CERN–TH/96–15
ENSLAPP–A–578/96
THU–96/04
hep–th/9601158

STRING EFFECTS AND FIELD THEORY PUZZLES WITH SUPERSYMMETRY *

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ABSTRACT

We investigate field theory puzzles occuring in the interplay between supersymmetry and duality in the presense of rotational isometries (also known as non–triholomorphic in hyper–Kahler geometry). We show that T–duality is always compatible with supersymmetry, provided that non–local world–sheet effects are properly taken into account. The underlying superconformal algebra remains the same, and T–duality simply relates local with non–local realizations of it. The non–local realizations have a natural description using parafermion variables of the corresponding conformal field theory. We also comment on the relevance of these ideas to a possible resolution of long standing problems in the quantum theory of black holes.

January 1996

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*Contribution to the proceedings of the 5th Hellenic School and Workshops on Elementary Particle Physics, Corfu, 3-24 September 1995

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Introduction

Attempts to describe phenomena using inadequate theories usually lead to paradoxes. A prototype example in elementary particle physics is the Klein paradox, which ceases to exist once field theory replaces quantum mechanics. It is our purpose in this note (based on two lectures given by the authors in Corfu) to describe a paradox of the effective field theory that occurs in the interplay between duality and supersymmetry, which on the other hand has a natural explanation within string theory. We will see that certain aspects of our investigation could also be relevant to string phenomenology, in connection with supersymmetry breaking mechanisms, which are usually considered at the level of the lowest order effective theory. Moreover, we will suggest at the very end a string theoretical framework for providing a plausible resolution to long standing problems in the quantum theory of black holes, using non–locally realized superconformal algebras.

The classical propagation of strings in a general target space with metric $G_{\mu\nu}(X)$ and antisymmetric tensor field $B_{\mu\nu}(X)$ is described by the 2–dim σ–model Lagrangian density $\mathcal{L} = Q^{+}_{\mu\nu}\partial_{+}X^{\mu}\partial_{-}X^{\nu}$, where $Q^{+}_{\mu\nu} \equiv G_{\mu\nu} + B_{\mu\nu}$; we also introduce $Q^{-}_{\mu\nu} \equiv G_{\mu\nu} - B_{\mu\nu}$ for later use. The natural time coordinate on the world–sheet is $\tau = \sigma^{+} + \sigma^{-}$, while $\sigma = \sigma^{+} - \sigma^{-}$ denotes the corresponding spatial variable. We consider string backgrounds with a Killing symmetry associated to a vector field $\partial/\partial X^{0}$, and denote the rest of the target space coordinates by X^{i}, $i = 1, \ldots, d-1$. We choose to work for convenience in an adapted coordinate system where the background fields are all independent of X^{0}. It was shown by Buscher some time ago [1] that under these circumstances a dual σ–model can be found, as a solution of the β–function equations, with background fields

$$\tilde{G}_{00} = \frac{1}{G_{00}} \ , \qquad \tilde{Q}^{\pm}_{0i} = \pm\frac{Q^{\pm}_{0i}}{G_{00}} \ , \qquad \tilde{Q}^{+}_{ij} = Q^{+}_{ij} \ - \ \frac{Q^{+}_{i0}Q^{+}_{0j}}{G_{00}} \ , \tag{1}$$

while conformal invariance at 1–loop also requires a shift in the dilaton field Φ by $\ln(G_{00})$.

We wish to investigate how the supersymmetric properties of σ-models behave under T–duality. This will turn out to be an intriguing problem, because geometrical objects in the target space (like Kahler forms and Killing spinors) are not necessarily independent of the Killing coordinate X^{0}, although the corresponding σ–model background fields always are. To achieve our goal we have to find a direct way to formulate the action of T–duality transformations on the target space coordinates themselves, from which the transformation properties of all other quantities in the target space can be easily deduced. We find that the relevant formulation here is the description of Abelian T–duality as a canonical transformation on the world–sheet. In this approach [2] one performs the transformation $(X^{0}, P_{0}) \rightarrow (\tilde{X}^{0}, \tilde{P}_{0})$, defined by $\tilde{P}_{0} = \partial_{\sigma}X^{0}$, $\partial_{\sigma}\tilde{X}^{0} = P_{0}$, where P_{0} is the conjugate momentum to X^{0} in the 2–dim σ–model. This amounts to a non–local redefinition of the target space variable associated with the Killing symmetry,

$$X^{0} = \int \tilde{Q}^{+}_{\mu 0}\partial_{+}\tilde{X}^{\mu}d\sigma^{+} \ - \ \tilde{Q}^{-}_{\mu 0}\partial_{-}\tilde{X}^{\mu}d\sigma^{-} \ , \tag{2}$$

which in the Hamiltonian description of the 2–dim σ–model yields (1); for earlier work on this subject see [3]. Therefore, despite non–localities, the dual background fields are locally related to the original ones by (1). However, other geometrical objects in target space that can generically depend on X^0 will become non–local in the dual face of the theory [4]. The canonical formalism offers the advantage to explore explicitly the transformation properties of such objects, including Kahler forms in particular.

When are we faced with a paradox in the effective field theory approach to quantum string dynamics? Only when we insist on having a consistent description in terms of local effective field theories and forget various non–local world–sheet effects generated by (2), when it is appropriate. Such non–local effects constitute the main theme of our work, and as we will see next by studying the interplay between T–duality and supersymmetry, they have a crucial role in understanding the string resolution to paradoxes of the effective theory.

Duality and world–sheet supersymmetry: generalities

It is well known that any background can be made $N = 1$ supersymmetric on the world–sheet [5], and there cannot possibly be a clash with duality (Abelian and non–Abelian as well as S–duality) in this case. In contrast, extended $N = 2$ supersymmetry [6, 7] requires that the background is such that an (almost) complex (hermitian) structure $F^{\pm}_{\mu\nu}$ exists in each sector associated to the right and left-handed fermions. Similarly, $N = 4$ extended supersymmetry [7, 8] requires that there exist three independent complex structures in each sector, $(F^{\pm}_I)_{\mu\nu}$, $I = 1, 2, 3$. The complex structures are covariantly constant with respect to generalized connections that include the torsion, they are represented by antisymmetric matrices, and in the case of $N = 4$ they obey the $SU(2)$ Clifford algebra. These conditions put severe restrictions on the backgrounds that can arise as supersymmetric solutions. For instance, in the absence of torsion, the metric should be Kahler for $N = 2$ and hyper–Kahler for $N = 4$ [7].

In order to proceed further we need to know the transformation properties of the complex structures under T–duality. If a complex structure is independent of X^0, then under duality it transforms as [9, 4]

$$(\tilde{F}^{\pm}_I)_{0i} = \pm \frac{(F^{\pm}_I)_{0i}}{G_{00}} , \qquad (\tilde{F}^{\pm}_I)_{ij} = (F^{\pm}_I)_{ij} + \frac{1}{G_{00}} \left((F^{\pm}_I)_{0i} Q^{\pm}_{j0} - (F^{\pm}_I)_{0j} Q^{\pm}_{i0} \right) , \quad (3)$$

and hence defines a locally realized extended supersymmetry in the dual model as well. If on the other hand it depends on X^0, then to obtain the right transformation one should replace X^0 by the non–local expression (2) in the corresponding Kahler form; in this case supersymmetry will be realized non–locally in the dual picture [4].

As a consequence of having non-local realizations of supersymmetry, many of the theorems established in the past for 2-dim σ–models with $N = 4$ extended supersymmetry are not strictly valid, since they were implicilty relying on the assumption that the complex structures are local functions of the target space variables. For instance, the transformed complex structures are not covariantly constant when non–local realizations come into play [4, 10]. Also, the generators of the holonomy group will no longer commute with the complex structures [11], in contrast with the case of locally realized $N=4$ (see, for instance, [8]). In the absence of torsion, in particular, non–local $N=4$ world–sheet supersymmetry does not imply that the manifold is hyper–Kahler, as for local $N=4$ [7]. If any of these theorems is used as a guiding principle, some supersymmetries will appear to be lost in the effective field theory approach after duality, as the non–local ones cannot be distinguished anymore. Then, an apparent paradox arises, as it was stated in [12]. If duality provides an equivalence between strings propagating in different spacetimes, then how can it destroy other genuine symmetries such as supersymmetry? The resolution to this paradox, as we will see, is simply that non–local world–sheet effects have to be taken into account in order to obtain a consistent picture with supersymmetry [4].

The criterion for having an X^0–dependent complex structure is ultimately connected with the way it fits into representations of the isometry group, which in our simple Abelian case is isomorphic to $U(1)$. In $N=2$ extended world–sheet supersymmetry we may arrange for the complex structure to be a $U(1)$ singlet, and hence independent of X^0, but for $N=4$ extended supersymmetry there are two distinct possibilities: either all three complex structures are $U(1)$ singlets, or one of them is a singlet and the other two form a doublet. In the former case the original locally realized $N=4$ supersymmetry remains local, and hence manifest after duality, whereas in the latter it becomes non-manifest by breaking to a local $N=2$ part, with the rest being realized non–locally.

A useful criterion for distinguishing when Abelian T–duality preserves manifest $N=4$ extended world–sheet supersymmetry is [11]

$$\partial_\mu Q^{\mp}_{0\nu}(F^{\pm}_I)^{\mu\nu} = 0 \ , \quad I=1,2,3 \ . \tag{4}$$

This condition is indeed satisfied by all three complex structures if they are $U(1)$ singlets, whereas in the case of a singlet plus a doublet it turns out that the right hand side of (4), instead of vanishing, takes the form [11]

$$\partial_\mu Q^{\mp}_{0\nu}(F^{\pm}_I)^{\mu\nu} = \frac{d}{2}\epsilon_{I12} \ . \tag{5}$$

Hence, the violation of (4) receives contribution from the singlet complex structure, which here and in the following will be labelled by $I=3$. Notice that the right hand side of (5) is just a constant, although not zero. Therefore, the validity of (4) may be checked in practice by expanding locally the right hand side around a reference target space point. In particular, let us briefly examine the case of 4-dim flat space with metric written in polar coordinates

$$ds^2 = d\rho^2 + \rho^2 d\varphi^2 + dx^2 + dy^2 \ . \tag{6}$$

This background has manifest $N=4$ world–sheet supersymmetry. The complex structure that remains invariant under φ–shifts by a constant is given by

$$F_3 \ = \ \rho d\rho\wedge d\varphi \ + \ dx\wedge dy \ . \tag{7}$$

It is a matter of short computation to verify that (5) (for $I=3$ and $d=4$) is satisfied. Thus, T–duality with respect to φ will break manifest $N=4$ supersymmetry. The same is true for all spaces that can be locally approximated by (6), in the vicinity of the fixed points of a Killing isometry. For obvious reasons such Killing vectors fields, and the corresponding string backgrounds, will be called rotational.

Complex structures and parafermions

Since the non–local world–sheet effects we have described are characteristic of the string theoretical nature of duality, we would like to see explicitly how they become manifest in the conformal field theory corresponding to a 2–dim σ–model. For this reason, we choose to demonstrate the aspects of our general framework in the special case of the 4–dim semi–wormhole solution, and its rotational dual background. An exact description is available for this background in terms of the $SU(2) \otimes U(1)$ WZW model and its cousins.

The semi–wormhole solution of 4–dim string theory provides an exact conformal field theory background with $N = 4$ world–sheet supersymmetry [13, 14]. The $N = 4$ superconformal algebra can be locally realized in terms of four bosonic currents, three non–Abelian $SU(2)_k$ currents and one Abelian current with background charge $Q = \sqrt{2/(k+2)}$, so that the central charge is $\hat{c} = 4$. There are also four free–fermion superpartners, and the solution is described by the $SU(2)_k \otimes U(1)_Q$ supersymmetric WZW model. It is convenient to parametrize the $SU(2)$ group element as

$$g = e^{\frac{i}{2}(\tau-\psi)\sigma_3} e^{i\varphi\sigma_2} e^{\frac{i}{2}(\tau+\psi)\sigma_3} . \tag{8}$$

Then the background fields of this model are given by

$$\begin{aligned} ds^2 &= d\rho^2 + d\varphi^2 + \sin^2\varphi \; d\psi^2 + \cos^2\varphi \; d\tau^2 \; , \\ B_{\tau\psi} &= \cos^2\varphi \; , \quad \Phi = 2\rho \; , \end{aligned} \tag{9}$$

where ρ corresponds to the $U(1)$ factor. For small φ this background approaches the flat space metric (6), with $\partial/\partial\psi$ playing the role of the rotational Killing vector field.

The following analysis was essentially performed in [4], using a different parametrization. The three complex structures for the right sector are

$$F_i^+ = 2d\rho \wedge \Sigma_i - \epsilon_{ijk}\Sigma_j \wedge \Sigma_k \; , \tag{10}$$

where the left invariant Maurer–Cartan forms of $SU(2)$, $\Sigma_i = -\frac{i}{2}Tr(g^{-1}dg\sigma_i)$, have been used:

$$\begin{aligned} \Sigma_3 &= \cos^2\varphi d\tau + \sin^2\varphi d\psi \; , \\ \Sigma_\pm &= \Sigma_1 \pm i\Sigma_2 = e^{\pm i(\tau+\psi)} \left(\pm i d\varphi + \frac{1}{2}\sin 2\varphi (d\tau - d\psi) \right) \; . \end{aligned} \tag{11}$$

The complex structures for the left sector can be similarly written down

$$F_i^- = 2d\rho \wedge \tilde{\Sigma}_i - \epsilon_{ijk}\tilde{\Sigma}_j \wedge \tilde{\Sigma}_k \; , \tag{12}$$

where $\tilde{\Sigma}_i = -\frac{i}{2}Tr(dgg^{-1}\sigma_i)$ are the right invariant Maurer–Cartan forms of $SU(2)$. Their explicit expressions can be obtained from (11) letting $(\tau, \psi, \varphi) \to (-\tau, \psi, -\varphi)$, up to an overall minus sign. It is easy to see that under constant shifts of ψ, F_3^+ is a singlet, whereas F_1^+ and F_2^+ form a doublet, and we may proceed similarly for the other chiral sector.

T–duality with respect to the Killing vector $\partial/\partial\psi$ gives the background [15]

$$\begin{aligned} d\tilde{s}^2 &= d\varphi^2 + \cot^2\varphi \, d\alpha^2 + d\beta^2 + d\rho^2 \ , \\ \tilde{\Phi} &= 2\rho + \ln(\sin^2\varphi) \ , \end{aligned} \tag{13}$$

with zero antisymmetric tensor, which corresponds to the $(SU(2)_k/U(1)) \otimes U(1) \otimes U(1)_Q$ model; to be precise we should make the redefinition of variables $\alpha = \tilde{\psi} - \frac{\tau}{2}$ and $\beta = \tilde{\psi} + \frac{\tau}{2}$. Note that although the torsion is zero, the Ricci tensor is not zero due to the presence of a non–trivial dilaton. This means that the manifold is not hyper–Kahler. According to our previous discussion, this would have been a paradox of the effective theory if non–local world–sheet effects were not properly taken into account in string theory.

The complex structure dual to $F_3^{\pm}$ is given by

$$\tilde{F}_3 = d\rho \wedge d\beta + \cot\varphi \, d\varphi \wedge d\alpha \ , \tag{14}$$

and clearly defines local supersymmetries in both chiral sectors. Since there is no torsion, there is also no distinction between the + and the – components. However, the dual of the complex structures $F_{1,2}^{\pm}$ are non–local due to the explicit appearance of ψ in (11), which after duality is given by (cf. (2))

$$\psi = \int (\cot^2\varphi \partial_+\alpha + \partial_+\beta) d\sigma^+ - (\cot^2\varphi \partial_-\alpha + \partial_-\beta) d\sigma^- \ . \tag{15}$$

We write the dual form of the complex structures as [4]

$$\begin{aligned} \tilde{F}_1^+ &= (d\rho + i d\beta) \wedge \Psi_+ + (d\rho - i d\beta) \wedge \Psi_- \ , \\ \tilde{F}_2^+ &= i(d\rho + i d\beta) \wedge \Psi_+ - i(d\rho - i d\beta) \wedge \Psi_- \ , \end{aligned} \tag{16}$$

for the right sector, and

$$\begin{aligned} \tilde{F}_1^- &= i(d\rho - i d\beta) \wedge \bar{\Psi}_+ - i(d\rho + i d\beta) \wedge \bar{\Psi}_- \ , \\ \tilde{F}_2^- &= (d\rho - i d\beta) \wedge \bar{\Psi}_+ + (d\rho + i d\beta) \wedge \bar{\Psi}_- \ , \end{aligned} \tag{17}$$

for the left sector. There appears to be a distinction between the + and the – components, although the torsion vanishes, and this is a novel characteristic of the non–local realizations of supersymmetry [11].

The parafermionic-type 1–forms are defined as

$$\begin{aligned} \Psi_{\pm} &= (d\varphi \pm i \cot\varphi \, d\alpha) e^{\pm i(\beta - \alpha + \psi)} \ , \\ \bar{\Psi}_{\pm} &= (d\varphi \mp i \cot\varphi \, d\alpha) e^{\pm i(\alpha - \beta + \psi)} \ , \end{aligned} \tag{18}$$

and they are non–local due to (15). They have a natural decomposition in terms of $(1,0)$ and $(0,1)$ forms on the string world–sheet

$$\Psi_\pm = \Psi_\pm^{(1,0)} d\sigma^+ \;+\; \Psi_\pm^{(0,1)} d\sigma^- \;, \qquad \bar\Psi_\pm = \bar\Psi_\pm^{(1,0)} d\sigma^+ \;+\; \bar\Psi_\pm^{(0,1)} d\sigma^- \;. \tag{19}$$

It can be easily verified using the classical equations of motion for the model (13) that they satisfy the chiral and anti–chiral conservation laws

$$\partial_- \Psi_\pm^{(1,0)} = 0 \;, \qquad \partial_+ \bar\Psi_\pm^{(0,1)} = 0 \;. \tag{20}$$

In this case, in fact, $\Psi_\pm^{(1,0)}$ and $\bar\Psi_\pm^{(0,1)}$ are nothing but the classical parafermions [16] for the $SU(2)_k/U(1)$ coset, with the field β providing the appropriate dressing to the full 4–dim model $(SU(2)_k/U(1)) \otimes U(1) \otimes U(1)_Q$. Thus, the original local $N = 4$ world–sheet supersymmetry breaks to a local part corresponding to (14), and the rest is realized non–locally using the non–local analogue of the complex structures (16), (17). At the (super)CFT level this manifests by replacing the three non-Abelian $SU(2)_k$ currents with two $SU(2)_k/U(1)$ parafermions and one Abelian current in the realization of the $N = 4$ superconformal algebra [14].

Restoration of manifest supersymmetry

Since Abelian T–duality acts as a Z_2 symmetry, it is obvious that by dualizing the background (13) with respect to the Killing vector field $\partial/\partial\tilde\psi$, where $\tilde\psi = \frac{1}{2}(\alpha+\beta)$, we will recover back (9). In the process, as it is expected, the "dressed" parafermions (18) will become the usual currents corresponding to the raising and lowering generators of the $SU(2)$ algebra. A suggestive way of thinking about it is that the duality provides a mechanism for restoring manifest supersymmetry. In the following we present a less trivial example based on the results reported in [11]. The same example was first considered in [17] from a different, though equivalent point of view, in connection with the restoration of manifest spacetime supersymmetry. The issues of spacetime supersymmetry will be discussed separately in some detail in the next section.

Let us consider a 4–dim string background with

$$\begin{aligned} ds^2 &= d\varphi^2 + \cot^2\varphi dx^2 + d\rho^2 + R^2(\rho) dy^2 \;, \\ \Phi &= \ln(\sin^2\varphi / R'(\rho)) \;, \end{aligned} \tag{21}$$

and zero antisymmetric tensor. The function $R(\rho)$ is dynamical, and it is constrained by requiring 1–loop conformal invariance to satisfy the differential equation

$$R' = C_1 R^2 + C_2 \;. \tag{22}$$

The constants C_1, C_2 completely classify the different solutions (see, for instance, [18]).

It turns out that (21) has $N = 4$ extended supersymmetry with the local $N = 2$ part corresponding to the complex structure

$$F_3 = \cot\varphi \; d\varphi \wedge dx + R(\rho) \; d\rho \wedge dy \;, \tag{23}$$

whereas the rest is realized non–locally. In order to present the analogue of the other two complex structures, it is convenient to introduce parafermionic-type 1–forms similar to (18). They are given by

$$\begin{aligned}\Psi^{(1)}_{\pm} &= (d\varphi \pm i\cot\varphi dx)\ e^{\pm i(-x+\theta_1)}\ , \qquad \bar\Psi^{(1)}_{\pm} = (d\varphi \mp i\cot\varphi dx)\ e^{\pm i(x+\theta_1)}\ ;\\ \theta_1 &\equiv \int \cot^2\varphi\partial_+ x d\sigma^+ - \cot^2\varphi\partial_- x d\sigma^-\ ,\end{aligned}\tag{24}$$

corresponding to the usual $SU(2)/U(1)$ classical parafermions, and

$$\begin{aligned}\Psi^{(2)}_{\pm} &= (d\rho \pm iRdy)\ e^{\pm i(c_2 y+\theta_2)}\ , \qquad \bar\Psi^{(2)}_{\pm} = (d\rho \mp iRdy)\ e^{\pm i(-c_2 y+\theta_2)}\ ,\\ \theta_2 &\equiv \int (c_2 - R')\partial_+ y d\sigma^+ - (c_2 - R')\partial_- y d\sigma^-\ ,\end{aligned}\tag{25}$$

where c_2 is an arbitrary constant. Then, the non–local complex structures assume the form [11]

$$F^+_1 = \Psi^{(1)}_+ \wedge \Psi^{(2)}_+ + \Psi^{(1)}_- \wedge \Psi^{(2)}_-\ , \qquad F^+_2 = i\Psi^{(1)}_+ \wedge \Psi^{(2)}_+ - i\Psi^{(1)}_- \wedge \Psi^{(2)}_-\ ,\tag{26}$$

and

$$F^-_1 = \bar\Psi^{(1)}_+ \wedge \bar\Psi^{(2)}_+ + \bar\Psi^{(1)}_- \wedge \bar\Psi^{(2)}_-\ , \qquad F^-_2 = -i\bar\Psi^{(1)}_+ \wedge \bar\Psi^{(2)}_+ + i\bar\Psi^{(1)}_- \wedge \bar\Psi^{(2)}_-\ ,\tag{27}$$

These define legitimate extended supersymmetries at the classical level, and they satisfy the general equations for having non–local complex structures [11].

We now address the question: under what circumstances is it possible, via a duality transformation of (21), to obtain a σ-model with manifest $N = 4$ supersymmetry, so that all the complex structures are local functions of the target space variables. Although we may consider a general $O(2,2)$ transformation on (21), it turns out that only for a particular choice of parameters we obtain the desired result, which is described below.

Let us introduce the coordinate change $x = \psi - \frac{\tau}{2}$, $y = \psi + \frac{\tau}{2}$ and perform a T–duality transformation with respect to the symmetry generated by the Killing vector $\partial/\partial\psi$. The resulting background is

$$\begin{aligned}d\tilde s^2 &= d\rho^2 + d\varphi^2 + \frac{1}{1+R^2\tan^2\varphi}(\tan^2\varphi d\tilde\psi^2 + R^2 d\tau^2)\ ,\\ \tilde B_{\tau\tilde\psi} &= \frac{1}{1+R^2\tan^2\varphi}\ , \quad \tilde\Phi = \ln\Big((\cos^2\varphi + R^2\sin^2\varphi)/R'\Big)\ .\end{aligned}\tag{28}$$

For constant R it can be interpreted as describing a continuous line of $J\bar J$-deformed $SU(2)\otimes U(1)_Q$ models [19] with R as the modulus. The value $R = 1$ corresponds to the WZW point. The model with R as a function of ρ was considered in its Minkowski version as a toy model for studying dynamical topology change in string theory [18]. We have also denoted the dual variable of ψ by $\tilde\psi$. Their explicit relation is of course non–local according to (2). What is important here is the relation of the corresponding world–sheet derivatives, which is found to be

$$\partial_\pm\psi = \frac{1}{1+R^2\tan^2\varphi}\left(\pm\tan^2\varphi\partial_\pm\tilde\psi + \frac{1}{2}((1-R^2\tan^2\varphi)\partial_\pm\tau)\right)\ .\tag{29}$$

This will be used in order to deduce the transformation of the functionals θ_1, θ_2 defined in (24), (25), and then the transformation of the non–local complex structures (26), (27). The phase factors in $F^{\pm}_{1,2}$ that are responsible for their non–local nature are

$$\theta_1 + \theta_2 \pm (c_2 y - x) = \pm(c_2 - 1)\psi \pm \frac{1}{2}(c_2 + 1)\tau$$
$$+ \int \Big((c_2 - R' + \cot^2\varphi)\partial_+\psi + \frac{1}{2}(c_2 - R' - \cot^2\varphi)\partial_+\tau \Big) d\sigma^+ - (+ \rightarrow -) \ . \quad (30)$$

We find generically that the non–localities persist after T–duality, except in a particular case where they completely cancel out. This happens if we choose $c_2 = 1$ and the function $R(\rho)$ to satisfy

$$R' = 1 - R^2 \quad \Rightarrow \quad R = 1 \quad or \quad \tanh\rho \quad or \quad \coth\rho \ . \quad (31)$$

Then, indeed, the phase factors (30) transform just to $\tilde{\psi} \pm \tau$, and the dual complex structures become local.

Among the three different solutions, the one with $R = 1$ corresponds to the WZW model for $SU(2) \otimes U(1)_Q$ given by (9). Hence we see that a marginal deformation away from the WZW point ($R = const. \neq 1$) leads to a loss of manifest $N = 4$. It can be shown [11] that this is a general statement valid for all WZW models based on quaternionic groups, with $SU(2) \otimes U(1)$ being the most elementary example.

The solutions $R = \tanh\rho$ and $R = \coth\rho$ correspond to a new 4-dim background with manifest $N = 4$ supersymmetry. In an appropriate coordinate system it assumes the form [11]

$$ds^2 = e^{-\Phi}\, dx_i dx_i \ , \qquad H_{ijk} = -\epsilon_{ijk}{}^{l}\partial_l\Phi \ ,$$
$$\Phi = \frac{1}{2}\ln\Big((x_i x_i + 1)^2 - 4(x_3^2 + x_4^2)\Big) \ . \quad (32)$$

The metric is conformally flat with the conformal factor satisfying the Laplace equation adapted to the flat space metric, i.e. $\partial_i\partial_i e^{-\Phi} = 0$, in agreement with a general theorem proved in [13]. The antisymmetric field strength solves the (anti)self–duality conditions of the dilaton–axion field and therefore the solution (32) is an axionic–instanton. The complex structures are

$$F_1^{\pm} = e^{-\Phi}(-dx_1 \wedge dx_3 \pm dx_2 \wedge dx_4) \ ,$$
$$F_2^{\pm} = e^{-\Phi}(\pm dx_1 \wedge dx_4 + dx_2 \wedge dx_3) \ ,$$
$$F_3^{\pm} = e^{-\Phi}(dx_1 \wedge dx_2 \pm dx_3 \wedge dx_4) \ . \quad (33)$$

In fact these are the complex structures for all 4–dim axionic instantons of the form (32) irrespectively of the particular dilaton field Φ [11], which is only needed for conformal invariance. Geometrically the metric represents the throat of a semi–wormhole. Notice that a true semi–wormhole is obtained only by shifting $e^{-\Phi}$ by a constant, since then asymptotically the space is Euclidean. In our case this corresponds to an S–duality transformation. We also note that the metric has singularities not at a single point, but in the ring $x_1 = x_2 = 0$, $x_3^2 + x_4^2 = 1$. Therefore, the throat never becomes infinitely thin. The background (32) is a generalization of the $SU(2) \otimes U(1)$ semi–wormhole background [13, 14] to which our solution approaches for large values of the x_i's.

It is important to emphasize that in trying to obtain a 2-dim σ-model with manifest $N = 4$ supersymmetry via a duality transformation from (21), at no point we required conformal invariance. The entire treatment was completely classical and the function $R(\rho)$ remained arbitrary. Both (21) and its dual (28) have non–locally realized $N = 4$ supersymmetry at the classical level. It turned out that the condition (31) that led to manifest $N = 4$ supersymmetry for the dual model is also a particular case of (22), with $C_2 = -C_1 = 1$, which guarantees 1–loop conformal invariance for both models. With these choices for $R(\rho)$, the parafermionic 1–forms $\Psi_{\pm}^{(2)}$ and $\bar{\Psi}_{\pm}^{(2)}$ correspond to the usual classical non–compact parafermions of the $SL(2, R)/U(1)$ coset. Then, the background (21) corresponds to the direct product $SU(2)/U(1)_k \otimes SL(2, R)_{-k-4}/U(1)$ and the $N = 4$ superconformal algebra is realized using as natural objects the compact and non-compact parafermions [14]. A similar CFT construction for the new background (32) remains an open problem. Since $N = 4$ is manifest at the classical level, it is expected that the realization of the corresponding superconformal algebra will be local, or at least it will become local in the classical regime of large k.

We conclude this section with a few comments on the cases where the isometry group of duality is non–Abelian [20] (for earlier work on the subject see [21]). We have seen that assigning the complex structures to representation of the isometry group is a useful way to determine the fate of supersymmetry under T–duality. Let us consider the effect of non–Abelian duality transformations on $SO(3)$–invariant hyper–Kahler metrics. In such cases the complex structures are either $SO(3)$ singlets, thus remaining invariant under the non–Abelian group action, or they form an $SO(3)$ triplet. The Eguchi–Hanson metric corresponds to the first case, while the Taub–NUT and the Atiyah–Hitchin metrics to the second [22]. It should be clear, then, that the dual version of the Eguchi–Hanson instanton with respect to $SO(3)$ will have an $N = 4$ world–sheet supersymmetry locally realized. On the other hand, applying non–Abelian $SO(3)$–duality to the Taub–NUT and the Atiyah–Hitchin metrics will result in a total loss of all the locally realized extended world–sheet supersymmetries. Instead, a non–local realization of supersymmetry will emerge in such cases, with three non–local complex structures that satisfy the general conditions of [11]. Also, in cases where non–Abelian duality is performed on a WZW model, the non–local realizations can be described in terms of non–Abelian parafermions. We hope to report some work along these lines elsewhere [23].

Duality and space–time supersymmetry

The conventional definition of a string background with unbroken target space supersymmetry requires the existence of solutions of the Killing spinor equations. Then, the quantum field theory of the fluctuations around this vacuum will be, at tree level, supersymmetric as well. The unbroken supersymmetries are in one to one correspondence with the independent solutions of these equations.

For concreteness consider $N = 1$ supergravity in $d = 10$ dimensions coupled to superYang–Mills, which is the low energy approximation to heterotic string theory. To simplify matters further, we will consider vacuum solutions with the gauge fields and their corresponding gluinos set equal to zero. Then the Killing spinor equations are

$$\begin{aligned} \delta\Psi_\mu &= \Big(\partial_\mu + \frac{1}{4}({\omega_\mu}^{\alpha\beta} - \frac{1}{2}{H_\mu}^{\alpha\beta})\gamma_{\alpha\beta}\Big)\,\xi = 0 \ , \\ \delta\lambda &= -\Big(\gamma^\mu\partial_\mu\Phi + \frac{1}{6}H_{\mu\nu\lambda}\gamma^{\mu\nu\lambda}\Big)\,\xi = 0 \ , \end{aligned} \tag{34}$$

where Ψ_μ and λ are the gravitino and dilatino fields respectively.

As for the world–sheet supersymmetry, the presence of rotational–type Killing vector fields also results to a breaking of manifest target space supersymmetry under Abelian T–duality, in the sense that no Killing spinors exist in the dual background [12, 24]. As an example consider a 10-dim background whose non-trivial part is given by the 4-dim $SU(2)\otimes U(1)$ model (9). Then, the solution of the Killing spinor equation is (34)

$$\begin{pmatrix} \xi_+ \\ \xi_- \end{pmatrix} = e^{-\frac{i}{2}\varphi\sigma_2} e^{-\frac{i}{2}(\tau+\psi)\sigma_3} \begin{pmatrix} 0 \\ \epsilon_- \end{pmatrix} \ , \tag{35}$$

where ϵ_- is the non–zero Weyl component of a constant spinor. However, for the dual background (13) there are no solutions of (34). This should be obvious from (35), as the Killing spinor has an explicit dependence on ψ and after duality it becomes non–local [10]. The crucial difference with the case of extended world–sheet supersymmetry is that here the lowest order effective field theory is not enough at all to understand the fate of target space supersymmetry under duality, since one has to generate the whole supersymmetry algebra and not just its truncated part corresponding to the Killing spinor equations. Nevertheless, it is believed that T–duality does not destroy target–space supersymmetry in an appropriate string setting. An approach to this problem has been made in [17], using CFT concepts, and will will not be discussed further.

We think that massive string modes play a crucial role in this game, as it is also apparent by making contact with the work of Scherk and Schwarz [25] on coordinate dependent compactifications. We propose a comparison between these two problems. In Scherk–Schwarz, dimensional reduction is performed not in the conventional way, by assuming that all fields and transformation parameters are independent of the Kaluza-Klein internal coordinates, but instead a special factorized dependence on the compactified coordinates is kept. It turns out that some fields acquire mass in the dimensionally reduced theory, although all the fields are massless in the unreduced theory, leading to a breaking of supersymmetry upon compactification. Recall that in our case the Killing spinors also depend on the "Kaluza–Klein" coordinate X^0 when the isometries are of rotational type. Since the Killing spinors are the supersymmetry transformation parameters, we expect by analogy with [25] that massive strings modes should play a role not only in the dimensionally reduced theory, but in the realization of the supersymmetry algebra after duality as well, which in turn renders the truncation to only the massless modes as inconsistent. The question we are really raising here is how to formulate correctly in a string framework the duality transformations with respect to Scherk–Schwarz "isometries", and more generally with respect the any other conceivable compactification of string theory. The results we have described so far can be viewed as a preliminary exercise towards this goal, which could bring many new ideas in the subject with numerous physical applications that seemed paradoxical in the effective field theory approach.

It is worth mentioning that the breaking of manifest target space supersymmetry occurs hand and hand with the breaking of local $N = 4$ extended world–sheet supersymmetry. However, although in the latter case the $N = 2$ part remains local, manifest target space supersymmetry appears to be completely broken. This can be attributed to the relation between Killing spinors and complex structures [26], using $F_{\mu\nu} = \bar{\xi}\Gamma_{\mu\nu}\xi$, thus making possible to construct local complex structures out of non–local Killing spinors. Such an example is precisely the background (13), which has manifest $N = 2$ world–sheet supersymmetry, but no manifest spacetime supersymmetry.

Finally, let us discuss briefly the mechanism of restoring manifest supersymmetry from the spacetime point of view. The starting point in [17] is also the background (28), with a dynamical modulus $R(\rho)$. Demanding that the dilatino equation in (34) is satisfied leads to the first order equation (31). The gravitino equation is also satisfied, and the explicit form of the Killing spinor can be found [11]. In fact, in the coordinate system (32) the solution of (34) is just the constant Weyl spinor. Notice that contrary to the restoration of manifest world–sheet supersymmetry, which required no quantum input at all (conformal invariance was not even an issue there), restoring manifest target space supersymmetry requires the use of the dilaton field Φ, which is a 1–loop quantum effect in the α'–expansion.

The various issues we have discussed here on the relation between duality and supersymmetry may also be relevant to string phenomenology in one way or another. If the duality can break or restore manifest supersymmetry, then this phenomenon should certainly be taken into consideration in various supersymmetry breaking scenarios relying on the effective field theory approach. "Apparently" non–supersymmetric backgrounds, such as (13) or (21), can qualify as vacuum solutions to superstring theory when it is possible to restore manifest supersymmetry through non–local world–sheet effects (of the type we have described) at the string level. In addition, this raises the question whether various solutions of physical interest in black hole physics or cosmology could have hidden supersymmetries in a string context. Since this possibility necessarily involves non–local world–sheet effects, it will be important to explore it further in our effort to understand ways that string theory can resolve fundamental problems in physics, in particular the quantum theory of black holes.

A speculation with black holes

The various solutions of the lowest order effective theory provide only a semiclassical approximation to the exact conformal field theories that correspond to different string vacua. Although there exist many solution generating techniques to lowest order in α', our present technology with CFT is comparatively limited to only a few exact constructions. For example, the familiar Schwarzchild metric describing 4–dim black holes in general relativity is a solution of the β–function equations, and for sufficiently large black holes with $(Gm)^2 > \alpha'$ the higher order corrections can be regarded as perturbation. An analysis of this problem was initially done in [27]. Finding the exact CFT of the Schwarzchild black hole, however, remains an outstanding problem up to this date. The successful solution will certainly help us to understand how string theory could cure some of the paradoxes associated with black hole evaporation. We would like to view these long standing problems as paradoxes of the low energy effective theory and there is hope to use supersymmetry for this purpose. Of course, we are thinking about non–local realizations of an underlying superconformal algebra, and we will present some speculations in that direction.

It is true that ordinary 4–dim black holes have no manifest space–time supersymmetry, which is consistent with their property of having a non–zero temperature T inversely proportional to their mass parameter m. Microscopic black holes are very hot, and it is for them that the α' expansion should not be trusted. Thus, stringy effects, if any, will manifest as paradoxes of the lowest order effective theory that has been used so far to describe black holes. Motivated from our previous results, we may forget now T–duality, and twist things around asking the following question: is the unknown CFT of black holes superconformal, but with a non–local realization, in which case non–local world–sheet effects might resolve the problems with their evaporation within string theory? The closest we can get to entertain this idea is by considering exact 4–dim CFT coset models built out of the 2–dim black hole coset $SL(2,R)/U(1)$. Our previous analysis explicitly demonstrates that in such toy models the lowest order geometry has no manifest space–time supersymmetry, but in the exact picture the parafermions provide the relevant realization of an underlying $N = 4$ superconformal algebra. For real black holes, we hope first to find a superconformal structure at the classical level, and when the exact CFT will be known to be able to promote it to an exact symmetry quantum mechanically, using non–local operators of the model analogous to the parafermions.

A possible way to proceed classically with the construction of non–local complex structures for the black hole geometry is motivated by the reinterpretation of T–duality as a non–local change of the target space variables. Recall at this point that for a large class of string backgrounds with Killing symmetries, T–duality is only one element of a bigger symmetry group of the β–function equations, also known as U–duality in its various discrete forms. A particularly interesting class of 4–dim backgrounds is provided by geometries with two commuting Killing symmetries. These include flat space and the black hole geometry, because it is a static and axisymmetric configuration. An interesting aspect of this class of geometries is the presence of an infinite dimensional group that acts as a complete solution generating symmetry, connecting any other solution with two isometries to the flat space geometry. This is the well known Geroch group in general relativity [28], but its generalizations to string theory (including anti–symmetric tensor and dilaton fields) also exist [29], unifying T with S–dualities as continuous groups of transformations. It is then natural to ask whether there is a reinterpretation of this huge symmetry as non–local change of the target space variables, thus extending the known result for T–duality. If this is not a formidable task, it will provide the right transformation rules of the complex structures of flat space. Since any other solution with two isometries can be generated from flat space in this fashion, non–local complex structures could be constructed according to the geometry.

We do not expect, however, that the transformed complex structures of flat space will always satisfy the right integrability conditions for having an underlying superconformal algebra to each geometry. It will be very useful in this regard to find those elements of the Geroch group that can lead to the proper non–local complex structures for defining a superconformal theory at the classical level, though non–locally realized. The black hole background is very special in this line of investigation. It is a celebrated result of Belinski and Sakharov that black holes (including also an arbitrary NUT parameter that determines their asymptotic behaviour) admit a solitonic interpretation, namely they arise as a double–soliton solution from flat space, using the integrability of the 2–dim reduced Ernst equation [30]. Thus, the Geroch group element that generates the black hole geometry from flat space is indeed special, and hopefully good enough to produce the wanted superconformal structure.

In conclusion, we have set up a framework, motivated from ideas arising in the interplay between supersymmetry and duality, in order to explore the possibility of having an extended superconformal algebra for the black hole (and possibly many other backgrounds as well) in string theory. We hope to report some encouraging results in this direction in the future.

Acknowledgments

We would like to thank the organizers for the warm hospitality and the enjoyable atmosphere they have created during the workshop.

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