Particle Accelerators, 1996, Vol. 52, pp. 201–219
Reprints available directly from the publisher
Photocopying permitted by license only

IMPEDANCES AND POWER LOSSES DUE TO PUMPING SLOTS IN B-FACTORIES

SERGEY S. KURENNOY* and YONG HO CHIN

KEK, 1-1 Oho, Tsukuba-shi, Ibaraki-ken, 305, Japan

(Received 1 December 1995; in final form 31 January 1996)

Contributions of pumping slots to the beam coupling impedances and power losses in a B-factory ring are considered. While their leading contribution is to the inductive impedance, for high-intensity machines with short bunches like e^+e^- B-factories the real part of the impedance and related loss factors are also important. Formulae for the coupling impedances and loss factors due to slots in a ring with an arbitrary cross section of the vacuum chamber are obtained. Effects of the slot tilt on the beam impedance are considered, and restrictions on the tilt angle are derived from limitations on the impedance increase. The power dissipated inside the vacuum chamber due to the fields scattered by the slots is evaluated using results for the real part of the coupling impedance. Estimates are made for the power flow through the slots to the pumping chamber. The analytical results obtained are applied to the KEK B-factory.

Keywords: Coupling impedance, power losses, pumping slots, B-factories

1 INTRODUCTION

The electron-positron colliders with unequal beam energies for study of B-meson physics, so called B-factories, are now in the design process at SLAC[1] and KEK.[2] The dominant issues in B-factories from the point of view of the beam stability are high beam currents (of the order of or above 1 A) and rather short bunches (with r.m.s. length $\sigma \leq 1$ cm) to achieve very high luminosity of 10^{34} cm^{-2}s^{-1}. For example, in the KEK B-factory (KEKB) the currents are 2.6 A in the Low Energy Ring (LER) and 1.1 A in the High Energy Ring (HER) and the bunch length is $\sigma_z = 4$ mm. Some other relevant parameters of the machine are listed in Table 1.

A common tendency in design of all modern high-intensity accelerators is to minimize beam-chamber coupling impedances to avoid beam instabilities and reduce wall heating. From this viewpoint, narrow slots oriented along the chamber axis are the best choice for pumping slots since they provide the highest ratio of their pumping area to the contribution of slots to the broad-band impedance,

*Permanent address: University of Maryland, Department of Physics, College Park, MD 20742, USA.

TABLE 1: KEKB machine parameters.

Case	*LER*		*HER*	
Circumference		3016.26		m
Energy	3.5		8.0	GeV
Particles	e^+		e^-	
Number of bunches		5120		
Bunch spacing		0.6		m
Betatron tune (h/v)	45.52/45.08		46.52/46.08	

e.g. One should note, however, that if a long slot has some small tilt with respect to the chamber axis (for example, due to manufacturing errors), its beam impedance can be much higher than that of an untilted slot. In the present paper limitations on the allowed value of the tilt angle are analyzed.

At low frequencies the coupling impedance of slots is mostly inductive. In the B-factories, because of the short bunch length, the beam sees coupling impedances up to very high frequencies, a few tens of GHz, and due to the high currents this can produce essential heat deposition via higher-order mode losses. Therefore, it is important to know the coupling impedances of the chamber elements, including the real part, up to rather high frequencies to calculate loss factors and beam power losses.

As a specific example, in the present paper we consider the pumping slots in the KEK B-factory. The LER vacuum chamber is a circular pipe with inner radius $b = 50$ mm and wall thickness $t = 6$ mm. About 1800 m of the beam pipe have long pumping slots to connect it with the pumping chamber containing the NEG pumps. The slots are of rectangular shape with rounded ends, with width $w = 4$ mm and length $l = 100$ mm. They are located in groups of 10 parallel slots near the bottom of the beam pipe, one group per meter, with the total number of pumping slots in the LER about 18000. The transverse distance between adjacent slots in the group is about 10 mm. Other types of slots in the LER vacuum chamber are narrow longitudinal slots 1.2 mm $\times$ 35 mm in RF connectors. The number of slots per connector is 40, with about 100 connectors in the ring.

The KEKB HER chamber has the rectangular cross section 104 mm $\times$ 50 mm with rounded ends of radius 25 mm, and the wall thickness is $t = 6$ mm. In the NEG part of the chamber the large dimension is the horizontal one, and 6 rounded-end slots 4 mm $\times$ 100 mm are located symmetrically with respect to the horizontal plane on the rounded part of the chamber in planes inclined to the horizontal one by the angles 4.4°, 13.3°, and 22.1°. The groups of 6 slots are separated by a distance of 15 cm, and the total number of NEG slots is 36000. In the Q part of the HER chamber the large dimension of the same cross section is oriented vertically, and

6 symmetric 4 mm × 120 mm slots are on the flat wall at angles 9.1°, 25.6°, and 38.7° from the horizontal plane. The total number of such Q slots is 1800.

The paper is organized as follows. In Section 2 the analytical approach used is shortly described, and then it is applied to analyze the slot tilt effects. A modification for long slots is also considered, and the existence of the trapped modes near the cutoff is examined. In Section 3 an expression for the loss factor of long slots on the vacuum chamber of an arbitrary cross section is derived. Specific results for the KEK B-factory are given mostly in Section 4, as well as a discussion of the power leakage through slots.

2 COUPLING IMPEDANCE OF SLOTS

2.1 Analytical Theory

An analytical theory for calculating coupling impedances of small discontinuities of the vacuum chamber including pumping holes and slots based on the Bethe theory of diffraction on small holes[4] has been developed in References 5–7.

Let us consider an infinite cylindrical pipe with an arbitrary cross section S and perfectly conducting walls. The z-axis is directed along the pipe axis. A slot in the chamber wall has width w and length l, $w \ll l$, and its center is located at the point $(\vec{b}, z = 0)$. We assume that a typical slot size h (i.e. its width w) satisfies $h \ll b$, where b is a typical size of the chamber cross section. Let $\hat{\nu}$ be an outward unit vector normal to the boundary ∂S, $\hat{\tau}$ be a unit vector tangent to ∂S in the chamber cross section S, and $\{\hat{\nu}, \hat{\tau}, \hat{z}\}$ form a right-handed basis.

In the frequency range where the wavelength is large compared to the typical hole size h, and at distances l such that $h \ll l \ll b$, the fields scattered by the hole into the beam pipe are equal to those produced by effective electric P and magnetic M dipoles[4]

$$P_\nu = -\chi \varepsilon_0 E_\nu^h/2; \quad M_\tau = (\psi_{\tau\tau} H_\tau^h + \psi_{\tau z} H_z^h)/2;$$
$$M_z = (\psi_{z\tau} H_\tau^h + \psi_{zz} H_z^h)/2, \tag{1}$$

where superscript h means that the external (beam[5,6] or corrected, self-consistent[7]) fields are taken at the hole. Then the coupling impedance produced by an arbitrary-shaped hole can be expressed in terms of its electric and magnetic polarizabilities, χ and ψ.[5–7]

These polarizabilities are purely geometrical factors and depend on the hole shape. For example, for a circular hole of radius a in a thin wall $\psi = 8a^3/3$ and $\chi = 4a^3/3$.[4] In general, ψ is a symmetric 2D-tensor, which can be diagonalized. If the hole is symmetric, and its symmetry axis is parallel to $\hat{z}$, the skew terms vanish, i.e. $\psi_{\tau z} = \psi_{z\tau} = 0$. In a more general case of a non-zero tilt angle α between the major symmetry axis and $\hat{z}$,

$$\begin{aligned}
\psi_{\tau\tau} &= \psi_{\perp} \cos^2\alpha + \psi_{\parallel} \sin^2\alpha \, , \\
\psi_{\tau z} &= \psi_{z\tau} = (\psi_{\parallel} - \psi_{\perp}) \sin\alpha \cos\alpha \, , \\
\psi_{zz} &= \psi_{\perp} \sin^2\alpha + \psi_{\parallel} \cos^2\alpha ,
\end{aligned} \tag{2}$$

where $\psi_{\parallel}$ is the longitudinal magnetic polarizability (for the external magnetic field along the major axis), and $\psi_{\perp}$ is the transverse one (the field is transverse to the major axis of the hole).

The longitudinal impedance at low frequencies is dominated by its inductive imaginary part[8]

$$Z(k) = -\frac{ikZ_0\tilde{e}_\nu^2}{2} \left(\psi_{\tau\tau} - \chi\right) , \tag{3}$$

where $Z_0 = 120\pi \ \Omega$ is the impedance of free space, $k = \omega/c$, and $\tilde{e}_\nu$ is merely a normalized electrostatic field produced at the hole location by a filament charge placed on the chamber axis. For an arbitrary single-connected chamber cross section S it can be expressed[8] as $\tilde{e}_\nu = e_\nu(0)$, where

$$e_\nu(\vec{r}\,) = -\sum_s \frac{e_s(\vec{r}\,)\nabla_\nu e_s^h}{k_s^2} . \tag{4}$$

Here $s = \{nm\}$ is a generalized index, k_{nm}^2, $e_{nm}(\vec{r})$ are eigenvalues and orthonormalized eigenfunctions (EFs) of the 2D boundary problem in S:

$$\left(\nabla^2 + k_{nm}^2\right) e_{nm} = 0 \, ; \qquad e_{nm}\big|_{\partial S} = 0, \tag{5}$$

where $\vec{\nabla}$ is the 2D gradient in plane S. The function $\tilde{e}_\nu^2$ gives the impedance dependence on the slot position in the cross section of the vacuum chamber. For a particular case of a circular pipe of radius b, $\tilde{e}_\nu = 1/(2\pi b)$. For a rectangular chamber of width a and height b with a hole located in the side wall at $x = a,\ y = y_h$

$$\tilde{e}_\nu = \frac{1}{b} \Sigma\left(\frac{a}{b}, \frac{y_h}{b}\right) , \tag{6}$$

where

$$\Sigma(u, v) = \sum_{l=0}^{\infty} \frac{(-1)^l \sin[\pi(2l+1)v]}{\cosh[\pi(2l+1)u/2]} \tag{7}$$

is a fast converging series.[8] The impedance decreases very quickly if the slot is displaced closer to the corners of the chamber, i.e. when $y_h \to b$ or $y_h \to 0$.

For a narrow slot with length l and width w such that $w \ll l$, the relation $\psi_{\|} \gg \psi_{\perp}$ holds. If the slot is untilted, i.e. $\alpha = 0$, the only contribution to the magnetic polarizability comes from $\psi_{\perp}$, cf. Equations (2):

$$\psi_{\tau\tau} = \psi_{\perp}; \quad \psi_{\tau z} = 0 .$$

The leading terms in χ and $\psi_{\perp}$ for $w \ll l$ are the same:

$$\chi \simeq \psi_{\perp} = C w^2 l + O(w^3) , \tag{8}$$

where the value of coefficient C depends on the slot shape (see Reference 3). Due to cancellation of these leading terms in the difference $(\psi_{\tau\tau} - \chi)$, the imaginary part of the impedance, Equation (3), is independent of the slot length for narrow long slots. For example, for a narrow rectangular slots with rounded ends in a thin wall (wall thickness $t \ll w$), $C = \pi/8$ in Equation (8) and

$$\psi_{\tau\tau} - \chi \simeq 0.267 w^3 - 0.1 w^4/l . \tag{9}$$

For the case of a thick wall, $t \gg w$, coefficient C is smaller, $C = 1/\pi$, and the impedance is also reduced, see Reference 6.

For a general cross section an analytical calculation of $\tilde{e}_\nu$ in Equation (4) is involved, but one can apply a 2D electrostatic code in region S. We have used a PC-version of the POISSON code.[7] Figure 1 shows the function $\tilde{e}_\nu^2$ for the KEKB HER chamber, normalized to that for a reference circular pipe of radius $b = 50$ mm (LER), versus the slot position (azimuthal angle $\varphi = 0$ corresponds to the horizontal plane). Note a strong dependence of the slot impedance on the slot position in the cross section of the chamber.

The real part of the slot impedance[7] is non-zero only above the lowest cut-off frequency of the chamber[a]

$$ReZ(k) = \frac{k^3 Z_0 \tilde{e}_\nu^2}{8} \left\{ \psi_{\tau z}^2 \sum_s^{<} \frac{k_s'^2 \left(h_s^h\right)^2}{k^2 \beta_s'} + \psi_{\tau\tau}^2 \left[\sum_s^{<} \frac{\left(\nabla_\nu e_s^h\right)^2}{\beta_s k_s^2} + \sum_s^{<} \frac{\beta_s' \left(\nabla_\tau h_s^h\right)^2}{k^2 k_s'^2} \right] \right.$$
$$\left. + \chi^2 \left[\sum_s^{<} \frac{\beta_s \left(\nabla_\nu e_s^h\right)^2}{k^2 k_s^2} + \sum_s^{<} \frac{\left(\nabla_\tau h_s^h\right)^2}{\beta_s' k_s'^2} \right] \right\} , \tag{10}$$

[a] As long as the radiation through the slot is neglected, cf. Reference 7.

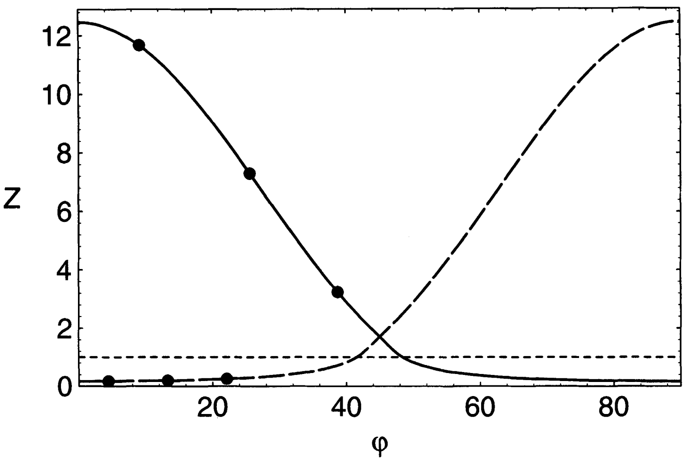

FIGURE 1: Longitudinal impedance of slot (in units of that for circular pipe of radius 50 mm) versus slot position (φ/degrees) in the HER chamber cross section. Solid line for Q-slots (upper thick points) and long-dashed for NEG-slots (lower thick points).

where $\beta_s = (k^2 - k_s^2)^{1/2}$, $\beta_s' = (k^2 - k_s'^2)^{1/2}$, and the sums include only a finite number of the eigenmodes propagating in the chamber at a given frequency, i.e. those with $k_s < k$ or $k_s' < k$. Here EFs h_{nm} satisfy the boundary problem in Equation (5) with the Neumann boundary condition $\nabla_\nu h_{nm}|_{\partial S} = 0$, and $k_{nm}'^2$ are corresponding eigenvalues.

At frequencies well above the chamber cutoff, the dependence of $Re\, Z$ on frequency can be derived as follows.[7] The average number $n(k)$ of the eigenvalues k_s (or k_s') which are less than k, for $kb \gg 1$, is

$$n(k) \simeq \frac{S}{4\pi} k^2 + O(k) \,, \tag{11}$$

where S is the area of the cross section. Using this property, one can replace sums in the RHS of Equation (10) by integrals as $\sum_s^< \to \int^k dk \frac{d}{dk} n(k)$. The result is

$$Re\, Z = \frac{Z_0 k^4 \tilde{e}_\nu^2}{12\pi} (\psi_{\tau\tau}^2 + \psi_{\tau z}^2 + \chi^2) \,. \tag{12}$$

The same answer has been obtained[8] simply by calculating the energy radiated by the dipoles into a half-space. The reason is clear: at high frequencies the dipoles radiate into the waveguide the same energy as into an open half-space.

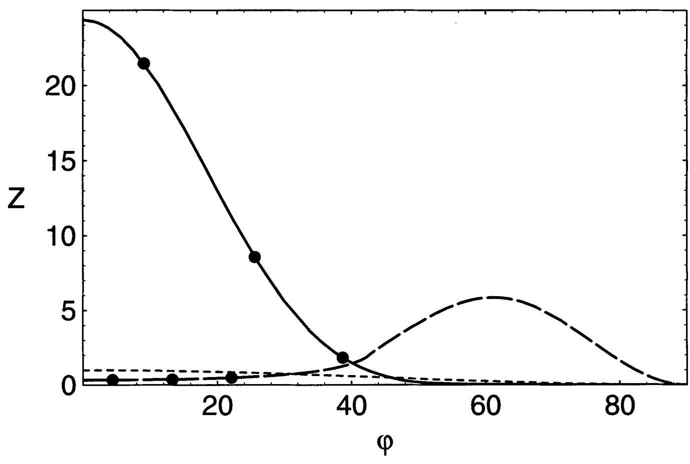

FIGURE 2: Transverse impedance for couple of symmetric slots (in units of the maximal value of that for circular pipe of radius 50 mm) versus slot position (φ/degrees) in the HER chamber cross section (notations are the same as in Figure 1). Short-dashed line shows the same for the circular pipe, $\cos^2\varphi$, cf. Equation (14).

The transverse coupling impedance of a slot has the same dependence on the polarizabilities as the longitudinal one, and it depends on the slot position via derivatives of function $e_\nu(\vec{r})$, defined by Equation (4).[7,8] In a chamber which is symmetric with respect to the horizontal plane, two slots symmetrically located at angles $\pm\varphi$ from this plane contribute only to the horizontal transverse impedance

$$Z_x(k) = -iZ_0 d_x^2 (\psi_{\tau\tau} - \chi) \, , \tag{13}$$

where $d_x = \frac{d}{dx} e_\nu(0)$. For a particular case of a circular pipe of radius b, $d_x(\varphi) = \cos\varphi/(\pi b^2)$, see Reference 8. It casts Equation (13) for a couple of symmetric slots in the circular pipe into

$$Z_x(k) = -\frac{iZ_0}{\pi^2 b^4} (\psi_{\tau\tau} - \chi) \cos^2\varphi \, . \tag{14}$$

For a more complicated cross section of the HER, we again apply the electrostatic code. The result is shown in Figure 2. The transverse impedance strongly depends on the slot position in the chamber cross section.

2.2 Effects of Slot Tilt on Impedance

2.2.1 Restrictions due to Im Z. As was discussed above, for an untilted narrow long slot the imaginary part of the impedance, Equation (3), is independent of the slot length due to the cancellation of the leading terms in $(\psi_{\tau\tau} - \chi)$, see Equations (8). However, if the slot is tilted by some small angle $\alpha \ll 1$, the situation changes drastically since its large longitudinal magnetic polarizability $\psi_{\parallel}$ also contributes, cf. Equation (2):

$$\psi_{\tau\tau} = \psi_{\perp} + (\psi_{\parallel} - \psi_{\perp}/2)\alpha^2 + O(\alpha^4). \tag{15}$$

The ratio of impedances of the tilted and untilted slot as a function of the tilt angle $\alpha \ll 1$ is

$$\frac{Z(k,\alpha)}{Z(k)} \equiv 1 + \delta(\alpha) \simeq 1 + \alpha^2 \frac{\psi_{\parallel}}{\psi_{\perp} - \chi}\,. \tag{16}$$

Respectively, if the maximal allowed fraction δ of the impedance increase is given, Equation (16) gives a restriction on the allowed tilt angle:

$$\alpha < \left(\delta\,\frac{\psi_{\perp} - \chi}{\psi_{\parallel}}\right)^{1/2}. \tag{17}$$

For a specific case of a narrow rectangular slots with rounded ends in a thin wall, the polarizabilities in Equation (16) are

$$\psi_{\parallel} = \frac{\pi l^3}{12}\left(\ln\frac{8l}{w} - \frac{7}{3}\right)^{-1}; \qquad \psi_{\perp} - \chi \simeq 0.267 w^3\,. \tag{18}$$

Then from Equation (16) follows

$$\delta(\alpha) \simeq \alpha^2 \left(\frac{l}{w}\right)^3 \left(\ln\frac{8l}{w} - \frac{7}{3}\right)^{-1}, \tag{19}$$

and restriction, Equation (17), takes the form

$$\alpha < \sqrt{\delta}\left(\frac{w}{l}\right)^{3/2}\left(\ln\frac{8l}{w} - \frac{7}{3}\right)^{1/2}. \tag{20}$$

Figure 3 shows this restriction for different values of the slot aspect ratio w/l.

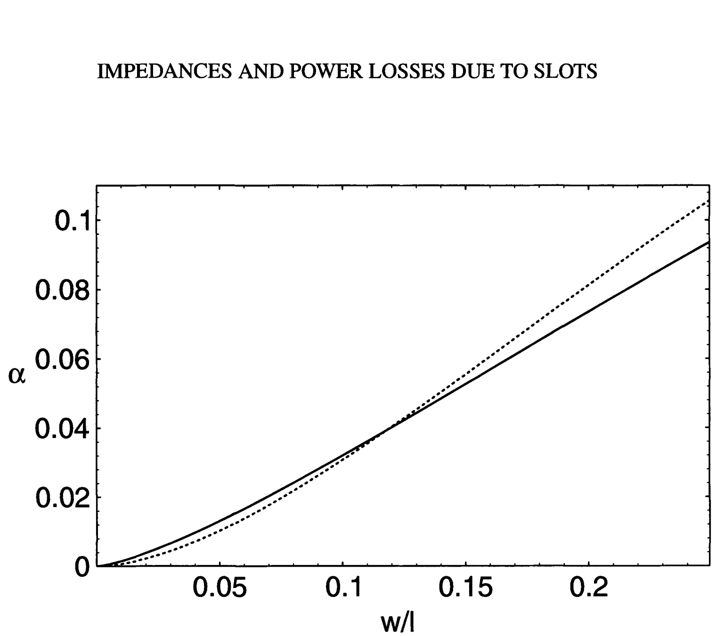

FIGURE 3: Allowed tilt angle (α/radian) versus slot aspect ratio w/l for $\delta = 0.5$: solid line is due to $Im\, Z$, dotted one is from $Re\, Z$.

2.2.2 Restrictions due to Re Z. In a similar way, considering an increase of the real part of the impedance due to a small tilt of the slot gives

$$\frac{Re\, Z(k,\alpha)}{Re\, Z(k)} \equiv 1 + \delta_R(\alpha) \simeq 1 + \alpha^2 \frac{\psi_\parallel^2}{\psi_\perp^2 + \chi^2}. \tag{21}$$

For the case of a narrow rounded-end slot, this leads to

$$\delta_R(\alpha) \simeq \frac{2}{9}\alpha^2 \left(\frac{l}{w}\right)^4 \left(\ln\frac{8l}{w} - \frac{7}{3}\right)^{-2}, \tag{22}$$

and the restriction on the tilt takes the form

$$\alpha < 3\sqrt{\delta/2}\left(\frac{w}{l}\right)^2 \left(\ln\frac{8l}{w} - \frac{7}{3}\right). \tag{23}$$

One can see that $Re\, Z$ gives the restrictions on the slot tilt similar to those due to $Im\, Z$, cf. Figure 3.

2.2.3 Averaged limitations. One should note that in practice the impedance increase $\delta(\alpha)$ has to be averaged with a corresponding tilt angle distribution. Assuming a Gaussian distribution due to manufacturing errors with average tilt $\bar{\alpha} = 0$ and r.m.s. tilt angle α_0, the average value of the increase $\bar{\delta}$ will be given again by Equation (19) with substitution $\alpha \to \alpha_0$, since $\delta \propto \alpha^2$. Figure 3 remains valid for this case also, if δ is considered now as an averaged value over the Gaussian distribution with r.m.s. α. Note also that the allowed tilt angle scales as $\sqrt{\delta}$.

The restrictions on the tilt angle of long slots from impedance requirements obtained above are rather tough for very long slots. They are shown in Figure 3 for the case of a narrow rectangular slot with rounded ends: the maximal allowed angle for a given impedance increase (taken to be $\delta = 0.5$, i.e. 50%) is plotted versus slot aspect ratio w/l. For the KEKB LER, where the ratio $w/l = 0.04$, it means that the limit on the allowed tilt angle is $\alpha \simeq 0.01$ rad $\simeq 0.6^\circ$ to prevent the slot impedance from increasing more than 50%.

2.3 Near Cutoff: Trapped Modes

It has been demonstrated[10] that a small discontinuity, such as a hole or slot, as well as an enlargement on a smooth waveguide can result in the appearance of trapped electromagnetic modes with frequencies slightly below the waveguide cutoff frequencies. The trapped modes produce narrow resonances of the coupling impedance near the cutoff. For a circular waveguide with many small discontinuities the phenomenon has been studied in Reference 11, and for the case of an arbitrary chamber in Reference 7. Using results[10,11] in this section we consider the trapped modes in the LER (in fact, only TM-modes since TE-waves easily leak out through the slots and therefore their trapping obviously fails to exist).

There are $M = 10$ pumping slots in one transverse cross section of the LER. With respect to the lowest TM_{01} trapped mode, all M slots in such a group work as a chamber enlargement with "effective" area A in its longitudinal cross section:[10]

$$A = \frac{M\psi_{\tau\tau}}{4\pi b} = \frac{Mw^2 l}{4\pi^2 b} = 8.1 \text{ mm}^2 .$$

The frequency shift down from the TM_{01} cutoff frequency $f_1 \simeq 2.3$ GHz for the trapped mode is

$$\frac{\Delta f}{f_1} = \frac{\mu_1^2}{2}\left(\frac{A}{b^2}\right)^2 \simeq 3 \cdot 10^{-5} , \tag{24}$$

where $\mu_1 \simeq 2.405$ is the first root of Bessel function J_0; i.e. $\Delta f \simeq 70$ kHz. This gap between the trapped mode frequency and the cutoff is rather small, being only marginally larger than the resonance width due to the energy dissipation in the walls

$$\gamma_1/\omega_1 = \delta/(2b) \simeq 1.35 \cdot 10^{-5},$$

where δ is skin-depth in the copper.

However, the length of the region which would be occupied by the field of the trapped mode for such a single discontinuity is $l_1 = b^3/(\mu_1^2 A) = 2.6$ m. Since this length is longer than the longitudinal separation $g = 1$ m between the adjacent groups of the pumping slots, the adjacent groups interact with one another. According to Reference 11, the number of discontinuities, which work as a single combined one, is $N_{\text{eff}} = \sqrt{2l_1/g} \simeq 2.3$, and the new "effective" length of interaction $L = \sqrt{l_1 g/2} = 1.15$ m. Due to this interaction the frequency gap increases $\Delta f \rightarrow N_{\text{eff}}^2 \Delta f \simeq 0.37$ MHz, which makes it large compared to the resonance width, so that the trapped mode can exist.

Should discontinuities be far separated, $g > l_1$, the total impedance of the ring would be just a sum of contributions $R_1 = Z_0 \mu_1^3 A^3/(\pi \delta b^5)$ from all $N = 2\pi R/g$ discontinuities on the ring (R is the machine radius): $Re\, Z/n = N R_1/n = 2\pi b R_1/(g\mu_1)$. The interaction of discontinuities changes the estimate above by replacements $N \rightarrow N/N_{\text{eff}}$ and $R_1 \rightarrow N_{\text{eff}}^3 R_1$

$$\frac{Re\, Z}{n} = N_{\text{eff}}^2 \frac{2\pi b}{g\mu_1} R_1 = \frac{4 Z_0 A^2}{\delta b g^2}. \tag{25}$$

It gives $Re\, Z/n \simeq 1.5\ \Omega$ as an upper estimate for the narrow-band impedance produced by the trapped modes in the LER. This value of the narrow-band coupling impedance at such a high frequency does not seem to be dangerous.

In fact, the pumping slots are not quite identical, they have some distribution of areas. It causes a frequency spread of resonances produced by different discontinuities, and can reduce the total narrow-band impedance due to the trapped modes essentially, see Reference 11 for details. The resonance damping can also be caused by small random variations of the beam pipe dimensions due to manufacturing errors.

2.4 Modification for Long Slots

Rigorously speaking, Equations (10)–(12) are restricted to wavelengths larger compared to the slot length. For higher frequencies or longer slots (but still for wavelengths larger than the slot width), one can modify the Bethe approach replacing the slot by a distribution of dipoles and taking into account phase shifts in excitation and radiation of these dipoles depending on their longitudinal position.[12] It does not change the result for the imaginary part, Equation (3), but modifies $Re\, Z$.

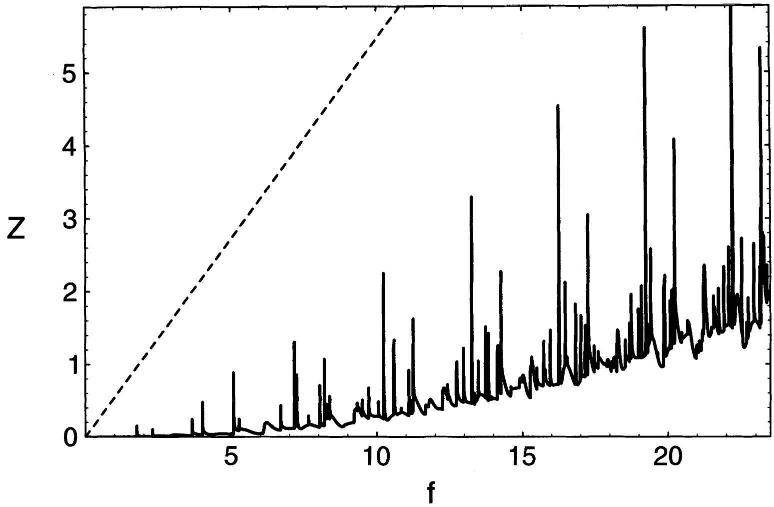

FIGURE 4: Longitudinal impedance ($Z/\mathrm{m}\Omega$) per slot versus frequency (f/GHz): the real part (solid line) is small compared to the imaginary one (dashed).

Taking into account that for long slots $\psi \simeq \chi$ ($\simeq w^2 l/\pi$ in a thick wall), we get

$$ReZ(k) = \frac{Z_0 k \tilde{e}_\nu^2 (\psi/l)^2}{4} \left\{ \sum_s^< \frac{\left(\nabla_\nu e_s^h\right)^2}{\beta_s k_s^2} \left[1 - \cos kl \cos \beta_s l\right] + \sum_s^< \frac{\left(\nabla_\tau h_s^h\right)^2}{\beta_s' k_s'^2} \left[1 - \cos kl \cos \beta_s' l\right] \right\}. \tag{26}$$

This equation is used in the next section to derive the loss factor. Figure 4 shows the real part of the slot impedance in the LER as a function of frequency.

3 BEAM POWER LOSSES

The power per unit length of the beam chamber dissipated due to beam fields scattered by the slots is

$$P' = \frac{N_{sl}}{S_b} f_{\mathrm{rev}} q_b^2 K , \tag{27}$$

where N_{sl} is the total number of slots, S_b is the bunch spacing, $f_{\rm rev} = c/(2\pi R)$ is the revolution frequency, q_b is the bunch charge. The loss factor per slot K is defined as

$$K = \frac{1}{\pi} \int_0^\infty d\omega Re\, Z(\omega) \exp\left[- \left(\frac{\omega\sigma}{c} \right)^2 \right] , \tag{28}$$

where $\sigma = \sigma_z$ is the r.m.s. bunch length.

Substituting Equation (26) into Equation (28), for long slots and short bunches, when $l/\sigma \gg 1$, one can replace fast oscillating functions in the integrand by their average values. Then the integrals are evaluated analytically, which yields a simplified expression for the loss factor of a long slot on the chamber of an arbitrary cross section:

$$K = \frac{Z_0 c\, \bar{e}_\nu^2 (\psi/l)^2}{8\pi^{1/2}\sigma} \left\{ \sum_s^\infty \frac{\left(\nabla_\nu e_s^h\right)^2}{\beta_s k_s^2} \exp(-k_s^2\sigma^2) + \sum_s^\infty \frac{\left(\nabla_\tau h_s^h\right)^2}{\beta_s' k_s'^2} \exp(-k_s^2\sigma^2) \right\} . \tag{29}$$

This expression for the particular case of a circular pipe of radius b becomes

$$K = \frac{Z_0 c (\psi/l)^2}{16\pi^{7/2} b^4 \sigma} \cdot \left\{ \sum_{n=0}^\infty \sum_{m=1}^\infty \frac{1}{1+\delta_{n,0}} \exp\left[- (\mu_{nm}\sigma/b)^2 \right] + \right. \tag{30}$$

$$\left. + \sum_{n=1}^\infty \sum_{m=1}^\infty \frac{n^2}{\mu_{nm}'^2 - n^2} \exp\left[- \left(\mu_{nm}'\sigma/b\right)^2 \right] \right\} ,$$

where μ_{nm} is mth zero of the Bessel function $J_n(x)$, and μ'_{nm} is mth root of its derivative, $J_n'(\mu'_{nm}) = 0$. Expression Equation (30) was obtained earlier[b] in Reference 13. Figure 5 shows the loss factor per slot for different values of the r.m.s. bunch length σ for the KEKB LER. The design value for the KEK B-factory is $\sigma = 4$ mm. The numbers in Figure 5 were calculated using a more complicated expression than Equation (30), which includes length-dependent terms. However, the results are very close to those obtained from Equation (30) for such large values of l/σ.

The dependence $K(\sigma)$ in Figure 5 can be roughly approximated as $K \propto \sigma^{-3}$, compare an analytical derivation in Reference 13. To check this dependence, we consider a rectangular pipe of cross section $a \times b$, in which case the eigenvalues and EFs have simple form, and summation in Equation (29) for short bunches can

[b]The overall factor in Reference 13 was incorrect: it should be multiplied by $1/(4\pi^2)$.

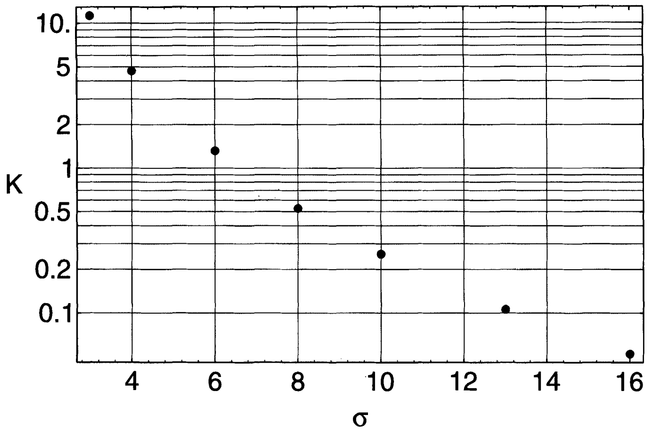

FIGURE 5: Loss factor per slot [$K/(10^{-6}$V/pC)] versus r.m.s. bunch length (σ/mm).

be carried out easily. The resulting loss factor is

$$K \simeq \frac{Z_0 c(\psi/l)^2}{16\pi^{3/2}\sigma^3}\tilde{e}_\nu^2 = \frac{Z_0 c(\psi/l)^2}{16\pi^{3/2}b^2\sigma^3}\Sigma^2\left(\frac{a}{b}, \frac{y_h}{b}\right), \tag{31}$$

where Σ is defined by Equation (7). In this case, $K \propto \sigma^{-3}$ explicitly. It can be proved also for an arbitrary cross section in the case of long slots, when $l/\sigma \gg 1$ but still $\sigma > w$, using the property, Equation (11) of the eigenvalues. Replacing sums in the RHS of Equation (29) by integrals as $\sum_s^< \to \int^k dk \frac{d}{dk} n(k)$ and performing integrations yields $K \propto \sigma^{-3}$. As follows from Equation (28), it means that effectively for long slots the averaged behavior of the real part of the impedance is

$$\overline{Re\, Z} \propto k^2 \, . \tag{32}$$

For comparison, for holes or short slots $Re\, Z \propto k^4$, see Equation (12), and according to Equation (28) the loss factor $K \propto \sigma^{-5}$.

4 RESULTS FOR KEK B-FACTORY

4.1 KEKB Low Energy Ring

The parameters of the LER and of the pumping slots in its vacuum chamber are cited in Introduction. As was mentioned in Section 2, for a thick wall, the impedance should be multiplied by a thickness correction factor.[6] This factor is about 0.6 for long elliptic holes,[6] but it is still unknown for the case of a very long slot with parallel sides and rounded ends. For a cautious estimate we take this factor equal to the ratio of the leading terms of the polarizabilities for an infinitely long slot in a thick and thin wall, namely $8/\pi^2$, Reference 3. Due to additivity of the impedances at frequencies below the chamber cutoff frequency which is 2.3 GHz, the analytical results of Section 2 give reliable estimates of the LER total coupling impedances due to slots in this frequency range, see Table 2.

TABLE 2: Reactive impedances of LER slots.

Slots	$\lvert Z/n \rvert$/ mΩ	$\lvert Z_\perp \rvert$/ (kΩ/m)
Pumping	1.0	0.6
RF connectors	0.1	0.035

Since the pumping slots in LER are located near the bottom of the chamber and symmetric with respect to the vertical plane, they contribute only to the vertical transverse impedance (the value in Table 2), as was discussed in Section 2.1.

Using results of Section 3, we calculate loss factors and beam power losses due to the beam interaction with the slots in the LER, see Table 3.

TABLE 3: Power losses due to LER slots.

Slots	K/(V/pC)	P' / (W/m)
Pumping	$4.7 \cdot 10^{-6}$ per slot	0.37
RF connectors	$1.5 \cdot 10^{-6}$ per conn.	–

One can conclude that the broad-band impedances of the slots are on the level of a few percent of the total estimated impedance budget of the LER, $|Z/n|_{\text{tot}} \simeq 15$ mΩ,[14] and the power loss is rather small compared to that due to synchrotron radiation.

4.2 KEKB High Energy Ring

The parameters of the KEKB HER chamber and pumping slots are cited in Introduction. The total coupling impedances of the slots are calculated as described in Section 2.1, and the results are shown in Table 4. All HER slots contribute only to the horizontal transverse impedance.

TABLE 4: Reactive impedances of HER slots.

Slots	$\|Z/n\|$/mΩ	$\|Z_\perp\|$/(kΩ/m)
NEG	0.4	1.5
Q	0.7	0.8

Due to the lower beam current in the HER (1.1 A instead of 2.6 A in the LER), the power loss due to HER slots is a few times lower than that in the LER, see above.

4.3 Power Flow Through Slots to the Pumping Chamber

A few possible sources of power leakage through the slots to pumping chambers are discussed, and estimates are given for the KEKB LER taken as an example.[13]

4.3.1 Power flow due to direct beam fields on the slots. When the beam field illuminates a slot, it produces scattered fields both inside the beam pipe and outside, in the pumping chamber. The energy radiated by these outside fields into the pumping chamber can be calculated in the same way as for the beam pipe. One should only use so-called "external" polarizabilities of slots instead of the "internal" ones which we used above, Reference 6. For the case of the thick wall (thickness t is larger than the slot width w) these "external" polarizabilities are much smaller than the "internal" ones:

$$(\psi, \chi)^{\text{ext}} \simeq \exp\left(-\pi t/w\right)(\psi, \chi). \tag{33}$$

Since the power loss is proportional to the polarizabilities squared, one can easily realize that the power flow to the pumping chamber due to this source is negligible:

$$P_1' \simeq \exp\left(-2\pi t/w\right) P_{\text{in}}' \simeq 0.04 \text{ mW/m}.$$

4.3.2 Power flow from transverse currents due to the beam betatron motion. One should note that the relation, Equation (33) works only for the electric and transverse magnetic polarizabilities. The term "transverse" here means that the beam magnetic field is directed perpendicular to the slot largest dimension, and this is correct for the case of the beam longitudinal motion when there is only the azimuthal component of the beam magnetic field. However, if the transverse motion of the beam is taken into account there is also some longitudinal magnetic field of the beam on the slot. In this case the longitudinal magnetic dipole moment is induced on the slot.

It is proportional to the longitudinal magnetic polarizability of the slot, which is equal to[3]

$$\psi_{\parallel} = \frac{\pi l^3}{12}\left(\ln\frac{8l}{w} + \frac{\pi t}{2w} - \frac{7}{3}\right)^{-1}, \tag{34}$$

and is much larger than the transverse one, $\psi_{\perp} = w^2 l/\pi + O(w^3)$, for long slots. In addition, its "internal" and "external" values are almost equal, because the magnetic field parallel to the slot easily penetrates through it. So, we have to estimate the power flow due to this effect.

Since the power radiated in TE-modes $P'_H \propto Re\, Z \propto M^2$, where $M = \psi H/2$ is the effective magnetic dipole moment induced by the beam magnetic field H on the slot, we will consider the ratio

$$r = \frac{M_z}{M_\varphi} = \frac{\psi_{\parallel} H_z}{\psi_{\perp} H_\varphi}. \tag{35}$$

The ratio of fields H_z/H_φ can be approximated by the ratio of the beam current components, $j_{\perp}/j \simeq \nu/n$, where $\nu \simeq 45$ is the betatron frequency, and $n = \omega/\omega_0$ is the longitudinal harmonic number, with $\omega_0 = c/R$ being the revolution frequency. Then

$$r = \frac{\pi^2 l^2}{12 w^2}\left(\ln\frac{8l}{w} + \frac{\pi t}{2w} - \frac{7}{3}\right)^{-1} \frac{\nu\omega_0}{\omega}, \tag{36}$$

which is about 0.2 at the beam pipe cutoff frequency.[c] We are interested in frequencies above the chamber cutoff, and because Equation (36) decreases with frequency increase, one can conclude that the power flow into the pumping chamber P'_2 from this source is less than 1/25 of the power radiated in TE-modes into the beam pipe, which is about one half of the total P', i.e. about 0.2 W/m, see Table 3. As a result,

$$P'_2 < 8 \text{ mW/m}.$$

4.3.3 Power flow due to the fields scattered by slots to the beam pipe. It is mentioned above that the total power P' radiated by slots into the beam pipe is divided almost evenly between TM- and TE-modes: $P'_E \simeq P'_H \simeq P'/2 \simeq$ 0.2 W/m. These modes propagate in the beam pipe and reach following slots. For TM-waves penetration through the longitudinal slots is exponentially small, see above. However, TE-waves have a longitudinal magnetic field (the wall currents

[c] A more accurate estimate for the ratio $j_{\perp}/j \simeq \nu/n$ would be $4a/L$, where a is the (vertical) betatron amplitude, and $L = 2\pi R/\nu$ is the length of one betatron oscillation. Replacing a with the pipe radius b to obtain an upper limit for the current ratio leads to the result which is close to Equation (36) at the cutoff.

are transverse to the slots), and contribute to the energy flow into the pumping chamber. The minimal (optimistic) estimate for that power flow would be

$$P_3'^{\,\min} = \frac{A_{sl}}{A_{\text{wall}}} P_H' \simeq 5 \text{ mW/m} ,$$

where $A_{sl}/A_{\text{wall}} = 0.025$ is the fraction of the wall surface occupied by slots. On the other hand, the most pessimistic (maximal) estimate is to assume that the total energy of TE-modes will leak out through slots:

$$P_3'^{\,\max} = P_H' \simeq 0.2 \text{ W/m} .$$

Of course, a part of the energy in TE-modes will be lost in the walls of the beam pipe, so the last number gives the upper estimate.

Concluding this section, one should mention that there is one more source of the power leakage through the slots to the pumping chamber: TE-modes generated in the vacuum chamber by the beam due to its interaction with large elements like RF chambers, etc. Such modes can easily leak out through long slots. This source is believed to be the most dangerous one; however, it is difficult to make estimates for the related power flow. A simple remedy to prevent the power flow to the pumping chamber (due to this or all other sources of HOM) would be placement of thin metallic grids on the outer part of the pumping slots.

5 SUMMARY AND CONCLUSIONS

The analytical methods are applied to analyze effects of the beam interaction with pumping slots in the vacuum chamber of a B-factory. Using earlier obtained analytical expressions, the slot contributions to the longitudinal and transverse broad-band coupling impedance are calculated for the LER and HER of the KEK B-factory. The existence of the trapped modes due to the slots and their contribution to the narrow-band impedance are considered.

New analytical results include an analysis of the influence of the slot tilt on the coupling impedance and derived restrictions on the tilt angle. Besides that, an expression for the loss factor of long slots for the case of short bunches is derived for an arbitrary cross section of the vacuum chamber. Using this result, the power losses due to the beam interaction with pumping slots in the KEK B-factory are calculated. The power flow through the slots to the pumping chamber is also discussed.

The specific calculations for the KEK B-factory show that the pumping slots contribute a few percent to the total broad-band coupling impedance budget. The trapped modes due to the slots (if any) can produce only a small narrow-band impedance at frequencies near the chamber cutoff. Calculated beam power losses are small compared to those from synchrotron radiation, and the power flow to the pumping chamber is well below the allowed level.

Acknowledgements

One of the authors (SK) would like to thank the KEK B-factory team, and especially Prof. Shin-Ichi Kurokawa and Prof. Yong Ho Chin, for their hospitality during his stay at KEK. His work was also supported in part by the U.S. Department of Energy.

References

[1] "PEP II An Asymmetric B factory", Report SLAC-418, Stanford, 1993.
[2] "KEKB B-factory Design Report", Report KEK 95-7, Tsukuba, 1995.
[3] Kurennoy, S.S (1993). SSC Lab Report SSCL-636, Dallas, in *Proceed. of the 4th EPAC*, edited by V. Suller *et al.* (London, 1994), p. 1286.
[4] Bethe, H.A. (1944). Phys. Rev. **66**, 163.
[5] Kurennoy, S.S. (1992). Part. Acc. **39**, 1.
[6] Gluckstern, R.L. (1992). Phys. Rev. A **46**, 1106, 1110.
[7] Kurennoy, S.S., Gluckstern, R.L. and Stupakov, G.V. (1995). *Phys. Rev. E* **52**, 4354.
[8] Kurennoy, S.S. (1992). In *Proceed. of the 3d EPAC*, edited by Henke, H. *et al.* (Berlin 1992), p. 871; more details in Institute for High Energy Physics (Protvino) Report No. IHEP 92–84 (unpublished).
[9] Halbach, K. (1967). In *Proc. of the Second Intl. Conf. on Magnet Technology*, Oxford, UK, p. 47.
[10] Stupakov, G.V. and Kurennoy, S.S. (1994). *Phys. Rev. E* **49**, 794.
[11] Kurennoy, S.S. (1995). Phys. Rev. E **51**, 2498.
[12] Stupakov, G.V. (1995). Phys. Rev. E **51**, 3515.
[13] Kurennoy, S.S. and Chin, Y.H. (1995). KEK Preprint 94-193, Tsukuba.
[14] Chin, Y.H. (1995). In *Proceed. of the Workshop on Collective Effects and Impedances in B-factories*, KEK, Tsukuba.