Opening the Window for Electroweak Baryogenesis *

M. Carena †, **M. Quirós** ‡ and **C.E.M. Wagner** †

† CERN, TH Division, CH–1211 Geneva 23, Switzerland
‡ Instituto de Estructura de la Materia, CSIC, Serrano 123, 28006 Madrid, Spain

Abstract

We perform an analysis of the behaviour of the electroweak phase transition in the Minimal Supersymmetric Standard Model, in the presence of light stops. We show that, in previously unexplored regions of parameter space, the order parameter $v(T_c)/T_c$ can become significantly larger than one, for values of the Higgs and supersymmetric particle masses consistent with the present experimental bounds. This implies that baryon number can be efficiently generated at the electroweak phase transition. As a by-product of this study, we present an analysis of the problem of colour breaking minima at zero and finite temperature and we use it to investigate the region of parameter space preferred by the best fit to the present precision electroweak measurement data, in which the left-handed stops are much heavier than the right-handed ones.

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The origin of the baryon asymmetry of the Universe remains one of the most intriguing open questions in high energy physics [1]. It was long assumed that this question could only be answered by the knowledge of the physics at very short distances, of the order of the Grand Unification or the Planck scale. This general assumption was challenged through the discovery that anomalous processes [2] can partially or totally erase the baryon asymmetry generated at extremely high energies [3]. Much attention was hence devoted to the possibility of generating the baryon asymmetry at the electroweak phase transition [4], assuming that no new physics beyond the Standard Model is present at the weak scale.

The Standard Model has all the required properties for the generation of the baryon asymmetry: CP violation, baryon number violating processes and, in addition, non-equilibrium processes that are generated at the first-order electroweak phase transition. To generate the required baryon asymmetry, the electroweak phase transition must be strongly first order. Quantitatively, the requirement is that the ratio of the vacuum expectation value of the Higgs field at the critical temperature to the critical temperature must be larger than 1 [5],

$$\frac{v(T_c)}{T_c} \gtrsim 1. \tag{1.1}$$

The Higgs potential of the Standard Model at finite temperature can be given by

$$V_{eff}^{\rm SM} = -m^2(T)\phi^2 - E_{\rm SM}\, T\, \phi^3 + \frac{\lambda(T)}{2}\phi^4 + \cdots, \tag{1.2}$$

where the coefficient of the cubic term is

$$E_{\rm SM} \sim \frac{2}{3}\left(\frac{2M_W^3 + M_Z^3}{\sqrt{2}\pi v^3}\right), \tag{1.3}$$

$\langle\phi(T)\rangle = v(T)/\sqrt{2}$, and the normalization of $\phi(T)$ is chosen such that its zero temperature vacuum expectation value is $\langle\phi(T=0)\rangle = v/\sqrt{2}$, with $v = 246.22$ GeV. The critical temperature is defined as that one for which the symmetry-breaking minimum has the same depth as the symmetry-preserving one. From Eq. (1.2), it is easy to show that

$$\frac{v(T_c)}{T_c} \simeq \frac{\sqrt{2}\, E_{\rm SM}}{\lambda}. \tag{1.4}$$

The effective quartic coupling λ at T_c is closely related to its zero temperature value, implying that the requirement of Eq. (1.1) puts an upper bound on the Higgs mass. This upper bound was estimated by the analysis of the improved one-loop effective potential to be of order 40 GeV [6]. It was subsequently shown that higher-loop effects can enhance the strength of the first-order phase transition [7]. The most recent non-perturbative studies [8] indicate that the real upper bound is still below the present experimental bound on the Higgs mass, $m_H \geq 65$ GeV.

Furthermore, in the Standard Model the source of CP-violation is associated with the CP-violating phase in the Cabbibo-Kobayashi-Maskawa matrix. Any CP-violating

M_V is the mass of the vector bosons. It was shown that these suppression factors are sufficiently strong to severely restrict the possible baryon number generation [9].

Thus, to generate the observed baryon asymmetry of the Universe at the electroweak phase transition, the presence of new physics at the weak scale is required. An interesting possibility is that the new physics be given by the minimal supersymmetric extension of the Standard Model (MSSM). In the MSSM, new sources of CP-violation are present [10, 11], which can serve to avoid the strong Standard Model suppression discussed above [10]. Preliminary results on the behaviour of the electroweak phase transition within this model [12, 13, 14] showed that the situation can only be improved slightly in comparison with the Standard Model case. This improvement was associated with the presence of light supersymmetric partners of the top quark (stops) and small values of $\tan\beta$.

In this article, we shall show that, in previously unexplored regions of parameter space, the phase transition can be more strongly first order than previously derived, without being in conflict with any phenomenological constraint. We shall follow the formalism and conventions of Refs. [13, 14], where some technical details relevant for this presentation can be found.

2 Light Top Squark Effects

In the following, we shall explain the reason why, as it was already observed in Ref. [13], the presence of light stops can help in enhancing the strength of the first-order phase transition. We shall work in the limit $m_A \gg M_Z$, which implies that only one Higgs doublet ϕ survives at scales of order T_c [1]. We shall also concentrate on the case that the light stop is predominantly right-handed, implying that $m_Q^2 \gg m_U^2,\ m_t^2$, where m_Q^2 and m_U^2 are the soft supersymmetry-breaking squared mass parameters of the left- and right-handed stops, respectively, and m_t is the running top quark mass. This hierarchy of masses is naturally expected in the small $\tan\beta$ regime, if supersymmetry is broken in the hidden sector [16, 17]. Moreover, this range of parameters is selected by the best fit to the precision electroweak data within the MSSM [15]. Indeed, large values of m_Q^2 assure a small supersymmetric contribution to the oblique corrections, while low or negative values of m_U^2 can help in enhancing the value of R_b, particularly in the presence of light charginos, with a dominant Higgsino component and close to the present experimental bounds.

Within the above framework, the stop masses are approximately given by

$$\begin{aligned} m_{\tilde{t}}^2 &\simeq m_U^2 + D_R^2 + m_t^2(\phi)\left(1 - \frac{\tilde{A}_t^2}{m_Q^2}\right) \\ m_{\tilde{T}}^2 &\simeq m_Q^2 + D_L^2 + m_t^2(\phi)\left(1 + \frac{\tilde{A}_t^2}{m_Q^2}\right), \end{aligned} \tag{2.1}$$

[1] This case is favoured by the strength of the phase transition [13] and by precision electroweak measurements in the low $\tan\beta$ regime [15].

stop masses, $D^2_{R,L}$ are the D-term contributions to the right- and left-handed stop squared masses, respectively, h_t is the top quark Yukawa coupling and $\tilde{A}_t = A_t - \mu/\tan\beta$ is the effective stop mixing mass parameter. The heaviest stop leads to a relevant contribution to the zero-temperature effective potential, which can be absorbed in a redefinition of the parameters m^2 and λ in Eq. (1.2). The contribution of the heavy stop to the quartic coupling is quite significant, growing with the fourth power of the top quark mass and logarithmically with m_Q [18, 19]. Large values of m_Q have hence the effect of increasing the Higgs mass. Although larger values of the Higgs mass are welcome to avoid the experimental bound, they necessarily lead to a weakening of the first-order phase transition. Indeed, the running Higgs mass is given by

$$m_H^2 = \lambda v^2, \tag{2.2}$$

and hence, any increase in the Higgs mass is associated with an increase of the quartic coupling λ, yielding lower values of $v(T)/T$. Therefore, very large values of m_Q, above a few TeV, are disfavoured from this point of view.

In the above discussion we have ignored the effect of operators of dimension higher than 4 in the effective potential. In the numerical computations, we include the full one-loop effective potential [19], which goes beyond the approximation of Eq. (1.2) [20]. For consistency, in the numerical evaluations, we neglect the two-loop effects on the Higgs mass. In this case, the Higgs mass expressions obtained in Ref. [20] reduce to the ones presented in Ref. [19], with the only difference that one-loop D-terms have been included in our computation. Observe, however, that the most important zero temperature two-loop contributions can be absorbed in a redefinition of the quartic coupling λ and hence, due to Eqs. (1.4) and (2.2), they will not modify the upper bound on the Higgs mass. The genuine two-loop finite-temperature contributions, instead, have a more relevant effect, making the phase transition more strongly first order (see Ref. [21]). This effect goes beyond the standard model contributions [7] discussed above.

The finite-temperature effects of the heaviest stop are exponentially suppressed and hence we shall ignore them in the discussion below. (They are, however, kept in the numerical evaluations.) The lightest stop, instead, plays an important role and we shall single out its most relevant effects. We have used a finite-temperature expansion for it and checked that the latter does not break down in our region of parameters. The improved one-loop finite temperature effective potential is given by

$$V_{\rm eff}^{\rm MSSM} = -m^2(T)\phi^2 - T\left[E_{\rm SM}\,\phi^3 + (2N_c)\frac{\left(m_{\tilde{t}}^2 + \Pi_R(T)\right)^{3/2}}{12\pi}\right] + \frac{\lambda(T)}{2}\phi^4 + \cdots \tag{2.3}$$

where

$$\Pi_R(T) = \frac{4}{9}g_3^2T^2 + \frac{1}{6}h_t^2\left[1 + \sin^2\beta\left(1 - \tilde{A}_t^2/m_Q^2\right)\right]T^2 + \left(\frac{1}{3} - \frac{1}{18}|\cos 2\beta|\right)g'^2T^2 \tag{2.4}$$

is the finite temperature self-energy contribution to the right-handed squarks (see section 4), g_3 is the strong gauge coupling and $N_c = 3$ is the number of colours. We have

persymmetric partners, in particular charginos, neutralinos and light stops [2]. Observe that, in general, as happens with the longitudinal components of the gauge bosons, the lightest stop contribution to the effective potential does not induce a cubic term. This is mainly related to the fact that the "effective finite-temperature stop mass" is not vanishing in the symmetric phase. At $\phi = 0$, this effective mass is given by

$$\left(m_{\tilde{t}}^{\rm eff}\right)^2 (\phi = 0) = m_U^2 + \Pi_R(T). \tag{2.5}$$

The second term in Eq. (2.5) is positive, and hence a small effective mass can only be obtained through a negative value of the soft supersymmetry-breaking parameter m_U^2.

Negative values of m_U^2 can hence enhance the strength of the phase transition, particularly if they are close to those for which $m_{\tilde{t}}^{\rm eff} = 0$. Indeed, if $m_{\tilde{t}}^{\rm eff}(\phi = 0, T = T_c) \simeq 0$, the strength of the cubic term in the effective potential receives a contribution proportional to the cube of the top quark Yukawa coupling,

$$E \simeq E_{\rm SM} + \frac{h_t^3 \sin^3\beta \left(1 - \tilde{A}_t^2/m_Q^2\right)^{3/2}}{2\pi}, \tag{2.6}$$

where the first term is the Standard Model contribution [$E_{\rm SM} \sim 0.018$]. Observe that, if $m_{\tilde{t}}^{\rm eff} \simeq 0$, the effective cubic term at T_c can be nine times as large as the Standard Model one. Since $v(T_c)/T_c \propto E/\lambda$, an enhancement by a factor 9 of E implies that the allowed Higgs mass values can be enhanced by as much as a factor three. Hence, in the case of zero mixing in the stop sector, $\tilde{A}_t \simeq 0$, and in the absence of additional phenomenological constraints, the bound on the Higgs mass within the MSSM can be of the order of 100 GeV. Large values of $\tilde{A}_t$, instead, reduce the induced cubic term coefficient and, for $\tilde{A}_t \simeq \pm m_Q$, the strength of the first-order phase transition is of the order of the Standard Model one.

For values of the Higgs mass $m_H \lesssim M_Z$ we should be concerned by the validity of the perturbative expansion for the thermal field theory. The usual argument in the Standard Model [7], which yields the condition $m_H^2 \ll M_W^2$, goes as follows. In the Standard Model the strength of the first-order phase transition is mainly dominated by gauge bosons [see Eq. (1.3)]. Therefore, in the region near the symmetry-breaking minimum of the potential (1.2), the value of the field is $\phi \sim g^3 T/\lambda$. Each additional loop of gauge bosons costs a factor of $g^2 T$ and the loop expansion parameter is obtained dividing $g^2 T$ by the leading mass of the problem, i.e. M_W. This gives the loop expansion parameter for the thermal perturbation theory of the Standard Model, as

$$\beta_{\rm SM} \sim \frac{g^2 T}{M_W} \sim \frac{\lambda}{g^2} \sim \frac{m_H^2}{M_W^2}. \tag{2.7}$$

The condition $\beta_{\rm SM} \ll 1$ provides the aforementioned condition on the Higgs mass.

In the MSSM, the strength of the phase transition at one-loop is dominated by the light stops [as can be seen from Eq. (2.6)]. Hence, as far as the phase transition is

[2] We shall work in the case of sufficiently heavy gluinos, right-handed sbottoms and first and second generation squarks, so that their contributions to Π_R are Boltzmann-suppressed.

the top Yukawa and strong gauge couplings. Using now Eqs. (2.3), (2.5) and (2.6) we can see that in the region near the symmetry breaking minimum, $\phi \sim (h_t \sin\beta)^3 T/\lambda$. Additional bosonic loops (on the one-loop light stop diagram) are dominated by the exchange of light stops and Higgs bosons, with an energetic cost $\sim (h_t \sin\beta)^2 T$ and by the exchange of gluons with a cost [3] $\sim g_3^2 T^2$. Since for the experimental range of the top quark mass $h_t \sin\beta \simeq g_3$ and considering the relevant mass of the problem to be $m_{\tilde{t}}^{\rm eff} \sim m_t$, we can write the loop expansion parameter for the thermal theory in the MSSM as

$$\beta_{\rm MSSM} \sim \frac{(h_t \sin\beta)^2\, T}{m_{\tilde{t}}^{\rm eff}} \sim \frac{\lambda}{(h_t \sin\beta)^2} \sim \frac{m_H^2}{m_t^2}. \tag{2.8}$$

In this way, the condition for the validity of the perturbative expansion $\beta_{\rm MSSM} \ll 1$ leads to the bound on the Higgs mass $m_H^2 \ll m_t^2$. In the above we have used Eq. (2.6), which is only valid if $m_{\tilde{t}}^{\rm eff}(\phi = 0)$ is close to zero. For larger values of $m_{\tilde{t}}^{\rm eff}(\phi = 0)$, as those associated with $m_U^2 \geq 0$, there is a significant decrease of the order parameter $v(T_c)/T_c$, with respect to the one used in Eq. (2.8), and hence the relative finite temperature QCD corrections may be larger than what is expected from $\beta_{\rm MSSM}$ in Eq. (2.8). The results of Ref. [21] confirm these expectations.

From the previous (qualitative) arguments one expects that for $m_{\tilde{t}}^{\rm eff}(\phi = 0) \simeq 0$, from the validity of the thermal perturbation theory, the upper bound on the Higgs mass in the MSSM will be softened with respect to that in the Standard Model by a factor $\sim m_t/M_W$ [4]. In the Standard Model, lattice calculations have shown that the electroweak phase transition is well described by perturbation theory for Higgs masses $m_H \lesssim 70$ GeV [8]. Similarly we can expect that in the MSSM, for the choice of supersymmetric parameters rendering the phase transition much stronger than in the Standard Model, the phase transition could be comfortably well described up to Higgs masses $m_H \lesssim M_Z$. Nevertheless, it is clear that a rigorous proof of this statement would require non-perturbative calculations, as previously stated.

In order to get $m_{\tilde{t}}^{\rm eff}(\phi = 0) \simeq 0$, the soft supersymmetry-breaking parameter m_U^2 must take negative values. Since $T_c = \mathcal{O}(100$ GeV) and Π_R is of order T^2, Eq. (2.4), relatively large negative values of m_U^2 must be phenomenologically acceptable. Such negative values of m_U^2 are associated with the presence of charge- and colour-breaking minima [23, 17]. As a conservative requirement, it should demanded that the physical vacuum state have lower energy than the color breaking minima. We shall present an analysis of the bounds obtained through such a requirement in the next section.

3 Colour-Breaking Minima at T = 0

Let us first analyse the case of zero stop mixing. In this case, since $m_Q^2 \gg |m_U|^2$ the only fields that acquire vacuum expectation values are ϕ and U. At zero temperature,

[3] We are considering here the case which strengthens as much as possible the phase transition and, therefore, leads to the largest possible values of the Higgs mass: negligible mixing in the stop sector, i.e. $\widetilde{A}_t/m_Q \simeq 0$, and heavy gluinos [13, 14].

[4] A similar observation was done in Ref. [22].

$$V_{eff}(\phi, U) = -m_\phi^2 \phi^2 + \frac{\lambda}{2}\phi^4 + m_U^2 U^2 + \frac{\widetilde{g}_3^2}{6} U^4 + \widetilde{h}_t^2 \sin^2\beta \phi^2 U^2 \tag{3.1}$$

where λ is the radiatively corrected quartic coupling of the Higgs field, with its corresponding dependence on the top/stop spectrum through the one-loop radiative corrections, $\widetilde{g}_3^2/3$ is the radiatively corrected quartic self-coupling of the field U and $\widetilde{h}_t^2$ is the bi-bilinear $\phi - U$ coupling. The latter couplings are well approximated by $\widetilde{g}_3 \simeq g_3$ and $\widetilde{h}_t \simeq h_t$. For convenience, we shall define

$$\widetilde{m}_U^2 = -m_U^2. \tag{3.2}$$

The minimization of this potential leads to three extremes, at: **(i)** $\phi = 0$, $U \neq 0$; **(ii)** $U = 0$, $\phi \neq 0$ and **(iii)** $\phi \neq 0$, $U \neq 0$. The corresponding expressions for the vacuum fields are:

$$\begin{aligned}
&\textbf{(i)} \quad U = 0, && \phi^2 = \frac{m_\phi^2}{\lambda}; \\
&\textbf{(ii)} \quad \phi = 0, && U^2 = \frac{3\widetilde{m}_U^2}{\widetilde{g}_3^2}; \\
&\textbf{(iii)} \quad \phi^2 = \frac{m_\phi^2 - 3\widetilde{m}_U^2 \widetilde{h}_t^2 \sin^2\beta/\widetilde{g}_3^2}{\lambda - 3\widetilde{h}_t^4 \sin^4\beta/\widetilde{g}_3^2}, && U^2 = \frac{\widetilde{m}_U^2 - m_\phi^2 \widetilde{h}_t^2 \sin^2\beta/\lambda}{\widetilde{g}_3^2/3 - \widetilde{h}_t^4 \sin^4\beta/\lambda}.
\end{aligned} \tag{3.3}$$

It is easy to show that the branch (iii) is continously connected with branches (i) and (ii). It can also shown that the branch (iii) defines a family of saddle-point solutions, the true (local) minima being defined by (i) and (ii). Hence, the requirement of absence of a colour-breaking minimum deeper than the physical one is given by

$$\widetilde{m}_U \leq \left(\frac{m_H^2 \, v^2 \, \widetilde{g}_3^2}{12} \right)^{1/4}. \tag{3.4}$$

For a typical Higgs mass $m_H \simeq 70$ GeV, the bound on $\widetilde{m}_U$ is of order 80 GeV.

In the case of stop mixing, $\widetilde{A}_t \neq 0$, the analysis is more involved, since the three fields Q, U and ϕ may acquire vacuum expectation values. Due to the large hierarchy between m_Q^2 and m_U^2, the vacuum expectation value of Q is always small with respect to that of U, unless the mixing parameter $\widetilde{A}_t$ is of order m_Q. We shall hence define

$$\phi = \alpha \, U, \qquad\qquad Q = \gamma \, U. \tag{3.5}$$

The effective potential is given by

$$\begin{aligned}
V_{eff} &= \left(-\widetilde{m}_U^2 - m_\phi^2 \alpha^2 + m_Q^2 \gamma^2 \right) U^2 + 2 h_t \sin\beta \widetilde{A}_t \alpha \gamma U^3 \\
&+ U^4 \left[\frac{\lambda}{2}\alpha^4 + h_t^2 \sin^2\beta \alpha^2 \left(1 + \gamma^2\right) + h_t^2 \gamma^2 + \frac{g_3^2}{6}\left(1 - \gamma^2\right)^2 \right. \\
&+ \left. \frac{1}{8} g^2 \gamma^2 (\gamma^2 + 2\alpha^2 \cos 2\beta) + \frac{1}{72} g'^2 (\gamma^2 - 4)(\gamma^2 - 4 - 6\alpha^2 \cos 2\beta) \right]
\end{aligned} \tag{3.6}$$

Higgs and squark-squark couplings. Contrary to what happens with the Higgs self-coupling, their tree-level values are proportional to either the strong gauge coupling or the top Yukawa coupling, and hence we expect the radiative corrections to be suppressed by typical one-loop factors. Hence, for the purpose of this paper, it is sufficient to keep their tree-level values. These corrections must be included, however, if a more precise quantitative study of the colour-breaking bounds is desired.

The effective potential can then be written as

$$V_{eff}(U,\gamma,\alpha) = F_1(\alpha,\gamma)\, U^2 + F_2(\alpha,\gamma)\, U^3 + F_3(\alpha,\gamma)\, U^4 \tag{3.7}$$

where the expressions of the functions F_i can be easily obtained from Eq. (3.6). In order to evaluate the depth of the color breaking minima, we shall use the following procedure: We first minimize the potential with respect to U. We find

$$U_{\min} = \frac{-3F_2 \; - \sqrt{9F_2^2 - 32F_1F_3}}{8F_3} \;, \tag{3.8}$$

where we have assumed that $\tilde{A}_t \geq 0$. Inserting this solution into the effective potential, Eq. (3.6), we find

$$V_{\min}(\gamma,\alpha) = U_{\min}^2 \left(\frac{F_1}{3} - U_{\min}^2 \frac{F_3}{3} \right) \tag{3.9}$$

The resulting function of α and γ, Eq. (3.9), may be evaluated numerically. For each value of α, we have performed a scanning over γ, looking for the minimum value of the effective potential, $V_{\min}(\alpha)$. Fig. 1 shows the plot of $V_{\min}(\alpha)$ for $m_Q = 500$ GeV, $m_t = 175$ GeV, $\tan\beta = 1.7$ and different values of $\tilde{A}_t$. The value of the potential at the minimum has been normalized to the absolute value of the potential at the physical expectation value $|V_{EW}|$, so that $V_{\min}/|V_{EW}| = -1$ for $\alpha \to \infty$. Due to the effective potential structure, Eq. (3.6), the $\tilde{A}_t$ effects are only relevant when the three fields U, Q and ϕ acquire a vacuum expectation value. It is easy to show that larger values of $\widetilde{m}_U$ have the effect of inducing lower colour breaking minima for both $\tilde{A}_t = 0$ and $\tilde{A}_t \neq 0$. Hence, in order to obtain a conservative upper bound on $\tilde{A}_t$, we have chosen the (fixed) value of $\widetilde{m}_U$, given by Eq. (3.4), such that the physical minimum (at $\alpha \to \infty$) is degenerate with the colour breaking one at $\alpha = 0$. We have explicitly checked that, as expected, for smaller values of $\widetilde{m}_U$, the upper bound on $\tilde{A}_t/m_Q$ moves to larger values.

For small and moderate values of $\tilde{A}_t$ [$\tilde{A}_t \lesssim 430$ GeV in Fig. 1], the saddle-point structure of the solutions (3.3) with $U \neq 0$ and $\phi \neq 0$ is clearly seen in the figure as a maximum, while the only (degenerate) minima are those at $\alpha = 0,\ \infty$. Hence, so far the condition (3.4) is fulfilled, the physical vacuum, with $U = Q = 0$ is the true vacuum of the theory. This behaviour is preserved for all values of $\tilde{A}_t$ such that the present experimental limit on the lightest stop is fulfilled. Indeed, the upper bound on $\tilde{A}_t$ is very close to the one obtained from the condition of avoiding a tachyon in the spectrum. In particular, for large values of $\tilde{A}_t$ [$\tilde{A}_t \gtrsim 430$ GeV in Fig. 1], a new global minimum with $\phi \neq Q \neq U \neq 0$ does appear, co-existing with the electroweak (local) minimum and the saddle point (maximum). When a tachyonic state appears in the stop spectrum [$\tilde{A}_t \sim 450$ GeV] the electroweak minimum and the saddle point collapse and

in the previous section, Eq. (2.6), large values of $\widetilde{A}_t$, close to the upper bound on this quantity, induce a large suppression of the potential enhancement in the strength of the first-order phase transition through the light top squark; they are hence disfavoured from the point of view of electroweak baryogenesis.

4 Phase Transition Results

As it follows from the discussion in sections 2 and 3, larger values of $\widetilde{m}_U$ can be helpful in inducing a strongly first-order phase transition, but one must be careful about the presence of charge- and colour-breaking minima. Since the question of vacuum stability is a delicate one, in this section we shall adopt the following strategy: we shall in general present results taking into account the vacuum stability constraint, Eq. (3.4). However, the possibility that the physical vacuum is a metastable state with a lifetime larger than the present age of the Universe [24] can also be considered. In this case, the bound Eq. (3.4) would be inappropriate as a phenomenological bound. In this article, we shall not address the question of the vacuum state lifetime in quantitative terms. We shall limit ourselves to also present the results obtained when the bound Eq. (3.4) is ignored in the phenomenological analysis. However, we shall always keep the constraint

$$-\widetilde{m}_U^2 + \Pi_R(T_c) > 0. \tag{4.1}$$

Indeed, if Eq. (4.1) were not fulfilled, the Universe would be driven to a charge- and colour-breaking minimum at $T \geq T_c$. Moreover, since the transition to the color breaking minimum is first order, one should also require the critical temperature for the transition to this minimum, T_c^U, to be below T_c. Because of the strength of the stop coupling to the gluon and squark fields, one should expect this transition to be more strongly first order than the electroweak one.

We shall assume that, as happens at zero temperature, it is sufficient to analyse the behaviour of the potential in the direction $U \neq 0$, $\phi = Q = 0$, to determine the conditions that assure the stability of the physical vacuum at finite temperature. In order to get a quantitative bound on the mass parameter $\widetilde{m}_U$, the effective finite temperature potential for the U field must be analysed. For this purpose, it is useful to compute the particle spectrum in a non-vanishing U-field background. The most relevant masses are:

a) The hypercharge (B) gauge boson with squared mass $8g'^2U^2/9$.

b) Four gluons with squared masses $g_3^2U^2/2$ and one gluon with squared mass $2g_3^2U^2/3$.

c) Five squarks (would-be Goldstones) with squared masses $-\widetilde{m}_U^2 + (g_3^2 + 4g'^2/3)U^2/3$ and one with squared mass $-\widetilde{m}_U^2 + (g_3^2 + 4g'^2/3)U^2$.

d) Four scalar (Higgs-left-handed squark) states with squared masses
$-m_H^2/2 + \left[h_t^2 \sin^2\beta \left(1 - \widetilde{A}_t^2/m_Q^2\right) - |\cos 2\beta| g'^2/3\right] U^2$.

e) Two Dirac fermion states (left quark-Higgsino) with squared masses $\mu^2 + h_t^2U^2$ and two Majorana fermion states (right top-bino) with masses $\sqrt{8g'^2/9U^2 + (M_1/2)^2} \pm M_1/2$.

$$V_U = \left(-\widetilde{m}_U^2 + \gamma_U T^2\right) U^2 - T E_U U^3 + \frac{\lambda_U}{2} U^4, \tag{4.2}$$

where

$$\begin{aligned} \gamma_U &\equiv \frac{\Pi_R(T)}{T^2} \simeq \frac{4g_3^2}{9} + \frac{h_t^2}{6}\left[1 + \sin^2\beta(1 - \tilde{A}_t^2/m_Q^2)\right]; \qquad \lambda_U \simeq \frac{g_3^2}{3} \\ E_U &\simeq \left[\frac{\sqrt{2}g_3^2}{6\pi}\left(1 + \frac{2}{3\sqrt{3}}\right)\right] \\ &+ \left\{\frac{g_3^3}{12\pi}\left(\frac{5}{3\sqrt{3}} + 1\right) + \frac{h_t^3 \sin^3\beta(1 - \tilde{A}_t^2/m_Q^2)^{3/2}}{3\pi}\right\}. \end{aligned} \tag{4.3}$$

The contribution to E_U inside the squared brackets comes from the transverse gluons, E_U^g, while the one inside the curly bracket comes from the squark and Higgs contributions [for simplicity of presentation, we have not written explicitly the small hypercharge contributions to E_U and γ_U.]. In the above, we have ignored the gluino and left-handed squark contributions since they are assumed to be heavy and, as we explained above, their contributions to the finite temperature effective potential is Boltzmann-suppressed.

The difference between T_0^U, the temperature at which $m_{\tilde{t}}^{\rm eff}(\phi = 0) = 0$, and T_c^U, is given by

$$T_c^U = \frac{T_0^U}{\sqrt{1 - E_U^2/(2\lambda_U\gamma_U)}}. \tag{4.4}$$

In order to assure a transition from the $SU(2)_L \times U(1)_Y$ symmetric minimum to the physical one at $T = T_c$, we should replace the condition (4.1) by the condition

$$-\widetilde{m}_U^2 + \Pi_R(T) > \widetilde{m}_U^2 \frac{\epsilon}{1 - \epsilon} \simeq \widetilde{m}_U^2 \epsilon, \tag{4.5}$$

with $\epsilon = E_U^2/2\lambda_U\gamma_U$, a small number. In Eq. (4.3) we have written the value of E_U that would be obtained if the field-independent effective thermal mass terms of the squark and Higgs fields were exactly vanishing at the temperature T. Although for values of $\widetilde{m}_U^2$, which induce a large cubic term in the Higgs potential, T_c is actually close to the temperature at which these masses vanish, an effective screening is always present. In the following, we shall require the stability condition, Eq. (4.5), while using the value of E_U given in Eq. (4.3). We shall also show the result that would be obtained if only the gluon contributions to E_U, E_U^g, would be considered. The difference between the two results quantifies the uncertainty in E_U due to the fact that the effective thermal masses of the squark and the Higgs fields are actually partially screened at T_c.

Let us first present the results for zero mixing. Fig. 2 shows the order parameter $v(T_c)/T_c$ for the phase transition as a function of the running light stop mass, for $\tan\beta = 2$, $m_Q = 500$ GeV and $m_t = 175$ GeV. For these parameters, the Higgs mass $m_H \simeq 70$ GeV, a result that depends weakly on $\widetilde{m}_U$. We see that for smaller (larger) values of $m_{\tilde{t}}$ ($\widetilde{m}_U$), $v(T_c)/T_c$ increases in accordance with the discussion of section 2. We have marked with a diamond the lower bound on the stop mass coming from the

bounds that would be obtained by requiring condition (4.5), while using the total and gluon-induced trilinear coefficients, E_U and E_U^g, respectively. We see that the light stop effect is maximum for values of $\widetilde{m}_U^2$ such that condition (4.5) is saturated, which leads to values of $m_{\tilde{t}} \simeq 140$ GeV ($\widetilde{m}_U \simeq 90$ GeV) and $v(T_c)/T_c \simeq 1.75$. To preserve Eq. (3.4) demands slightly larger stop mass values. Still, there is a large region of parameter space for which $v(T_c)/T_c \geq 1$ and is not in conflict with any phenomenological constraint.

Figure 3 shows the results for zero mixing and $m_Q = 500$ GeV as a function of $\tan\beta$ and for the values of $\widetilde{m}_U$ such that the maximum effect on $v(T_c)/T_c$ is achieved. We also plot in this figure the corresponding values of the stop and Higgs masses. As in Fig. 2, the solid [dashed] line represents the result when the bound (3.4) [the stability bound of Eq. (4.5)] is preserved. We see that $v(T_c)/T_c$ increases for lower values of $\tan\beta$, a change mainly associated with the decreasing value of the Higgs mass or, equivalently, of the Higgs self-coupling. For values of $\tan\beta \simeq 2.7$, $v(T_c)/T_c \simeq 1$, and hence the value of the Higgs mass yields the upper bound consistent with electroweak baryogenesis. This bound is approximately given by $m_H \simeq 80$ GeV. If the bound on color breaking minima, Eq. (3.4), is ignored, the upper bound on m_H is close to 100 GeV, in accordance with our qualitative discussion of section 1.

Due to the logarithmic dependence of m_H on m_Q, larger values of m_Q have the effect of enhancing the Higgs mass values. It turns out that, for zero mixing, the results for $v(T_c)/T_c$ depend on the Higgs mass and on the value of m_U, but not on the specific value of m_Q. Hence, different values of m_Q have the only effect of shifting (up or down) the preferred values of $\tan\beta$. In particular, the fixed-point solution, which corresponds to values of $\tan\beta \simeq 1.6$ for $m_t \simeq 175$ GeV, leads to values of $m_H \geq 65$ GeV and $v(T_c)/T_c \gtrsim 1$ so far m_Q is above 750 GeV and below a few TeV.

Finally, let us discuss the effect of mixing in the stop sector. For fixed values of m_Q and $\tan\beta$, increasing the values of $\tilde{A}_t$ has a negative effect on the strength of the first-order phase transition for three reasons. First, large values of $\tilde{A}_t$ lead to larger values of the Higgs mass m_H. Secondly, as shown in Eq. (2.6) they suppress the stop enhancement of the cubic term. Finally, there is an indirect effect associated with the constraints on the allowed values for $\widetilde{m}_U$. This has to do with the fact that for larger values of $\tilde{A}_t$, the phase transition temperature increases, rendering more difficult an effective suppression of the effective mass $m_{\tilde{t}}^{\rm eff}$, Eq. (2.5). Of course, this third reason is absent if the bound (3.4) is ignored. As we have shown above, for zero mixing the bounds (1.1), (3.4) and (4.5) are only fulfilled for values of the stop mass larger than approximately 140 GeV. Light stops, with masses $m_{\tilde{t}} \lesssim 100$ GeV, can only be consistent with these constraints for larger values of the mixing mass parameter $\tilde{A}_t$. This can be relevant for physical processes, which demand the presence of such light sparticles in the spectrum. For instance, it is important in getting corrections to R_b [15, 17, 25].

Figure 4 shows the result for $v(T_c)/T_c$ as a function of $\tilde{A}_t$ for $\tan\beta = 1.7$, $m_Q = 500$ GeV, and values of m_U such that the maximal light stop effect is achieved. The same conventions as in Fig. 3 have been used. Due to the constraints on $\widetilde{m}_U$, light stops with $m_{\tilde{t}} \lesssim M_W$ may only be obtained for values of $\tilde{A}_t \gtrsim 0.6\, m_Q$. For these values of $\tilde{A}_t$, however, the phase-transition temperature is large and induces large values of $m_{\tilde{t}}^{\rm eff}$, for all values of $\widetilde{m}_U$ allowed by Eq. (3.4). In Fig. 4, we have chosen the parameters such

$v(T_c)/T_c \geq 1$ and the Higgs mass bound. As we mentioned above, lower values of $\tan\beta$ can be achieved for larger values of m_Q. Since the stop spectrum depends only slightly on $\tan\beta$, we obtain that, as far as the bounds on color breaking minima are preserved, the mixing effects are not very helpful to obtain lower stop masses compatible with a sufficiently strong first order phase transition. If the weaker bound, Eq. (4.5), were required (thin and thick dashed lines in Fig. 4), light stops, with masses of order M_Z would not be in conflict with electroweak baryogenesis.

5 Conclusions

In this article we show that, contrary to what was suggested by previous analyses, there are large regions of phenomenologically acceptable parameter space, that are consistent with the present experimental bound on the Higgs mass and with a sufficiently strong electroweak first-order phase transition, Eq. (1.1). This region of parameter space is associated with low values of $\tan\beta$, low values of the lightest stop mass, $m_{\tilde{t}} \lesssim m_t$, and low values of the Higgs mass, $m_H \lesssim M_Z$. It can hence be tested by experimental Higgs and stop searches at the Tevatron and LEP2 colliders. Interestingly enough, this region is also consistent with the unification of the bottom and τ Yukawa couplings at the grand unification scale, and consequently with the quasi-infrared fixed point solution for the top quark mass. The hierarchy of soft supersymmetry breaking parameters $m_Q^2 \gg m_U^2$ is naturally obtained at the fixed-point if supersymmetry is broken in a hidden sector. Furthermore, as has been discussed in Ref. [17], negative values of m_U^2 are associated with non-universal boundary conditions for the scalar soft supersymmetry-breaking terms at the scale M_{GUT}. For these values of m_U^2, the bounds on the colour breaking minima are decisive in defining the allowed parameter space and, for a top quark mass $m_t \simeq 175$ GeV, light stop masses below 130 GeV turn to be disfavoured. If these constraints are ignored, while assuming that we live in a metastable vacuum, light stops, with masses of the order of the Z^0 mass become consistent with a strongly first-order electroweak phase transition.

Three additional remarks are in order: i) First, we have always considered the case of very large m_Q. A stronger first-order phase transition may be obtained by considering values of m_Q such that the left handed stop finite temperature contribution is not negligible. Since the Higgs mass value may be controlled through $\tan\beta$, the strongest bounds on m_Q come from preserving a good fit to the electroweak precision measurements. As we discussed above, for the experimentally preferred values of the top quark mass, large values of m_Q are preferred. ii) We have only analysed the case of large values of the CP-odd Higgs mass. For the CP-violating effects associated with the supersymmetric particles to lead to an efficient baryon generation at the electroweak phase transition, the ratio of vacuum expectation values must change along the bubble walls [10]. This in turn means that the CP-odd mass cannot be much larger than the critical temperature. It is difficult to derive a quantitative upper bound on the CP-odd Higgs mass from these considerations. However, since there is no significant change of the order parameter $v(T_c)/T_c$ up to CP-odd Higgs masses as low as $\simeq 2T_c$, we do

found in the present analysis. iii) Throughout this paper we have ignored higher-loop corrections. These corrections tend to make the phase transition more strongly first order [21] and enlarge the allowed parameter space. A non-perturbative study will be useful to check the validity of the perturbative bounds on the MSSM parameters consistent with electroweak baryogenesis.

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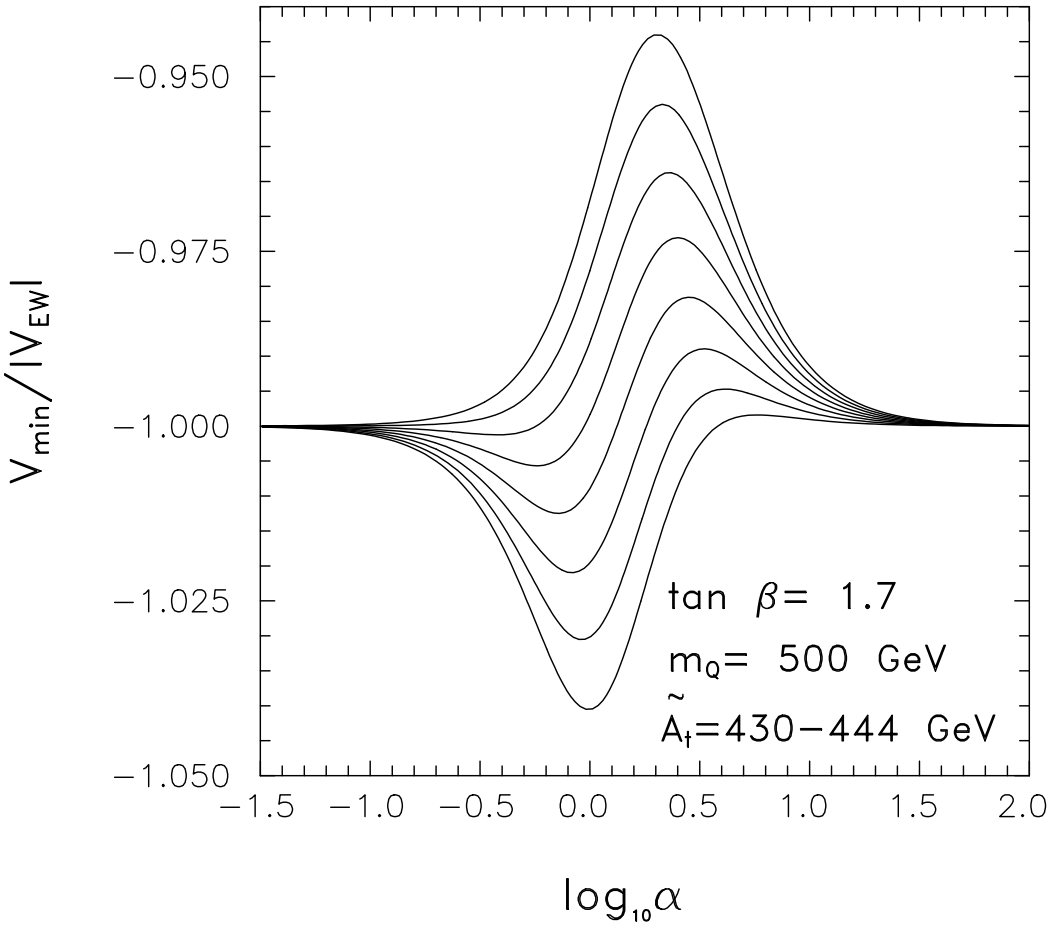

Figure 1: Plot of $V_{\rm min}/|V_{EW}|$ for m_t = 175 GeV, m_Q = 500 GeV, $\tan\beta$ = 1.7 and $\tilde{A}_t$=430 GeV [upper curve]–444 GeV [lower curve], step=2 GeV.

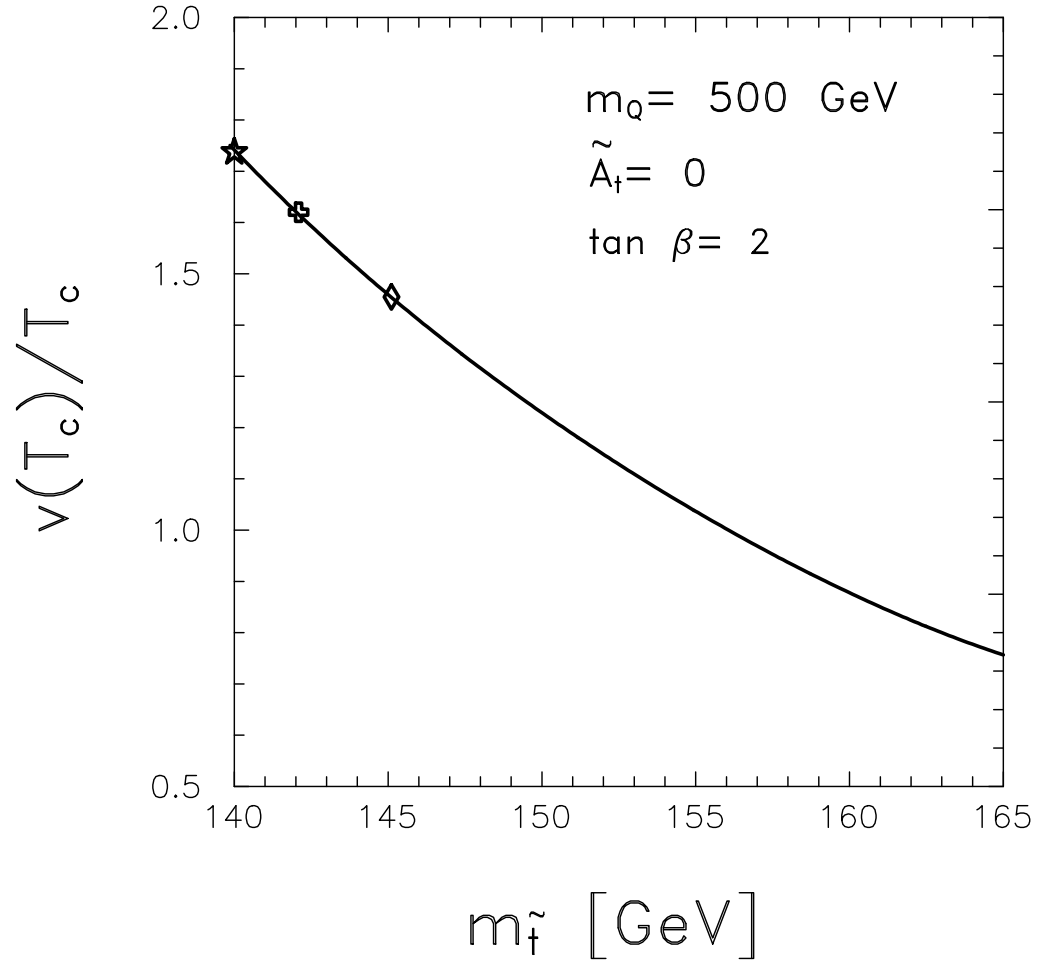

Figure 2: Plot of $v(T_c)/T_c$ as a function of $m_{\tilde{t}}$ for m_Q and m_t as in Fig. 1, $\tilde{A}_t = 0$ and $\tan\beta = 2$. The diamond [cross, star] denotes the value of $\widetilde{m}_U$ for which the bound, Eq. (3.4) [Eq. (4.5) with E_U given by Eq. (4.3), Eq. (4.5) with $E_U = E_U^g$] is saturated.

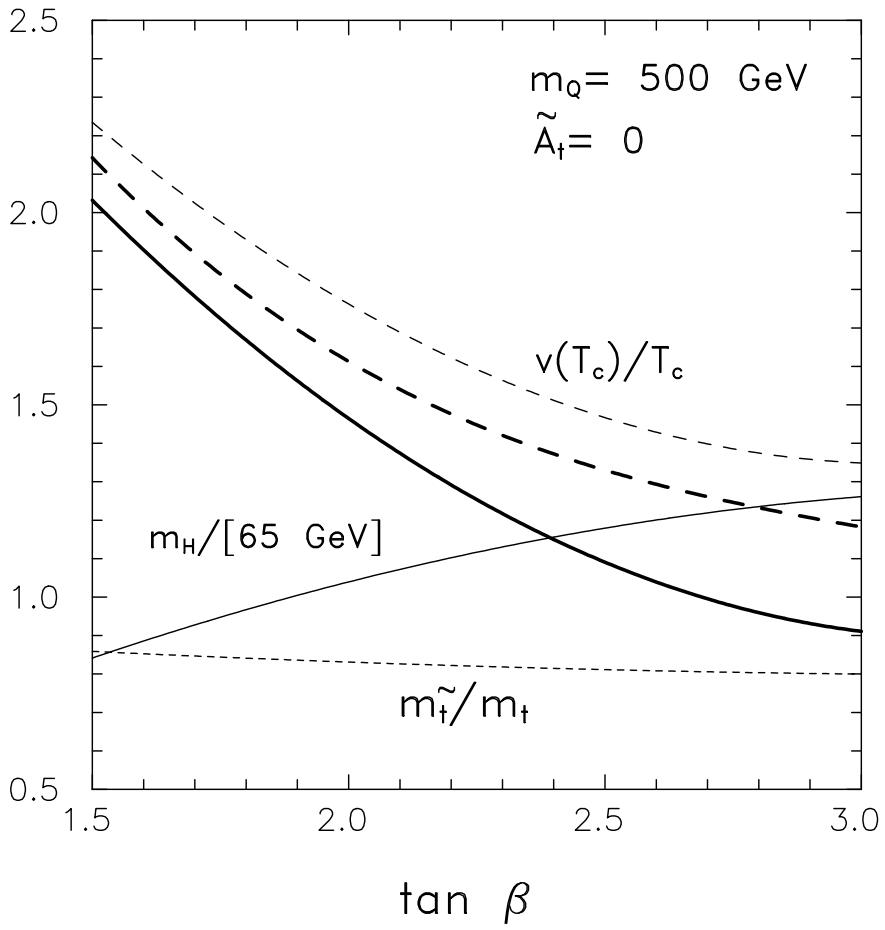

Figure 3: Plot of $v(T_c)/T_c$ as functions of $\tan\beta$ for m_Q and $\tilde{A}_t$ as in Fig. 2, and m_U saturating Eq. (3.4) [solid] and Eq. (4.5) [thick dashed line for E_U given by Eq. (4.3) and thin dashed line for $E_U = E_U^g$]. The additional thin lines are plots of m_H in units of 65 GeV [solid] and $m_{\tilde{t}}$ in units of m_t [short-dashed], corresponding to the values of $\widetilde{m}_U$ associated with the solid line.

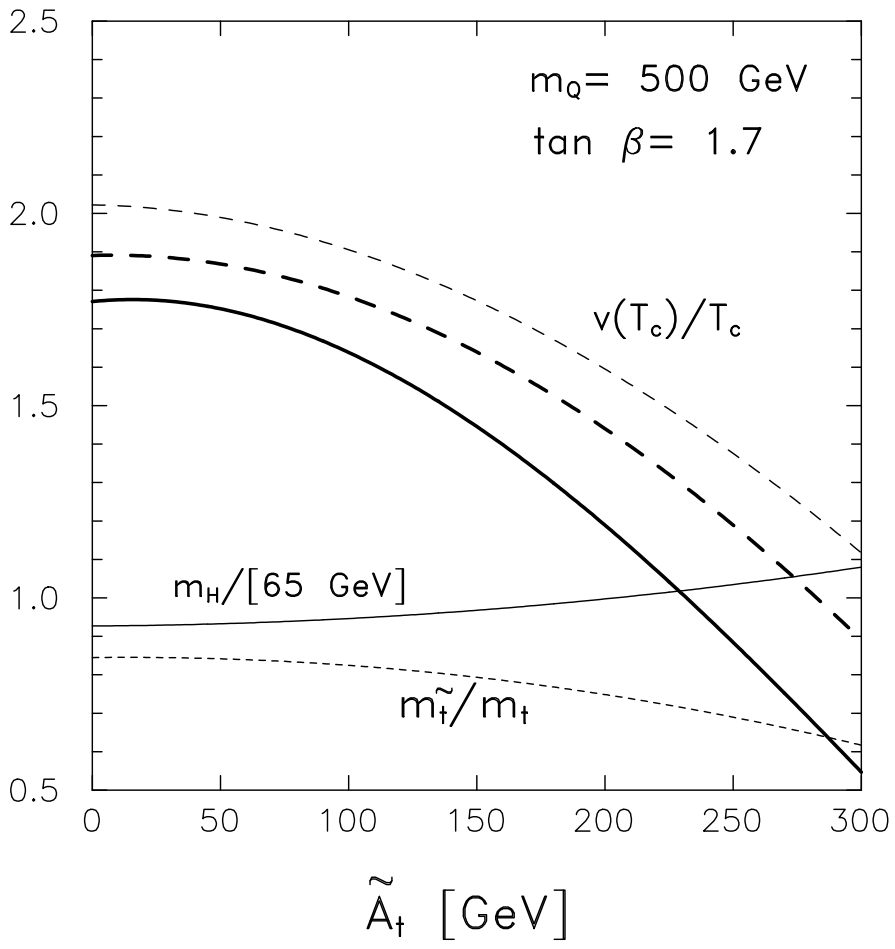

Figure 4: The same as in Fig. 3, but as functions of $\tilde{A}_t$, for $\tan\beta = 1.7$.