme charged black hole solutions is a generalization of the one found [57] between extreme vacuum domain walls ( $\alpha = 0$ ) and ordinary extreme black holes ( $\alpha = \infty$ ). Interestingly, only for the  $N = 1$  supergravity with the coupling  $\alpha = 1$ , which corresponds to an effective tree level theory from superstrings, both extreme dilatonic walls *and* extreme charged dilatonic black holes are void of naked singularities. Such a complementarity seems to exist only in the case of *extreme* configurations. In the case of the non- or ultra-extreme configurations the complementarity is not carried over. For one thing, the domain walls are not static configurations anymore, while non- and ultra-extreme black holes remain static configurations. Nevertheless, the connection between the space-times of domain walls and black holes deserves further study.

In the following subsection we shall summarize results about supersymmetric embedding of some of the above extreme black holes within  $N = 4$  superstring vacua.

### 8.1.2 Relationship to the BPS-saturated black holes of $N = 4$ superstring vacua

Effective  $N = 4$  superstring vacua in four dimensions can be parameterized in terms of massless fields of heterotic string compactified in six-torus  $T^6$  (and equivalently, due to string-string duality, in terms of massless fields of type IIA string theory compactified on  $K3 \times T^2$  where  $K3$  is the two-complex-dimensional Calabi–Yau manifold).<sup>27</sup> For a review see Ref. [201].

The bosonic part of the effective  $N = 4$  Lagrangian is parameterized in terms of the graviton, 28 U(1) gauge fields and 134 scalar fields, with complex field  $S$ , parameterizing the gauge coupling and 132 scalar-moduli, parameterizing the six-torus of the compactified space. The solutions of the theory possess the  $T$ -duality  $O(6, 22)$  symmetry, associated with the symmetries of the compactified space, and  $S$ -duality  $SL(2, \mathbb{R})$  relating the strong and weak couplings of string vacua. The general BPS-saturated spherically symmetric solutions of this effective Lagrangian, parameterized by 28 electric and 28 magnetic charges have been obtained in Ref. [66, 71, 67], while general non-extreme solutions (compatible with the corresponding Bogomol’nyi bound) have been constructed in Ref. [68].<sup>28</sup>

Here we will quote only a special example of the BPS-saturated spherically symmetric static solutions, which are parameterized by two electric charges  $Q_1$  and  $Q_2$  of the respective Kaluza–Klein and the ‘two-form’ U(1) gauge fields associated with the first compactified direction of the six-torus, and two magnetic

<sup>27</sup>An analogous embedding exists also in the case of  $N = 8$  superstring vacua, parameterized in terms of the massless fields of toroidally compactified Type IIA superstring.

<sup>28</sup>See also Ref. [68] for the references for certain special solutions of this effective theory.

charges  $P_1$ ,  $P_2$  of the respective Kaluza–Klein and the ‘two-form’ U(1) gauge fields associated with the second compactified direction of the six-torus first obtained in Ref. [71].<sup>29</sup> In this case the space–time metric (8.1a), the dilaton field  $\Phi$  and the two scalar fields  $g_{11}$  and  $g_{22}$ , i.e. the respective moduli for the circles of the first and the second compactified direction, are of the form:

$$\lambda = \frac{r^2}{[(r + Q_1)(r + Q_2)(r + P_1)(r + P_2)]^{\frac{1}{2}}}, \quad R\lambda = r^2, \quad (8.3)$$

$$e^{2\Phi} = \left[ \frac{(r + P_1)(r + P_2)}{(r + Q_1)(r + Q_2)} \right]^{\frac{1}{2}}, \quad g_{11} = \frac{r + P_2}{r + P_1}, \quad g_{22} = \frac{r + Q_1}{r + Q_2}, \quad (8.4)$$

with the ADM mass:

$$M = \frac{1}{4}(Q_1 + Q_2 + P_1 + P_2). \quad (8.5)$$

Interestingly, for special values of the above four charge assignments the solution reduces to the solution (of the consistently) truncated  $N = 4$  bosonic Lagrangian with only one U(1) gauge field, coupled to one scalar field  $\phi$  with the specific value of the coupling  $\alpha$ . These non-zero values of the coupling  $\alpha$  and the corresponding non-zero charge assignments are of the following form [71, 79]:

$$\alpha = \infty, \quad P_1 = P_2 = Q_1 = Q_2 \equiv P, \quad (8.6a)$$

$$\alpha = 3, \quad P_1 = P_2 = Q_1 \equiv \frac{2}{\sqrt{3}}P, \quad (8.6b)$$

$$\alpha = 1, \quad P_1 = P_2 \equiv \sqrt{2}P, \quad (8.6c)$$

$$\alpha = \frac{1}{3}, \quad P_1 \equiv \frac{\sqrt{3}}{2}P. \quad (8.6d)$$

With the charge assignments (8.6), Eqs. (8.3) and (8.4) reproduce the solution for the extreme magnetically charged black holes (discussed in the previous Section) whose value of  $\alpha$  is also given in (8.6), while the radial coordinates related as  $r \rightarrow r + P\sqrt{(1 + \alpha)/\alpha}$ . Thus, within  $N = 4$  superstring vacua there are BPS-saturated black hole solutions with four specific charge assignments. Each of these assignments falls within the four specific classes of the scalar couplings  $\alpha$ , i.e.  $\alpha = \infty$ ,  $\alpha = 3 > 1$ ,  $\alpha = 1$ , and  $\alpha = 1/3 < 1$ , each of them with its distinct space–time structure (see Figure 23) and thermal properties, which are complementary to the corresponding extreme Type I domain walls.

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<sup>29</sup>The special case with  $Q_1 = Q_2$  and  $P_1 = P_2$  was obtained in Ref. [140].

## 8.2 Domain walls within $N = 1$ superstring vacua

In this Section we would like to discuss how the supergravity domain wall configurations, arising within general effective  $N = 1$  supergravity Lagrangian (described in Section 2), could arise within effective  $N = 1$  supergravity of superstring vacua, with or without inclusion of non-perturbative effects. For a review of the structure and in particular constraints on the field dependence of the couplings for the  $N = 1$  effective Lagrangian from superstring theory, see Ref. [141] (see also Ref. [195]).

### 8.2.1 Stringy domain walls without inclusion of non-perturbative effects

Perturbative  $N = 1$  supersymmetric four-dimensional string vacua are specified by the  $N = 1$  supergravity Lagrangian (2.4) of Section 2 where the linear multiplet  $\mathcal{S}$  has a tree level Kähler potential (2.3b) with  $\alpha = 1$ . The matter chiral superfields split into moduli  $\mathcal{T}_i^{\text{mod}}$ , parameterizing the symmetries of the compactified space, which do not have any self-interaction in the superpotential (2.2), and the matter chiral superfields  $\mathcal{T}_j^{\text{matt}}$  which are in general charged under the gauge group and have non-trivial superpotential  $W$ . In certain cases, if some of the moduli  $\mathcal{T}_i^{\text{mod}}$ , do not couple to the (gauge neutral) matter fields which can acquire non-zero vacuum expectation value (and form the topological defect), these moduli could act as effective “dilaton-type” fields with a separable Kähler potential of the form (2.3). In some cases the toroidal moduli would play the rôles of dilaton-type fields with the separable Kähler potential  $-\alpha \ln(\mathcal{T}_i^{\text{mod}} + \mathcal{T}_i^{\text{mod}*})$  with  $\alpha = 1, 2, 3$ .

In principle, the superpotential for the matter fields can possess discrete symmetries. Sometimes they are consequences of discrete symmetries of the compactified space, i.e. Calabi–Yau compactified space. Therefore one can (at least) in principle allow for the existence of isolated minima of the matter potential, and thus for the existence of dilatonic domain walls discussed in Section 7 with an effective coupling  $\alpha$  which assumes a discrete values  $\geq 1$ .

Thus, it is in principle possible that perturbative string vacua allow for the existence of dilatonic domain walls with effective discrete values of  $\alpha \geq 1$  [54, 69] at energy scales that are larger than the energy scale where the non-perturbative effects, e.g. gaugino condensation, take place. However, we should point out that the existence of isolated superstring vacua with non-zero vacuum expectation of the (neutral) matter fields would be an indication of the perturbative *instability* of such string vacua, which is an unlikely possibility.<sup>30</sup>

### 8.2.2 Domain walls with inclusion of non-perturbative effects

A more likely possibility is that specific supergravity walls exist as solutions within  $N = 1$  superstring vacua after the (non-perturbative) supersymme-

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<sup>30</sup>For the study of domain walls (with fixed dilaton values) due to spontaneously broken discrete symmetry of the compactified Calabi–Yau manifolds see Refs. [46, 42].

try breaking effects are taken into account. However, at present the non-perturbative effects in string theory are not well understood. One scenario is based on gaugino condensation in the hidden sector of the string theory, which would in turn provide a dilaton dependent, and due to genus-one threshold corrections, also a moduli dependent superpotential.<sup>31</sup> In this case it is believed that the dilaton could be stabilized and the non-perturbatively induced superpotential for the (toroidal) moduli respects a discrete non-compact (generalized) target space duality symmetry also referred to as  $T$ -duality, e.g. the  $\text{SL}(2, \mathbb{Z})$ , which is the symmetry associated with the toroidally compactified space of the string theory.  $\text{SL}(2, \mathbb{Z})$  is specified by:

$$T^{\text{mod}} \rightarrow \frac{aT^{\text{mod}} - ib}{icT^{\text{mod}} + d}, \quad ad - bc = 1, \quad a, b, c, d \in \mathbb{Z}, \quad (8.7)$$

where the real part of  $T^{\text{mod}}$  specifies the radius of the compactified space. Non-perturbative potentials for the modulus field  $T^{\text{mod}}$ , respecting  $\text{SL}(2, \mathbb{Z})$  symmetry, a discrete noncompact symmetry, were extensively classified in Ref. [58].

The physics of the moduli fields is an intriguing generalization of the well known axion physics introduced to solve the strong CP problem in QCD.<sup>32</sup> Namely, perturbatively the effective Lagrangian for the modulus field  $T^{\text{mod}}$ , only, possesses a non-compact symmetry  $\text{SL}(2, \mathbb{R})$ , which is broken down to its discrete subgroup  $\text{SL}(2, \mathbb{Z})$ . Thus, prior to the non-perturbative effects taking place, the modulus Lagrangian allows for the existence of “stringy” cosmic strings [107].

In general, non-perturbative moduli potentials for the moduli fields, respecting  $\text{SL}(2, \mathbb{Z})$  symmetry, allow for discrete isolated vacua, and thus for the existence of supergravity vacuum domain walls of the type discussed in Section 6. In the cosmological context such domain walls are bounded [64] by stringy cosmic strings. Further cosmological implications of such domain walls were studied in Ref. [56].

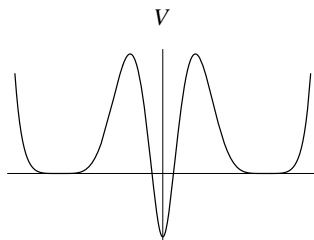


Figure 24: Modular invariant potential along the geodesic  $T(z) = e^{i\Phi(z)}$  for the Kähler potential and superpotential of Eq. (8.8).

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<sup>31</sup>For a recent review of the status of non-perturbative effects due to gaugino condensation see for example Ref. [194]. For another non-perturbative phenomenon within Liouville (non-critical) string theory, which induce a periodic non-perturbative potential see Ref. [186].

<sup>32</sup>For a review see [146].

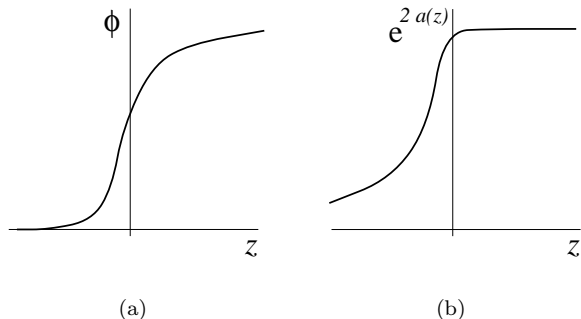


Figure 25: The solutions for  $\Phi(z)$  and  $e^{2a(z)}$  for the potential of Fig. 24.

An illustrative example of an explicit solution [60] for the (finite size) domain wall (with the  $SL(2, \mathbb{Z})$  symmetry due to one modulus field  $T = T^{\text{mod}}$ ) is given for the potential with the following choice of Kähler potential and superpotential [60]:

$$K = -3 \ln [(T + T^*) |\eta(T)|^4], \quad W = \Omega^3 J(T), \quad (8.8)$$

where  $J$  and  $\eta$  are the absolute modular invariant and Dedekind function (the modular function with the modular weight  $-1/2$ ), respectively [200].<sup>33</sup> Here  $\Omega$  corresponds to the scale where non-perturbative effects, e.g. gaugino condensation, which stabilize the dilaton, take place. In this case the potential has two supersymmetric isolated minima, one at  $T = 1$  ( $\mathbb{Z}_2$  symmetric point of the fundamental domain, corresponding to the anti-de Sitter space-time) and the other one at  $T = e^{i\pi/6}$  ( $\mathbb{Z}_3$  symmetric point of the fundamental domain, corresponding to the Minkowski space-time). The domain wall is then the extreme Type I vacuum domain wall with the geodesic for  $T(z) = e^{i\Phi(z)}$  (i.e. on the boundary of the fundamental domain). In Figure 24 the potential along  $\Phi(z)$  is given, while the solution for  $\Phi(z)$  and the conformal (metric) factor  $e^{2a(z)}$  as a function of  $z$  are given in Figure 25. Note that those correspond to explicit numerical solutions of the Bogomol'nyi equations (5.8) (without dilaton field).

Another example [56] is provided by choosing  $W = \Omega^3 \eta(T^{\text{mod}})^{-6}$ . In this case the potential has the property [58] that the discrete *degenerate* minima with negative cosmological constant break supersymmetry. The type of walls in this case are the non-extreme (reflection symmetric) vacuum domain walls between anti-de Sitter vacua. For further discussion of cosmological implications, including the possibility of inflation due to non-perturbative moduli potential, see Ref. [56].

Another type of domain wall configuration could arise if the non-perturbatively induced potential for the dilaton field  $S$  preserves the the weak-strong

<sup>33</sup> For a review of the modular group  $SL(2, \mathbb{Z})$  and its modular functions see, e.g. Ref. [160]. A large class of  $SL(2, \mathbb{Z})$  invariant potentials are given in Ref. [58].



coupling duality symmetry, also referred to as  $S$ -duality symmetry. The strong-weak coupling duality conjecture assumes that the string vacua possess the  $SL(2, \mathbb{Z})$  symmetry associated with the dilaton  $S$ . Thus, in this case the non-perturbatively induced potential for the dilaton field respects the  $SL(2, \mathbb{Z})$  symmetry, and thus one can have vacuum domain walls associated with the dilaton field, whose features are analogous to those associated with the modulus field  $T^{\text{mod}}$ . The possibility of such walls (due to dilaton field) and their physical implications, including the possibility to account for inflation within domain walls [170, 228], have been recently studied in Refs. [131, 13].

Another possibility may be that the non-perturbative effects in the superpotential are negligible, but the Kähler potential for the dilaton is modified [14] so that it preserves the  $SL(2, \mathbb{Z})$  symmetry. In this case the domain wall cannot be formed due to the dilaton field. However, another field (with non-zero) superpotential forms the wall, while the dilaton would modify the solution in such a way that it would describe the domain wall with the self-dual dilaton coupling discussed in Section 7.2.3. In this case the non- and ultra-extreme solutions can reduce to the singularity-free vacuum domain wall solutions [65].

Further study of non-perturbative effects in string theory would shed light not only on aspects of supersymmetry breaking, but also on the physical implications of domain walls in string theory.

## 9 Conclusions

In this paper we have systematically reviewed domain wall configurations as solutions of general  $N = 1$  supergravity theory. We have given a thorough analysis of the space–time structure of such domain walls and emphasized the special rôle that supersymmetry is playing in determining the nature of such configurations. Detailed results were given for both the vacuum domain walls, i.e. configurations where on either side of the wall all the matter fields assume constant vacuum expectation values, as well as for dilatonic domain walls, where the dilaton field, which couples to the scalar potential, varies with a spatial separation from the wall.

The vacuum domain walls can be classified according to the energy density stored in the wall. It turns out that domain walls interpolating between supersymmetric minima of the matter potential correspond to the *static* planar configurations, whose energy density (in the thin wall approximation) is  $\sigma = \sigma_{\text{ext}}$ , specified by the value(s) of the cosmological constant(s) on each side of the wall. The non-extreme walls (with  $\sigma > \sigma_{\text{ext}}$ ) correspond to expanding bubbles with two insides, while ultra-extreme wall solutions with ( $\sigma < \sigma_{\text{ext}}$ ) correspond to the false vacuum decay bubbles. The extreme solution (with supersymmetric embedding) therefore provides a dividing line between the two classes of domain wall solutions.

Vacuum domain walls between vacua with non-positive cosmological constants can belong to any of the three classes. The domain walls with at least one of the cosmological constant positive correspond to the false vacuum decay walls, only.

Dilatonic domain walls can be classified analogously with extreme dilatonic domain walls corresponding to static walls which interpolate between supersymmetric vacua with varying dilaton field and thus a space-time structure different from that of extreme vacuum domain walls. The nature of the space-time crucially depends on the value of the dilaton coupling to the matter potential. The non-extreme and ultra-extreme dilatonic walls again correspond to expanding bubbles, however, now one always encounters one or more naked singularities. The results due to non-perturbative corrections, which induce the dilaton potential were also analysed.

We also reviewed the properties of supergravity domain walls as they appear within effective  $N = 1$  supergravity from superstring theory with or without inclusion of non-perturbative effects. Perturbative string vacua may allow for appearance of dilatonic domain walls. On the other hand, a non-perturbatively induced potential for the moduli fields, which preserve discrete non-compact symmetry of the compactification space ( $T$ -duality), as well as a non-perturbatively induced potential for the dilaton field, which preserves the strong-weak coupling duality ( $S$ -duality), allow for vacuum domain walls due to the modulus and/or dilaton field. Such domain walls may have interesting cosmological implications. However, before a more detailed analysis of the physics of domain walls within  $N = 1$  string vacua can be carried out, a better understanding of the non-perturbative phenomena in string theory is needed.

Interestingly, the space-time structure of certain extreme (supersymmetric) domain walls has a space-time structure that is closely related to that of certain extreme magnetically charged dilatonic black holes, some of them corresponding to the BPS-saturated states of  $N = 4$  (or  $N = 8$ ) superstring vacua. Further study of the connection between supergravity walls and other topological defects in supergravity (and superstring theory) is also needed.

While this review provides a systematic analysis of the domain wall solutions within  $N = 1$  supergravity theory, little has been said beyond the general statements, about their implications for cosmology, in particular about mechanisms by which they can be formed in the early universe and their implications for the early universe evolution. Also the stability of such solutions as well as related dynamical questions have to be studied in more details. All of these questions await further investigations.

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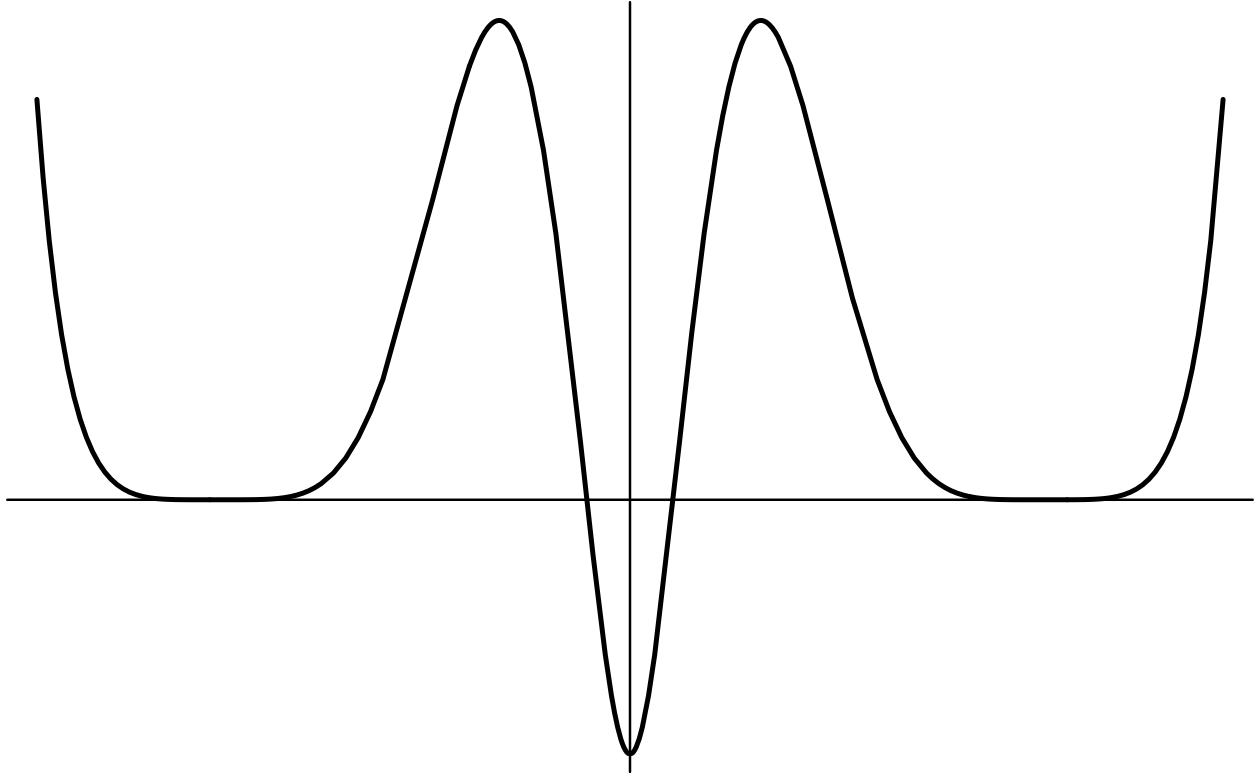
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