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On the Problem of Boosting Nonleptonic b Baryon Decays

N.G. Uraltsev

TH Division, CERN, CH-1211 Geneva 23, Switzerland
and
St.Petersburg Nuclear Physics Institute, Gatchina, St.Petersburg 188350, Russia[*]

Abstract

The constituent picture of hadrons implies certain quantum mechanical inequalities which must hold in the potential models. Basing on this qualitative consideration I argue that it is not easy to increase significantly the scale of the flavor-dependent $1/m_b^3$ effects within the heavy quark expansion preserving the conventional constituent picture of heavy flavor hadrons. I briefly address the physical consequences one might expect if the effects of weak scattering and interference are attempted to be pushed above the 10% level within $1/m_b$ expansion not invoking qualitatively different mechanisms including violations of duality.

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[*]Permanent address

Heavy quark expansion allows one to address systematically the inclusive widths of the heavy flavor hadrons based genuinely on QCD with minimal – though rather important – qualitative information supplied by experiment about the behavior of QCD in the strong interaction regime. The essential elements of the present theoretical technology were set up in mid 80's [1] and were applied already then to estimate the preasymptotic effects in charmed and beauty particles. Later, the systematic study of the $1/m_Q$ expansion for the inclusive decay rates has been done with special attention to the subtleties involved in the application of the OPE to the Minkowsky decay processes. In particular, it was shown [2, 3] that the power corrections to the inclusive widths (both semileptonic and nonleptonic, as well as radiative ones) are absent at the $1/m_Q$ level and start with $1/m_Q^2$ terms. These leading corrections do not depend explicitly on the flavor of the spectator; they were calculated in [3] (for the review see [4]). Although these leading effects differentiate lifetimes of mesons and baryons, their effects are not large in the individual parton level decay channels of b particles, and appear to be additionally suppressed numerically in the total decay width.

The flavor-dependent effects emerge at the $1/m_b^3$ level but are numerically enhanced and, in general, constitute several per cent of $\Gamma_{\rm tot}$ in beauty. They are given by the expectation values of the four-fermion operators

$$O_{\alpha\beta} \;=\; \bar{b}\gamma_\alpha(1-\gamma_5)q\,\bar{q}\gamma_\beta(1-\gamma_5)b \qquad ; \qquad O \;=\; O_{\alpha\alpha} \tag{1}$$

(with two possible color contraction schemes), where q is the appropriate light quark. The Lorentz scalar operators O emerge when one integrates out the diquark loop and describe interference (PI) in the decays of B mesons as well as weak scattering (WS) in baryons; the different combination of the components of $O_{\alpha\beta}$ results due to $q\bar{q}$ intermediate pair and is responsible for weak annihilation (WA) in mesons and PI in baryons [1]. The tree level coefficient functions are well known in the general case:

$$c^{qq} \;=\; \frac{G_F^2 P^2}{2\pi}\,|{\rm KM}|^2\, p\left(1-\frac{m_1^2+m_2^2}{P^2}\right) \quad , \qquad c_{\alpha\beta}^{q\bar{q}} \;=\; -\frac{G_F^2 P^2}{6\pi}\,|{\rm KM}|^2\,\left(A\delta_{\alpha\beta}-B\frac{P_\alpha P_\beta}{P^2}\right)$$

$$A \;=\; p\left(1-\frac{m_1^2+m_2^2}{2P^2}-\frac{(m_1^2-m_2^2)^2}{2P^4}\right) \quad , \qquad B \;=\; p\left(1+\frac{m_1^2+m_2^2}{P^2}-\frac{2(m_1^2-m_2^2)^2}{P^4}\right)$$

$$p \;=\; \left[\left(1-\frac{(m_1+m_2)^2}{P^2}\right)\left(1-\frac{(m_1-m_2)^2}{P^2}\right)\right]^{\frac{1}{2}} \tag{2}$$

where m_1, m_2 are the quark masses in the loop, P is the total momentum flowing into it (normally identified with the momentum of the heavy quark p^b or of the hadron P^H) and $|{\rm KM}|^2$ symbolically denotes the product of the quark mixing angles. The QCD corrections to the coefficient functions are also known [1] and include color traces depending on N_c and $c_\pm$ and, in particular, the so-called "hybrid" renormalization coming from the scales below m_b. The corrections to the width are given [1, 2, 5] by the forward matrix element of the corresponding generic sum $c_i \cdot O_i$ over the particular hadron, B, Λ_b etc.:

$$\Delta\Gamma_{H_Q} \;=\; \frac{1}{2M_{H_Q}}\,\langle H_Q|\,c_i\cdot O_i\,|H_Q\rangle \;\; . \tag{3}$$

The situation with the matrix elements is less clear and is the subject of the present discussion. In the case of mesons one employs factorization:

$$\langle B_{q'}|\,\bar{b}\gamma_\alpha(1-\gamma_5)A^a q\;\bar{q}\gamma_\beta(1-\gamma_5)A^b b\,|B_{q'}\rangle \;=\; f_B^2 P_\alpha^B P_\beta^B\cdot\frac{1}{9}\mathrm{Tr}\,A^a\,\mathrm{Tr}\,A^b\,\delta_{q'q} \tag{4}$$

where $A^{a,b}$ are color matrices and q, q' are light quark flavors. WA appears to be strongly suppressed *in the factorization approximation* by the ratio m_c^2/m_b^2 (for the dedicated discussion see [2, 5]).

The baryonic matrix elements are even less certain. Their estimates rely so far mostly on simple potential quark models of heavy flavor baryons. Ignoring for a moment the color indices, one uses the Fierz identities and the equation of motion for the b field to write [1]

$$O \;=\; \bar{b}\gamma_\alpha(1-\gamma_5)b\;\bar{q}\gamma_\alpha(1-\gamma_5)q\;\;;\qquad P_\alpha P_\beta\,O_{\alpha\beta}\;=\;-\frac{1}{2}\,P^2\;\bar{b}\gamma_\mu(1+\gamma_5)b\;\bar{q}\gamma_\mu(1-\gamma_5)q\;\;. \tag{5}$$

In the current $\bar{b}\gamma_\alpha(1-\gamma_5)b$ the vector part is nonzero for $\alpha=0$ and the axial part survives for the spacelike components. If the color singlet b quark current is considered, the former represents the heavy quark density, whereas the latter is the b quark spin density which decouples from the light degrees of freedom as m_b goes to infinity. Therefore, in the matrix element over the Λ_b state the axial current does not contribute in the heavy quark limit [1] (it does for Σ_b and B states). Strictly speaking, this general statement holds only for the particular color structure of the four-fermion operator.

Further simplification arises when one applies the description of the baryon relying on ordinary quantum mechanics of only the constituent quarks. Then the single (antisymmetric) color structure survives and one has to know the unique matrix element

$$\frac{1}{2M_{H_Q}}\langle\Lambda_b|\,\bar{b}^i\gamma_0 b^i\;\bar{q}^j\gamma_0 q^j\,|\Lambda_b\rangle \;=\; |\Psi^d(0)|^2 \tag{6}$$

where Ψ^d denotes the heavy-light diquark wave function. Collecting all coefficients together (see, e.g., [1, 6]) one arrives at the following expressions for the effects in Λ_b:

$$\frac{\Gamma_{WS}}{\Gamma_0}\;\simeq\;96\pi^2c_-^2\,\frac{|\Psi^d(0)|^2}{m_b^3} \tag{7}$$

$$\frac{\Gamma_{PI}}{\Gamma_0}\;\simeq\;-96\pi^2c_+(c_- - \frac{c_+}{2})\,\frac{|\Psi^d(0)|^2}{m_b^3}\;\;; \tag{8}$$

Γ_0 denotes the (phase space uncorrected) bare semileptonic width $\Gamma_0 = G_F^2 m_b^5|V_{cb}|^2/(192\pi^3)$ and $c_\pm$ are the standard short distance coefficients. I neglected here minor corrections due to the final state quark masses, eq. (2), which anyway do not exceed the effect of the higher dimension operators, and more essential hybrid renormalization effects (to be briefly addressed later). These expressions are the standard starting point [1] for the numerical evaluation of the flavor-dependent preasymptotic corrections.

Before proceeding to the specific subject of the current paper, let me note that the factorization eq. (4) clearly holds in the constituent quark ansatz, with [7]

$$f_B^2 \;=\; 12\,\frac{|\Psi(0)|^2}{M_B} \tag{9}$$

where $\Psi(0)$ now denotes the light quark wavefunction at zero separation and the factor 12 comes from the color and spin traces.

The wavefunctions are governed by the strong interaction dynamics and thus seem to be very uncertain, depending crucially on the quark interaction even within the potential description. Therefore, it is tempting to allow for the larger effects of PI and WS in Λ_b pushing $|\Psi^d(0)|^2$ up to meet the (not firmly established yet, though) experimental evidence [8] that the Λ_b lifetime can be noticeably smaller than that of B. I will argue that such an option would require certain revision of the simple constituent models for heavy baryons, in particular, applicability of the heavy flavor symmetry at a quantitative level.

In the QM description one has the following constraints on the wavefunctions:

$$\Psi(0) \;=\; \int \frac{d^3\,p}{(2\pi)^3}\,\Psi(p) \tag{10}$$

$$\int d^3\,x\,|\Psi(x)|^2 \;=\; \int \frac{d^3\,p}{(2\pi)^3}\,|\Psi(p)|^2 \;=\; N \;=\; 1 \;\; . \tag{11}$$

One must also assume that $p^2|\Psi(p)|^2$ falls off rapidly above certain characteristic hadronic scale μ, for the physics of the harder modes is absorbed into the coefficient functions of the effective low energy operators. Assuming for simplicity that $\Psi(p) = 0$ at $|\vec{p}\,| > \mu$, one uses the Cauchy-Bunyakowski-Schwartz inequality to get

$$|\Psi(0)|^2 \;\le\; \int_{\vec{p}^{\,2}<\mu^2} \frac{d^3\,p}{(2\pi)^3}\cdot N \;=\; \frac{\mu^3}{6\pi^2}\;\;, \qquad f_B^2 \;\le\; \frac{2\mu^3}{\pi^2 M_B}\;\; . \tag{12}$$

Thus there is a simple upper bound on the wavefunction in terms of the phase space allocated for the system. The inequality saturates when $\Psi(p) = \mathrm{const}$ which yields the "finite size δ-function" $\Psi(x) \;=\; \mu^{3/2}\frac{1}{\pi}\sqrt{\frac{3}{2}}\,(\sin(\mu|x|)/(\mu|x|) \,-\, \cos\mu|x|)/(\mu^2x^2)$ in the coordinate space.

Inequality (12) appears to be rather restrictive: taking μ as large as 1 GeV one has

$$f_B \lesssim 200\,\mathrm{MeV}\;\;, \quad f_D \lesssim 330\,\mathrm{MeV}\;\; . \tag{13}$$

This upper bound for f_B is close to the existing theoretical estimates and maybe even somewhat lower than the expected values for f_{B_s}. It is worth noting the important role of

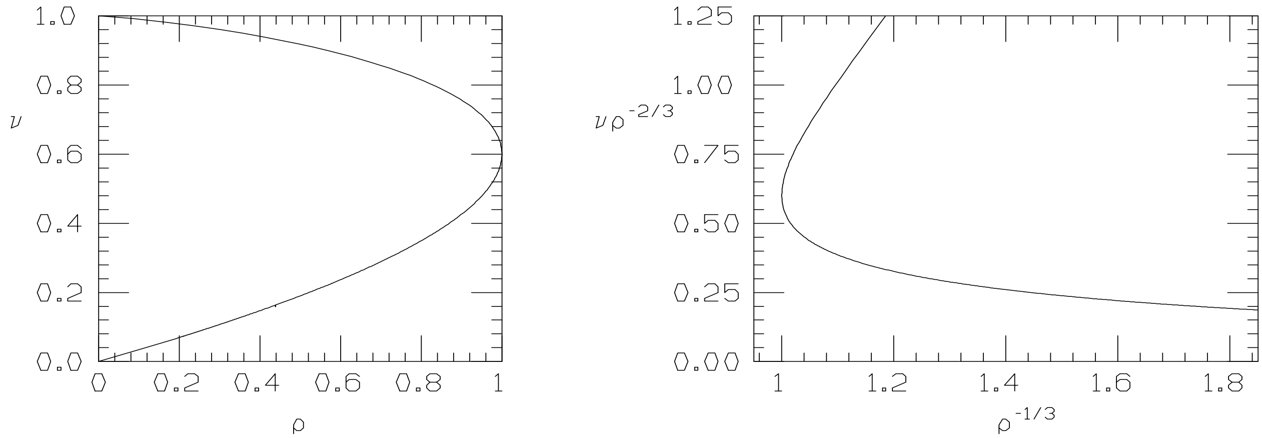

Figure 1: **Left:** possible values of $6\pi^2|\Psi(0)|^2$ (ρ) and μ_π^2 (ν) in units of μ. **Right:** the range of variation of the dimensionless ratio $\mu_\pi^2/(6\pi^2|\Psi(0)|^2)^{2/3}$ versus the degree of saturation of inequality (12).
The allowed regions lie between the upper and lower branches.

the color factor $N_c = 3$ in f_B^2 in accommodating such, naively quite moderate, magnitude of f_B. Eq. (12) demonstrates that, for the dimensional estimates, the quantity πf_B rather than plain f_B gives the proper scale; this strongly offsets, for example, apparently huge enhancement factor $24\pi^2$ of the two body phase space in PI, WA or WS compared to the three-body one in the free decay kinematics.

Adopting $\mu = 1\,\mathrm{GeV}$ as the reference point we obtain numerically

$$|\Psi^d(0)|^2 \lesssim 0.017\,\mathrm{GeV}^3 \quad . \tag{14}$$

This estimate thus sets the scale of what seems to be the maximal "natural" diquark density. It is justified to state that the values significantly larger than this and, correspondingly, the related $1/m_b^3$ corrections to the inclusive widths, call for a specific underlying mechanism to be added to the conventional picture.

One such dynamical mechanism is known – the perturbative short distance enhancement in B mesons due to the hybrid gluon exchange [1]:

$$f_B^2 = \left[\frac{\alpha_s(\mu^2)}{\alpha_s(m_b^2)}\right]^{4/b} \cdot 12\,\frac{|\Psi(0)|^2}{M_B} \quad , \qquad b = \frac{11}{3}N_c - \frac{2}{3}n_f \simeq 9 \quad . \tag{15}$$

Although for large μ this perturbative factor cannot be large, it literally enhances f_B^2 by only a factor of $1.3 \div 1.5$ (I use here the V scheme α_s as a physically adequate one), it is sufficient to move $|\Psi(0)|^2$ in B to the "comfortable" zone near $0.01\,\mathrm{GeV}^3$ and to allow for a less extreme value of $\mu \gtrsim 0.8\,\mathrm{GeV}$, in particular adopting the theoretically preferable values of $f_B \simeq 160\,\mathrm{MeV}$ [9].

Large scale of the essential momenta of the constituents in B imply relatively high μ_π^2, the expectation value of the kinetic operator in B mesons: assuming, as before, that Ψ vanishes above μ one gets, for example, $\mu_\pi^2 = 3/5\mu^2$ if μ is the minimal cutoff capable to accommodate given $|\Psi(0)|^2$ and, therefore,

$$\mu_\pi^2 \;=\; \frac{3}{5}\left(6\pi^2|\Psi(0)|^2\right)^{2/3} \;\simeq\; \frac{3}{5}\left(\frac{\pi^2}{2}\right)^{2/3} f_B^{4/3} M_B^{2/3}\left[\frac{\alpha_s(m_b^2)}{\alpha_s(\mu^2)}\right]^{8/27} \;\simeq\; 0.4\,\mathrm{GeV}^2\;\;; \qquad (16)$$

for larger μ somewhat smaller μ_π^2 is also possible, although the decrease cannot be dramatic unless μ is really large. In Fig. 1 I show the allowed values of the dimensionless ratios

$$\nu \;=\; \frac{\mu_\pi^2}{\mu^2} \quad , \quad \rho \;=\; \frac{6\pi^2|\Psi(0)|^2}{\mu^3}$$

relevant for the absolute values of the hadronic parameters μ_π^2 and f_B^2. The range of the direct ratio of μ_π^2 to $(6\pi^2|\Psi(0)|^2)^{2/3}$ which is given by $\nu\rho^{-2/3}$ is also interesting and shown in the right plot. Smaller values of μ_π^2 are possible only beyond the two particle picture of B mesons. Relaxing any constraint on μ one can have, in principle, arbitrary value of $\nu\rho^{-2/3}$, i.e. in this case no lower bound on μ_π^2 emerges. Still, one can obtain the μ-independent lower bound on the expectation value of the fourth power of momentum, $\langle\vec{p}^{\,4}\rangle$:

$$2^{1/2}3^{1/4}\,\pi\;\left(\langle\vec{p}^{\,4}\rangle\right)^{3/4} \;\geq\; 6\pi^2|\Psi(0)|^2\;\;. \qquad (17)$$

This follows from one of the Sobolev's family of inequalities occurring in the so-called *embedding theorems*, namely,

$$|f(y)|^2 \;\leq\; \frac{1}{2^{1/2}3^{3/4}\pi}\;\left(\int d^3x\,|f(x)|^2\right)^{1/4}\cdot\left(\int d^3x\,|\nabla^2 f(x)|^2\right)^{3/4}\quad . \qquad (18)$$

We see, therefore, that the QCD sum rule determination of f_B and μ_π^2 [9] looks consistent from this perspective; they are also in agreement with another, more rigorous QCD lower bound on the kinetic operator [10] and with the more phenomenological estimates [11].

An attempt to boost $|\Psi^d(0)|^2$ in Λ_b requiring larger μ, however, does not imply necessarily the large expectation value of the kinetic operator: the momentum of one of the light quarks can be balanced by another light quark rather than by b if the two light quarks are strongly correlated. In other words, in b baryons the moments of $|\Psi^d(p)|^2$ do not coincide with the expectation values of the operators $\bar{b}(i\vec{D})^k b$ appearing in the $1/m_b$ expansion, and under certain assumptions can be much smaller. In particular, the possibility remains that $\mu_\pi^2(\Lambda_b)$ is very small. Although this would uniformly enhance the Λ_b decay rate compared to B due to the Lorentz dilation [12], this effect can hardly reach even the two percent level.

Let us briefly examine the consequences of the hypothesis that the essential momenta of the light quarks are large and μ noticeably exceeds 1 GeV. I do not try to speculate here whether any QM-type potential model can be formulated in a self-consistent fashion if the light quark momenta are of such high scale; it is more reliable to discuss what would be the expected model-independent features for heavy flavors from the general QCD perspective. Clearly, it would destroy the applicability of the heavy quark expansion to the corresponding charm hadrons, including the spectrum and exclusive formfactors. It is not natural to expect that in such a case all traces of the symmetry relations are wiped away; rather, one would think that the symmetry pattern still persists at a qualitative level, whereas quantitative model-independent QCD predictions cannot be done. Some static characteristics can still survive large diquark density in baryons if, as mentioned above, the two light quarks form a very compact color-antitriplet configuration which is only softly bound to the heavy quark. It is not clear how natural this option is, but such a peculiarity must manifest itself in a number of other processes.

Large intrinsic momenta of the spectators would hardly justify the applicability of the standard expressions for the spectator-dependent $1/m_Q^3$ inclusive corrections to the widths. It is most transparent in the case of interference: the final expressions for the interference term via $|\Psi(0)|^2$ clearly imply that the typical momenta of the light decay quarks are larger than the momenta of the spectators – and the former are typically about 1.5 GeV even in beauty decays. Moreover, it is this ratio of the intrinsic to the final state quark momenta that controls the importance of the higher order power corrections and the significance of the "exponential" terms signaling the duality violation. The trend of such effects can hardly be predicted *a priori* in actual QCD.

While the intrinsic momenta of the spectators somewhat above 1 GeV can still allow for semi-quantitative analysis of the nonleptonic decays in beauty, the charm decay rates would at best offer a possibility to discuss only the qualitative pattern, with all spectator-dependent corrections being not suppressed at all. Even the semileptonic width of charmed particles can be seriously affected if such a scenario represents reality.

Turning back to the effects of PI and WS in Λ_b, let us assume that $\mu \simeq 1\,\mathrm{GeV}$ and set, according to eq. (14), the diquark density $0.017\,\mathrm{GeV}^3$. We then get numerically

$$\frac{\Gamma_{WS}}{\Gamma_{B_0}} \simeq 0.067 \quad , \qquad\qquad \frac{\Gamma_{PI}}{\Gamma_{B_0}} \simeq -0.028 \quad . \tag{19}$$

The expressions for the net effect of the hybrid renormalization has been given in the second paper [1] and, applied literally, amount to the additional factors 0.62 for WS and 1.1 for PI (I used here the value of the strong coupling in the V scheme $\alpha_s(2.3\,\mathrm{GeV}) = 0.336$ [13]). Needless to say, the accuracy of the estimates (19) relying on the simple model for the four-fermion matrix elements is not high, nor even of their ratio $\Gamma_{\mathrm{WS}}/\Gamma_{\mathrm{PI}}$ from which the unknown wavefunction naively drops out. Therefore it would be unjustified, in my opinion, to state the significant cancellation which is literally suggested by eqs. (19), and, in particular, when the hybrid renormalization is accounted for. It is more reasonable to expect merely that PI somewhat decreases the possible effect of WS in Λ_b [6], at least in the framework of the leading terms in the $1/m_b$ expansion. Since there is no large enough room for the (logarithmic) perturbative physics for large μ, the effect of the hybrid renormalization on WS can be viewed, conservatively, as the uncertainty of the simple estimates which typically neglect such effects originating in the domain below m_b.

Very recently the heavy-light diquark density at zero separation in Λ_b was estimated in the potential model-motivated way using the information on the hyperfine splitting in beauty mesons and Σ states [14]. The value suggested by that consideration agreed with the numerical bounds I discussed above and, thus, the resulted effects of WS and PI were only at a few percent level.

To summarize, I argued that the conventional constituent models of heavy flavor hadrons have restrictive intrinsic limitations on the possible size of the matrix elements governing $1/m_Q^3$ corrections to widths. These qualitative arguments are quantified by inequalities (12) and (17). Attempts to boost these effects in the straightforward manner above the scale of 10% in Λ_b require an essential revision of the assumptions set up in such models and would lead to important consequences, in particular, for the applicability of the heavy flavor symmetry to charmed particles. One can also guess that such localized hadrons' wavefunctions maybe not easy to extract reliably from the existing lattice simulations, requiring rather big lattice to correctly reproduce both relatively large momenta and "usual" soft components.

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