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QUADRUPOLE DESIGN FOR HIGH ACCEPTANCE

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1. Introduction

When more than one beam emerges from an internal or external target, the width of the first quadrupole in the beam often limits acceptance. Special high power magnets, their parameters specifically chosen to improve acceptance, may either provide an increase in the particle flux available in a given beam, or allow more beams of useful intensity to be exploited from the same target.

The relation between the optical parameters of a doublet and its acceptance have been analysed using thin lens theory by Geibel and Auberson (1), and, using thick lens theory by King (2). A new kind of quadrupole, small in the horizontal dimension, (the "Figure of Eight" quadrupole) has been described by the author (3). In this report the relative merits of this and other types of design are compared using King's theory. As a basis for comparison the situation is considered where the quadrupoles are used in a doublet arrangement, turning the beam parallel.

In addition to this comparison the more general problem of the choice of aperture, length and current density of quadrupoles and its effect on the acceptance of doublets is discussed. The choice of quadrupole parameters for maximum acceptance depends on the momentum of the beam and the angle subtended at the target by the space available for transport magnets. While it is impractical to provide special magnets for each beam, a compromise design is possible which leads to an improvement over the present standard elements in most cases.

It is found that a "Figure of Eight" design (Fig. 1) is capable of larger acceptances than other designs. This is true even though, because the "Figure of Eight" design is not capable of as high a field gradient as other designs, it must be longer to achieve the same focussing power.

Parameters of a "Figure of Eight" quadrupole of high power consumption and appropriate to the present needs of CERN are mentioned (they are set out in detail in another note)[4]. It is suggested that such a quadrupole might allow acceptances to be increased by factors 2 - 3 and at the same time allow 60 o/o more beams to be taken from the same target.

2. Doublet Design for Maximum Acceptance

The calculations of acceptance in the following sections are based on the curves calculated by King for a quadrupole doublet in which each magnet is treated as a thick lens. Only doublets will be considered which turn a beam, diverging from a target, into a parallel beam. The most relevent of these curves are reproduced in this report (Figs. 2, 3, 4). It should be noted that acceptance Ω is defined as $\alpha_H \alpha_V$, the product of the semi-angles in the horizontal and vertical plane at the target. It can be seen from Fig. 4 that acceptance may be gained by reducing z, the distance of the first quadrupole from the target. Acceptance may finally be limited in one of two ways :

a) Space Limited Case

The physical width of the magnet may restrict z/d to a minimum value. This situation is shown in Fig. 5. Two adjacent beams coming from the same target are shown. There may of course be other beams adjacent to these, as for instance in the South Hall. z is limited by the condition that the outer edge of the magnets must not cross the lines of distinction, AB, AB', with the adjacent beams. In certain practical cases z may be limited by other physical obstacles. For instance if the target is an internal one its virtual position as seen through the fringe field of the PS may be a long way inside the machine and z may then be limited by the physical obstacle of the machine itself. Alternatively if a bending magnet lies between the doublet and either external or internal target, this may limit z . However, these limitations are peculiar to specific beams and not within the scope of the present study.

From Fig. 5 it can be seen that :

$$z/d = w/2d \tan \Psi \tag{1}$$

where

d = magnetic effective length of quadrupole,

w = width of quadrupole,

Ψ = angle between axis of beam and the line of symmetry with adjacent beam. (Note that Ψ is not the production angle which may be chosen independently of Ψ).

The maximum acceptance is then achieved at the point A in Fig. 6(a). The two magnets are placed as close together as possible ($x = x_{min}$ where x_{min} is the distance between the hard edged equivalents of the two quadrupoles when the magnets are physically touching).

b) Strength Limited Case

$$\theta_1 = d\ (ke/p)^{1/2} \tag{2}$$

where

k = field gradient of the magnet,

p = momentum of the beam.

If the lenses are weak the value of θ_1 given by (2) may be insufficient, even when the first magnet is at its maximum gradient, to allow the doublet to be placed as close to the target as the limit defined by (a). In this case the maximum acceptance is achieved at point B in Figure 6(b). x is now greater than its minimum value.

It is possible that both limitations may operate simultaneously (point C in Fig.6 (c).

Given complete freedom of choice in the length d, the situation to be sought after for optimisation of acceptance is that θ_1 should be just sufficient at maximum field for the doublet to be both strength and space limited and that x/d should equal zero. Such a doublet will be referred to as the optimum doublet and corresponds to point D in Figure 6(d).

The variation of acceptance Ω on either side of an optimum situation is shown in Figure 7.

3. Quadrupoles of High Current Density and Small Overall Width

The overall width, volume and cost of a quadrupole magnet depends, not only on its length and aperture, but also on the current density in its coil. In principle it is possible to design a quadrupole of given length and aperture with any desired overall width. However, increasing the current density, although reducing the overall width and cost of the magnet, implies a higher power consumption.

Almost all existing quadrupoles at various laboratories have been designed with modest power consumption and a variety of current densities, often determined by the available power supplies. In comparing the acceptance of different types of design, it is useful to consider magnets of the same power consumption, operating in equally favourable situations. Accordingly some simple scaling relations (Appendix I) have been applied to the three basic quadrupole designs studied in a previous report [3] and also to the Asner Design [5]. The overall width w, maximum strength θ, and minimum x/d for a wide range of lengths and apertures of quadrupole of each design type are considered. All the quadrupoles have the same power consumption and current rating, 330 kW, 860 Amps, (that of the most powerful generator used at CERN). They all have 10 water circuits per pole and a cooling water pressure drop of 10 atmospheres. The maximum temperature in their windings is 50° C above ambient.

These parameters would not normally be chosen for a large number of general purpose magnets but they could be justified, on the grounds of reduction of overall width, for a few magnets available for particular applications.

A large number of cooling circuits, high water pressure and temperature rise, contribute to a reduction in overall width without affecting power consumption.

It is realised that the choice of these particular parameters must necessarily limit the generality of the study, but the qualitative conclusions may be widely applicable.

4. The Optimum Quadrupole

The three quadrupole designs, "Figure of Eight", "Conventional" and "DESY", described in reference 3 and the Asner design described in reference 5 can be scaled using the relations deduced in Appendix I. It is possible to calculate the following functions :

$$\left.\begin{aligned} w &= w\,(L,\ a,\ J) \\ d &= d\,(L,\ a,\ J) \\ x_{min} &= x\,(L,\ a,\ J) \end{aligned}\right\} \quad (3)$$

where

w	=	width of quadrupole,
d	=	effective magnetic length,
L	=	physical length of quadrupole poles,
a	=	aperture radius,
x_{min}	=	distance of closest approach of the hard edged equivalent quadrupole to its nearest neighbour,
J	=	current density

However, by making the restriction that the quadrupole should have a particular power consumption we remove one of the degrees of freedom. In fact it is convenient to remove the variable J from the expressions above. The next step is simply to transform the physical parameters given by these expressions into the optical parameters of the doublet.

The optical parameters required are :

$$\left.\begin{aligned} z/d &= z/d\ (L,\ a)_{\psi} \\ \theta &= \theta\ (L,\ a)_{p} \\ x/d &= x/d\ (L,\ a) \\ (a/d)^2 &= (a/d)^2\ (L,\ a) \end{aligned}\right\} \quad (4)$$

where

$$\begin{aligned} p &= \text{particle momentum} \\ \psi &= \text{angle defined in section 2(a)} \end{aligned}$$

z/d is calculated for various values of ψ using expression (1) and θ_1 , for different momenta using expression (2). (x/d) and $(a/d)^2$ are independent of p and ψ . These quantities have been computed for each type of quadrupole and examples are plotted in Figs. 8 to 11 for a 10 cm aperture radius "Figure of Eight" quadrupole.

Looking at a particular combination of ψ , a, p, and quadrupole type and varying L, the quantities z/d, θ, and x/d all vary. By a process of iteration a value of L whose combination of z/d, θ_1 and z/d corresponds to an optimum doublet may be found. The acceptance Ω may then be read from King's curves (Fig. 2 - 4). So, for a given aperture and quadrupole type it is possible to find a unique length of quadrupole which gives maximum acceptance in a given beam situation defined by p and ψ. This has been done for many different values of p and ψ and the results are plotted in figures 12 - 19.

It is appropriate at this point to give a qualitative interpretation of these results. Suppose the optimum length of quadrupole is known for a given angle Ψ and momentum p, and also for a given type of quadrupole of aperture a. If this magnet were to be used at a higher p, its strength θ would be insufficicient to allow the design of an optimum acceptance doublet with its members in contact. To do so the length L would have to be increased and, in order to preserve the same power consumption, so would w. The ratio z/d changes to a new, and in practice lower value according to expression (1). This new value of z/d requires a larger θ, requiring a further increase in L, and hence a higher w and d. However, the process is convergent and eventually a value of L is found where z/d, θ_1 , and x/d are that of an optimum acceptance design. This is in fact the process of iteration used to compute the optimum value of L as a function of Ψ and p and it explains qualitatively why longer magnets are required to give maximum acceptance at higher momenta.

During this iteration the acceptance of the doublet changes. Since, at the higher momentum z/d is smaller, a higher point on Figure 4 is reached, i.e. $\Omega/(a/d)^2$ is larger. But it turns out that this is more than offset by the reduction in the value $(a/d)^2$ and acceptance decreases with increasing momentum (Fig. 12). This is to be expected because doublets if they are to be longer at high momentum must be wider and therefore cannot be placed as close to the target.

Similarly, a reduction in Ψ causes an increase in z/d, and if the optimum condition is to be preserved L, d, and hence θ must be reduced. This too leads to a convergent iteration and it is found that the acceptances of doublets which are optimum for smaller values of Ψ is smaller. The length of magnet required is also smaller. It is interesting to note that the value of Ω/Ψ is roughly constant for a given type of quadrupole and so the acceptance is roughly proportional to the angle occupied by the beam. (Figs. 16 to 19).

It is important to note that the optimum value of L depends strongly on the values of Ψ and p.

5. Comparison Between Different Kinds of Quadrupole

The comparison is made on the basis that the limiting aperture of all types is simply the circle which passes through the pole tips. In the case of the "Conventional" and "Figure of Eight" designs the region of good field ($\Delta B/B \leqslant 0.001$) extends to less than this radius, usually to $r = 0.8a$. It is claimed that the "DESY" and "Asner" designs have good field regions extending to $r = 1.3a$ in the region between adjacent poles. However, the following points should be borne in mind :

a) If significant use is to be made of that part of the good field which lies outside the region $r = a$, the ratio of beam width to height must be large ($\simeq 2$ to 3) at its point of maximum excursion from the origin. This is not the case in a maximum acceptance doublet where the elements are touching (Fig. 20).

b) In estimating the region of good field in the "Asner" quadrupole the criterion applied was $\Delta B/B \leqslant 0.01$, which is insufficient for accurate optics.

c) Until prototypes of the "DESY" and "Asner" designs have been made and measured the inhomogeneities in the end field are still unknown.

By assuming that all the quadrupoles have the same nominal aperture, the errors introduced should be less than the differences in acceptance which are found to occur. The calculated acceptance need only be scaled according to r^2 if different regions of good field are assumed.

Comparison between the optimum acceptance of the four kinds of quadrupole is presented graphically in Figs. 12, 13, 17 and 19 for two values of ψ, ($\psi = 50$ and 150 mr) over a range of momenta; and again for $p = 4$ and 10 GeV/c over a range of ψ. It can be seen that if $p \leqslant 10$ and / or $\psi \leqslant 100$ mr the value of Ω for the "Figure of Eight" design is about twice that for the "Conventional" design which in turn is larger than that for the "Asner" and "DESY" designs.

The points marked "CERN 1 m" on Figs. 14 and 15 are the acceptance values for this magnet working at optimum combinations of Ψ and p. This indicates that the new quadrupoles gain a factor 2 in Ω by using 330 kW instead of 100 kW.

It will be argued below that for quite independent reasons a high acceptance quadrupole would be most useful at Ψ = 50 mr. and p = 4 GeV/c and under these conditions the "Figure of Eight" quadrupole gives considerably larger acceptances than the other designs considered.

6. The Choice of Aperture

In examining the variation of acceptance with aperture, qualitative arguments similar to those of section 4 are inconclusive. As a is increased for fixed Ψ and p the length L must be extended since the field gradient k varies as 1/a. This forces a reduction in z/d on account of the increase in d but the associated increase in w may cause the optimum value of z/d at the new aperture to be either less or more than before. It is not clear therefore whether $\Omega/(a/d)^2$ is improved. However, the effects may be examined quantitatively, and typical results are presented in Figures 14 and 15.

The optimum acceptances of 15 cm and 10 cm aperture magnets appear to be not significantly different, and it would seem that acceptance varies only slowly with aperture. At CERN, the large quantity of existing beam handling components tailored to the 10 cm aperture 1 m and 2 m quadrupoles suggests that new magnets should be also of this aperture. However, in planning equipment for a completely new laboratory the negligable variation of acceptance with aperture might lead one to small aperture systems. Limitations would be the widths of beams emerging from the machine, the tolerances on the construction of components which reduction of size implies and the difficulties of cooling small magnets of high power dissipation. Advantages would be much cheaper components and shorter paths for decay when unstable particles are being used.

A complete beam handling system based on say 5 cm aperture magnets may be an attractive future proposition.

7. The Choice of Optimum Length

As has been mentioned above, this must depend on the values of Ψ and p considered. At present at CERN values of Ψ for beams in the South Hall vary from 30 to 120 mr. with an average of 80 mr. If present beam intensities are acceptable it may be argued that the value of Ψ should be chosen to give the same acceptances with new magnets. In this way a Ψ of less than 30 mr. could be used and 3 times the number of beams might be accommodated.

However, the prospect of 3 times as many experiments sharing the same target may be impractical; moreover, bending magnets and other existing components would limit Ψ seriously in this situation.

It has already been mentioned that the ratio Ω/Ψ is roughly constant for a given type of quadrupole and so a more modest reduction in Ψ would afford propo·tionaly higher beam intensities. It would seem that a reasonable compromise would be to assume Ψ = 50 mr. as a representative requirement. This would allow room for about 60 o/o more beams ; each with an acceptance 2 or 3 times that at present possible with 1 m magnets operating under optimum conditions.

The momentum requirements are less easy to define. It might be thought reasonable to choose 10 GeV/c as the representative momentum. With Ψ = 50 mr. this would require a "Figure of Eight" design of length, L = 1·5 m. The existing 1 m and 2 m magnets provide optimum doublet conditions at 10 GeV/c and above, although, because of their low power consumption, the acceptances achieved are lower than could be provided by 330 kW "Figure of Eight" magnets. The need seems to be more for a magnet which can give high acceptances in the region 0 to 8 GeV/c. (The wide 50 cm quadrupoles now on order cannot be said to cover this requirement). A "Figure of Eight" quadrupole, optimised in length to Ψ = 50 mr. and p = 4 GeV/c is therefore considered. It would also have the advantage that two such units could be placed one behind the other to form the first element of a doublet if a higher momentum, high acceptance beam channel was urgently required.

Summarising, the parameters listed in Table 1 would seem a reasonable choice for a new quadrupole. In Fig. 21 the variation of acceptance of such a magnet is compared with the CERN 1 m and 2 m magnets over a wide momentum range.

REFERENCES

1. J.A. Geibel and G. Auberson — Note sur l'acceptance angulaire d'un doublet. CERN MPS/Int./DL 62-10

2. N.M. King — Some focussing properties of quadrupole doublets. CERN MPS/EP/22.

3. A. Asner and E.J.N. Wilson — Different types of 30 cm aperture quadrupole lenses. CERN ENG/Int./ EE 62-20

4. E.J.N. Wilson — A quadrupole magnet of large acceptance. CERN MPS/EP/29.

5. A. Asner — Quadrupoles with a 40 x 10 cm^2 field region. CERN ENG/Int./DL 61-16

TABLE 1

PARAMETERS OF RECOMMENDED FIGURE OF EIGHT QUADRUPOLE

Aperture (a)		=	10 cm Radius
Pole length (L)		=	73 cm
Power Consumption (W)		=	330 kW
Max Current (I)		=	860 Amps
Weight (approximate)		=	2 tons
Field Gradient (k)		=	10 web/m^3 (1010 gauss/cm)
Magnetic length (d)		=	83 cm
Conductor dimensions	(D)	=	7.4 mm
	(Dm)	=	3.72 mm
	(α)	=	21.5 mm^2
Overall width (w)		=	41.8 cm
Current Density (J)*		=	15.8 A/mm^2
Distance of approach in doublet (hard edged) (x)		=	12.6 cm
Optimum momentum (p)		=	4 GeV/c
Optimum Ψ		=	50 mradians

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* $J = I/D^2$

T A B L E 2

PARAMETERS OF SOURCE DESIGNS

PARAMETER	FIGURE OF 8	CONVENTIONAL	ASNER	DESY
a (cm)	15	15	15	15
L (cm)	122.7	105.7	90.0	91.9
k (w/m^3)	0.675	0.810	0.900	0.950
Δp (π)	10	10	10	10
ΔT (^oC)	50	50	50	50
W (kW)	330	330	330	330
I (A)	860	860	860	860
R (Ω)	0.435	0.435	0.435	0.435
n	10	10	10	10
α (mm^2)	45.3	49.65	55.4	60.25
Dm (mm)	5.03	4.57	4.6	4.58
D (mm)	9.84	9.84	10.14	10.46
N	84	101	105	120
n	12	15	11	12
$\bar{R}$ (cm)	15.9	16.6	22.0	23.1($4 r_o$)
R_{ext} (cm)	20.6	22.6	33.0	--
w(a) (cm)	40.0	64.6	68.0	57.4
w(c) (cm)	25.0	39.4	67.0	91.7
w(c) + w(a)	65.0	104.0	135.0	149.1

APPENDIX I

The Scaling Functions for Quadrupoles

It is necessary to compute the properties of four families of quadrupole. Each family is of a different design type and all the quadrupoles have certain fixed parameters in common, namely :

Δp	=	water pressure (atmospheres)	=	10
ΔT	=	temperature rise (^{o}C)	=	50
W	=	power consumption (kW)	=	330
I	=	maximum current (amps)	=	860
R	=	resistance at 20^{o}C (ohms)	=	0·435
n	=	number of water circuits/pole	=	10

The family is generated by varying the parameter α (cross section of copper in a single turn of conductor (mm^2)). As α is increased the length and overall width of the magnet increase ; the quadrupole becomes magnetically stronger but, in order to have the same power consumption, must be more bulky.

The parameters of each family are calculated by scaling four designs, one of each type, whose parameters are given in Table II. These will be referred to as the "Source Designs" and their parameters will be distinguished by the suffix "o".

The source designs are not identical with those given in Reference 3. Small modifications have been made so that all four source designs have the same water pressure, number of water circuits, peak current and resistance. Also, since the publication of Reference 3, more exact estimates of the maximum field gradient of each design have become available as a result of Asner's measurements with a conducting sheet analogue.

The notation used is summarised below and in Fig. 22 :

a	=	aperture radius (cm)
L	=	length of the poles of the magnet defined in a particular way for the DESY design (cm) (Fig. 22)
k	=	maximum field gradient (Webers/ m^3)
α	=	cross section of copper in a single turn of conductor (mm^2)
Dm	=	hydraulic diameter of the cooling hole (mm)
	=	4 x area/perimeter
D	=	side of square occupied by a single conductor with insulation and an allowance for winding build up (mm)
N	=	number of turns/pole
n	=	number of turns in longest water circuit
$\bar{R}$	=	mean radius of bend of the conductor at the end of the quadrupole (cm) (Fig. 22)
Δr	=	equivalent of $\bar{R}$ for"DESY"
R_{ext}	=	radius of bend of outermost conductor at the end of the quadrupole (cm) (Fig. 22)
w(a)	=	length of that portion of the horizontal width which scales linearly with aperture (cm)
w(c)	=	(total horizontal width) - w(a)
G	=	flow of cooling water in the longest water circuit (ℓ/min)
ℓ	=	mean length of one turn of conductor (m)

In deriving the scaling functions (3) which describe the family the first step is to find the relation between D, Dm and α.

We use the empirical relations :

$$I^2 n \ell \rho \quad = \quad 0{\cdot}007 \Delta T G \alpha \tag{5}$$

$$G = 0{\cdot}133 \; (\Delta p / n \ell)^{0{\cdot}57} \; Dm^{2{\cdot}71} \tag{6}$$

where

$$\rho \quad = \quad \text{conductivity of copper} \quad = \quad 1{\cdot}9 \mu\Omega \, \text{cm.}$$

(it is assumed that the mean temperature of the copper is 25°C above ambient)

Also

$$R \quad = \quad \frac{400 \; N \ell \rho}{\alpha} \quad \text{ohms} \tag{7}$$

Combining (5), (6) and (7) by eliminating G and ℓ we have, when the appropriate fixed parameters are inserted :

$$Dm \quad = \quad 6{\cdot}06 \quad \alpha^{0{\cdot}21} \; (n/N)^{0{\cdot}58} \tag{8}$$

it is convenient to write

$N/n \; = \; W$. This would be the number of water circuits per coil if all were as long as the longest. W should be constant for a particular design. So :

$$Dm \quad = \quad 6{\cdot}06 \; \alpha^{0{\cdot}21} / \; (W_o)^{0{\cdot}58} \tag{9}$$

Dm is a simple function of α for each type of design.

Having fixed α and determined Dm, D may be calculated. Making an allowance of 2 o/o for build-up of the coil during winding and adding a further 1·25 mm to the width of each conductor for insulation :

$$D = 1{\cdot}27 + 1{\cdot}02 \, (\alpha + Dm_o{}^2) \tag{10}$$

And the current density :

$$J = I/D^2 \tag{11}$$

It is interesting to note that Dm, D, and J are independent of aperture for a given α.

The next step in deriving the scaling relations is to calculate from the values of D determined by α the length, magnetic length and overall width of the quadrupole. The way in which this is done depends on the type of quadrupole.

a) Conventional, Figure of Eight, Asner

We assume an idealized model for the quadrupole coil (Fig. 22). The length of the straight portion of the coil is assumed to be extended by 0·5 a at each end to accommodate cylindrical end shims. The ends of the coil are semi-circular so that :

$$\ell = 2\pi\bar{R} + 2a + 2L$$

But $$\ell = \alpha R/400 \; N\rho$$

$$L = 0{\cdot}5 \, (\alpha R/400 \; N\rho - 2a + 2\pi\bar{R}) \tag{12}$$

Now, since the field gradient (k) varies as 1/a and :

$$NI = ka^2/0{\cdot}8\pi$$

$$N = N_o\ (a/a_o) \qquad (13)$$

The mean radius of bend at the end of the magnet increases with the number of turns and the width of the conductor. In fact :

$$\bar{R} \propto (ND^2)^{1/2}$$

So from (13) :

$$\bar{R} = \bar{R}_o\ (D/D_o)(a/a_o)^{1/2} \qquad (14)$$

Substituting (13) and (14) in (12) we have :

$$L = (\alpha R/800\ \rho N_o)(a_o/a) - a - \pi\bar{R}_o(D/D_o)(a/a_o)^{1/2} \qquad (15)$$

Within the required accuracy the magnetic length is given by

$$d = L + a \qquad (16)$$

The outer coil radius R_{ext} scales in the same way as $\bar{R}$ (espression 14)

$$x_{min} = L + a + R_{ext} - d$$

$$= R_{ext}$$

$$= R_{ext_o}\ (\frac{D}{Do})(\frac{a}{a_o})^{1/2} \qquad (17)$$

In scaling the overall width of the magnet one assumes that the linear dimensions of the coil scale as $D\sqrt{N}$ i.e.

$$w(c) = (D\sqrt{N}/D_o\sqrt{N_o})\, w(c)_o = (D/D_o)(a/a_o)^{1/2} w(c)_o$$

With the coil must be included a portion of the yoke, shaded in Fig. 23, whose dimensions are dependent on those of the coil.

The rest of the horizontal width (w(a)) scales linearly with aperture. Therefore :

$$w = w_{(a)_o}\,(a/a_o) + w_{(c)_o}\,(D/D_o)(a/a_o)^{1/2} \qquad (18)$$

b) <u>DESY Design</u>

L is defined as the length of the pole including the rounded ends. (see Fig. 22)

Assuming that the field falls linearly between the edge of the straight part of the poles and the end plate

$$d = L - 0{\cdot}28a \qquad (16a)$$

The principal difference between the DESY and other designs is that the width of the coil space and also the addition to the length of the magnet due to coils and rounded pole ends scale linearly with aperture.

Without going into too much detail, the changes to expressions (15) (17) and (18) are :

$$L = (\alpha R/800\rho N)\ (a_o/a) - 2{\cdot}23a - (\pi/4)\Delta r_o (D/D_o)^2 \qquad (15a)$$

$$x_{min} = 1{\cdot}32a \qquad (17a)$$

$$w = w_{(a)_o}\ (a/a_o) + w_{(c)_o}\ (D^2/D_o{}^2) \qquad (18a)$$

Δr_o is the difference between the inner and outer radii of bend of the flattened end of the coil which lies against the mirror plate.

A Mercury Computer program has been written to generate the physical parameters(3)and optical parameters(4)of each family of quadrupoles using the above expressions. The close agreement between the parameters predicted by the above theory (Table 1) and the practical design developed from them [4] demonstrates the reliability of the theory.

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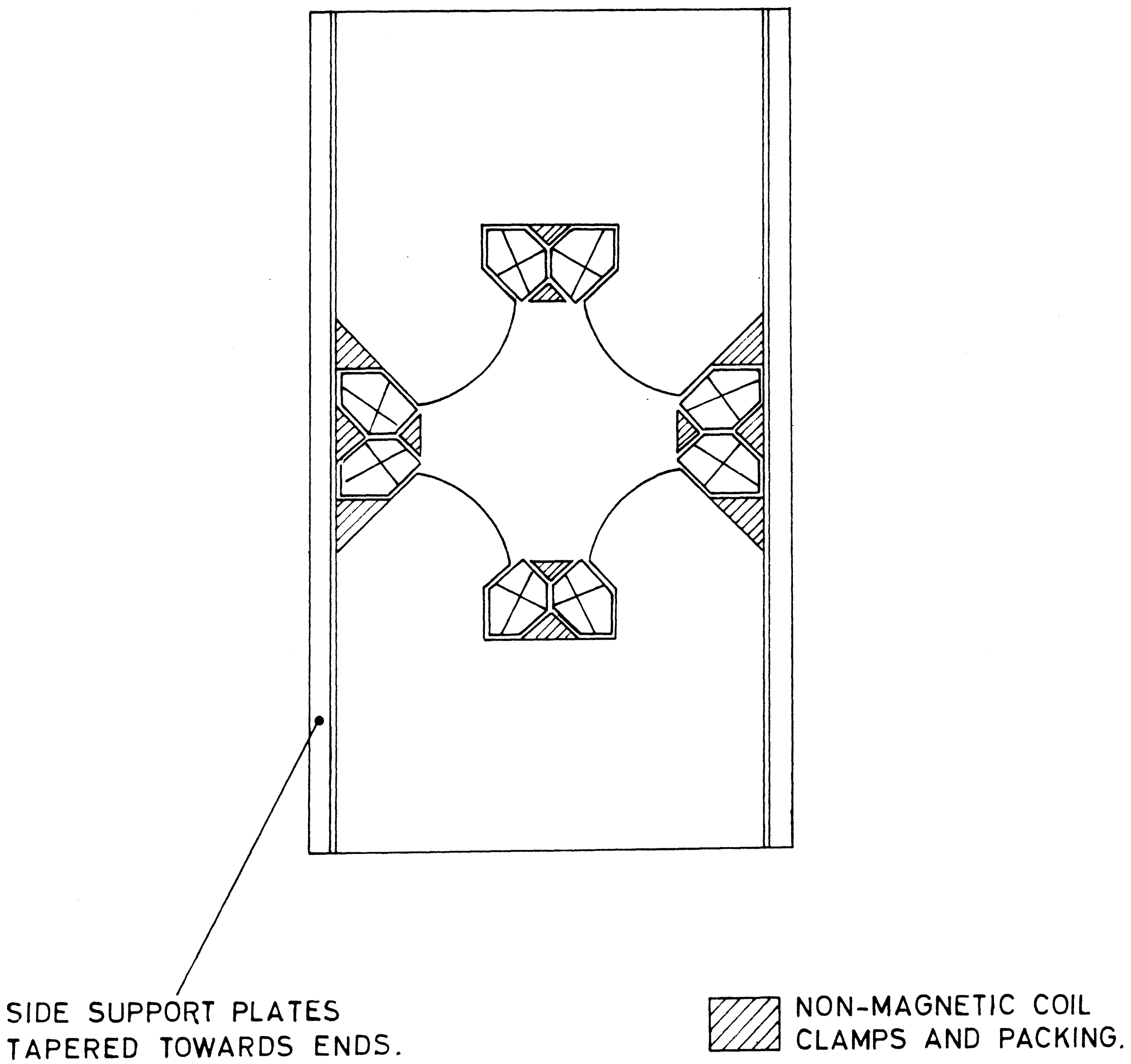

DIAGRAM OF FIGURE OF EIGHT DESIGN.

Fig. 1.

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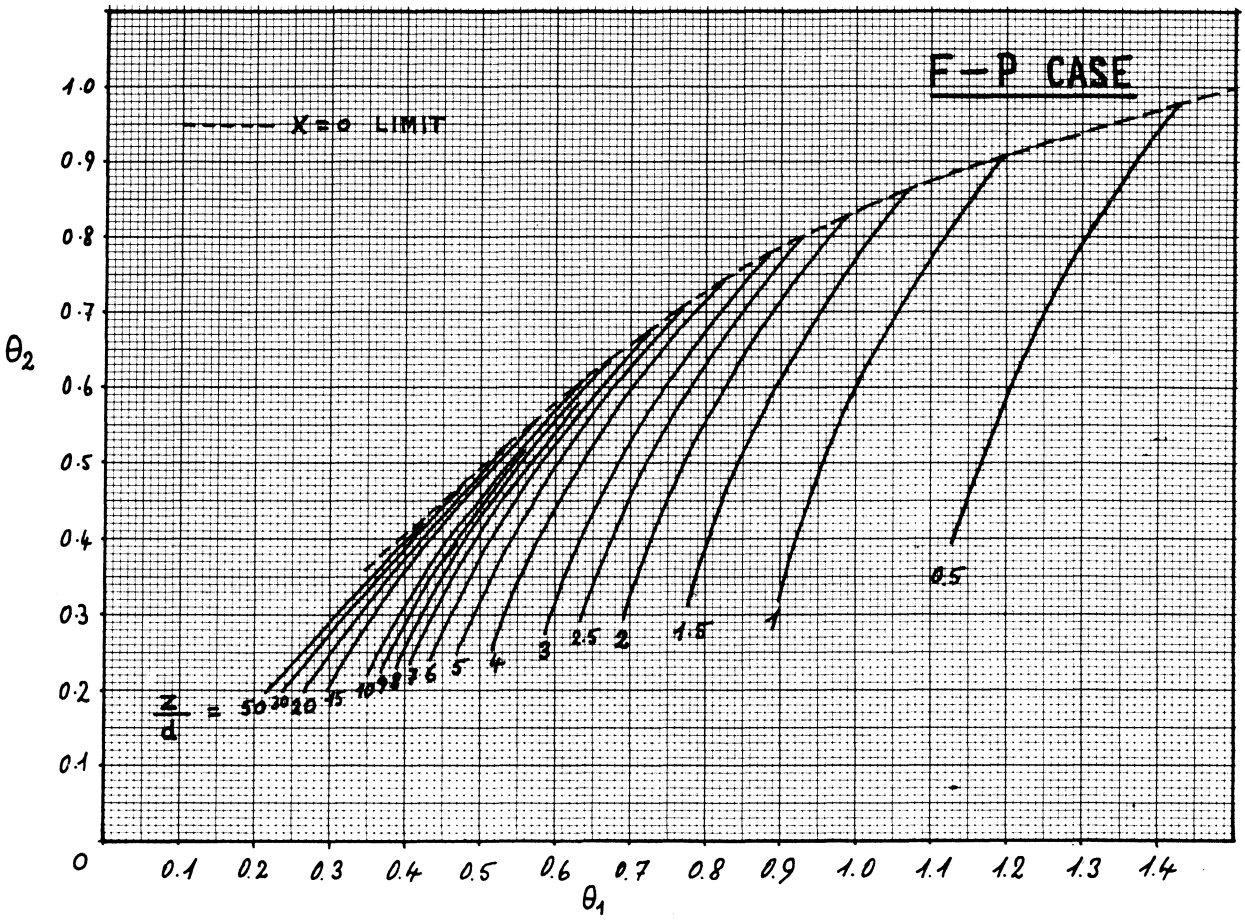

FIG. 2 ELEMENT STRENGTHS

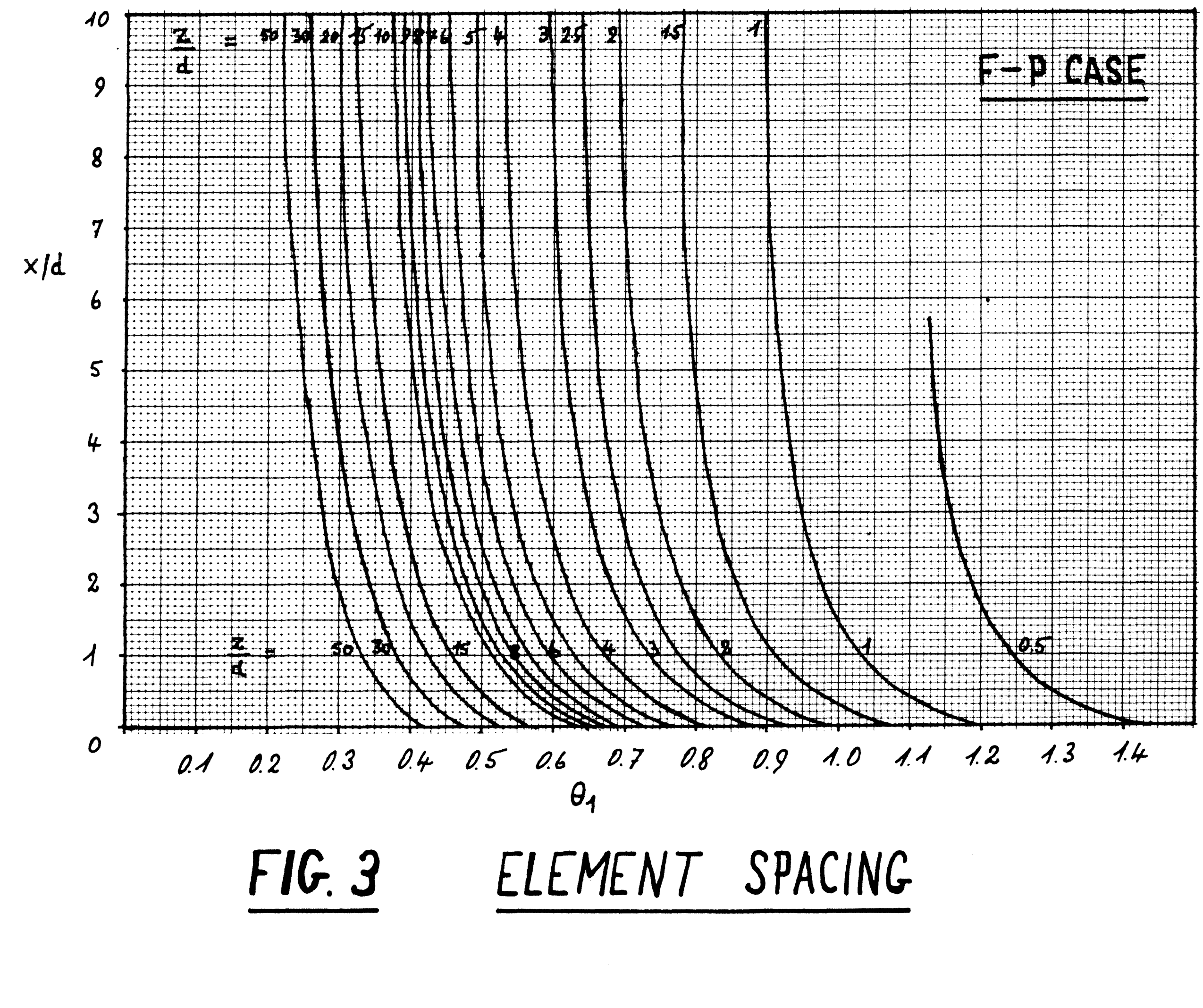

FIG. 3 ELEMENT SPACING

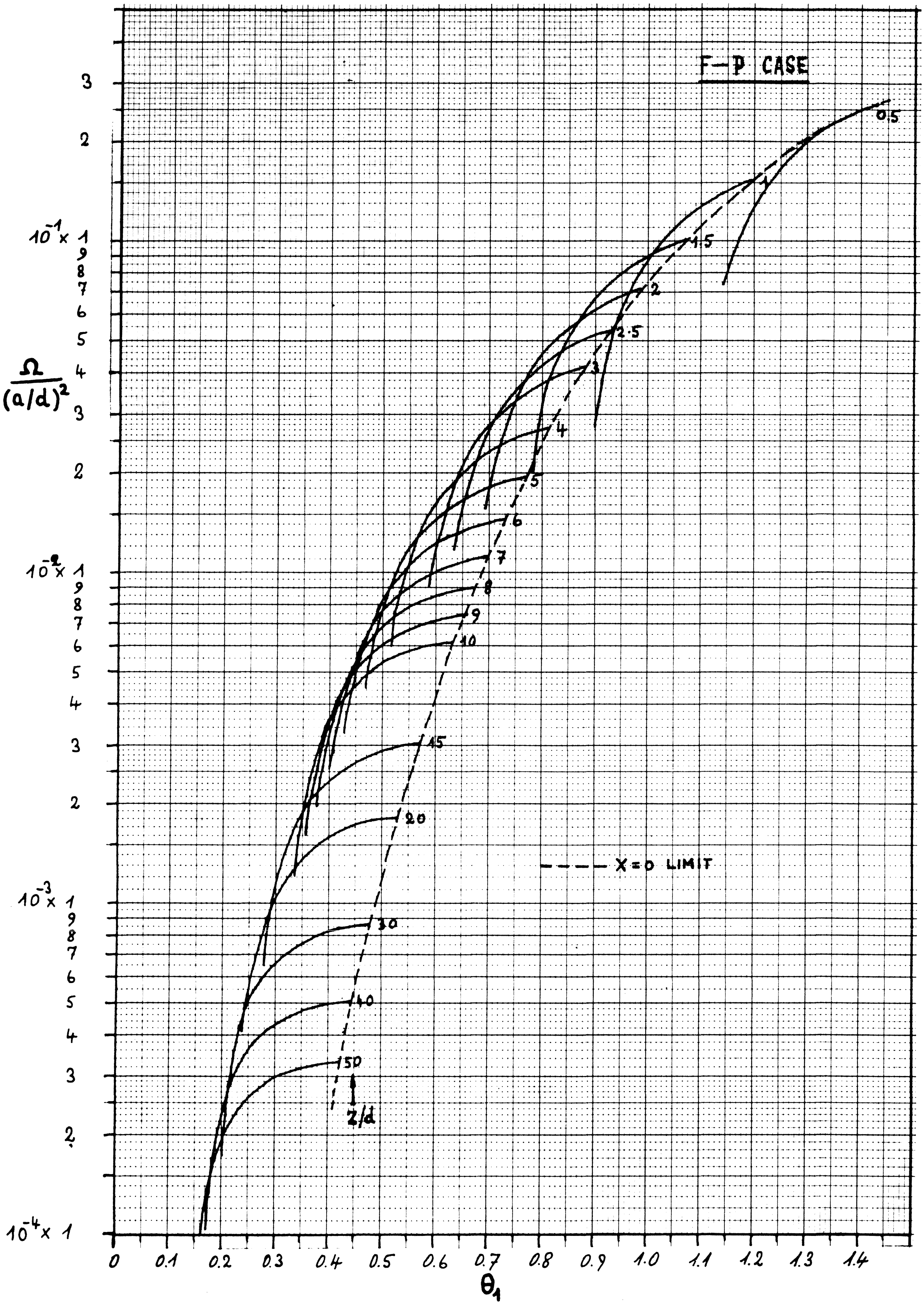

FIG. 4 ACCEPTANCE

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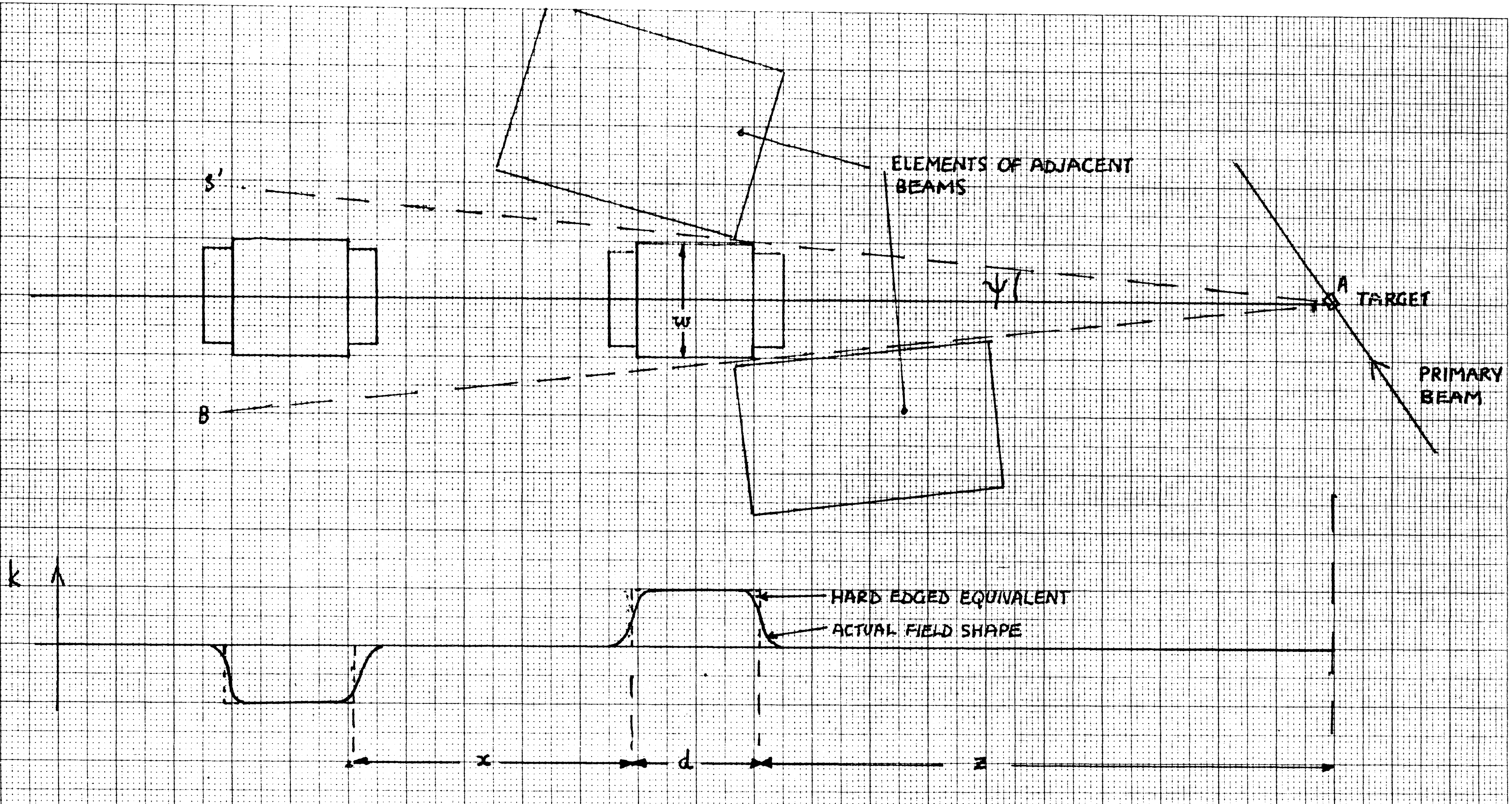


NOTATION FOR A POST-TARGET DOUBLET

FIG 5

SIS/R/5567

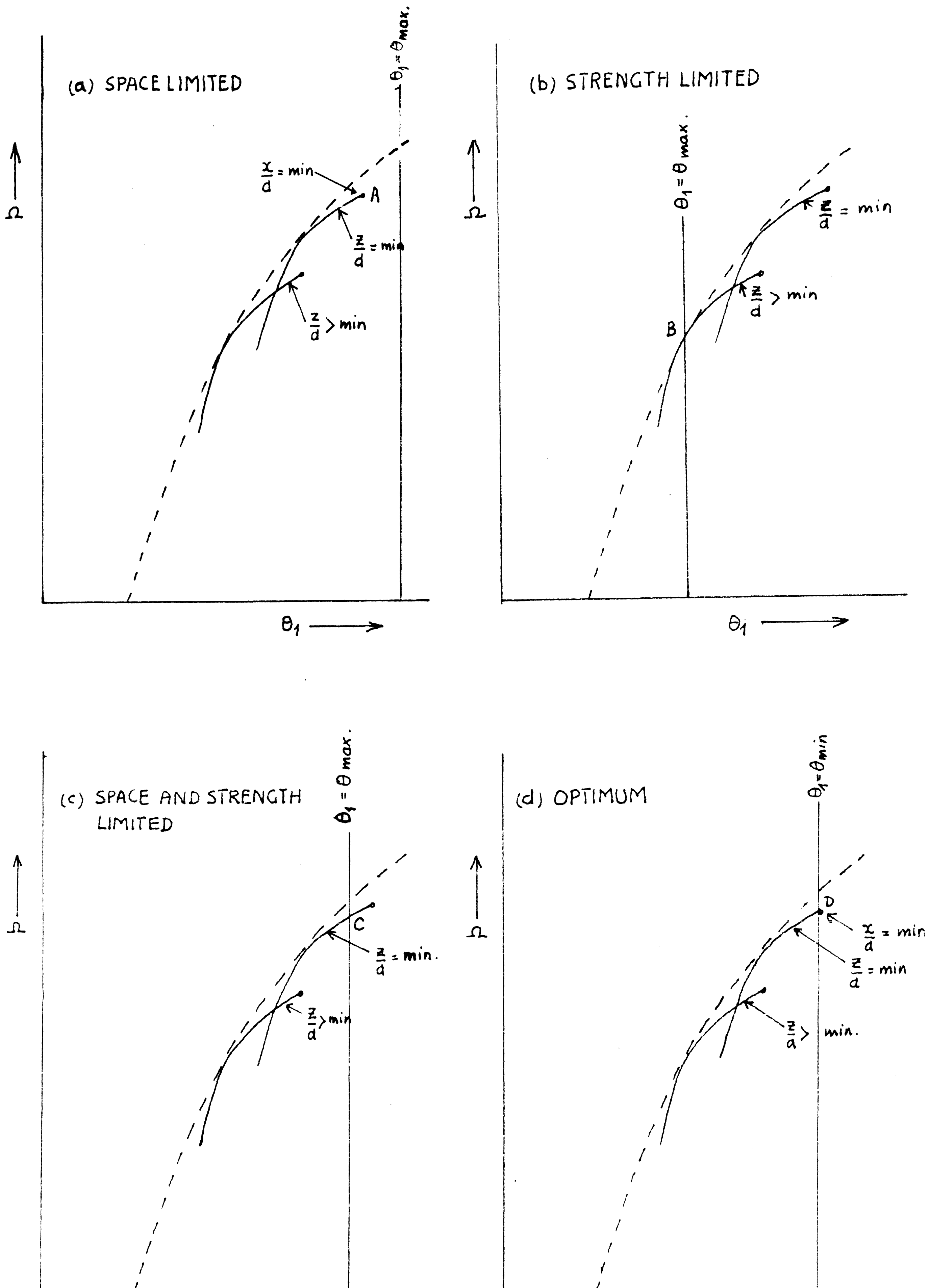

(a) SPACE LIMITED
$\theta_1 = \theta_{max}$
$\frac{x}{d}$ = min
A
$\frac{z}{d}$ = min
$\frac{z}{d}$ > min
Ω
θ_1
(b) STRENGTH LIMITED
$\theta_1 = \theta_{max}$
$\frac{z}{d}$ = min
$\frac{z}{d}$ > min
B
Ω
θ_1
(c) SPACE AND STRENGTH LIMITED
$\theta_1 = \theta_{max}$
C
$\frac{z}{d}$ = min.
$\frac{z}{d}$ > min
Ω
θ_1
(d) OPTIMUM
$\theta_1 = \theta_{min}$
D
$\frac{x}{d}$ = min
$\frac{z}{d}$ = min
$\frac{z}{a}$ > min.
Ω
θ_1

FIG. 6.

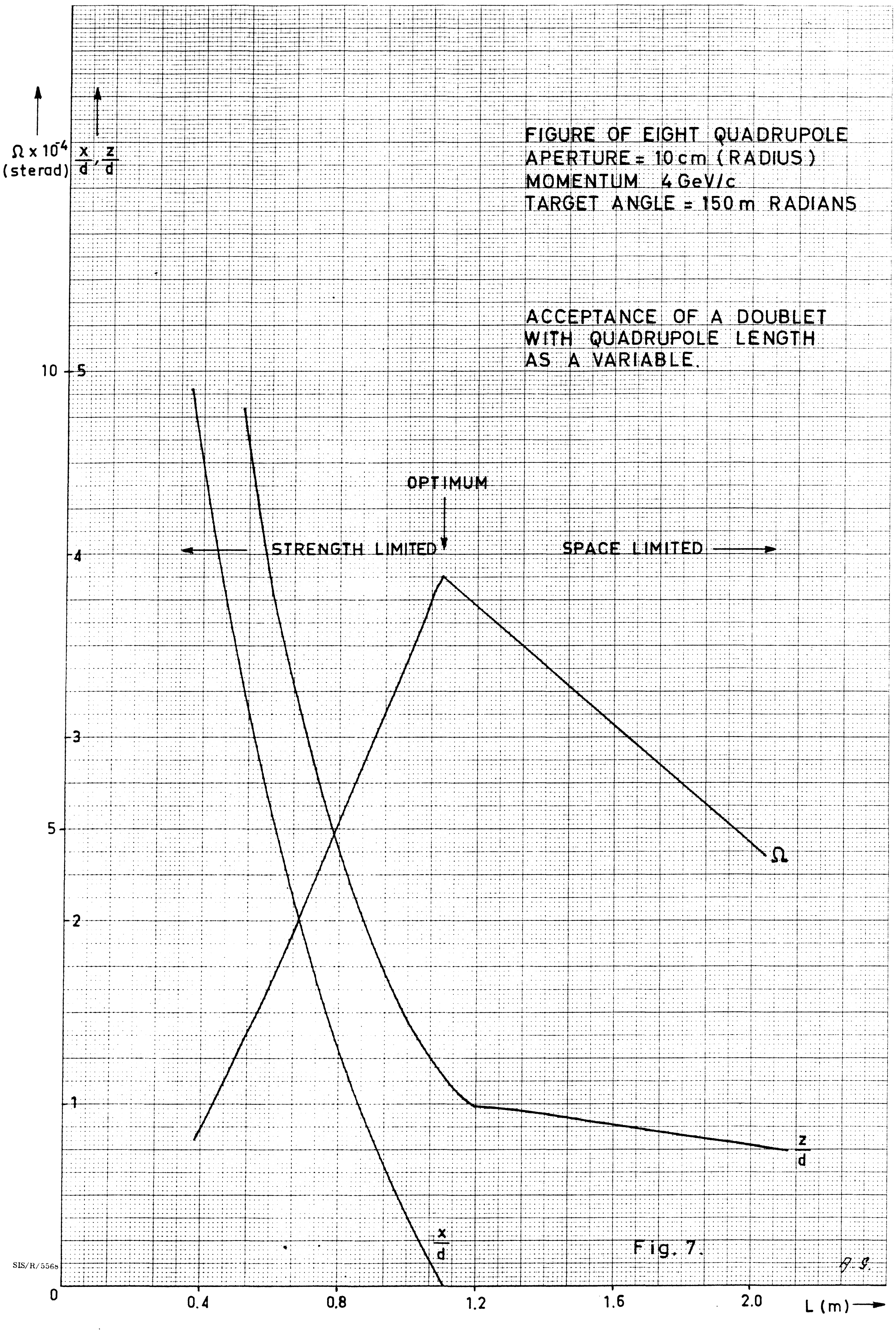


FIGURE OF EIGHT QUADRUPOLE
APERTURE = 10 cm (RADIUS)
MOMENTUM 4 GeV/c
TARGET ANGLE = 150 m RADIANS
ACCEPTANCE OF A DOUBLET
WITH QUADRUPOLE LENGTH
AS A VARIABLE.
$\Omega \times 10^{-4}$ (sterad)
$\frac{x}{d}, \frac{z}{d}$
OPTIMUM
STRENGTH LIMITED
SPACE LIMITED
10
5
0
5
4
3
2
1
0.4
0.8
1.2
1.6
2.0
L (m)
Ω
$\frac{z}{d}$
$\frac{x}{d}$
SIS/R/5568

Fig. 7.

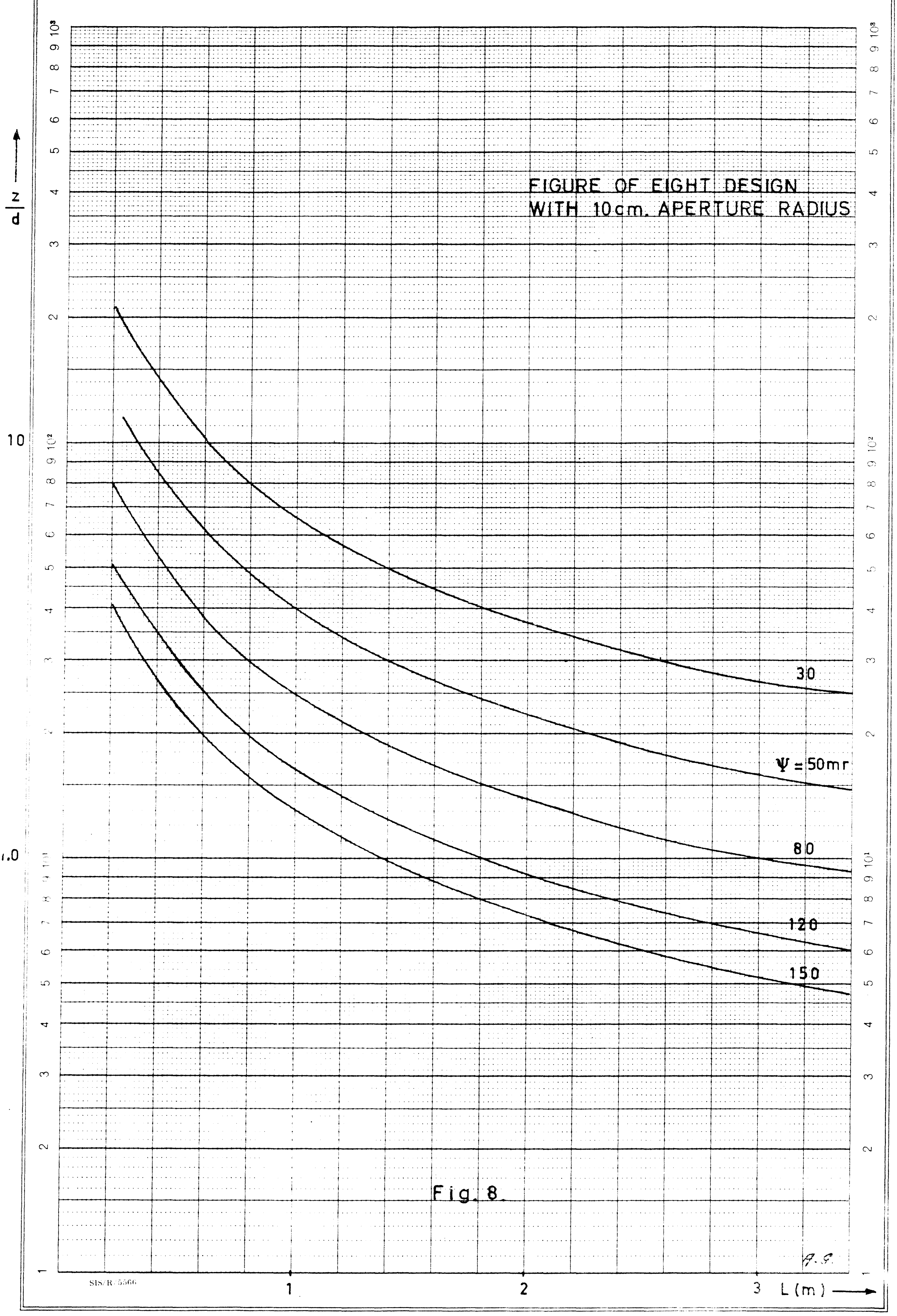


FIGURE OF EIGHT DESIGN
WITH 10cm. APERTURE RADIUS
Z/d
10
1.0
1
2
3
L(m)
30
Ψ = 50mr
80
120
150
SIS/R 5566

Fig. 8.

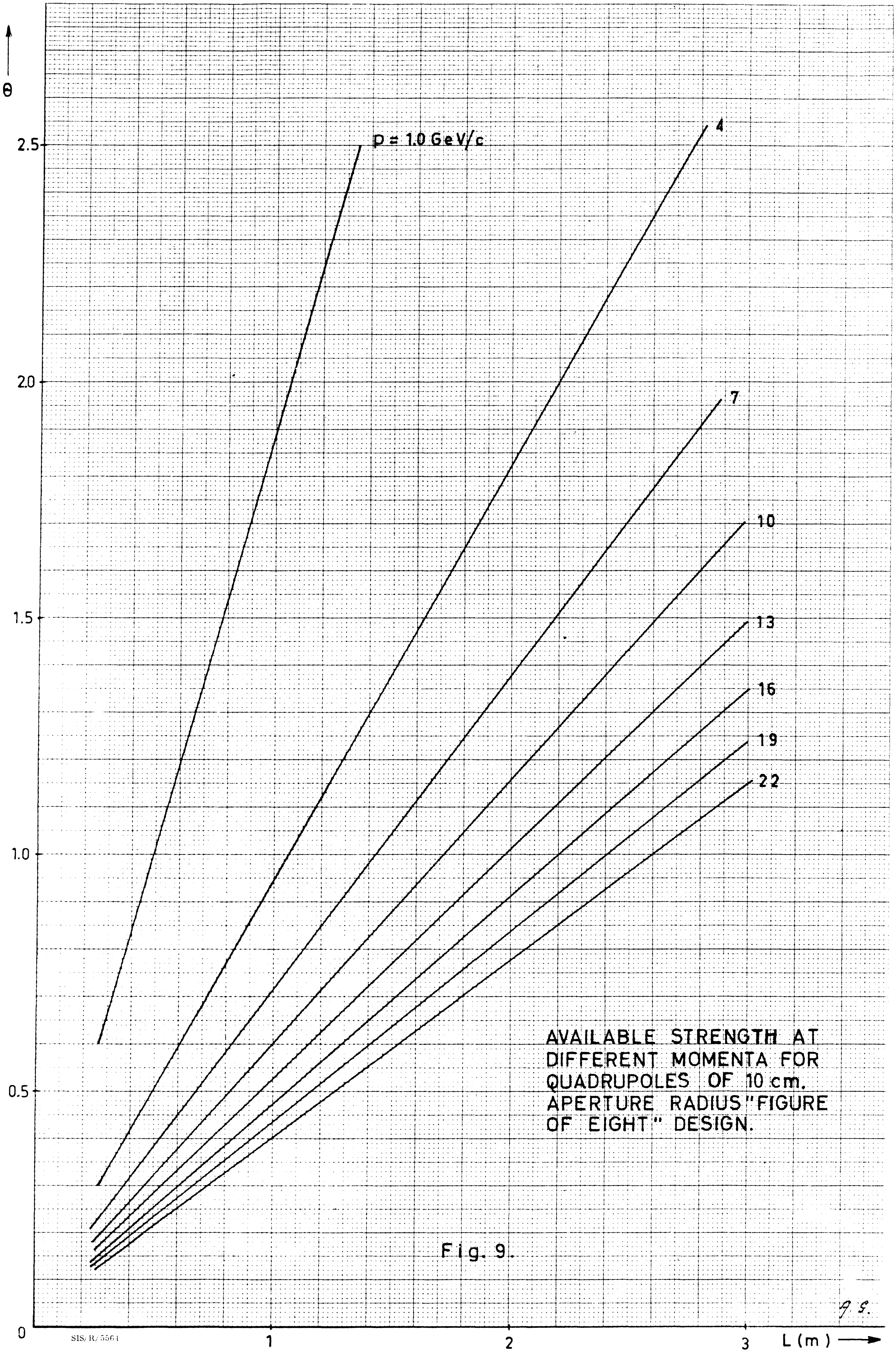
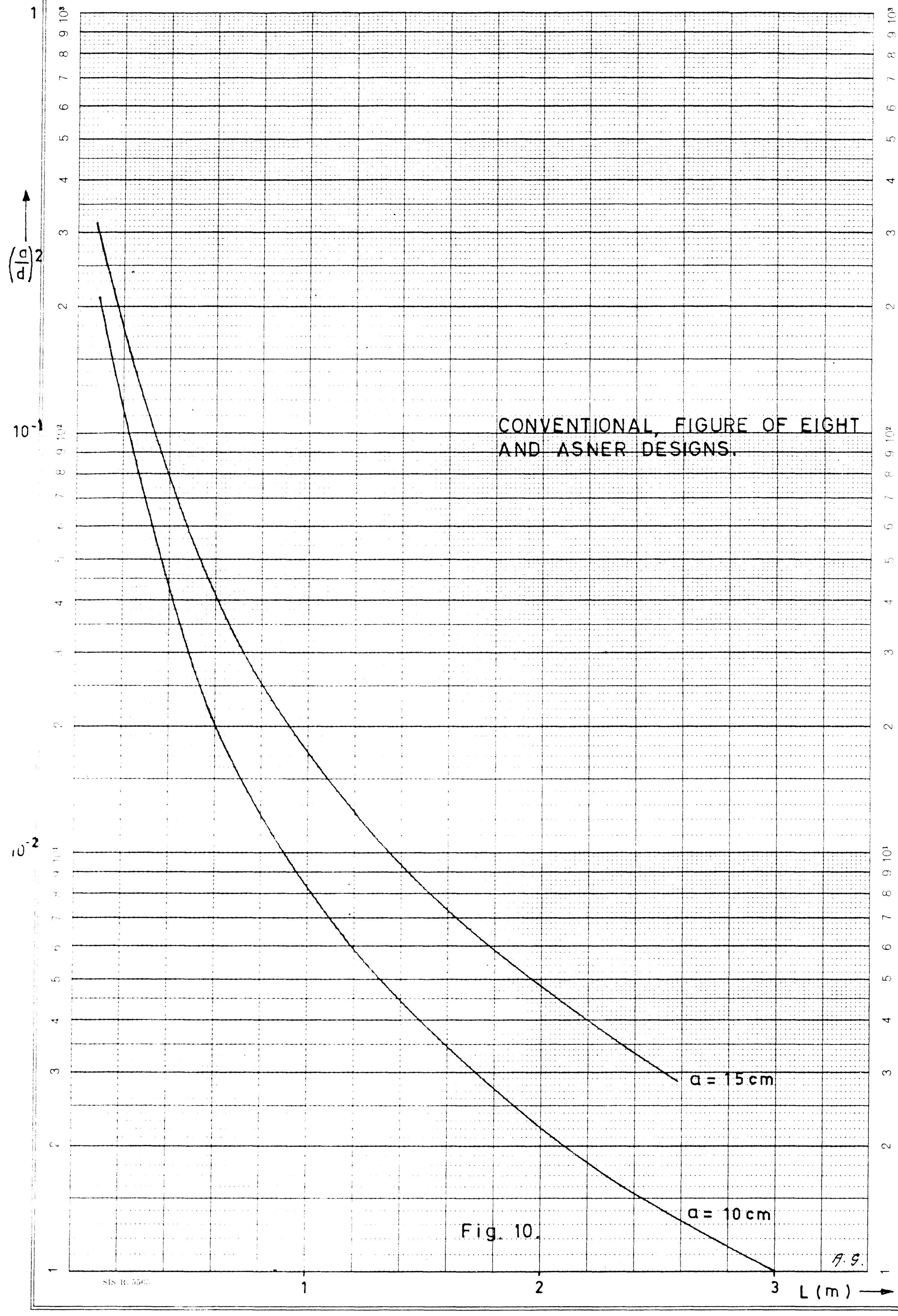


Fig. 9.

SIS/R/5564



CONVENTIONAL, FIGURE OF EIGHT AND ASNER DESIGNS.
$\left(\frac{a}{d}\right)^2$
$L\ (m)$
a = 15 cm
a = 10 cm
1
10^{-1}
10^{-2}
1
2
3
A. S.

Fig. 10.

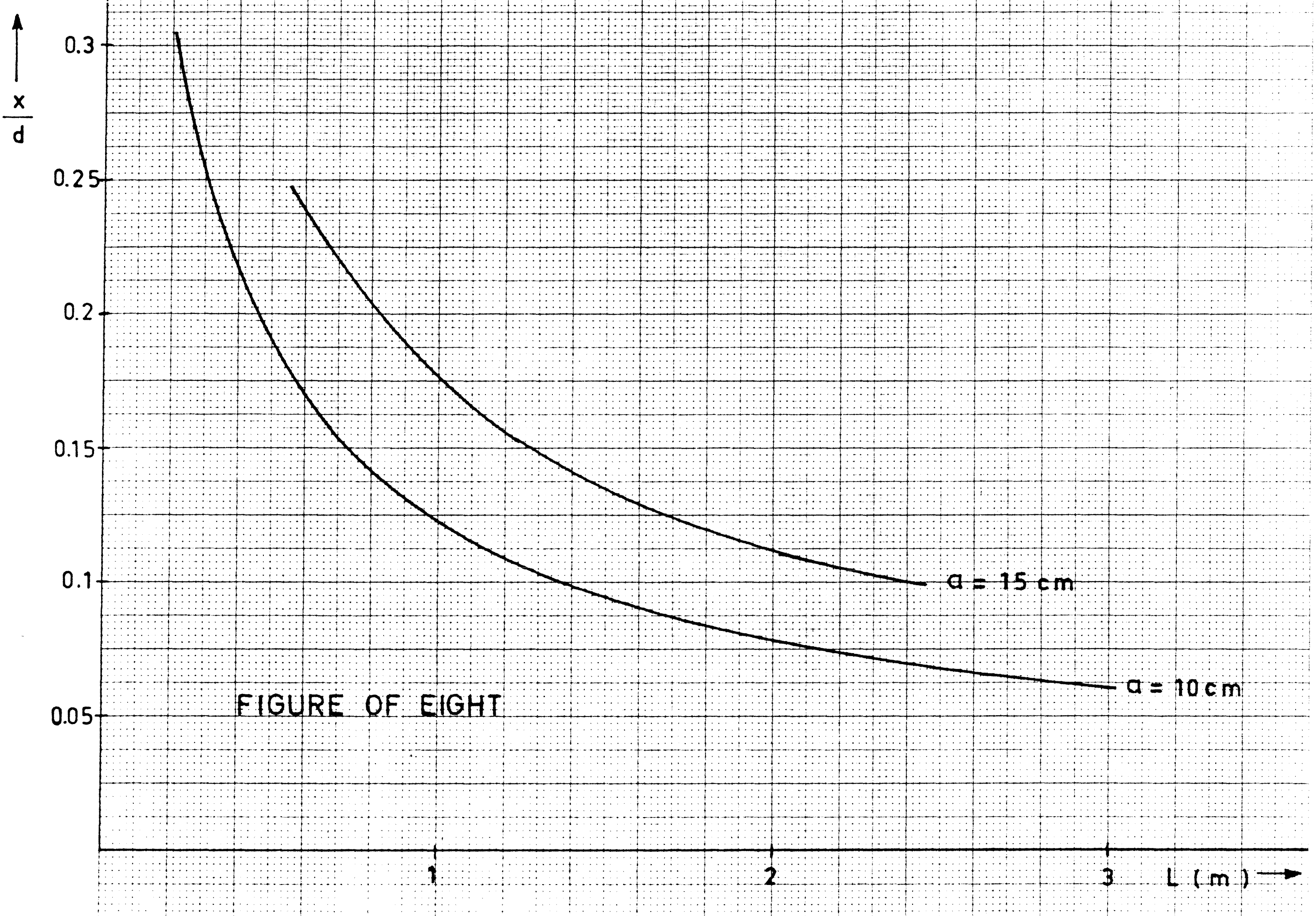


Fig. 11.

A.S.

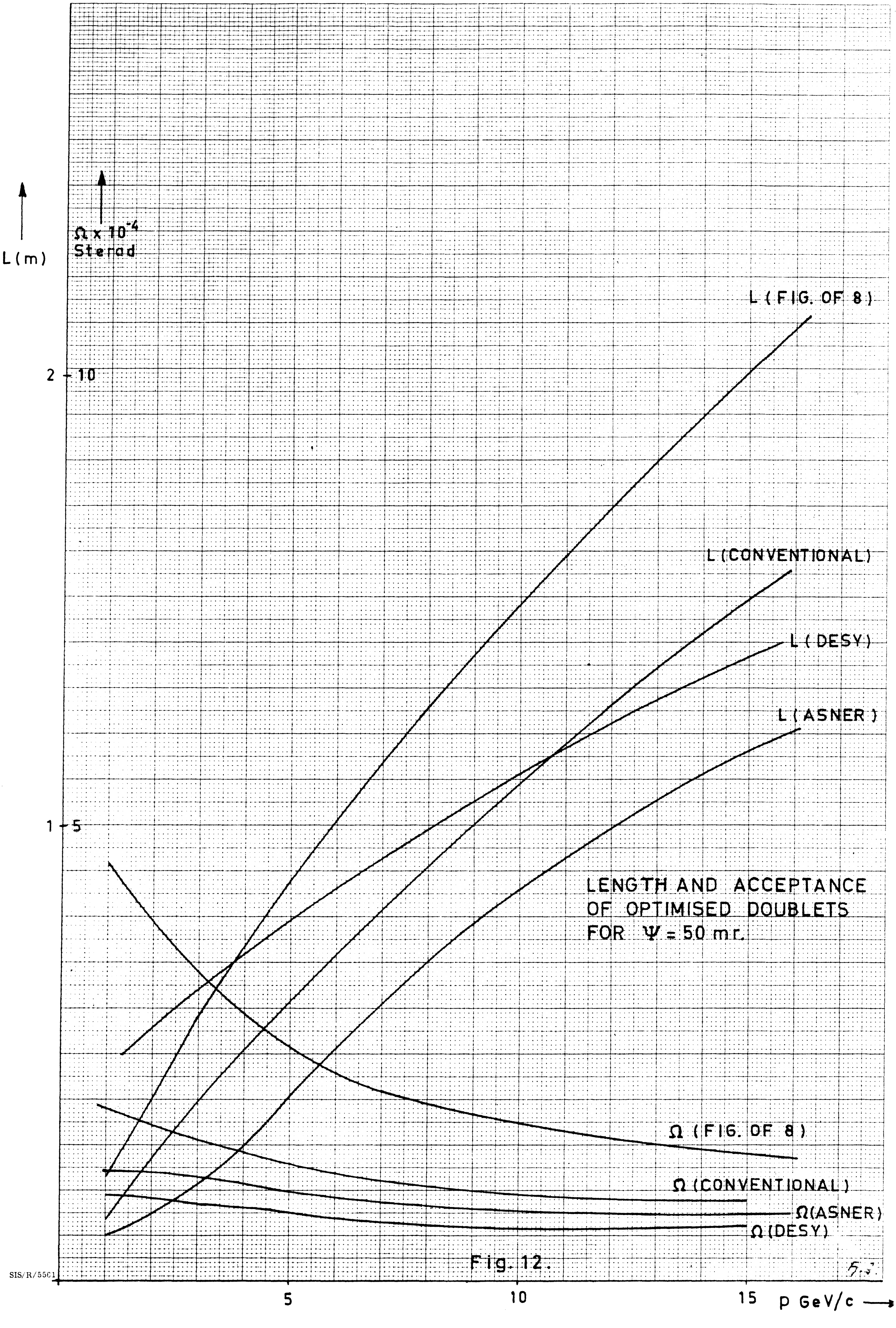
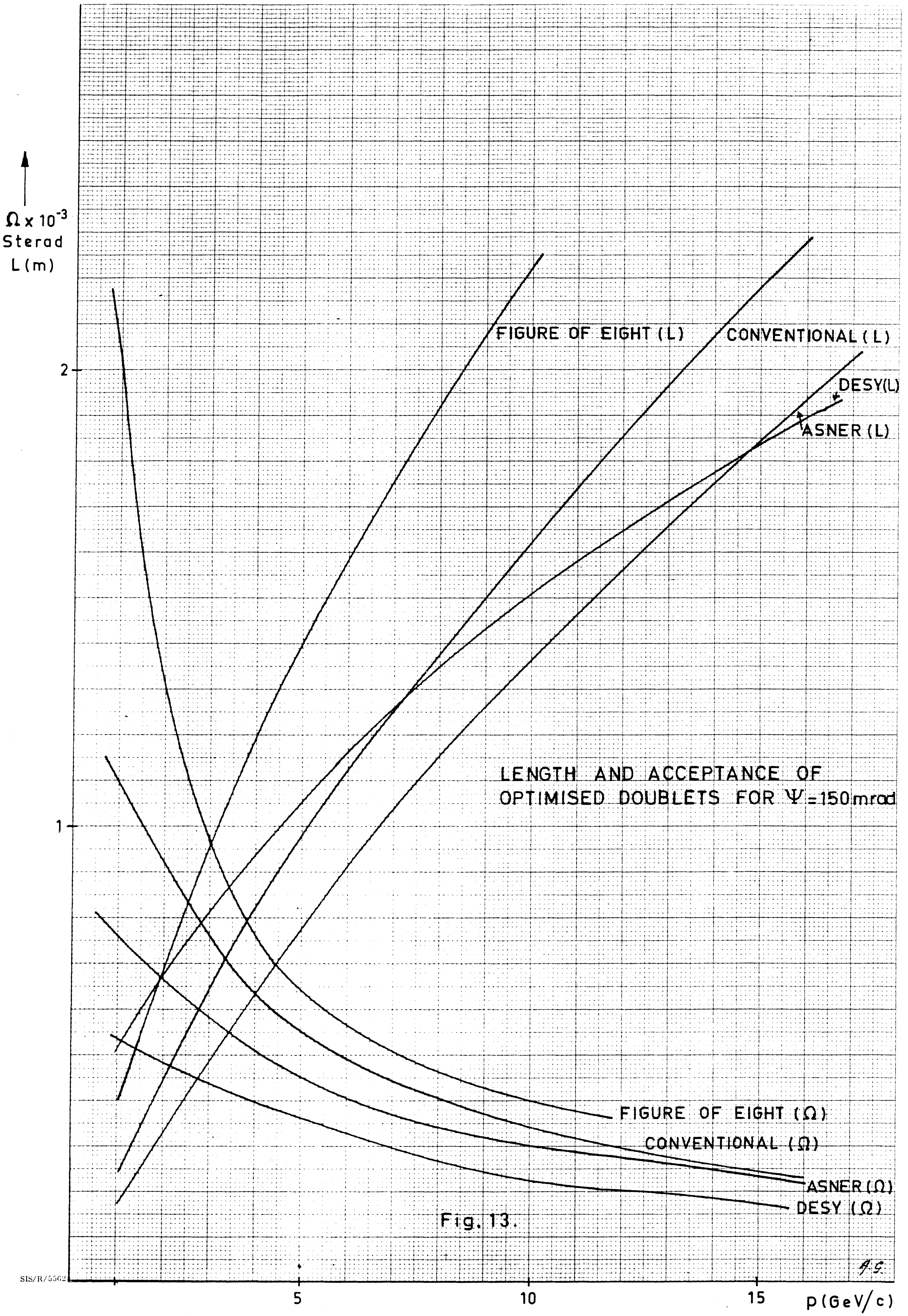


Fig. 12.

SIS/R/5501



Ω x 10⁻³
Sterad
L(m)
2
1
FIGURE OF EIGHT (L)
CONVENTIONAL (L)
DESY(L)
ASNER (L)
LENGTH AND ACCEPTANCE OF
OPTIMISED DOUBLETS FOR Ψ=150mrad
FIGURE OF EIGHT (Ω)
CONVENTIONAL (Ω)
ASNER (Ω)
DESY (Ω)
5
10
15
p (GeV/c)
A.G.
SIS/R/5562

Fig. 13.

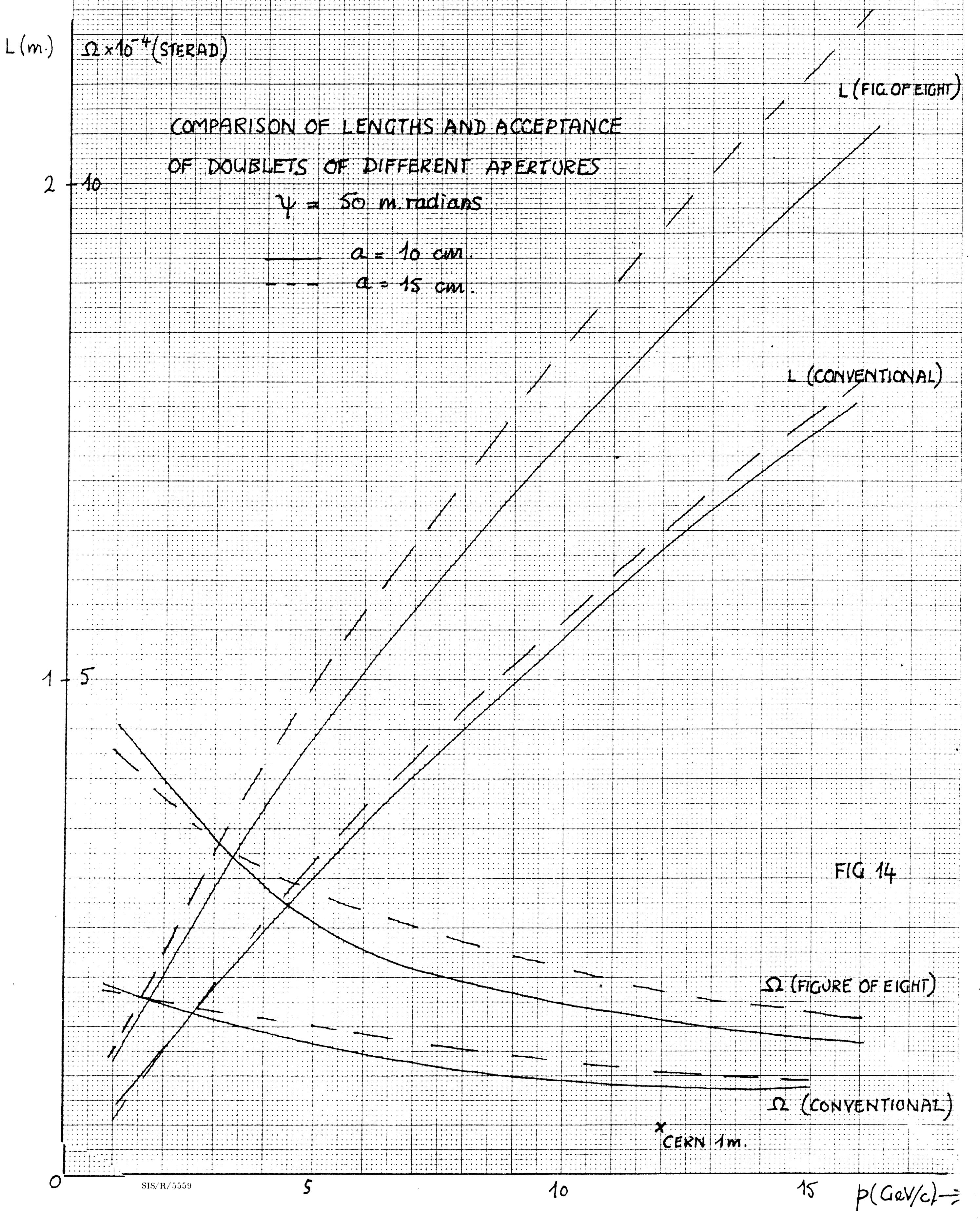


FIG 14

SIS/R/5559

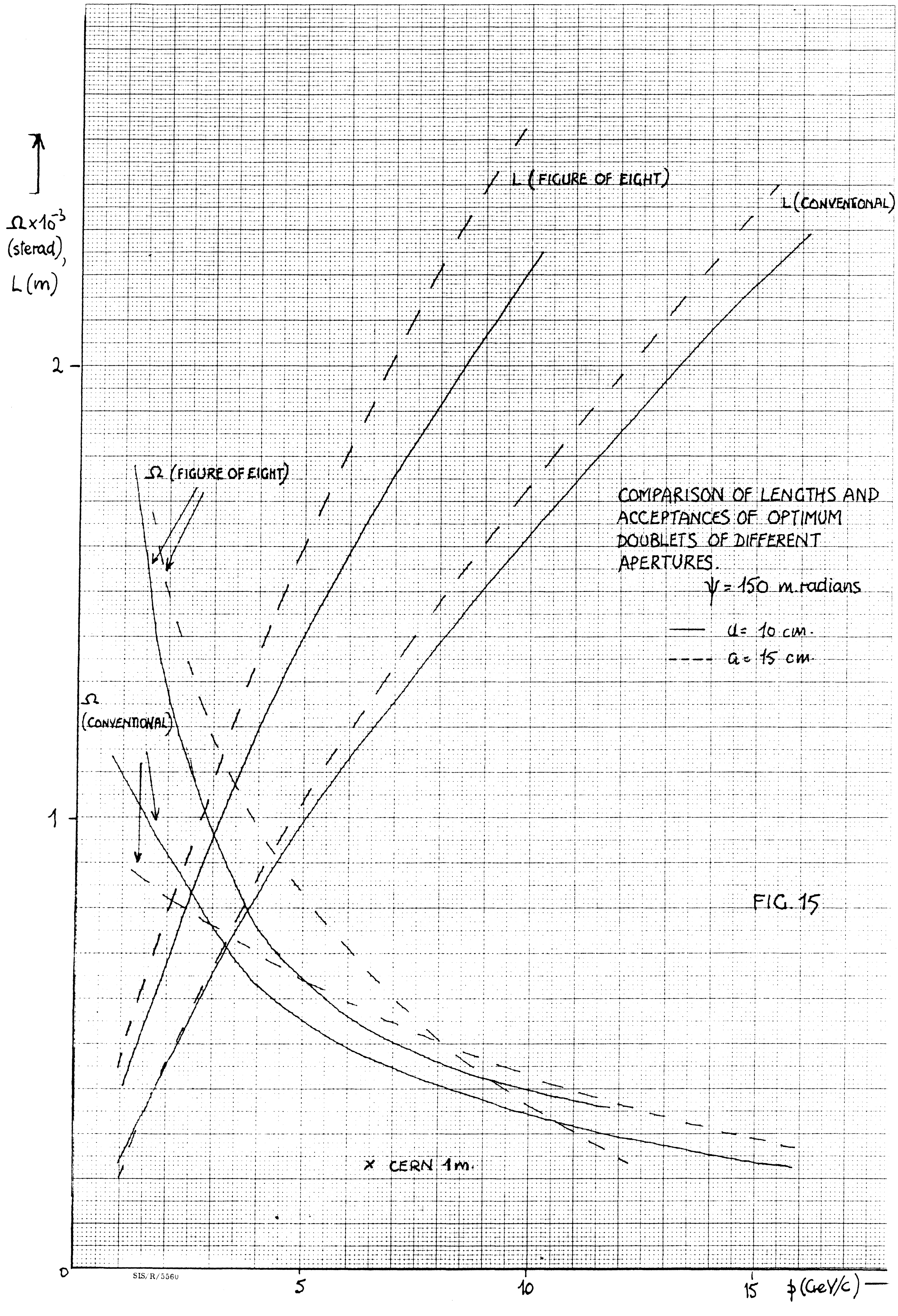


FIG. 15

SIS/R/5560

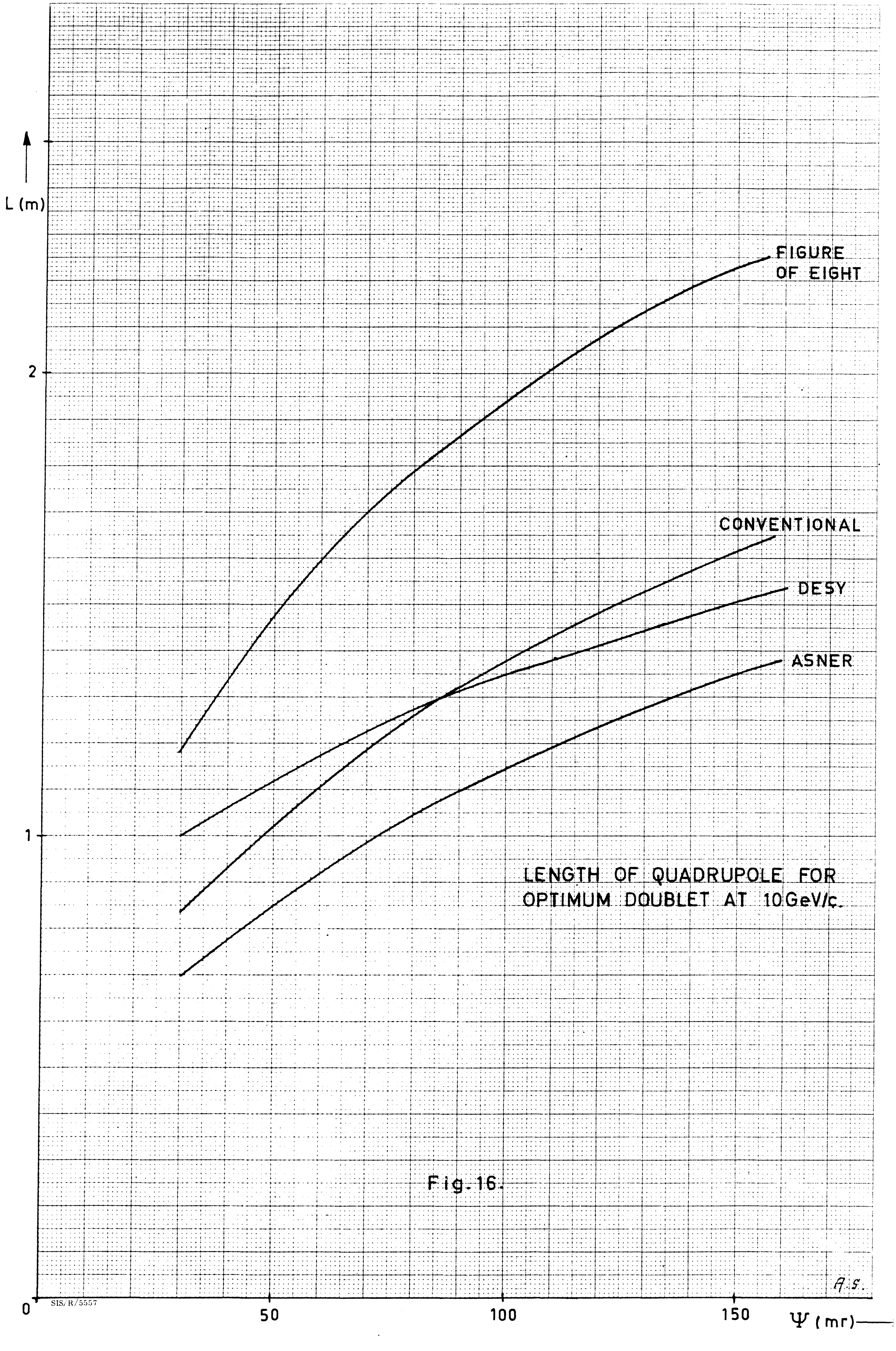
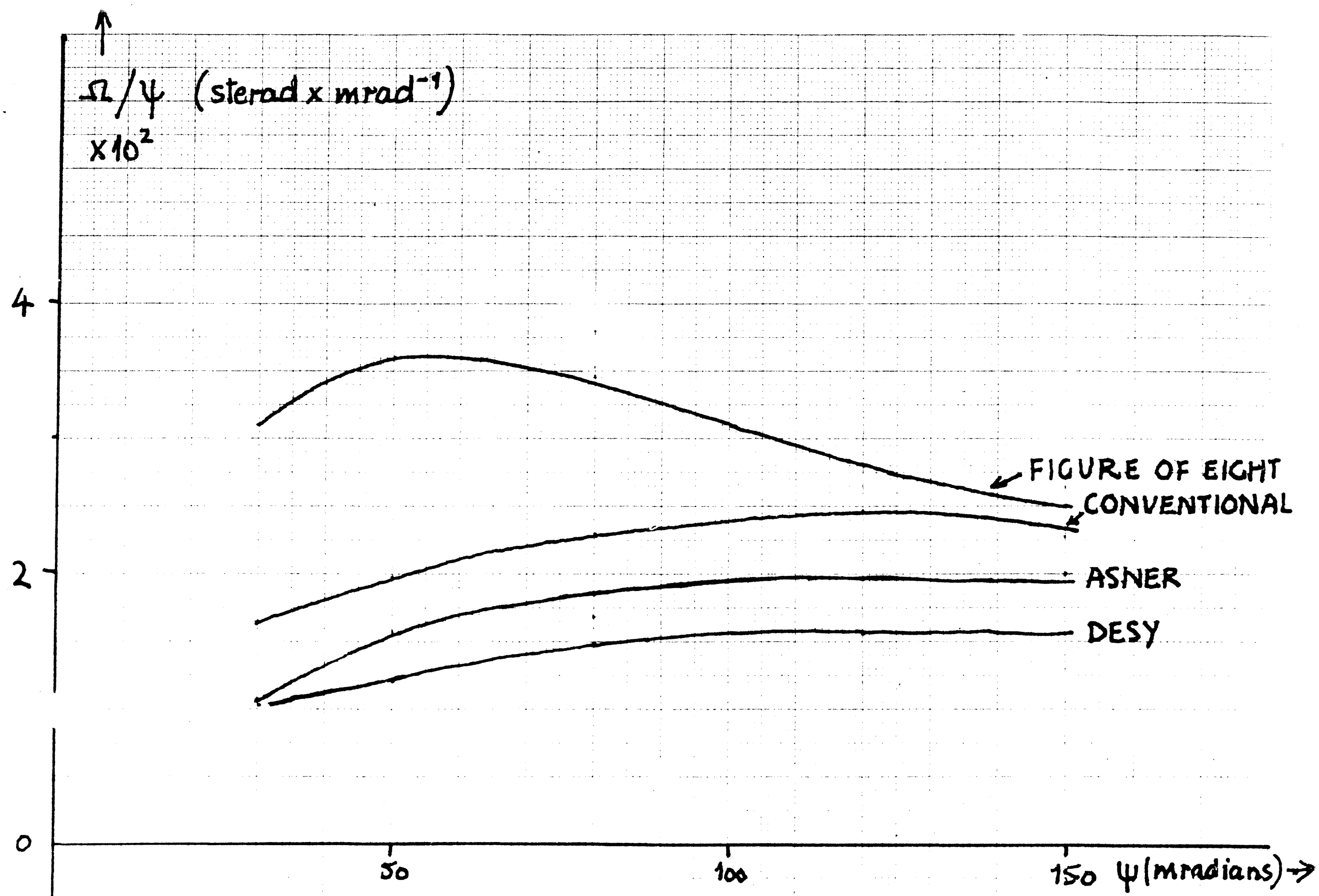


Fig. 16.

SIS/R/5557



ACCEPTANCE CURVES FOR
OPTIMUM DOUBLETS AT 10 GeV/c

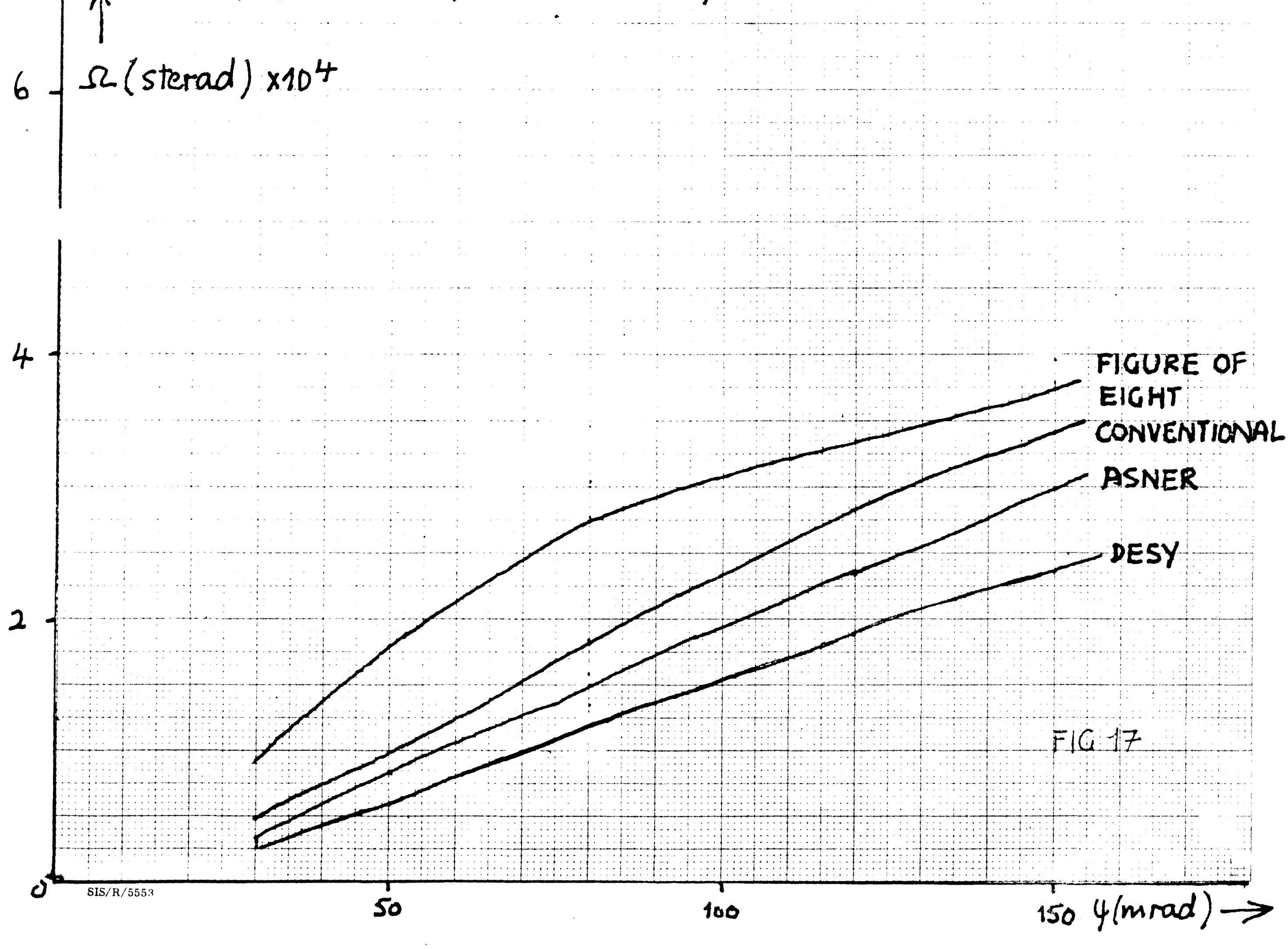


FIG 17

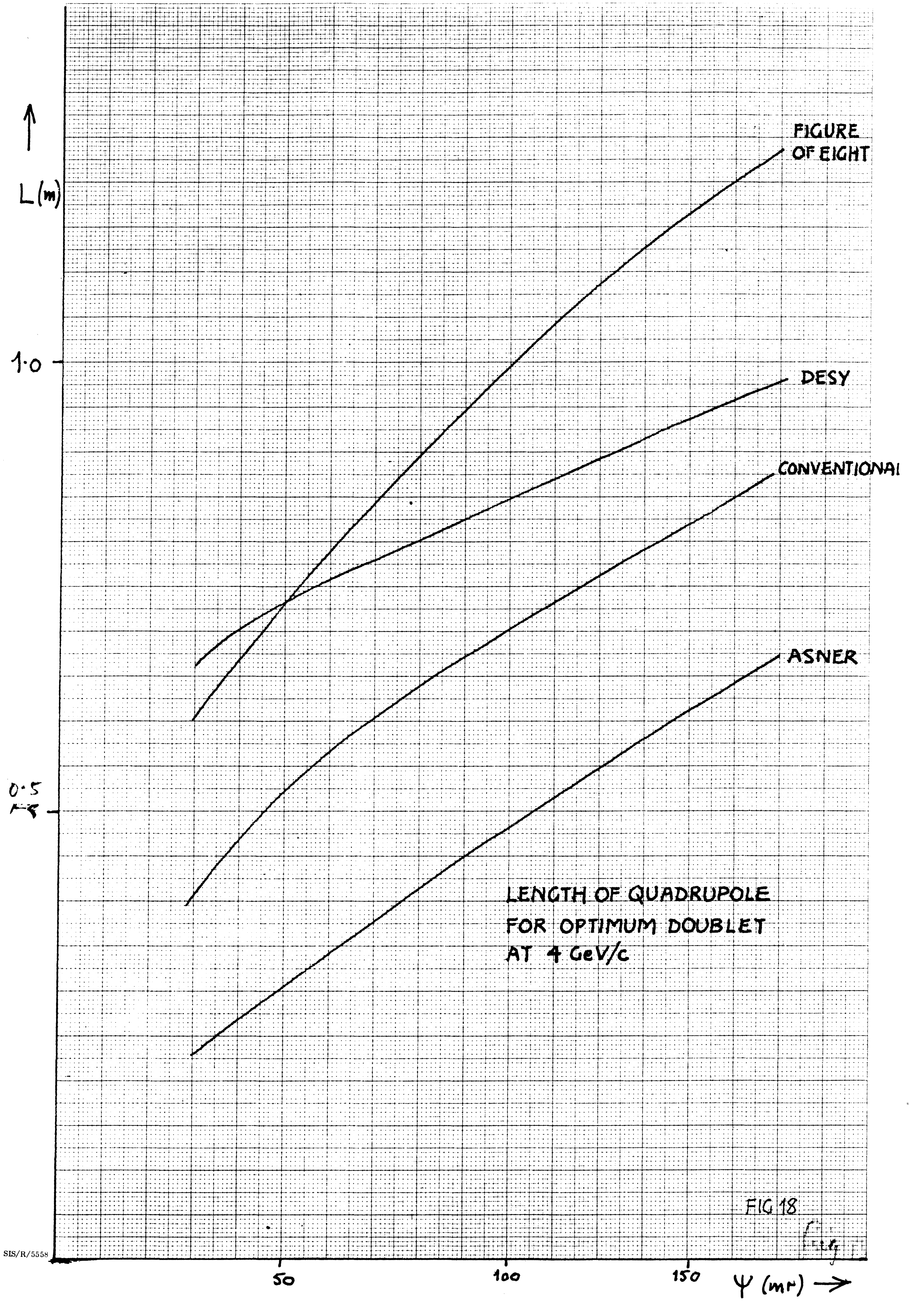
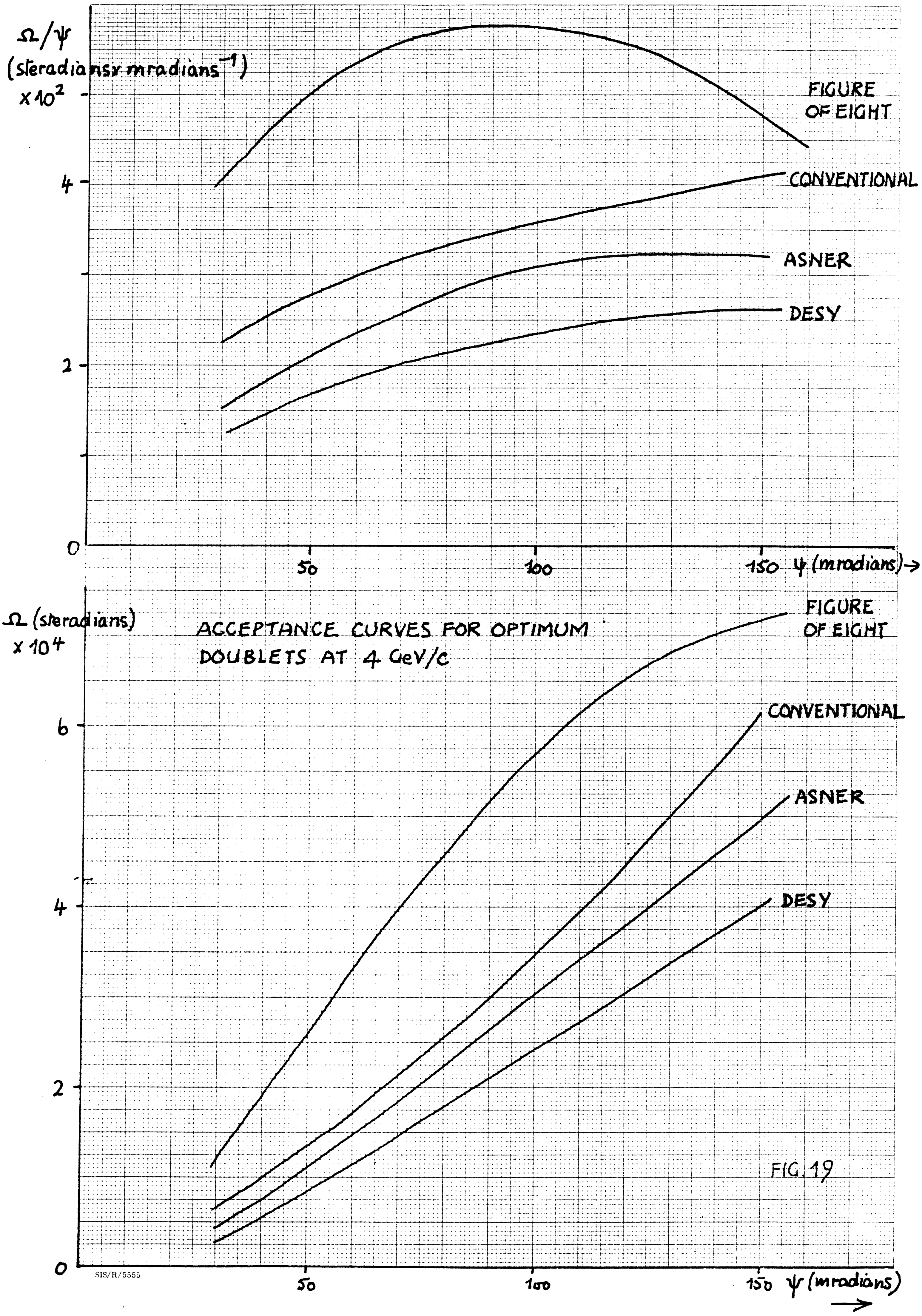
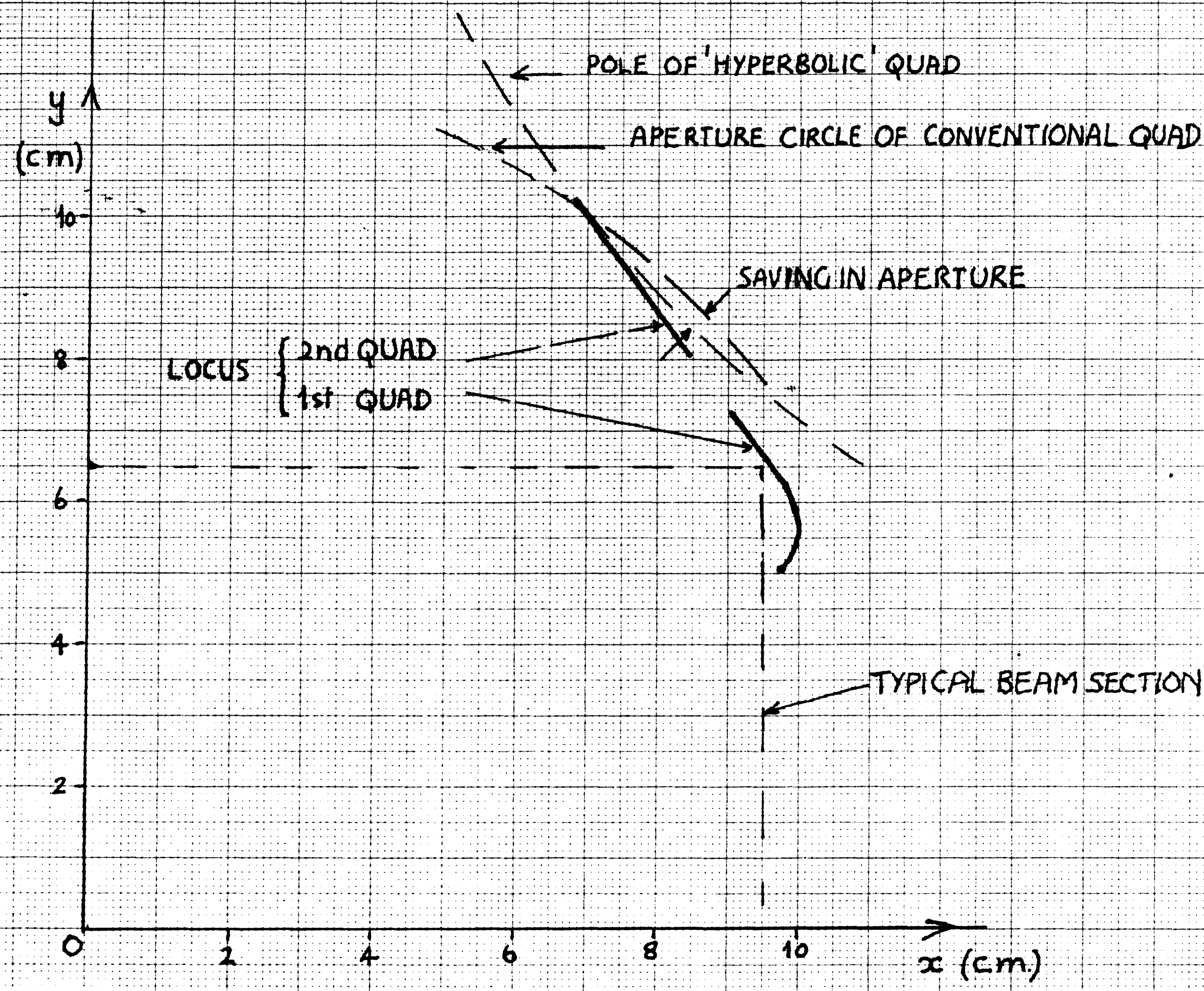


FIG 18

SIS/R/5558


Ω/ψ
(steradians x mradians⁻¹)
x 10²
4
2
0
50
100
150
ψ (mradians) →
FIGURE OF EIGHT
CONVENTIONAL
ASNER
DESY
Ω (steradians)
x 10⁴
ACCEPTANCE CURVES FOR OPTIMUM DOUBLETS AT 4 GeV/c
6
4
2
0
50
100
150
ψ (mradians) →
FIGURE OF EIGHT
CONVENTIONAL
ASNER
DESY
FIG. 19
SIS/R/5555



LOCUS OF EXTREME EXCURSION OF BEAM SECTION IN AN OPTIMUM DOUBLET

FIG 20

OPTIMUM CONDITIONS R_4

$p = 4$ GeV/c

$d = 83$ cm

$z = 4{\cdot}48$ cm

$x = 12{\cdot}6$ cm

SIS/R/5556

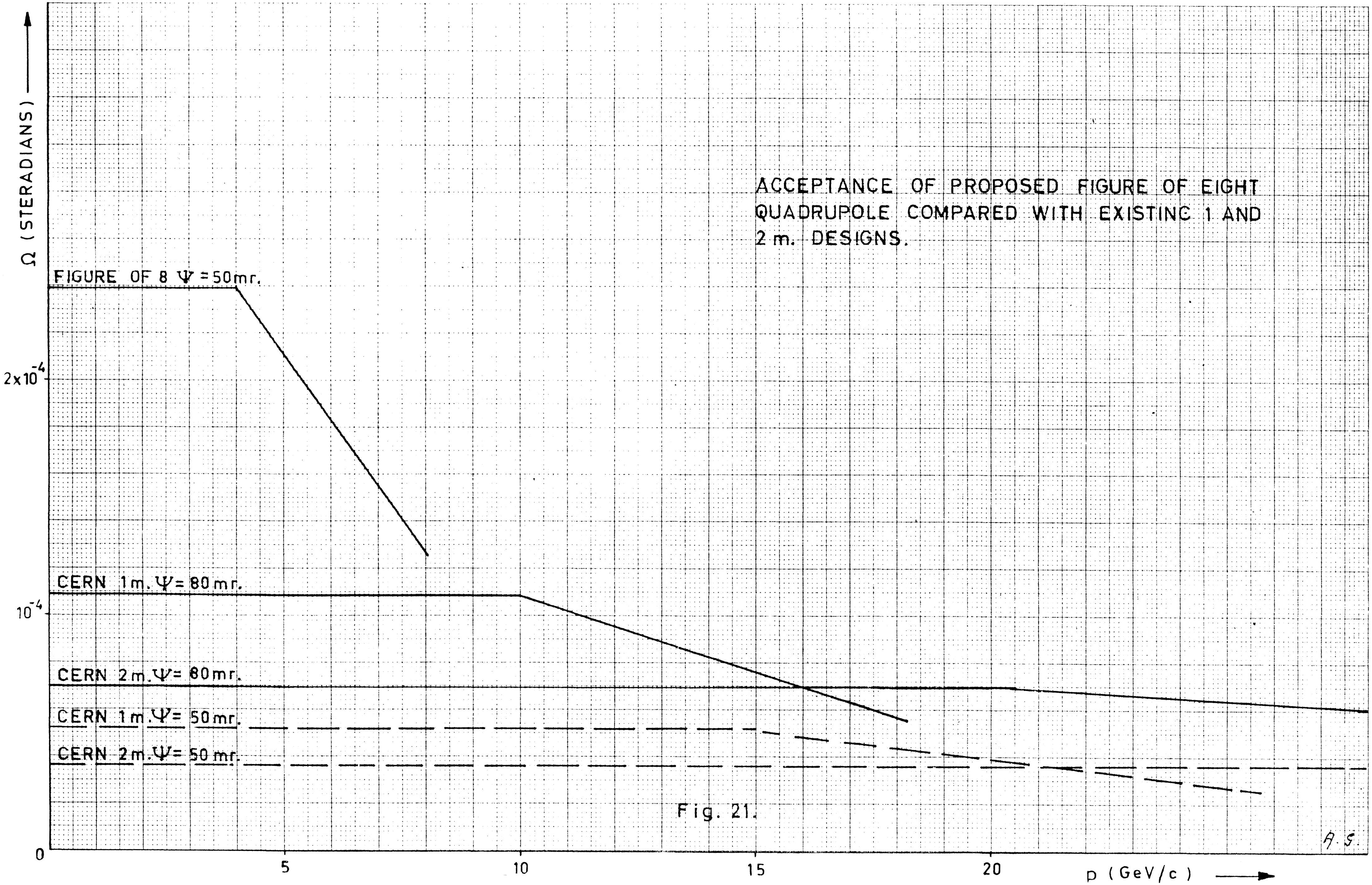
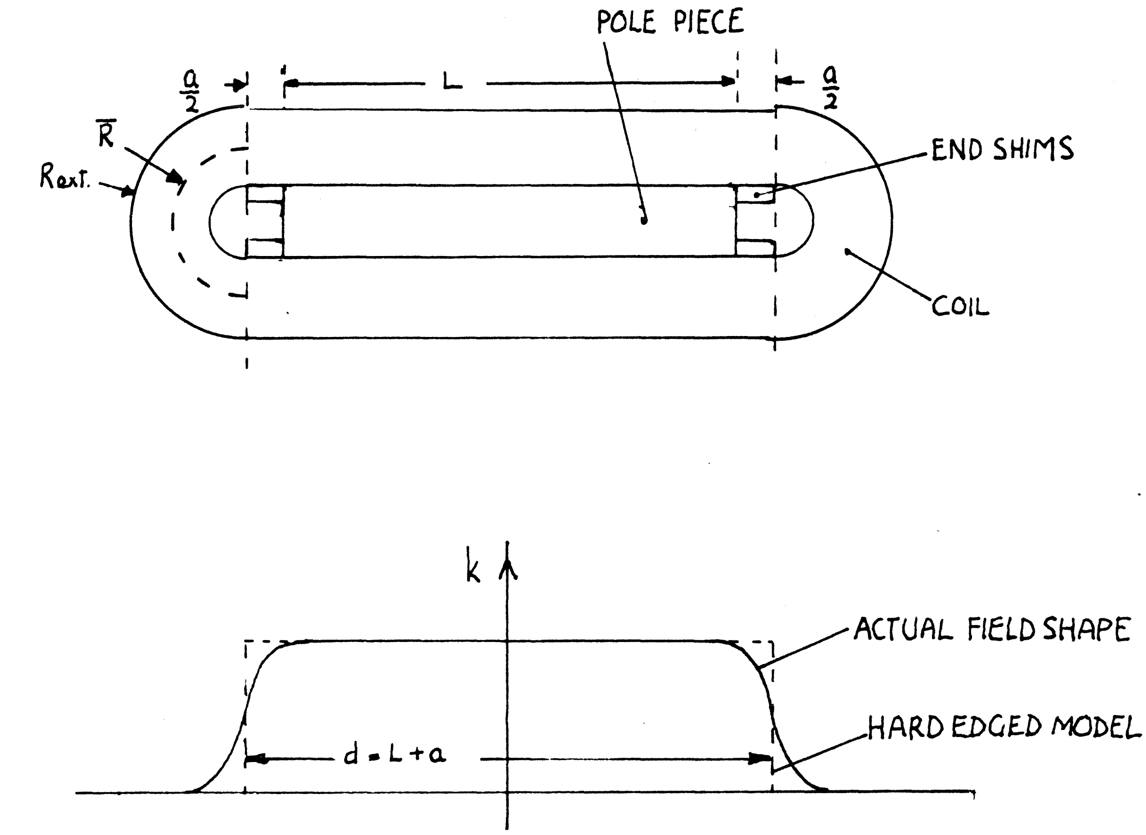
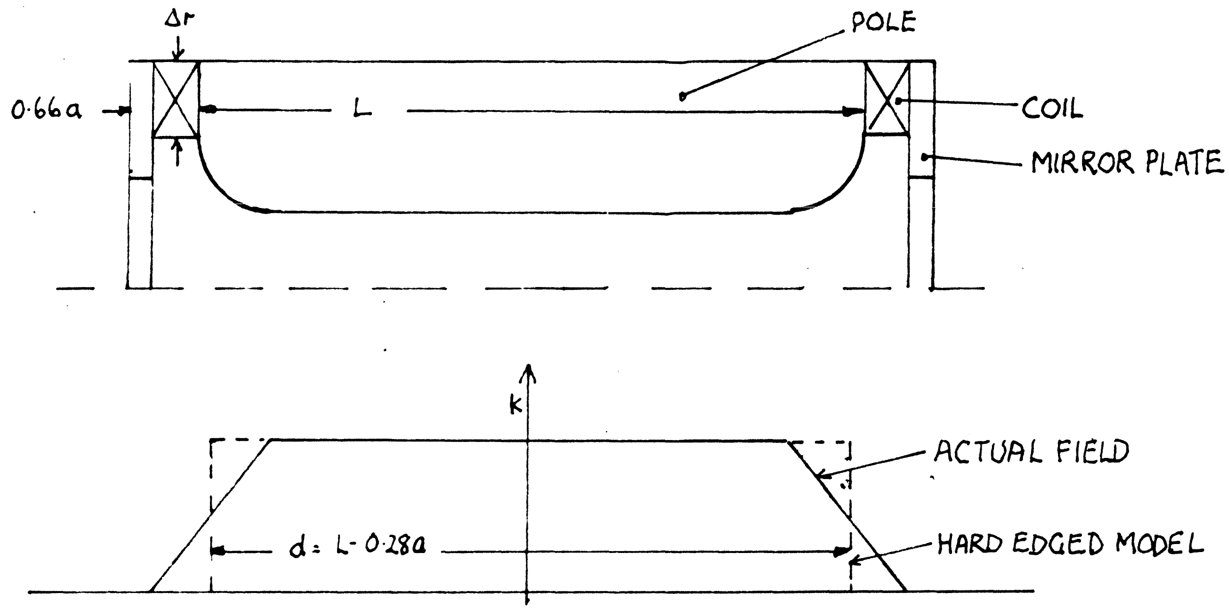

ACCEPTANCE OF PROPOSED FIGURE OF EIGHT QUADRUPOLE COMPARED WITH EXISTING 1 AND 2 m. DESIGNS.
Ω (STERADIANS)
2x10^-4
10^-4
0
FIGURE OF 8 Ψ = 50mr.
CERN 1m. Ψ = 80 mr.
CERN 2m. Ψ = 80 mr.
CERN 1m. Ψ = 50 mr.
CERN 2m. Ψ = 50 mr.
5
10
15
20
p (GeV/c)
A. S.

Fig. 21.



NOTATION FOR FIGURE OF EIGHT, CONVENTIONAL AND ASNER DESIGNS.



NOTATION FOR DESY DESIGN.

FIG. 22

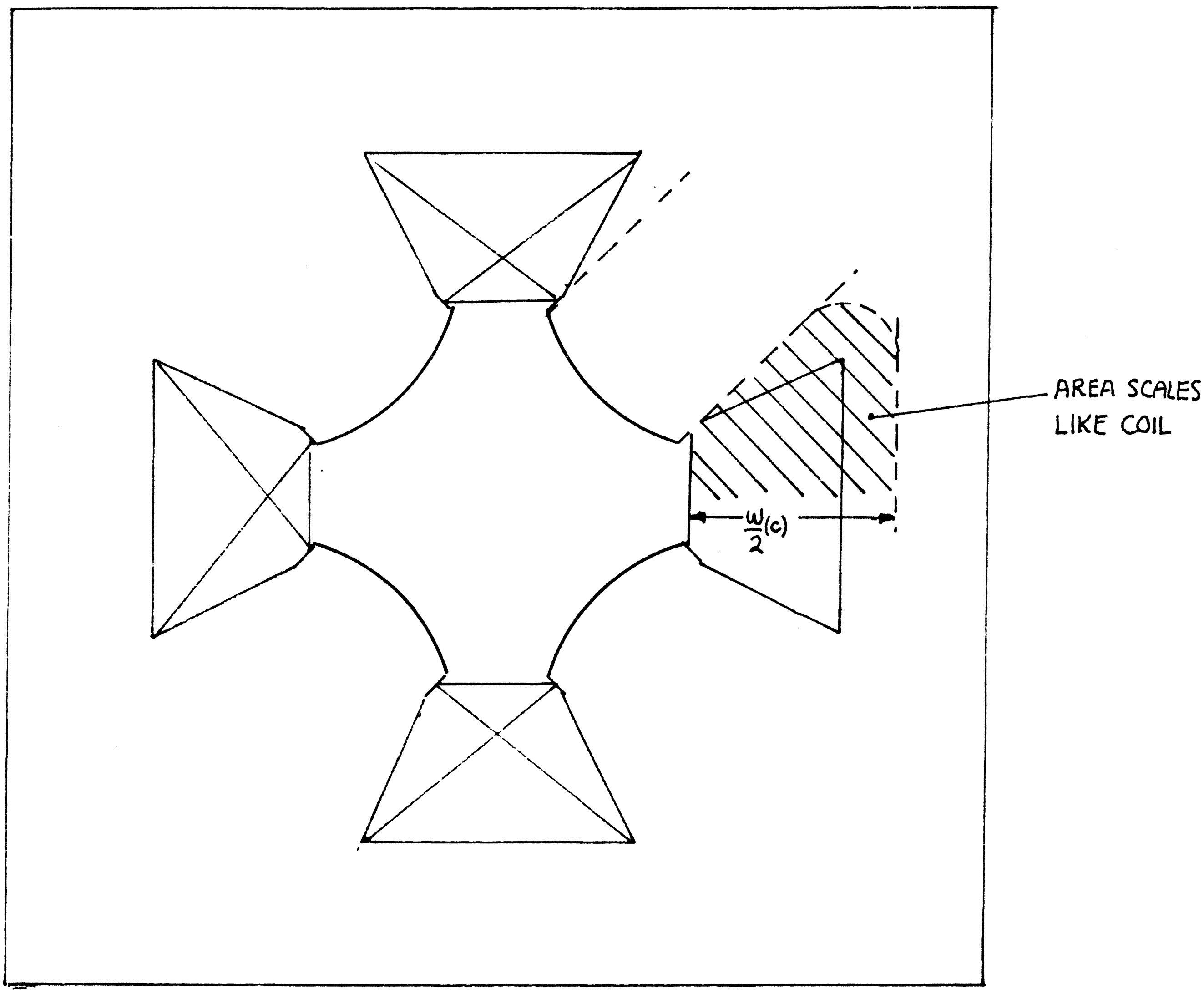


FIG 23.