

Model Independent Predictions of Big Bang Nucleosynthesis from ^4He and ^7Li : Consistency and Implications

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ABSTRACT

We examine in detail how BBN theory is constrained, and what predictions it can make, when using only the most model-independent observational constraints. We avoid the uncertainties and model-dependencies that necessarily arise when solar neighborhood D and ^3He abundances are used to infer primordial D and ^3He via chemical and stellar evolution models. Instead, we use ^4He and ^7Li , thoroughly examining the effects of possible systematic errors in each. Via a likelihood analysis, we find near perfect agreement between BBN theory and the most model-independent data. Given this agreement, we then *assume* the correctness of BBN to set limits on the single parameter of standard BBN, the baryon-to-photon ratio, and to predict the primordial D and ^3He abundances. We also repeat our analysis including recent measurements of D/H from quasar absorption systems and find that the near perfect agreement between theory and observation of the three isotopes, D, ^4He and ^7Li is maintained. These results have strong implications for the chemical and stellar evolution of the light elements, in particular for ^3He . In addition, our results (especially if the D/H measurements are confirmed) have implications for the stellar depletion of ^7Li . Finally, we set limits on the number N_ν of neutrino flavors, using an analysis which carefully and systematically includes all available experimental constraints. The value $N_\nu = 3.0$ fits best with BBN and a 95% CL upper limit of $N_\nu \lesssim 4$ is established.

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1 Introduction

Because of the central role big bang nucleosynthesis (BBN) plays in the standard cosmology, it is crucial to understand how robust the BBN results are. Consequently there has recently been intense scrutiny of different possible sources of uncertainty in the BBN analysis. There has been attention to refining the theoretical calculations, particularly the ^4He yields [1, 2], as well as quantifying their uncertainties due to the errors in their input parameters [3, 4, 5, 6, 7], and there has been examination of the observational procedures and results for possible systematic errors [8, 9]. Finally, there has been a focus on the model-dependence that arises in making the link between the observables and the theory predictions; in this paper we explore in detail an analysis[10] that minimizes this model-dependence.

It is useful to situate our present concerns in the context of the subject of BBN analysis, which consists of three conceptually independent features. (1) The theory itself rests upon first-principles calculations and/or detailed laboratory results; the only really free parameter to the theory calculation is the baryon-to-photon ratio $\eta \equiv n_{\text{B}}/n_{\gamma}$, and the output for each value of η consists of the primordial abundances of the light elements D, ^3He , ^4He , and ^7Li . (We will consider throughout only homogeneous BBN, and furthermore we will first study the “standard” model with $N_{\nu} = 3$; later we will relax the latter assumption and look at possible constraints on N_{ν} .) (2) The observations pertinent to BBN are determinations of the light element abundances in various astrophysical settings; for the most part, these environments are at low or zero redshift, i.e., contemporary to our own, with the exception of QSO absorption line system (QSOALS) D/H measurements which we discuss in more detail below. As the observable abundances are contemporary, one must invoke (3) the final facet of BBN analysis, namely some account of galactic chemical evolution that models the evolution of elemental and isotopic abundances from their primordial state to the present. In this paper we emphasize that the chemical evolution results needed for each of the light isotopes have very different degrees of model-dependence; consequently we will retain only the results which are the most model-independent, and pursue their implications, both for BBN and for chemical evolution.

It is necessary to distinguish between the general implications chemical evolution gives and its detailed results. Generally, metal abundances (i.e., abundances of elements with $Z \geq 6$) increase with time, while the specific histories require the detailed, quantitative modeling of abundance evolution. Turning to the light elements, one finds that to infer the primordial D and ^3He from local and recent abundances (i.e., solar system and the ISM) requires detailed modeling. Of the two, D has the fewest complications and depends only on the chemical evolution model features and not at all on stellar evolutionary data, since D/H is entirely destroyed in the pre-main sequence phases of stars. ^3He is subject to considerable uncertainties regarding not only chemical evolutionary parameters but also its stellar processing; the possible production in low mass stars and the possible destruction through mixing effects in the giant branch stars. On the other hand, to infer primordial abundances for ^4He and ^7Li requires no detailed modeling, not because their evolution is simpler, but because we may determine their evolution empirically. Specifically, we observe ^4He and ^7Li in systems with different, and in particular low metallicities. Since metals increase with time, we may trace the evolution and infer the primordial abundances by extrapolating the observed trends to zero metallicity. This relatively simple procedure is, however, complicated by the very likely presence of systematic errors in the data, and so we will address in detail the possible instances of and distributions

of systematic errors.

Given that the inferred primordial D and ^3He abundances suffer from large chemical evolution uncertainties, there are several approaches one may take when trying to constrain BBN. The traditional strategy has been to use the chemical evolution results for primordial D and ^3He , and either trying to quantify the concomitant uncertainty this introduces [11, 12, 13], or to use a generalized form of chemical evolution which is designed to be “generic” [14, 1, 15]. This approach has the particular appeal that, as it uses all of the light elements, it strongly overconstrains the one-parameter BBN theory. On the other hand, there remains a concern about the chemical evolution framework itself, which plays a central role. As no first-principles calculations exist for this, it is hard to be certain that the idealizations in the available models can capture all relevant aspects of the (unavailable) full solution. This procedure breaks down when the data begin to be refined enough to show inconsistencies between theory and observation and the conclusions for BBN become overly sensitive to highly uncertain modeling.

An alternative means of addressing the chemical evolution uncertainties is to minimize the reliance on chemical evolution by using only its most general features [10]. That is, one uses only the inferred primordial ^4He and ^7Li abundances, rather than all four light element isotopes. Of course, one always has to use chemical evolution at some level, but for ^4He and ^7Li we need only assume that metal abundances (specifically C, N, O, and Fe) increase with time. With only this assumption, and a prescription for including possible systematic errors, one may constrain BBN. Note that in the case of ^7Li , which is observed in old low metallicity stellar populations, we also include an uncertainty on the degree to which these stars did not deplete their ^7Li abundance.

In this paper, we will first review the observational status of all four light element isotopes. In the cases of D and ^3He , we will also review briefly the arguments which limit their implementation in an analysis of BBN. Thus we assess the validity of BBN using the most model-independent data. We quantify the success of BBN using likelihood analysis, including theoretical uncertainties as well as the observational statistical and systematic errors. This analysis is discussed in detail, and attention is given to the significance of the amplitudes of the likelihood functions as indicators of the goodness of fit. With this analysis we find, using only ^4He and ^7Li , that BBN theory is consistent with the observations, and we determine the best values for η . We also attempt to quantify the reliability of these predictions.

Though we are privileging ^4He and ^7Li , this is not to say that these abundances are known with absolute certainty. We take as a starting point the abundances directly from the observations. These observations are in remarkable agreement for a baryon-to-photon ratio which is rather low, $\eta \simeq 1.8 \times 10^{-10}$. A similar range has been also suggested in [16]. We will also show the effect of modifying the primordial abundances. For example, it is sometimes argued that the lithium abundance observed in halo stars is not the primordial value but represents a depleted primordial abundance [17, 18]. We will show how our results change when the effects of ^7Li depletion are included.

As we have noted, the traditional use of solar and ISM data on D/H requires the application of chemical evolution before one may use it to constrain BBN; however, there is a real chance that the recent measurements of D/H in quasar absorption systems [19, 20] are in fact true measures of the primordial D/H abundance. Therefore, we will also show results of the likelihood analysis which includes ^4He , ^7Li and D. In fact we find that the agreement is remarkably good, suggesting perhaps that the putative D in quasar absorption line systems is real and very

close to primordial levels.

Having determined the success of BBN, we may then use it as has long been done, as a powerful tool in particle astrophysics. On the astrophysics side, we predict the abundances of primordial D and ^3He on the basis of the ^4He and ^7Li data. These results are (in addition to being available for testing against quasar absorption line systems) then useful as inputs in chemical evolution models. We also determine the potential for BBN to set limits to ^7Li depletion; these constraints become considerably stronger, when D is included in the BBN analysis. On the particle side, we let N_ν vary, and quantify the allowed extra relativistic degrees of freedom. As we will show, if we were to use BBN, to “predict” the value of N_ν we would find, again remarkably, that the best value is $N_\nu = 3.0$, independent of whether or not we include D in the analyses. To wit, the uncertainty in this “prediction” depends primarily on the uncertainty in the ^4He observations, which at the the 2σ level translates to a BBN upper limit of about 4 on N_ν .

The paper is organized as follows: The observational data on each of the four light element isotopes is discussed in section 2, as is the role of chemical evolution in deriving primordial abundances of each isotope. We will carefully describe our choices of the input abundances and their statistical and systematic uncertainties. In section 3, we describe our likelihood analysis method and present results for BBN consistency in section 4. Included in section 4, is our discussion of the potential for constraining ^7Li depletion in stars, and our analyses including D with an assumed primordial value taken from the QSOALS measurements [20]. In section 5, we discuss the implication of these results on particle astrophysics constraints, particularly, the value of N_ν . Finally, in section 6, we draw our final conclusions.

2 Data

The literature on light element abundance measurements is large, and is well-reviewed in, e.g., references [1, 21, 22]. Here we will give only a brief summary of the current best abundances for each light isotope, with emphasis on new developments, particularly the possible detection of D/H in quasar absorption line systems (QSOALS). We will also discuss the issue of systematic errors in the data, and the possible forms such errors might take.

2.1 Deuterium

As discussed in §1, until recently D abundances were only available for the solar system and the local ISM. The solar value for $\text{D}/\text{H} = (2.6 \pm 0.6 \pm 1.4) \times 10^{-5}$ (see [23] for a recent discussion) has a considerable uncertainty. Individual ISM observations are much more accurate [24] but they appear to show dispersion along different lines of sight through the ISM [25, 26]. This result is unexpected in standard models of chemical evolution, and again suggests their incompleteness. Moreover, neither of these measurements is directly attributable to a primordial value. The D (and ^3He) in these systems has suffered a considerable degree of processing, and so determining a primordial abundance from these requires detailed chemical evolution modeling. As we have argued, our approach is to avoid such modeling, and so we will not employ these results.

The observational situation for D is rapidly changing however, and is sparking a renewed interest in D evolution. Observers have employed high-resolution spectra of quasar absorption

line systems to determine the D/H abundance via its isotope-shifted Lyman- α line. Initial reports [19] gave a high abundance, $D/H \sim 2 \times 10^{-4}$, but there remained a significant chance of D being mimicked by an interloping H cloud, which would lead to an overestimate of the D abundance. Recently, new observations [20] of this same system have in fact resolved this original system into two components each with an abundance comparable to the old result. The weighted average of these two measurements is [20],

$$\left(\frac{D}{H}\right)_{z=3.32} = (1.9 \pm 0.4) \times 10^{-4} \quad (1)$$

The resolution into two components dramatically reduces the likelihood of an interloper, strongly suggesting that the observations are really measuring D. Caution is still warranted however, (as it is with any new observation) until the systematic uncertainty in these observations is clarified and more importantly until these results can be confirmed in the QSOALS in other directions. For these reasons, we do not include the D/H results in our first BBN analysis. We do however, repeat our analyses using the primordial value for D/H from Eq. (1).

2.2 Helium-3

Abundances of ^3He are only available for the solar system and the ISM. The solar value for $^3\text{He}/\text{H}$ is $(1.5 \pm 0.2 \pm 0.3) \times 10^{-5}$ and varies between $(1 \text{ to } 5) \times 10^{-5}$ in galactic HII regions [27] (see also [23] for a general discussion). Thus as in the case of D, one must use detailed chemical evolution models to extrapolate a primordial abundance. This procedure is particularly uncertain at the moment since it is even unclear, on the basis of stellar evolution theory, whether one expects ^3He to increase or decrease with time. On the one hand, ^3He has long been thought to be produced by low mass stars [28, 29], an idea apparently supported by the observation of high ^3He abundances in several planetary nebulae [30]. On the other hand, chemical evolution models including such ^3He production lead to overproduction of ^3He relative to the observations [31, 32, 23, 33].

There has been some recent progress on this question as the now venerable models for ^3He production have been updated [34, 35]. The different groups agree in finding that there is net production in the standard model. It has been suggested [36, 37, 38, 39], however, that in fact stellar mixing might destroy ^3He . The inclusion of these effects certainly goes a long way in alleviating the problems caused by a high primordial D/H abundance [23, 40, 41]. Indeed, a thorough study of ^3He evolution along with other isotopes produced by low-mass stars would help to clarify the situation.

Given the compounded uncertainty regarding the evolutionary history of ^3He from both stellar and chemical evolution, we know of no model-independent way to obtain a statistically significant primordial abundance for ^3He and therefore we do not include it in our analysis. We emphasize, however, that ^3He remains an important tool for chemical evolutionary models and perhaps also for stellar evolutionary models as the destruction of ^3He has important consequences on the abundances of other isotopes.

2.3 Helium-4

The most useful site for obtaining ^4He abundances has proven to be H II regions in irregular galaxies. Such regions have low (and varying) metallicities, and thus are presumably more

primitive than such regions in our own Galaxy. Because the ${}^4\text{He}$ abundance is known for regions of different metallicity one can trace its evolution as a function of metal content, and by extrapolating to zero metallicity we can estimate the primordial abundance.

There is a considerable amount of data on ${}^4\text{He}$, O/H, and N/H in low metallicity extragalactic HII regions [42, 43, 44]. In fact, there are over 50 such regions observed with metallicities ranging from about 2–30% of solar metallicity. The data for ${}^4\text{He}$ vs. either O/H or N/H is certainly well correlated and an extensive analysis [8] has shown this correlation to be consistent with a linear relation. While individual determinations of the ${}^4\text{He}$ mass fraction Y have a fairly large uncertainty ($\Delta Y \gtrsim 0.010$), the large number of observations lead to a *statistical* uncertainty that is in fact quite small. A recent calculation [8, 45] gives

$$Y_p = 0.234 \pm 0.003(\text{stat}) \pm 0.005(\text{syst}) \quad (2)$$

with similar central values and statistical errors obtained by other groups.

We have claimed that the primordial abundance of ${}^4\text{He}$ used below is devoid of any serious dependence on galactic chemical evolution. The role chemical evolution plays here is simply to verify that the linear extrapolation to zero metallicity is a meaningful estimate of the primordial abundance Y_p . The basic assumption that metal abundances increase linearly with time has already proven to be a secure ansatz; in addition to the strong correlation of the data between different metallicities, we are further assured of the accuracy of the estimate because the source of the data is derived from very low metallicity environments.

Despite the very small statistical uncertainty in the fit shown in Eq. (2), more serious complications arise in the process of extracting abundances from linestrengths. As pointed out elsewhere [9], assumptions about the H II region and model-dependencies could introduce a significant systematic error. While the value of $\sigma_{\text{sys}} = 0.005$ we quote above attempts to estimate this error (see e.g. [46, 8]), its magnitude and distribution are not well understood. Thus in our analysis, we will examine the effect of different assumptions concerning the systematic errors.

2.4 Lithium–7

The primordial ${}^7\text{Li}$ abundance is best determined by studies of the Li content in various stars as a function of metallicity (in practice, the Fe abundance). At near solar metallicity, the Li abundance in stars decreases with decreasing metallicity, dropping to a level an order of magnitude lower in extremely metal poor Population II halo stars with $[\text{Fe}/\text{H}] \lesssim -1.3$ ($[\text{Fe}/\text{H}]$ is defined to be the \log_{10} of the ratio of Fe/H relative to the solar value for Fe/H). At lower values of $[\text{Fe}/\text{H}]$, the Pop II abundance remains constant down to the lowest metallicities measured, (some with $[\text{Fe}/\text{H}] < -3$!) and form the so-called ‘‘Spite plateau.’’ With Li measured for nearly 100 such stars, the plateau value is well established. We use the recent results of [47] to obtain the ${}^7\text{Li}$ abundance in the plateau

$$\frac{{}^7\text{Li}}{\text{H}} = (1.6 \pm 0.1) \times 10^{-10} \quad (3)$$

where the errors are statistical only. Again, if we employ the basic chemical evolution conclusion that metals increase linearly with time, we may infer this value to be indicative of the primordial Li abundance.

One should be aware that there are considerable systematic uncertainties in the plateau abundance. First there are uncertainties that arise even if one assumes that the present Li abundance in these stars is a faithful indication of their initial abundance. The actual ${}^7\text{Li}$ abundance is dependent on the method of deriving stellar parameters such as temperature and surface gravity, and so a systematic error arises due to uncertainties in stellar atmosphere models needed to determine abundances. While some observers try to estimate these uncertainties, this is not uniformly the practice. To include the effect of these systematics, we will introduce the asymmetric error range $\Delta_1 = {}^{+0.4}_{-0.3}$ which covers the range of central values for ${}^7\text{Li}/\text{H}$, when different methods of data reduction are used (see eg. [45]).

Another source of systematic error in the ${}^7\text{Li}$ abundance arises due to uncertainty as to whether the Pop II stars actually *have* preserved all of their Li. It has been suggested that, e.g., rotational effects could reduce the initial Li abundance to a much lower level, and models have been advanced which claim to do so while maintaining the plateau behavior with respect to metallicity and temperature [17, 18]. While the detection of the more fragile isotope ${}^6\text{Li}$ in two of these stars may argue against a strong depletion [48], it is difficult to exclude depletion of the order of a factor of two. Furthermore there is the possibility that the primordial Li has been supplemented, by the time of the Pop II star’s birth, by a non-primordial component arising from cosmic ray interactions in the early Galaxy [49, 50, 51]. While such a contribution cannot dominate, it could be at the level of tens of percent [49]. Note that this effect acts only to correct *downwards* the true primordial Li abundance from the plateau level. To allow for these possibilities, we will investigate the effect of a second Li systematic error, having the range $\Delta_2 = {}^{+1.6}_{-0.3}$. In fact, in §4.2, we will turn the problem around, and by using the D measurements in QSOALS we will offer BBN *constraints* on the level of Li depletion.

3 Likelihood Analysis

Monte Carlo and likelihood analyses have proven to be useful tools in testing the consistency of BBN [3, 4, 5, 6, 7, 10, 52]. As the observational determinations of the light element abundances improve, there is little justification for neglecting the uncertainties in the BBN predictions. For ${}^4\text{He}$, these uncertainties are dominated by the uncertainty in the neutron half-life which have been dramatically reduced in the last several years, and lead to deviations (1σ) $\lesssim 0.001$. For D and ${}^3\text{He}$, the uncertainty in the BBN yields are less than 10%. Of the four light element isotopes, the largest uncertainty resides with ${}^7\text{Li}$, where the 1σ deviations remain as large as 20–25% [7].

If we restrict our attention to the standard big bang model, in the context of the standard electroweak model with three neutrino flavors, there is one single unknown parameter in the standard model of big bang nucleosynthesis, the baryon-to-photon ratio, η . For a given value of η , the uncertainties in the BBN calculation of the light element abundances stem from the uncertainties in the nuclear (and weak) interaction rates employed. When we consider deviations from the standard electroweak theory, which can sometimes be parameterized by changing the number of light neutrino degrees of freedom, the abundances will be sensitive to the choice of N_ν . As we stated earlier, it will be sufficient to use only two of the light element abundances to test this one-parameter theory (one is enough to constrain it). If and when they are applicable, the others will even more strongly test/constrain the theory.

The Monte Carlo calculations in BBN make available a distribution of abundances at each value of η , based on the uncertainties in the nuclear and weak interaction rates, which we take to be Gaussian distributed. As in [10], we will use as the starting point the Monte Carlo results from Hata et al. [7]. Thus for each of the light elements we obtain a theoretical distribution function, which depends on η and on the element abundance. For example, we can begin with a likelihood distribution from the BBN calculation for ${}^4\text{He}$:

$$L_{\text{BBN}}(Y, \eta) = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-(Y - Y_{\text{BBN}}(\eta))^2 / 2\sigma_1^2} \quad (4)$$

where $Y_{\text{BBN}}(\eta)$ is the central value for the ${}^4\text{He}$ mass fraction produced in the big bang, and $\sigma_1(\eta)$ is the uncertainty in that value derived from the Monte Carlo calculations. Note that there is a mild dependence on η in σ_1 .

There is also a likelihood distribution based on the observations. In this case we have two sources of errors as discussed above, a statistical uncertainty, σ_2 and a systematic uncertainty, σ_{sys} . Unfortunately, there is no well defined way to treat the systematic errors. One possibility is to assume that the systematic error is described by a top hat distribution [7, 53]. In this case, the convolution of the top hat distribution and the Gaussian (to describe the statistical errors in the observations) results in the difference of two error functions

$$L_{\text{O}}(Y, Y_{\text{O}}) = \frac{1}{2(\sigma_{\text{sys}+} + \sigma_{\text{sys}-})} \left[\text{erf} \left(\frac{Y - Y_{\text{O}} + \sigma_{\text{sys}-}}{\sqrt{2}\sigma_2} \right) - \text{erf} \left(\frac{Y - Y_{\text{O}} - \sigma_{\text{sys}+}}{\sqrt{2}\sigma_2} \right) \right] \quad (5)$$

where in this case, Y_{O} is the observationally determined value for the primordial ${}^4\text{He}$ mass fraction and we have allowed for the possibility of asymmetric systematic uncertainty. The distribution (5) is normalized to one with respect to integration over Y .

In addition to the top-hat distribution for the systematic uncertainty, we have also derived the likelihood functions assuming that the systematic errors are Gaussian distributed. In this case, the convolution also leads to a Gaussian, with an error $\sigma^2 = \sigma_2^2 + \sigma_{\text{sys}}^2$. Finally, as a third case, we have also simply shifted the mean value Y_{O} by an amount $\pm\sigma_{\text{sys}}$. In this case L_{O} is also a Gaussian, with spread σ_2 . These functions were similarly derived for ${}^7\text{Li}$.

For ${}^4\text{He}$, we constructed a total likelihood function for each value of $\eta_{10} \equiv 10^{10}\eta$, convolving for each the theoretical and observational distributions

$$L_{\text{total}}^{{}^4\text{He}}(\eta) = \int dY L_{\text{BBN}}(Y, \eta) L_{\text{O}}(Y, Y_{\text{O}}) \quad (6)$$

An analogous calculation was performed for ${}^7\text{Li}$. At this point we could use the two likelihood functions, and their product, to calculate (say) 68% CL intervals in η . This would tell us the values of the parameter (η) for which the theory (BBN) best fits the data. It tells us nothing however about whether that “best” fit is a good fit. For example, if the observed ${}^7\text{Li}$ abundance were several standard deviations below the minimum value in the ${}^7\text{Li}$ vs η curve, then clearly the data fit the theory very poorly. The likelihood function for ${}^7\text{Li}$ would still however show a peak at a value of η near that minimum, and would lead to a 68% CL interval in η similar to that obtained for a much higher observed value. If the only errors we had to concern ourselves

with were Gaussian then we could calculate a χ^2 from the likelihood, and this would then give us the probability that the theory fits the data (or more correctly, that the theory fails to fit the data). Unfortunately of course, the errors are not all Gaussian and there is no standard technique for calculating the goodness of fit. In this paper we estimate the goodness of fit from the height of likelihood functions at their peak as described below.

We begin by defining a renormalized probability distribution in η by a simple rescaling, such that

$$\int L_{\text{total}}^{4\text{He}}(\eta)d\eta = 1. \quad (7)$$

Likelihood functions like this, derived also for lithium, form the basis of our subsequent analysis. We should note that, while this procedure in principle throws away the information on whether the observed lithium abundance is below the minimum value in the lithium vs η curve (as in our example above) this is never an issue in practice.

We will see that each of the abundances can individually be reconciled with the one-parameter theory, each predicting a distinct concordance region in η . Because of the independence of the observational distributions folded with the (dependent) theoretical distributions to give our likelihood distributions, these “measurements” of η are all independent of each other. Demanding that two or more, in our case in particular ^4He and ^7Li , be fit simultaneously, constitutes a test for the theory. To do this in practice, one examines the product of the individual likelihoods, $\mathcal{L} = L_{\text{total}}^{4\text{He}}(\eta)L_{\text{total}}^{7\text{Li}}(\eta)$ which yields information on the goodness of fit based on the magnitude of this quantity at the peak of the combined distribution (if any) and of the spread in the allowed values in η_{10} .

Before we move on to discuss specific results, it will be useful to discuss the issue of goodness of fit, and how it is to be quantified. In the case of BBN the number of “measurements” never increases; we will always have only 2 to 4 of them. So there is no hope of ever being able to apply the central limit theorem to obtain a well defined mean and spread in the predicted range of η in the usual sense of finding “world averages” of experimental quantities from ensembles of independent experimental results. On the contrary, in BBN one can always go back and refine the errors of the individual “measurements”, thus getting an improved test of the theory that way. Nevertheless, the statistical likelihood analysis will always be hampered by the small number of data points. Indeed, even if all of the errors are Gaussian, the usual χ^2 analysis only marginally applies for $N = 2 - 4$. However, in our case the errors and distributions are in fact far from Gaussian. Though we can still define a quantity $\chi^2 = -2 \ln \mathcal{L}$, its utility is unclear. However, the combined likelihood function does carries certain information about the goodness of fit; we will choose to access this by examining the significance of the amplitude.

To introduce our approach, we first consider some simple cases to determine what the magnitude at the peak of the combined distribution will tell us. For example, suppose that we have two normalized Gaussian distributions, $L_1(x) = (1/\sqrt{2\pi}\sigma_1)e^{-x^2/2\sigma_1^2}$, and $L_2(x) = (1/\sqrt{2\pi}\sigma_2)e^{-(x-\mu)^2/2\sigma_2^2}$. (Think of these as toy models for $L_{\text{total}}^{4\text{He}}(\eta)$ and $L_{\text{total}}^{7\text{Li}}(\eta)$ respectively.) The product of these distributions is also a Gaussian

$$L_1(x)L_2(x) = (1/2\pi\sigma_1\sigma_2)e^{-\mu^2/2\sigma^2}e^{-\frac{\sigma^2}{2\sigma_1^2\sigma_2^2}(x-\frac{\sigma_1^2}{\sigma^2}\mu)^2} \quad (8)$$

where $\sigma^2 = \sigma_1^2 + \sigma_2^2$. In the event that the two distributions have the same mean value, $\mu = 0$, then the peak of our combined distribution has a magnitude $1/2\pi\sigma_1\sigma_2$. When the distributions

are offset, even though the product is still a Gaussian, the magnitude at the peak is now suppressed by a factor $e^{-\mu^2/2\sigma^2}$. Clearly for $\mu \gg \sigma$, this suppression can be significant and indicates a lack of goodness of fit. It would seem appropriate, therefore, to compare the value of the peak of the combined likelihood distribution (6) with the quantity $(1/2\pi\sigma_1\sigma_2)$. Despite the fact that the suppression factor in (8) is exponential, unless $\mu \gg \sigma$, it will take values in the tens of percent. For example, if $\sigma_1 \simeq \sigma_2$ and if our distributions are offset by 1 (2) σ , the suppression factor is still relatively mild, 0.78 (0.37). Even for a distribution whose peaks are separated by 3σ , i.e. distributions that we would generally judge as being inconsistent, the suppression is only about 0.1. Thus when we make the comparison at the peak of our combined distribution to $(1/2\pi\sigma_1\sigma_2)$, any value significantly less than 1 will indicate a poor fit.

Thus far in our example, we have only considered the case in which the likelihood functions, $L(\eta)$ are Gaussian. However, in general our likelihood distributions for ${}^4\text{He}$ and ${}^7\text{Li}$ will *not* be Gaussians. In particular, as we will see, in the case of ${}^7\text{Li}$, our distribution is double peaked. We can however still make the comparison to $(1/2\pi\sigma_1\sigma_2)$ if we define the respective σ 's as the half-width at half-maximum of a local peak. This will be made clear when we consider specific examples below.

In considering the bimodality of the Li likelihood function, it is useful to adopt a larger perspective. Over an extended range of η , beyond the canonical $\eta_{10} \sim 1 - 10$ range,¹ all of the light element abundances are *non-monotonic* functions of η . Thus, including a larger range of η in one's analysis would yield multiple peaks in each isotope's individual likelihood function. Of course, the "new" peaks would not overlap between different isotopes, and the predicted range for η as given by the combined likelihood function would remain unchanged. Hence it is reasonable to independently normalize each well-isolated peak in an isotopic likelihood. This procedure is straightforward for all of the isotopes other than Li, whose peaks are not isolated and can overlap to some extent. Thus we normalize over both of these peaks and therefore expect a reduced amplitude in the combined distribution.

4 Model-Independent Odds on BBN

4.1 Standard Cases

The procedure that we set up in the preceding section would be straightforward to carry out if it were not for the complication of the treatment of the systematic uncertainties in the data. We will therefore present results for various different assumptions regarding the data. As described in section 2, we will always make comparisons with respect to our standard case in which the data is described by Eqs. (2) and (3), namely $Y_p = 0.234 \pm 0.003 \pm 0.005$ and ${}^7\text{Li}/\text{H} = (1.6 \pm 0.1 \text{ }^{+4}_{-3}) \times 10^{-10}$. The effects of lithium depletion are neglected at this stage, and the systematic errors are described by a top-hat distribution.

With these assumptions, we have calculated the likelihood functions for ${}^4\text{He}$ and ${}^7\text{Li}$ [10] which are shown in figure 1. The shapes of these curves are characteristic, with one peak for ${}^4\text{He}$ (whose abundance rises monotonically with η), and two for ${}^7\text{Li}$ (whose abundance goes

¹Indeed, if one is testing BBN itself, one should in principle consider a large range in η , but of course experience with BBN calculations, as well as other available bounds on Ω_{B} and hence on η , lead one to focus straightaway on the canonical range.

through a minimum). In this case (and most others) the minimum theoretical Li is somewhat below most of the observational values and so the sides of the minimum are favored, leading to the two peaks; i.e., for a given observational value of ${}^7\text{Li}$, there are two values for η at which this may be achieved. By glancing at figure 1, it does not take very much statistical machinery to see that the BBN predictions for ${}^4\text{He}$ is consistent with the low- η value of the ${}^7\text{Li}$ prediction when compared with the observations. These distributions are clearly consistent.

The combined likelihood, for fitting both elements simultaneously, is given by the product of the two functions in figure 1, and is shown in figure 2. As we discussed in the previous section, we have scaled the combined distribution by a factor $2\pi\sigma_4\sigma_7$ where $\sigma_4 = .66$ is the half-width at half-maximum of $L_{\text{total}}^{4\text{He}}(\eta)$ and $\sigma_7 = .38$ is the corresponding quantity for $L_{\text{total}}^{7\text{Li}}(\eta)$. In this latter case, the distribution $L_{\text{total}}^{7\text{Li}}(\eta)$ is clearly far from Gaussian and the value chosen for σ_7 corresponds to the low- η peak only. Because the ${}^7\text{Li}$ distribution is nearly equally divided into two peaks we should expect completely overlapping distributions to yield a peak value of only ~ 0.5 in the combined distribution². This is what one finds in figure 2, which does show concordance, the peak is indeed close to 0.5 at $\eta_{10} = 1.8$.

The allowed 68% CL and 95% CL ranges are

$$\begin{aligned} 1.6 < \eta_{10} < 2.8 & \quad 68\% \text{CL} \\ 1.4 < \eta_{10} < 3.8 & \quad 95\% \text{CL} \end{aligned} \tag{9}$$

Note that these intervals are based on standard statistical techniques, and are not dependent on our chosen method of assigning a goodness-of-fit to the height of the likelihood peak.

Thus, for this ‘‘standard’’ case, we find that the abundances of ${}^4\text{He}$ and ${}^7\text{Li}$ are consistent, and select an η_{10} range which overlaps with (at the 95% CL) the longstanding favorite range around $\eta_{10} = 3$. Further, by finding concordance (in this case) using only ${}^4\text{He}$ and ${}^7\text{Li}$, we infer that if there is problem with BBN analysis, it must arise from D and ${}^3\text{He}$ and is thus tied to chemical evolution. The most model-independent conclusion is that standard BBN with $N_\nu = 3$ is not in jeopardy, but there are problems with our detailed understanding of D and particularly ${}^3\text{He}$ chemical evolution.

The concordance range for η given in Eq.(9) allows us to make definite predictions for the primordial abundances of D and ${}^3\text{He}$. The 95 (68) % CL ranges in (9) corresponds to $5.5(8.9) < (\text{D}/\text{H}) \times 10^5 < 28(22)$ with a best value for $\text{D}/\text{H} = 1.8 \times 10^{-4}$ at $\eta_{10} = 1.8$. As we have already noted, this value for D/H agrees incredibly well with the QSOALS observations in [20]. For ${}^3\text{He}$, we have, $1.4(1.7) < ({}^3\text{He}/\text{H}) \times 10^5 < 2.7(2.5)$, with a best value ${}^3\text{He}/\text{H} = 2.3 \times 10^{-5}$ which, it should be noted, is larger than the solar value of ${}^3\text{He}/\text{H}$.

In the preceding analysis, we have described the systematic uncertainty by a top-hat distribution. This prescription gives a tight set of errors which provides a stringent test to BBN, however, it also gives the most restrictive bounds on η . Thus when we relax this assumption and treat the systematics as Gaussian distributed, we will expect concordance over a greater range. In figure 3, we show the ${}^4\text{He}$ and ${}^7\text{Li}$ likelihood functions when the systematic errors are Gaussian. It should not be surprising that the concordance is present at the same level though

²Note that the resulting peak value of 0.5 does carry some statistical information. For example, suppose our ${}^7\text{Li}$ distribution takes a form in which the peaks were different from one another (e.g. if the left peak were a factor of 10 smaller than the right peak). While we could argue that we would expect a combined fit of only 0.1, we should also conclude a low confidence level for consistency. Thus it is not appropriate to scale out this factor in the combined distribution function. However, the ${}^7\text{Li}$ peaks are nearly always about equal.

the widths of the distributions are larger. In this case, $\sigma_4 = .86$ and $\sigma_7 = .53$. The combined distribution shown in figure 4 is also similar to that in figure 2, the peak is again close to 0.5 and the width of the distribution is larger, placing greater weight at higher η . This distribution leads to a predicted range for η , $1.4(1.7) < \eta_{10} < 4.3(3.4)$. The peak of the distribution is unchanged at $\eta_{10} = 1.8$. This range in η corresponds to $4.6(6.5) < D/H \times 10^5 < 28(20)$ and $1.3(1.5) < ({}^3\text{He}/\text{H}) \times 10^5 < 2.7(2.4)$.

Having found concordance for BBN in the standard model, we now vary the parameters of the distributions—in particular, those for the systematic errors—and examine the results.

4.2 ${}^7\text{Li}$ Depletion

Next we would like to test the uncertainty in ${}^7\text{Li}$ given by Δ_2 . That is, what is the effect of ${}^7\text{Li}$ depletion and cosmic-ray production? We will first assume that the ${}^7\text{Li}/\text{H}$ abundance is shifted up by Δ_2 , that is, we assume a factor of 2 depletion in the Pop II halo stars. We will treat the error in Δ_1 , as well as the systematic error in ${}^4\text{He}$ as Gaussian, this being a more conservative approach than the top-hat method.

By shifting the observational primordial ${}^7\text{Li}$ abundance upwards, we are in effect separating the two peaks in the ${}^7\text{Li}$ likelihood distribution. The results for this case is shown in figure 5. There is still some overlap between the ${}^4\text{He}$ and ${}^7\text{Li}$ distributions. In this case however, there is a distinction between the two lithium peaks. The half-width of the low- η peak is 0.19 while for the high- η peak it is 0.68. In the combined distribution, we have scaled the overlap regions according to these respective peaks, which can be justified in this case since the combined distribution now shows two distinct distributions as seen in figure 6. This combined likelihood distribution is in clear contrast to the previous cases considered. Note the peak value at low- η is only about 0.15 (it is even lower for the high- η peak). We do not consider this to be a good fit of theory to data; yet on the basis of this result relying ${}^4\text{He}$ and ${}^7\text{Li}$ alone, we cannot quite exclude depletion at this level.

To salvage the consistency of BBN with a factor of 2 depletion in Li one might consider an increase the systematic uncertainty in ${}^4\text{He}$ by a factor of 2, to $\sigma_{\text{sys}} = 0.01$. By doing so, we would flatten and greatly broaden the ${}^4\text{He}$ likelihood distribution. This would of course produce a greater overlap with the high- η peak in the ${}^7\text{Li}$ distribution. However, we can not place any great significance to the improved overlap in this case, because by increasing the systematic uncertainty to this extent, we have in effect taken all of the predictive power away from ${}^4\text{He}$. (The ${}^4\text{He}$ distribution in this case is now mildly peaked at $\eta_{10} \simeq 1.8$ at a value of 0.2, and is only slowly decreasing out past $\eta_{10} = 10$.) Thus we are only really considering ${}^7\text{Li}$ in this case, and some level of consistency is almost trivially guaranteed.

Another possibility to salvage BBN consistency with ${}^7\text{Li}$ depletion would be to shift up the primordial ${}^4\text{He}$ abundance. However, a shift up by an amount $\sigma_{\text{sys}} = 0.005$ to a primordial value $Y_p = 0.239 \pm 0.003$ results in a terrible fit in which the combined distribution peaks at a value of 0.05! Only if the primordial abundance is shifted by at least 0.01 is the overlap between ${}^4\text{He}$ and ${}^7\text{Li}$ significant enough to give a reasonable combined distribution. It is our opinion, however, that in this case we are straying far from the raw observations which show the remarkably good compatibility in figures 1-4.

Let us also comment that if one assumes that perhaps 20% of the observed ${}^7\text{Li}$ were produced by cosmic-ray nucleosynthesis rather than BBN, then compatibility is further improved. If we

shift the ${}^7\text{Li}$ down by Δ_2 , the two ${}^7\text{Li}$ peaks begin to merge and there is a strong overlap in the distribution functions. The combined distribution peaks at $\eta_{10} = 2.0$ in this case.

Finally, we point out that in order to bring the probability of overlap of the theoretical and observational values of the Li abundance³ below the 5 per cent level, would require reducing the observationally inferred abundance by an order of magnitude. This would be the lowest observational value BBN could be in agreement with, but at present the limit does not seem at all interesting.

4.3 Primordial D/H

It is interesting to note that the central (and strongly) peaked value of η_{10} determined from the combined ${}^4\text{He}$ and ${}^7\text{Li}$ likelihoods is at $\eta_{10} = 1.8$. The corresponding value of D/H is 1.8×10^{-4} , very close to the value of D/H in quasar absorbers in the published set of observations [19, 20]. It is not clear if this is a coincidence or if we really have evidence that three of the light element abundances point to the same value of η_{10} .

Noting the perhaps still preliminary nature of the QSOALS D/H measurements, we move ahead and perform the likelihood analysis for the three light elements D, ${}^4\text{He}$ and ${}^7\text{Li}$. To include D/H, we proceed in much the same way as with the other two light elements. We compute likelihood functions for the BBN predictions as in Eq. (4) and the likelihood function for the observations using $\text{D/H} = (1.9 \pm 0.4) \times 10^{-4}$. In this case, since the systematic error is not stated, it is neglected here and we adopt a simple Gaussian form for the statistical errors. These are then convolved as in Eq. (6). In figure 7, the resulting normalized distribution, $L_{\text{total}}^{\text{D}}(\eta)$ is super-imposed on distributions appearing in figure 1. It is indeed startling how the three peaks, for D, ${}^4\text{He}$ and ${}^7\text{Li}$ are literally on top of each other. In figure 8, we show the combined distribution which has now been scaled by $((2\pi)^{3/2}\sigma_2\sigma_4\sigma_7)^{-1}$ with $\sigma_2 = .29$. We have a very clean distribution and prediction for η :

$$\begin{aligned} 1.65 < \eta_{10} < 2.06 & \quad 68\% \text{CL} \\ 1.50 < \eta_{10} < 2.37 & \quad 95\% \text{CL} \end{aligned} \tag{10}$$

with the peak of the distribution at $\eta_{10} = 1.75$. The absence of any overlap with the high- η peak of the ${}^7\text{Li}$ distribution has considerably lowered the upper limit to η . Overall, the concordance limits in this case are dominated by the deuterium likelihood function, which, we should caution again, is based on an observation along a single line of sight.

A high value of D/H, as measured in the QSOALS, requires a significant amount of D destruction over the history of the Galaxy. Although this alone is not necessarily problematic, since chemical evolution models can be constructed to account for such factors of deuterium destruction [31], it compounds the problem of ${}^3\text{He}$ overproduction. The fact that ${}^3\text{He}$ was problematic even at higher $\eta_{10} \sim 3$, was our motivation for neglecting ${}^3\text{He}$ to begin with.

Finally, we consider the effects of the D/H measurements when ${}^7\text{Li}$ is assumed to be depleted. In figures 9 and 10, we show the individual and combined likelihood functions which can be compared with those in figures 5 and 6 with D/H. In the combined distribution, the high- η peak is now gone, and the low- η peak is hardly significant. Recall (§4.2) that ${}^7\text{Li}$ depletion could be

³This is given by an integral equation similar to (7) for lithium with the lower limit of integration set to the theoretical minimum and letting the observational value vary until the value of the integral becomes 0.05.

tolerated by either an enlargement of the ${}^4\text{He}$ systematic error or a shift upwards in the ${}^4\text{He}$ abundance, which would allow significant overlap with the high- η peak. However, when the D/H observations are included in the analysis, the high- η peak is absent in the combined distribution and the low- η peak takes a value of less than 0.05. Once the D/H measurements are confirmed, they will provide a strong constraint on the degree of ${}^7\text{Li}$ depletion in halo stars when the consistency of big bang nucleosynthesis is assumed.

5 Constraints on N_ν

In the previous section we have demonstrated the health of BBN when model-independent constraints are applied; we arrived at these conclusions without first assuming the validity of BBN. Now we assume its validity and, as has been done traditionally, use it to constrain new physics. In particular, we will consider the effects of our analyses on the limit to the number of neutrino flavors, N_ν .

The light element isotope which is most sensitive to the number of neutrino flavors is ${}^4\text{He}$. In fact, it has been common [1] to express (by means of a fit) the ${}^4\text{He}$ mass fraction as a function of η_{10} , N_ν , and the neutron meanlife. For example, near $\eta_{10} = 2$ and $N_\nu = 3$, we have,

$$Y = 0.2262 + 0.0131(N_\nu - 3) + 0.0135 \ln \eta_{10} + 2 \times 10^{-4}(\tau_n - 887) \quad (11)$$

Therefore, given an upper limit to the ${}^4\text{He}$ mass fraction *and* a lower limit to η one can derive an upper bound to N_ν . Indeed, unless the lower bound to η_{10} is greater than 0.4 (something we are assured of now by the D/H measurements), there is no upper limit to N_ν [54]. In [14], it was argued that the solar value of $\text{D} + {}^3\text{He}$ could provide a reasonable lower bound to η . Recall that this argument was made before the evidence from planetary nebulae pointed towards the need for ${}^3\text{He}$ production in low mass stars and before the QSOALS measurements of extremely high D/H. Therefore, the role of chemical evolution was not believed to be critical. Upper limits to N_ν were then derived by inserting the 2σ upper limit to Y , the lower bound on η from $\text{D} + {}^3\text{He}$, and the lower bound on τ_n into Eq. (11). The result found in [1] was $N_\nu < 3.3$.

The methods for deriving N_ν have since been refined incorporating a more statistical meaning to the bound. In [8], using the best central values and their associated uncertainties in an expression similar to (11) gave a best estimate, $N_\nu = 2.2 \pm 0.3 \pm 0.4$, (where the statistical error is taken from the observational determination of Y and the neutron mean life, and the systematic error from ${}^4\text{He}$ and from η , assuming that $\eta_{10} = 3.0 \pm 0.3$). Since one could well imagine theories that would effectively *lower* the value of N_ν as well as increase it, one has to, in the broadest sense take the bound seriously, and accept that it might show preference for some extension of the standard model predicting less helium. The $(2\sigma_{\text{stat}} + \sigma_{\text{sys}})$ upper limit was found to be $N_\nu < 3.1$ [8]. A similar bound, $N_\nu < 3.04$ was obtained from a Monte Carlo analysis [5] and an even stronger bound was obtained using a more refined treatment of the solar $\text{D} + {}^3\text{He}$ argument, $N_\nu = 2.0 \pm 0.3$ [52]. If we assumed a priori that $N_\nu > 3$, say, then we could use a Bayesian approach to relax the upper limit to N_ν [53, 55]. An alternative argument for relaxing the constraint based on a stochastic approach to chemical evolution was made in [56]. Also, for a critical review on the history of the bound on N_ν see also [57].

All of the above methods for placing a bound on N_ν rely on ${}^3\text{He}$ and in particular, the $\text{D} + {}^3\text{He}$ argument for a lower bound to η . However, as we have emphasized repeatedly in this

work, the uncertainties in the evolutionary history of ${}^3\text{He}$ make it a poor indicator of N_ν or of the validity of BBN. If instead we use the results found in §4, for η based on ${}^4\text{He}$ and ${}^7\text{Li}$, we find as the best estimate for N_ν ,

$$N_\nu = 2.99 \pm 0.23 \pm 0.38 \begin{matrix} +0.11 \\ -0.57 \end{matrix} \quad (12)$$

showing no particular preference to $N_\nu < 3$, in fact preferring the standard model result of $N_\nu = 3$ and leading to $N_\nu < 3.90$ at the 95 % CL level when adding the errors in quadrature. (Since the central value in Eq. (12) is dramatically close to 3, we show two decimal places, but of course the errors imply that the precision of the agreement is somewhat fortuitous.) In (12), the first set of errors are the statistical uncertainties primarily from the observational determination of Y and the measured error in the neutrino half life τ_n . The second set of errors is the systematic uncertainty arising solely from ${}^4\text{He}$, and the last set of errors from the uncertainty in the value of η and is determined by the combined likelihood functions of ${}^4\text{He}$ and ${}^7\text{Li}$, ie taken from Eq. (9). We view Eq. (12) as a further (and remarkable) confirmation of standard BBN.

Since we have shown our likelihood results when quasar D/H is included, we can also derive the best value for N_ν when D, ${}^4\text{He}$ and ${}^7\text{Li}$ are all included in the analyses. In this case,

$$N_\nu = 3.02 \pm 0.23 \pm 0.38 \begin{matrix} +0.06 \\ -0.18 \end{matrix} \quad (13)$$

leading again to a 95 % CL limit of $N_\nu < 3.9$. In Eq. (13), the central value is slightly higher due to the central value for η_{10} being slightly lower in this case (1.75 rather than 1.80) and the error due to the uncertainty is considerably smaller especially on the low side (corresponding to the high side on η).

6 Conclusions

We have argued that because D and ${}^3\text{He}$ are observed only in the solar system or the local ISM, determinations of their primordial abundances are particularly uncertain and dependent on models of galactic chemical evolution. It is therefore important to determine the status of BBN without them. Using a likelihood analysis we find that the compatibility of the ${}^4\text{He}$ and ${}^7\text{Li}$ constraints depends on the assumptions one makes about the size and distribution of the possible systematic errors. Nevertheless, for the most “standard” assumptions, i.e. taking the data on ${}^4\text{He}$ and ${}^7\text{Li}$ at face value, we find that these two isotopes are quite compatible and give BBN concordance to a high degree of confidence. The baryon-to-photon ratio selected by this concordance is centered around $\eta_{10} \sim 1.8$ and ranges up to 3.8 (or 4.3 if all errors are assumed to be Gaussian).

While the putative determinations of D in quasar absorption line systems are still in their infancy, the technique can be a powerful probe. Upon introducing into our analysis the relatively high D abundance reported for several systems, we find that the agreement with ${}^4\text{He}$ and ${}^7\text{Li}$ is striking. It is crucial to further observe and better understand these systems, which can provide a decisive constraint on BBN.

Having found BBN theory to be consistent with observation, we turn to its implications. In the realm of astrophysics, we find that the needed primordial D and ${}^3\text{He}$ abundances challenge

our understanding of the chemical evolution of these nuclides, particularly ${}^3\text{He}$. We find that the presence of ${}^7\text{Li}$ depletion does not improve the BBN agreement, and further, if one adopts the high D abundance suggested by the quasar measurements, one can determine the primordial ${}^7\text{Li}$ abundance and thus tightly constrain the degree of depletion. In the realm of particle physics, we revisit the N_ν counting argument, with a careful treatment of all the data giving a best value for $N_\nu = 3.0$ and a 95% CL upper limit of $N_\nu \lesssim 3.9$.

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FIGURE CAPTIONS

1. Normalized likelihood distribution for each of ${}^4\text{He}$ and ${}^7\text{Li}$, shown as a function of η . The one-peak structure of the ${}^4\text{He}$ curve corresponds to its monotonic increase with η , while the two-peaks for ${}^7\text{Li}$ arise from its passing through a minimum in the theoretical calculation. Systematic uncertainties have been assumed to be top-hat distributions.
2. Combined likelihood for simultaneously fitting ${}^4\text{He}$ and ${}^7\text{Li}$, as a function of η . Plotted is the product of the normalized likelihood distributions shown in figure 1 multiplied by $2\pi\sigma_4\sigma_7$.
3. As is Figure 1, where the systematics have been assumed to be Gaussian distributions.
4. As in figure 2, using the likelihood distributions in figure 3.
5. As is Figure 3, where a factor of 2 depletion for ${}^7\text{Li}$ has been assumed.
6. As in figure 2, using the likelihood distributions in figure 5.
7. As in figure 1, adding the likelihood for D/H as given by the QSOALS observations [19].
8. As in figure 2, using the likelihood distribution in figure 7.
9. As in figure 5, assuming the observed Pop II ${}^7\text{Li}$ to be depleted by a factor of 2, and including the D/H as observed in QSOALS.
10. As in figure 6, using the likelihood distribution of figure 9.

