CERN-TH/96-47

Macroscopic Forces from Supersymmetry[1]

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Abstract

We argue that theories in which supersymmetry breaking originates at low energies often contain scalar particles that mediate coherent gravitational strength forces at distances less than a cm. We estimate the strength and range of these forces in several cases. Present limits on such forces are inadequate. However new techniques, such as those based on small cryogenic mechanical oscillators, may improve the present limits by ten orders of magnitude or discover new forces as weak as 1 % of gravity at distances down to 40 microns.

CERN-TH/96-47
February 1996

[1]Based on talk given at the Santa Barbara Workshop on SusyGUTs, December 6-9, 1995.
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1 Light Scalars and Low-Energy Supersymmetry Breaking

A trademark of superstring theories is the occurrence of gravitationally coupled massles scalars called moduli. To avoid conflict with Newtonean gravity moduli must obtain mass. One possibility is that stringy non-perturbative phenomena create a potential which gives them a mass $\sim M_{\rm PL}$. A second possibility is that Planckean physics leaves the moduli massless and they obtain mass only as a result of supersymmetry breaking. In this case, since they are gravitationally coupled, they are expected to get a mass proportional to $\sim F/M_{\rm PL}$, where F is the scale where supersymmetry breaking originates. In theories with gravity-mediated supersymmetry breaking F is about $(10^{11}~{\rm GeV})^2$ and the moduli masses are at the weak scale. Since they are only gravitationally coupled, they are not directly relevant for experiment. Recently the supersymmetry flavour problem [1] has renewed interest in theories with low-energy supersymmetry breaking[1], where F can be as small as $(10~{\rm TeV})^2$. Moduli in these theories are so light that they can have macroscopic Compton wavelenghts and mediate macroscopic forces of gravitational strength. In this paper we estimate the range and magnitude of the force mediated by moduli in theories where supersymmetry breaking originates at low energies. We separate the moduli in two categories: the Yukawa moduli, which determine the Yukawa couplings, and the gauge moduli which fix the gauge couplings. We find that both types of moduli can mediate potentially measurable forces especially in the millimeter range.

In addition we study, in section 3, the forces mediated by (pseudo-)Goldstone bosons of the broken flavour group. We find that these forces are potentially observable only if the flavour group is broken at energies smaller than M_{GUT}. In the last section we comment on the present limits and a possible future search for sub-cm range forces; these results are summarized in fig. 2 together with some of our predictions. Our results for the range and magnitude of the moduli and Goldstone mediated forces are summarized in the tables.

2 Moduli

All the parameters of the supersymmetric standard model —Yukawas, gauge couplings, soft terms, the μ term— may depend on moduli which are undetermined by Planckean physics. All such moduli can mediate macroscopic forces. In this paper we shall focus on two classes of moduli which are of special interest because they couple directly to ordinary matter and therefore mediate the strongest forces. These are the Yukawa moduli and the gauge moduli. Consider a Yukawa modulus ϕ, coupled as follows to up-type quarks and the Higgs boson H:

$$\mathcal{L} = \lambda(\phi) q_L \bar{u}_R H + {\rm h.c.} \tag{1}$$

[1]The possibility that supersymmetry is broken at low energies with gauge interactions serving as messengers was considered by a number of authors in the early 1980's. These models are reviewed in [2]. More recently, models with dynamical breaking of supersymmetry at low energies have been proposed in ref. [3].

For definitness we assume that the field ϕ appears only in a single, but arbitrary, Yukawa coupling and we drop the flavour indices. Interactions analogous to eq. (1) can also occur for down-type quarks and charged leptons. As discussed in sect. 1, ϕ is a field with Planck-suppressed couplings which is postulated to have no potential until supersymmetry breaking turns on.

Supersymmetry breaking is expected to give ϕ a mass proportional to the gravitino mass, $m_\phi \sim F/M_{\rm PL}$, with a coefficient that depends on the couplings of ϕ to other fields. We do not know the complete form of the effective potential $V(\phi)$, but at least we can compute the contribution coming from the operator in eq. (1). Including explicitly the dominant contribution from fig. 1a , $V(\phi)$ is given by

$$V(\phi) = \frac{k}{(16\pi^2)^2}\lambda^\dagger(\phi)\lambda(\phi)m_s^2\Lambda^2 + V_0(\phi) \ . \tag{2}$$

Here $V_0(\phi)$ is the unknown part of the potential and k is a coefficient, expected to be of order one, which parametrizes the loop integration. Λ is the scale above which the soft terms shut off and supersymmetry is recovered. In ordinary gravity-mediated supersymmetry breaking $\Lambda \sim M_{\rm PL}$. In theories with low-energy gauge-mediated supersymmetry breaking Λ is of the order of the mass of the messengers which communicate supersymmetry breaking to the ordinary sector. Contributions from figs. 1b-1c are typically less important, as they have at most a logatithmic dependence on Λ. Finally, m_s^2 is a measure of supersymmetry-breaking in the ordinary particle sector; it is the larger between the squark mass squared and the higgs-higgsino mass squared splitting. For definitness we assume it is equal to the squark mass squared which, in models where supersymmetry breaking is communicated by gauge interactions, can be related to the messenger mass Λ [3, 4]:

$$m_{\tilde{q}}^2 \simeq \frac{8N}{3}\left(\frac{\alpha_s}{4\pi}\right)^2\frac{F^2}{\Lambda^2} \ . \tag{3}$$

Here N is the number of messenger multiplets and F is the scale where supersymmetry breaking originates or, more precisely, a measure of the messengers' mass splittings.

The requirement that all gauge couplings remain perturbative below the GUT scale implies $N < 4$. A lower bound on the scale Λ can be obtained from the experimental limit on the right-handed selectron mass,

$$m_{\tilde{e}_R}^2 \simeq \frac{10N}{3}\left(\frac{\alpha}{4\pi\cos^2\theta_W}\right)^2\frac{F^2}{\Lambda^2} \ , \tag{4}$$

and the consistency condition that messengers do not receive a negative mass squared, *i.e.* $F < \Lambda^2$. We obtain that Λ can be as small as $30/\sqrt{N}$ TeV.

Replacing m_s^2 in eq. (2) with eq. (3), we find

$$V(\phi) = \frac{8kN\alpha_s^2}{3(16\pi^2)^3}\lambda^\dagger(\phi)\lambda(\phi)F^2 + V_0(\phi) \ . \tag{5}$$

To extract the mass of ϕ, it is convenient to Taylor-expand $\lambda(\phi)$ around its minimum $\langle\phi\rangle \sim M$, where M is expected to be of the order of the string scale 5×10^{17} GeV,

$$\lambda(\phi) = \lambda^{(0)} + \lambda^{(1)}\frac{(\phi - \langle\phi\rangle)}{M} + \frac{1}{2}\lambda^{(2)}\frac{(\phi-\langle\phi\rangle)^2}{M^2} + \dots , \tag{6}$$

$$\lambda^{(i)} \equiv \frac{d^i\lambda}{d\phi^i}(\phi = \langle\phi\rangle) . \tag{7}$$

The ϕ mass is now extracted from eq. (5)

$$m_\phi^2 = \frac{16kN\alpha_s^2}{3(16\pi^2)^3}(\lambda^{(1)2} + \lambda^{(0)}\lambda^{(2)})\frac{F^2}{M^2} + \frac{d^2V_0}{d\phi^2}(\phi = \langle\phi\rangle) , \tag{8}$$

and the minimization of the potential implies:

$$\lambda^{(1)} = -\frac{3(16\pi^2)^3}{16kN\alpha_s^2\lambda^{(0)}}\frac{M_{\rm PL}}{F^2}\frac{dV_0}{d\phi}(\phi = \langle\phi\rangle) . \tag{9}$$

Assuming no accidental cancellation among the different terms in eq. (8), from the known part of the potential we can at least establish a lower bound on the ϕ-mass or an upper bound on its Compton wavelength

$$\lambda_\phi \simeq 90~\mu{\rm m}~\frac{(100~{\rm TeV})^2}{F}\left(\frac{M}{5\times 10^{17}~{\rm GeV}}\right)\left(\frac{\rm GeV}{m_q}\right)\left(\frac{H_q}{1/\sqrt{2}}\right)\left[kN\left(\frac{\lambda^{(1)2}}{\lambda^{(0)2}} + \frac{\lambda^{(2)}}{\lambda^{(0)}}\right)\right]^{-1/2} . \tag{10}$$

Here m_q is the mass of the corresponding quark and H_q is equal to $\sin\beta$ for up-type quarks and $\cos\beta$ for down-type quarks, where $\tan\beta$ is the ratio of the two Higgs vacuum expectation values.

In order to compute the long-range force potential, we first need to relate the scalar ϕ-quark coupling of eq. (1) to the scalar ϕ-nucleon coupling. The matrix element of the light-quark current can be obtained from the measurements of the pion-nucleon “sigma term”. However, to extract the separate matrix elements for up and down quarks, we need to rely on particular model calculations. Following ref. [5], we take the proton and neutron matrix elements to be

$$\langle p|m_d\bar{d}d|p\rangle = 0.034~m_N \qquad \langle n|m_d\bar{d}d|n\rangle = 0.041~m_N \tag{11}$$

$$\langle p|m_u\bar{u}u|p\rangle = 0.023~m_N \qquad \langle n|m_u\bar{u}u|n\rangle = 0.019~m_N \tag{12}$$

where m_N is the nucleon mass. It is interesting to notice that the fields ϕ corresponding to up and down quarks have different couplings to the proton and the neutron. This leads to forces which depend on the atomic number of the test material and therefore to small violations of the equivalence principle at macroscopic scales.

The strange-quark current has a larger matrix element. We use the result from ref. [6]

$$\langle N|m_s\bar{s}s|N\rangle = 0.14~m_N , \tag{13}$$

although values as large as 0.4 m_N [5] and as small as 0.08 m_N [7] are quoted in the literature.

The matrix element of the heavy-quark current can be computed by relating it to the gluon matrix element through the anomaly [8]. Finally the gluon matrix element is computed by expressing the nucleon mass in terms of the trace of the QCD energy-momentum tensor:

$$\langle N|m_Q\bar{Q}Q|N\rangle = \frac{2}{27}\left(m_N - \sum_{q=u,d,s}\langle N|m_q\bar{q}q|N\rangle\right) = 0.06\ m_N \tag{14}$$

Equation (14) holds for each heavy quark, $Q = c, b, t$.

The scalar coupling of the field ϕ to the nucleon N is

$$\mathcal{L} = \mathcal{G}_\phi \frac{m_N}{M_{\rm PL}}\phi\bar{\psi}_N\psi_N\ , \tag{15}$$

where

$$\mathcal{G}_\phi = \frac{\lambda^{(1)}}{\lambda^{(0)}}\ \frac{M_{\rm PL}}{M}\ \frac{\langle N|m_q\bar{q}q|N\rangle}{m_N}\ . \tag{16}$$

From eq. (15) we can now derive the potential between two particles with masses m_1 and m_2 at a distance r, which is generated by a one-ϕ exchange:

$$V(r) = G_N m_1 m_2 \mathcal{G}_\phi^2 \int \frac{d^3k}{(2\pi)^3}\frac{e^{i\vec{k}\cdot\vec{r}}}{\vec{k}^2+m_\phi^2}\ . \tag{17}$$

When added to gravity, eq. (17) describes an additional attractive force:

$$V(r) = -G_N\frac{m_1 m_2}{r}\left(1+\frac{\mathcal{G}_\phi^2}{4\pi}e^{-r/\lambda_\phi}\right)\ . \tag{18}$$

The results for the Compton wavelengths and the strengths of the moduli forces relative to gravity are summarized in tables 1 and 2 respectively. From table 2 we see that the moduli forces can easily reach gravitational strength. The strange-modulus ϕ_s for example mediates a force of gravitational strength at a distance of 0.5 mm. The magnitude of the forces and Compton wavelengths are quite sensitive to the various parameters as well as the uncertainties in the matrix elements of the quark currents. For example, if $\lambda^{(1)}/\lambda^{(0)} = 10$, the strength of the moduli forces become 100 times larger and their range 10 times smaller. In particular the strange-modulus force is 100 times stronger than gravity and all other moduli-forces are also stronger than gravity. Similarly, if the scale M is lowered to coincide with the grand unification mass, $M_{\rm GUT} = 10^{16}$ GeV, then all moduli-forces increase by a factor of 600. Typical ranges of λ_ϕ and $\mathcal{G}_\phi/4\pi$ for several different ϕ are shown in fig. 2. The areas are obtained by taking $kN = 1$, $\tan\beta = 1$, $\lambda^{(2)} = 0$, by varying $\sqrt{F}$ between 30 and 100 TeV, and by varying $\lambda^{(0)}/\lambda^{(1)} \times M/(5\times 10^{17}{\rm GeV})$ between 10^{-2} and 10^2.

The dilaton couples to nucleons with a stength which can be as large as 80 times gravity [9]. However since it couples to all fields in the theory, it is expected to receive a mass $\sim F/M$

from its strong coupling to the primordial supersymmetry-breaking sector. This would make its Compton wavelength less than 10^{-2} μm $(100 \text{ TeV})^2/F$, which is too short to be experimentally observed.

However another interesting possibility is the coupling of a modulus to gluons. We are envisaging here a field ϕ which does not universally couple to all gauge bosons in the theory (in particular to a possible strongly interacting sector responsible for supersymmetry breaking), but only to the standard model gauge bosons. We assume an effective coupling given by

$$\mathcal{L} = \frac{\lambda_g}{8\pi^2}\frac{\phi}{M}G^a_{\mu\nu}G^{a\mu\nu} \ . \tag{19}$$

Here λ_g is an undetermined coupling constant and the factor of $8\pi^2$ is there to account for the fact that gauge couplings depend on moduli only at higher order.

Proceeding as before, we can estimate an upper bound on the ϕ Compton wavelength by computing the contribution to its mass coming from the interaction in eq. (19),

$$m_\phi^2 = k\frac{\alpha_s}{4\pi}\frac{\lambda_g^2}{(8\pi^2)^2}\frac{M_{\tilde{g}}^2\Lambda^2}{M^2} \ . \tag{20}$$

The gluino mass $M_{\tilde{g}}$ is related to the supersymmetry-breaking scale F by [3, 4]

$$M_{\tilde{g}} = \frac{\alpha_s}{4\pi}N\frac{F}{\Lambda} \ . \tag{21}$$

The final expression for the Compton wavelength of the field ϕ is

$$\lambda_\phi = 8\times 10^{-4} \text{ m } \left(\frac{M}{5\times 10^{17} \text{ GeV}}\right)\frac{(100 \text{ TeV})^2}{F} \ . \tag{22}$$

With the help of eq. (14), we obtain the following nucleon matrix element of the gluon operator:

$$\langle N| - \frac{9\alpha_s}{8\pi}G^a_{\mu\nu}G^{a\mu\nu}|N\rangle = 0.8 \ m_N \ . \tag{23}$$

Therefore the ϕ-nucleon coupling can be still expressed by eq. (15), where $\mathcal{G}_\phi$ is now given by

$$\mathcal{G}_\phi = \frac{\lambda_g}{8\pi^2}\frac{M_{\rm PL}}{M}\frac{\langle N|G^a_{\mu\nu}G^{a\mu\nu}|N\rangle}{m_N} \simeq -6 \ \lambda_g \left(\frac{5\times 10^{17} \text{ GeV}}{M}\right) \ . \tag{24}$$

Figure 2 shows the range of λ_{ϕ_g} and $\mathcal{G}_{\phi_g}/4\pi$ obtained by taking $\sqrt{k}N = 1$ and by varying $\sqrt{F}$ between 30 and 100 TeV and $\lambda_g^{-1}\times M/(5\times 10^{17} \text{ GeV})$ between 10^{-2} and 10^2.

3 Pseudo-Goldstone Bosons

In this section we consider long-range forces mediated by pseudo-Goldstone bosons. Pseudo-Goldstone bosons originating from the spontaneous breaking of an *abelian* symmetry have only pseudoscalar couplings to fermionic matter fields. Thus they mediate spin-dependent forces with potential falling off with the distance r as $1/r^3$, because, in the non-relativistic limit, pseudoscalar couplings involve spin-flip transitions. However long-range forces induced by scalar couplings of the pseudo-Goldstone bosons to the matter fields can be present if CP is violated. Examples of such an effect in the case of CP violation from the Kobayashi-Maskawa matrix [10], the θ-term in QCD [11], or new kind of interactions [12] have already been presented in the literature.

Here we want to show that pseudo-Goldstone bosons of *non-abelian* broken symmetries can mediate long-range forces, even in the absence of any CP violation. Let us consider for instance the case of a unitary non-abelian symmetry and let us choose the generators to be either purely real or purely imaginary. From the interaction Lagrangian one observes that Goldstone bosons corresponding to transformations along real (imaginary) generators are CP-odd (CP-even) particles. The CP symmetry does not exclude therefore scalar couplings of some of the pseudo-Goldstone bosons to matter. These pseudo-Goldstone bosons, even under CP, correspond to imaginary generators and therefore only couple to matter fields with different unitary symmetry indices. Since the unitary symmetry is explicitly broken, non-diagonal couplings can be converted into diagonal couplings and ultimately into long-range coherent interactions.

We will illustrate this phenomenon in a class of supersymmetric models introduced in ref. [13], where the dynamics can solve the flavour problem[2], *i.e.* it can appropriately suppress all flavour-changing neutral currents. These theories have a global $U(3)^5$ flavour symmetry group, which is spontaneously (but not explicitly) broken in the limit of vanishing Yukawa couplings. The corresponding Goldstone bosons acquire small masses from the explicit source of symmetry breaking (the Yukawa couplings) and can potentially mediate long-range forces. The fundamental assumption underlying these theories is that the soft supersymmetry-breaking masses for the different squarks and sleptons $\tilde{m}_A^2$ $(A = Q, \bar{U}, \bar{D}, L, \bar{E})$ are promoted to fields:

$$\tilde{m}_A^2 \to \Sigma_A \equiv U_A^\dagger \bar{\Sigma}_A U_A \ , \tag{25}$$

where $\bar{\Sigma}_A$ are diagonal matrices with real, positive parameters ordered according to increasing magnitude. Each $\bar{\Sigma}_A$ corresponds to the vacuum expectation value of the field Σ_A, which spontaneously break a $U(3)$ factor of the flavour group. U_A are 3×3 unitary matrices in flavour space containing the Golstone bosons σ_A^α. They can be written explicitly as

$$U_A = \exp(i \sum_\alpha \lambda^\alpha \sigma_A^\alpha) \ , \tag{26}$$

where λ^α are the Gell-Mann matrices. The sum in eq. (26) extends over the generators of the flavour group broken by $\bar{\Sigma}_A$, in short the six generators of $SU(3)/U(1)^2$. The supersymmetry-breaking trilinear terms A can also be interpreted as fields in an analogous fashion [13]. Here we ignore these terms for simplicity, as they do not affect the essential properties of the Goldstone-mediated long-range forces.

[2]The dynamical determination of the flavour parameters is much in the same spirit as the dynamical determination of the top-quark mass, proposed in ref. [14].

In order to estimate the masses of the pseudo-Goldstone bosons, we need to specify the flavour symmetry breaking part of the potential. This was done in ref. [13], under the assumption that the Yukawa couplings h_e, h_u, and h_d are the only source of explicit flavour breaking. The dominant contribution to the σ_A^α masses come from the following terms in the effective potential:

$$V_{\rm eff} = -\frac{2}{(4\pi)^4}\Lambda^2 {\rm Tr}\left[h_e^\dagger h_e \Sigma_L + h_e h_e^\dagger \Sigma_{\bar E} + \frac{7}{4}(h_u^\dagger h_u + h_d^\dagger h_d)\Sigma_Q + \frac{7}{4}h_u h_u^\dagger \Sigma_{\bar U} + \frac{7}{4}h_d h_d^\dagger \Sigma_{\bar D}\right] . \quad (27)$$

Λ is the cut-off scale, which roughly corresponds to the mass of the messenger particles which communicate supersymmetry breaking to the observable sector. Equation (27) exhibits a strong sensitivity on the physics near Λ caused by a two-loop quadratic divergence in the σ_A^α mass (or, in other words, in the zero-point energy of a conventional softly-broken supersymmetric theory [15, 13]).

Each term in eq. (27) can be expanded in series of σ_A^α; the first term gives, aside from a σ_A^α-independent constant,

$${\rm Tr}(h_e^\dagger h_e \Sigma_L) = -\sum_\alpha (\sigma_L^\alpha)^2 \sum_{i>j} |\lambda_{ij}^\alpha|^2 (\bar\Sigma_{Li} - \bar\Sigma_{Lj})(h_{ei}^2 - h_{ej}^2) + \mathcal{O}(\sigma_L^{\alpha 3}) , \quad (28)$$

and analogous expressions hold for the other terms. Here h_{ei} is the charged-lepton Yukawa coupling of the i-th generation. Equations (27) and (28) show explicitly the dynamical alignment of the soft masses and Yukawa couplings in flavour space, as found in ref. [13]. Notice that as an effect of the non-trivial Kobayashi-Maskawa matrix, h_d and h_u cannot be simultaneously diagonalized and the alignment in the left quark-squark sector is not complete. Nevertheless flavour-changing neutral current processes are adequately suppressed. This provides a solution of the flavour problem in supersymmetric theories very different than the usual assumption of soft-terms universality. Squark masses have here a high degree of non-degeneracy, but their mixing angles are closely related to the quark mixing angles[3].

Before we can identify the physical masses of the pseudo-Goldstone bosons, we need to define the canonical fields. Dimensional analysis and the hypothesis that flavour symmetry is broken spontaneously at a scale f suggest that the field Σ' defined by

$$\Sigma = \frac{m_s^2}{f}\Sigma' \quad (29)$$

is canonically normalized. Here f can be identified with the Planck mass or possibly with some lighter scale connected with flavour breakdown, as $M_{\rm GUT}$. In eq. (29) m_s is the typical mass scale of the soft supersymmetry-breaking terms. We want to stress however that our choice of Σ' being the canonical field is arbitrary and different choices can lead to different masses and couplings for the physical particles. The properly normalized kinetic term is

$$\frac{1}{2}\ {\rm Tr}\ \partial_\mu \Sigma'_A \partial^\mu \Sigma'_A = \left(\frac{f}{m_s^2}\right)^2 \sum_\alpha \partial_\mu \sigma_A^\alpha \partial^\mu \sigma_A^\alpha \sum_{i>j} |\lambda_{ij}^\alpha|^2 (\bar\Sigma_{A_i} - \bar\Sigma_{A_j})^2 . \quad (30)$$

[3] A similar solution of the flavour problem, based however on family symmetries rather than a dynamical principle, has been proposed in ref. [16].

From this and eqs. (27-28) we can read the physical masses squared of the σ_A^α particles[4]:

$$m^2_{\sigma_A^\alpha} = \frac{2c\Lambda^2}{(4\pi)^4} \frac{m_s^4}{f^2} \sum_{i>j} |\lambda^\alpha_{ij}|^2 \left(\frac{h_i^2 - h_j^2}{\bar{\Sigma}_{A_i} - \bar{\Sigma}_{A_j}} \right) , \tag{31}$$

where h are the corresponding Yukawa couplings and $c = 1$ for leptons, $c = 7/4$ for quarks. From eq. (31), we see that $\sigma_A^4, \sigma_A^5, \sigma_A^6, \sigma_A^7$ get masses proportional to the third generation fermion mass of species A, while σ_A^1, σ_A^2 get masses proportional only to the second generation fermion mass of species A. A convenient expression for the masses of the σ_A^α $(A = Q, \bar{U}, \bar{D})$ is

$$m_{\sigma_A^\alpha} \simeq 3 \times 10^{-4} \text{ eV} \left(\frac{10^{16} \text{ GeV}}{f} \right) \left(\frac{\Lambda}{100 \text{ TeV}} \right) \left(\frac{m^A_{f\alpha}}{1 \text{ GeV}} \right) \left(\frac{1/\sqrt{2}}{\cos\beta} \right) \left(\frac{m_s}{300 \text{ GeV}} \right) \sqrt{\frac{m_s^2}{\Delta m_s^2}} \ . \tag{32}$$

Here $\Delta m_s^2/m_s^2$ is the relevant sparticle mass splittings and $m^A_{f_\alpha}$ denotes the third (second) generation fermion mass of species A if $\alpha = 4, 5, 6, 7$ $(\alpha = 1, 2)$. The Compton wavelengths of the σ^α particles are

$$\lambda_{\sigma_A^\alpha} \simeq 6 \times 10^{-4} \text{ m} \left(\frac{f}{10^{16} \text{ GeV}} \right) \left(\frac{100 \text{ TeV}}{\Lambda} \right) \left(\frac{1 \text{ GeV}}{m^A_{f\alpha}} \right) \left(\frac{\cos\beta}{1/\sqrt{2}} \right) \left(\frac{300 \text{ GeV}}{m_s} \right) \sqrt{\frac{\Delta m_s^2}{m_s^2}} \ . \tag{33}$$

Thus, unless the squark mass splitting is very small, the σ can mediate forces between two objects separated by a macroscopic distance and lead to deviations from the equivalence principle.

Let us now turn to discuss the couplings of the pseudo-Goldstone bosons to matter. As we have previously discussed, the CP properties of the σ^α are essential in determining the nature of the forces. The σ^α that correspond to imaginary λ^α, namely σ_2, σ_5 and σ_7, are CP-even scalars and they can mediate $1/r^2$ forces. Diagonal couplings of σ_2, σ_5 and σ_7 to ordinary matter will arise because of the mismatch between mass and interaction eigenstates. Since for leptons the mixing angles vanish, there are no diagonal long-range forces coupled to lepton number. Within our approximation of neglecting left-right squark mixings, these forces can only be mediated by σ_Q, as flavour violation resides in the left quark-squark sector.

We will work in the basis defined by $h_u = \hat{h}_u$ and $h_d = \hat{h}_d K^\dagger$, where $\hat{h}_{u,d}$ are diagonal real matrices and K is the unitary Kobayashi-Maskawa matrix. This basis is particularly convenient because it approximately corresponds to the mass eigenbasis for all squarks[5]. The coupling of the properly normalized σ to squarks ϕ is given by

$$\mathcal{L}_{\sigma\phi\phi} = \frac{i}{\sqrt{2}} \frac{m_s^2}{f} \sum_\alpha \sum_{i>j} \phi_i^\star \lambda^\alpha_{i,j} \phi_j \sigma^\alpha + \text{h.c.} \tag{34}$$

[4]The non-linear Goldstone parametrization used here makes sense only if the explicit symmetry breaking $(h_i^2 - h_j^2)\langle H \rangle^2$ is not larger than the spontaneous breaking $(\bar{\Sigma}_i - \bar{\Sigma}_j)$. This is why eq. (31) apparently blows up as $\bar{\Sigma}_i - \bar{\Sigma}_j \to 0$.

[5]This is true unless the splittings of the squarks soft masses are smaller than the corresponding quark mass splittings. Here we are interested in a case where there is a significant departure from universality.

The interactions of the pseudo-Goldstone bosons with squarks, eq. (34), can be converted into a diagonal coupling to ordinary matter exploiting the Kobayashi-Maskawa angles which rotate the down quarks from the basis we are working into their mass eigenbasis. This can be done via one-loop diagrams mediated either by gluinos (for the coupling to down quarks) or by charginos (for both up and down quarks). It is reasonable to expect that strong interactions make the gluino exchange dominant over the chargino, although this may depend on the various parameters. The gluino-exchange produces a scalar effective coupling between σ_Q^α and a pair of down quarks d_k, with identical flavour index k, given by

$$\mathcal{L}_{\sigma\bar{d}d} = \frac{\sqrt{2}\alpha_s}{9\pi}\frac{m_s^2}{M_{\tilde{g}}^2}\frac{m_{d_k}}{f}\ \bar{d}_k d_k \sigma_Q^\alpha \sum_{i>j} \mathrm{Im}(\lambda_{ij}^\alpha K_{jk} K_{ik}^\star)\ g\left(\frac{\bar{\Sigma}_i}{M_{\tilde{g}}^2}, \frac{\bar{\Sigma}_j}{M_{\tilde{g}}^2}\right)\ , \tag{35}$$

where

$$g(x,y) = \frac{3}{2(x-y)}\left[\frac{1}{x-1} + \frac{x(x-2)}{(x-1)^2}\log x - (x \to y)\right] \tag{36}$$

is normalized so that $g(1,1) = 1$ and $M_{\tilde{g}}$ is the gluino mass. It is apparent from eq. (35) that if CP is conserved, or in other words if K is real, only imaginary λ^α can generate scalar couplings. Equation (35) is also proportional to the down quark mass, m_{d_k}. This was to be expected because, as $m_{d_k} \to 0$, K looses its meaning and no flavour transitions are allowed.

The effective σ_Q-nucleon coupling is

$$\mathcal{L}_{\sigma_Q^\alpha \bar{N}N} = \frac{m_N}{M_{\mathrm{PL}}}\mathcal{G}_{\sigma_Q^\alpha}\sigma_Q^\alpha \bar{\psi}_N \psi_N\ , \tag{37}$$

where $\mathcal{G}_{\sigma_Q^\alpha}$ measures the strength of the σ_Q^α-coupling relative to gravity[6],

$$\mathcal{G}_{\sigma^\alpha} = \frac{\sqrt{2}\alpha_s}{9\pi}\frac{m_s^2}{M_{\tilde{g}}^2}\frac{M_{\mathrm{PL}}}{f}\sum_k\sum_{i>j}\mathrm{Im}(\lambda_{ij}^\alpha K_{jk}K_{ik}^\star)\ g\left(\frac{\bar{\Sigma}_i}{M_{\tilde{g}}^2}, \frac{\bar{\Sigma}_j}{M_{\tilde{g}}^2}\right)\ \frac{\langle N|m_{d_k}\bar{d}_k d_k|N\rangle}{m_N}\ . \tag{38}$$

The Goldstone forces' range and strength relative to gravity are shown in tables 1 and 2 respectively. The Goldstone forces are significantly weaker than the moduli forces because of the associated global symmetries. As a result prehaps the only Goldstone that has a chance to be observable is σ_Q^2, provided that the flavour scale f is around M_{GUT} or lower.

Finally we want to recall that, in simple cosmologies, the Goldstones as well as the moduli suffer from the usual "cosmological moduli problem" [17]. We have nothing to add to that except to hope that either inflation or some other mechanism solves this problem.

4 Prospects

Present limits on new forces at scales larger than 1 cm are reviewed in ref. [18] . Existing experimental limits at shorter distances come from two sources, the electromagnetic Casimir

[6]We have neglected here an effective σ_Q^α-gluon coupling coming from the integration of a squark loop. Its contribution is expected to be smaller than the contribution given in eq. (35).

force measurements [19] and the 2 cm Cavendish experiment [20] which are shown in fig. 2. From this we see that the Casimir force measurement allows for new forces up to 10^9 times gravity in the range between $\sim 10^{-4}$ cm and $\sim 10^{-1}$ cm. This constraint is a steep function of distance and becomes even weaker at short distances where the Casimir force is larger; at 10 μm it allows a force up to 10^{12} times gravity. The Cavendish experiment at 2 cm gives a constraint that decays exponentially in significance at distances below a cm; at 0.5 mm it allows a new force up to 10^8 times gravity. Cryogenic mechanical oscillator techniques have been proposed [21] that can improve the existing limits by up to 10^{10} in the range between 40 microns and 1 cm and can detect forces 10^{-2} times gravity with a range greater than 40 microns. The dotted lines in fig. 2 indicate the sensitivity of these techniques. The region to the left of the steep dotted line is inaccessible because the background electrostatic force from the surface potential dominates. The region below the dotted line is swamped by the Newtonian background due to edge effects arising from the finite size of the parallel plates. Another proposal under consideration involves atomic beams [22]; it is not shown in the figure because its domain of sensitivity is still being studied. In the figure we also show bands that correspond to some of our predictions as we vary the unknown parameters of our theory. We see that a broad range is accessible to the cryogenic oscillator techniques.

To summarize, there are two essential ingredients that led to the qualitative conclusions of this paper. First, the existence of scalars with gravitational couplings which get mass only from supersymmetry breaking. Second, the hypothesis that supersymmetry breaking originates at low energies, not too far from the weak scale. These simple hypotheses led us to the possibilities discussed here. Of course, the numerical uncertainties associated with the magnitude and range of the forces are large. Nevertheless, we hope that these estimates will motivate renewed efforts for searches of new sub-cm forces. It seems hard to overestimate the importance of discovering such a force. It would provide us with a rare window into Planckean Physics and the scale of supersymmetry breaking. It is even possible that a study of the material and distance dependence of these forces could give us a more detailed picture of how flavour symmetry emerges from the Planck scale.

5 Acknowledgements

It is a pleasure to thank Costas Kounnas, Michael Peskin, Leonard Susskind, and Gabriele Veneziano for valuable discussions and our experimental colleagues Steve Chu, Mark Kasevich, Peter Michelson and Doug Osheroff for teaching us about the experimental feasibility of testing these ideas over the last few months.

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Table 1: Estimates for the Compton wavelengths of the light scalar particles discussed in the text.

Particle	λ [m]
ϕ_u	$2 \times 10^{-2} \quad \frac{(100\ \mathrm{TeV})^2}{F} \left(\frac{M}{5\times10^{17}\ \mathrm{GeV}}\right) \left(\frac{\sin\beta}{1/\sqrt{2}}\right) \left[kN\left(\frac{\lambda_u^{(1)2}}{\lambda_u^{(0)2}} + \frac{\lambda_u^{(2)}}{\lambda_u^{(0)}}\right)\right]^{-1/2}$
ϕ_d	$1 \times 10^{-2} \quad \frac{(100\ \mathrm{TeV})^2}{F} \left(\frac{M}{5\times10^{17}\ \mathrm{GeV}}\right) \left(\frac{\cos\beta}{1/\sqrt{2}}\right) \left[kN\left(\frac{\lambda_d^{(1)2}}{\lambda_d^{(0)2}} + \frac{\lambda_d^{(2)}}{\lambda_d^{(0)}}\right)\right]^{-1/2}$
ϕ_s	$5 \times 10^{-4} \quad \frac{(100\ \mathrm{TeV})^2}{F} \left(\frac{M}{5\times10^{17}\ \mathrm{GeV}}\right) \left(\frac{\cos\beta}{1/\sqrt{2}}\right) \left[kN\left(\frac{\lambda_s^{(1)2}}{\lambda_s^{(0)2}} + \frac{\lambda_s^{(2)}}{\lambda_s^{(0)}}\right)\right]^{-1/2}$
ϕ_c	$7 \times 10^{-5} \quad \frac{(100\ \mathrm{TeV})^2}{F} \left(\frac{M}{5\times10^{17}\ \mathrm{GeV}}\right) \left(\frac{\sin\beta}{1/\sqrt{2}}\right) \left[kN\left(\frac{\lambda_c^{(1)2}}{\lambda_c^{(0)2}} + \frac{\lambda_c^{(2)}}{\lambda_c^{(0)}}\right)\right]^{-1/2}$
ϕ_b	$2 \times 10^{-5} \quad \frac{(100\ \mathrm{TeV})^2}{F} \left(\frac{M}{5\times10^{17}\ \mathrm{GeV}}\right) \left(\frac{\cos\beta}{1/\sqrt{2}}\right) \left[kN\left(\frac{\lambda_b^{(1)2}}{\lambda_b^{(0)2}} + \frac{\lambda_b^{(2)}}{\lambda_b^{(0)}}\right)\right]^{-1/2}$
ϕ_t	$5 \times 10^{-7} \quad \frac{(100\ \mathrm{TeV})^2}{F} \left(\frac{M}{5\times10^{17}\ \mathrm{GeV}}\right) \left(\frac{\sin\beta}{1/\sqrt{2}}\right) \left[kN\left(\frac{\lambda_t^{(1)2}}{\lambda_t^{(0)2}} + \frac{\lambda_t^{(2)}}{\lambda_t^{(0)}}\right)\right]^{-1/2}$
ϕ_g	$8 \times 10^{-4} \quad \frac{(100\ \mathrm{TeV})^2}{F} \left(\frac{M}{5\times10^{17}\ \mathrm{GeV}}\right) \left(\sqrt{k}N\lambda_g\right)^{-1}$
σ_Q^2	$4 \times 10^{-3} \quad \left(\frac{f}{10^{16}\ \mathrm{GeV}}\right) \left(\frac{100\ \mathrm{TeV}}{\Lambda}\right) \left(\frac{\cos\beta}{1/\sqrt{2}}\right) \left(\frac{300\ \mathrm{GeV}}{m_s}\right) \sqrt{\frac{\Delta m_s^2}{m_s^2}}$
σ_Q^5	$1 \times 10^{-4} \quad \left(\frac{f}{10^{16}\ \mathrm{GeV}}\right) \left(\frac{100\ \mathrm{TeV}}{\Lambda}\right) \left(\frac{\cos\beta}{1/\sqrt{2}}\right) \left(\frac{300\ \mathrm{GeV}}{m_s}\right) \sqrt{\frac{\Delta m_s^2}{m_s^2}}$
σ_Q^7	$1 \times 10^{-4} \quad \left(\frac{f}{10^{16}\ \mathrm{GeV}}\right) \left(\frac{100\ \mathrm{TeV}}{\Lambda}\right) \left(\frac{\cos\beta}{1/\sqrt{2}}\right) \left(\frac{300\ \mathrm{GeV}}{m_s}\right) \sqrt{\frac{\Delta m_s^2}{m_s^2}}$

Table 2: Estimates for the couplings relative to gravity of the new forces mediated by the light scalar particles discussed in the text.

Particle	$\mathcal{G}^2/(4\pi)$	
ϕ_u	2×10^{-2}	$\left(\frac{5\times10^{17}\ \mathrm{GeV}}{M}\right)^2 \left(\frac{\lambda_u^{(1)}}{\lambda_u^{(0)}}\right)^2$
ϕ_d	8×10^{-2}	$\left(\frac{5\times10^{17}\ \mathrm{GeV}}{M}\right)^2 \left(\frac{\lambda_d^{(1)}}{\lambda_d^{(0)}}\right)^2$
ϕ_s	1	$\left(\frac{5\times10^{17}\ \mathrm{GeV}}{M}\right)^2 \left(\frac{\lambda_s^{(1)}}{\lambda_s^{(0)}}\right)^2$
ϕ_c	2×10^{-1}	$\left(\frac{5\times10^{17}\ \mathrm{GeV}}{M}\right)^2 \left(\frac{\lambda_c^{(1)}}{\lambda_c^{(0)}}\right)^2$
ϕ_b	2×10^{-1}	$\left(\frac{5\times10^{17}\ \mathrm{GeV}}{M}\right)^2 \left(\frac{\lambda_b^{(1)}}{\lambda_b^{(0)}}\right)^2$
ϕ_t	2×10^{-1}	$\left(\frac{5\times10^{17}\ \mathrm{GeV}}{M}\right)^2 \left(\frac{\lambda_t^{(1)}}{\lambda_t^{(0)}}\right)^2$
ϕ_g	3	$\left(\frac{5\times10^{17}\ \mathrm{GeV}}{M}\right)^2 \lambda_g^2$
σ_Q^2	5×10^{-3}	$\left(\frac{10^{16}\ \mathrm{GeV}}{f}\right)^2 \left[\frac{m_s^2}{M_{\tilde{g}}^2}\ g\left(\frac{\bar{\Sigma}_2}{M_{\tilde{g}}^2}, \frac{\bar{\Sigma}_1}{M_{\tilde{g}}^2}\right)\right]^2$
σ_Q^5	5×10^{-6}	$\left(\frac{10^{16}\ \mathrm{GeV}}{f}\right)^2 \left[\frac{m_s^2}{M_{\tilde{g}}^2}\ g\left(\frac{\bar{\Sigma}_3}{M_{\tilde{g}}^2}, \frac{\bar{\Sigma}_1}{M_{\tilde{g}}^2}\right)\right]^2$
σ_Q^7	5×10^{-5}	$\left(\frac{10^{16}\ \mathrm{GeV}}{f}\right)^2 \left[\frac{m_s^2}{M_{\tilde{g}}^2}\ g\left(\frac{\bar{\Sigma}_3}{M_{\tilde{g}}^2}, \frac{\bar{\Sigma}_2}{M_{\tilde{g}}^2}\right)\right]^2$

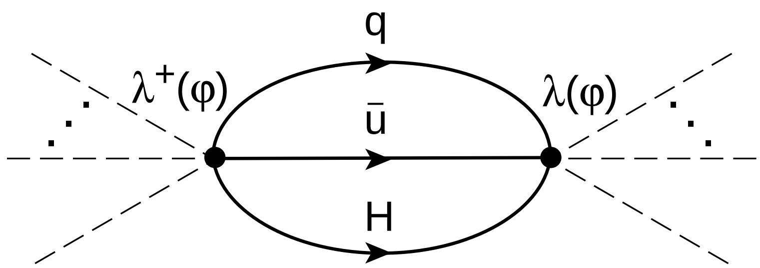

Figure 1a

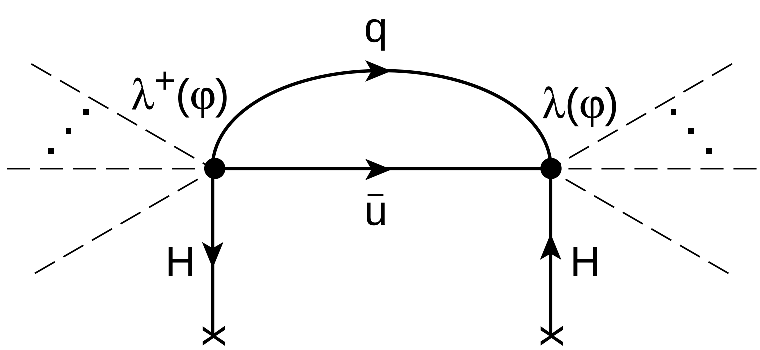

Figure 1b

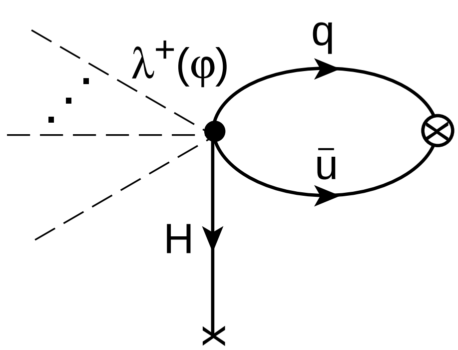

Figure 1c

Figure 1: Feynman diagrams contributing to the effective potential $V(\phi)$. The black dot represents the effective λ interaction and the cross represents the QCD condensate.

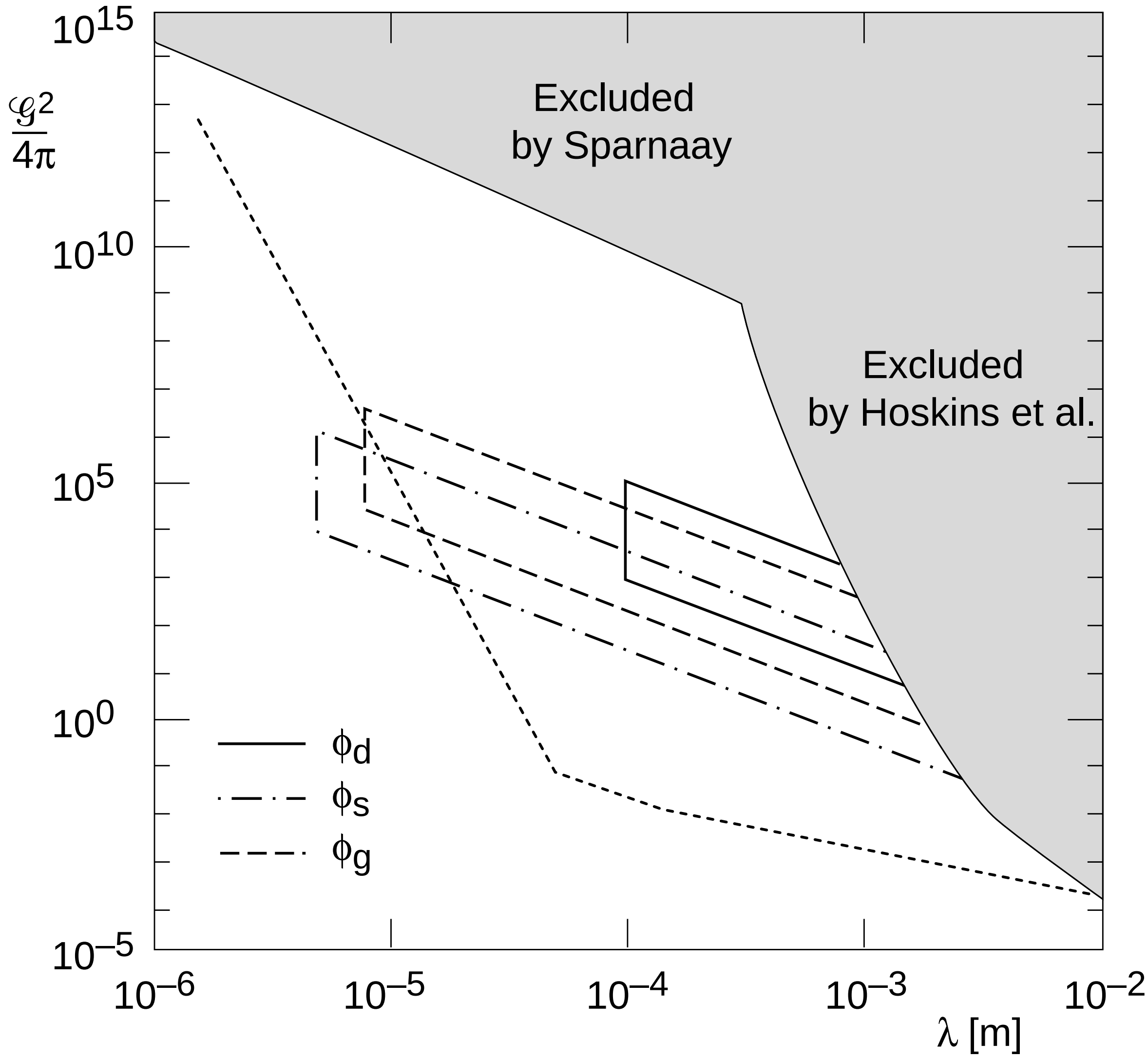

Figure 2

Figure 2: Estimates for the region of Compton wavelength (λ) and force strength relative to gravity ($\mathcal{G}/4\pi$) corresponding to strange- and down-Yukawa moduli (ϕ_s, ϕ_d) and gluon modulus (ϕ_g). The shaded region is experimentally excluded by the searches described in refs. [19, 20]. The dotted line shows the expected sensitivity of the planned experiment described in ref. [21].