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THE DYNAMIC APERTURE EXPERIMENT AT THE CERN SPS

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Abstract

The Dynamic Aperture Experiment at the CERN Super Proton Synchrotron (SPS) was aimed at finding the relevant effects that limit single-particle stability in hadron storage rings. These effects were studied in the SPS and compared with long-term particle tracking to determine to what extent computer simulations can predict the dynamic aperture under well-known conditions. Such investigations are very important for future hadron colliders like the Large Hadron Collider (LHC) as the design of these machines relies heavily on simulations. Besides this practical goal it was of utmost interest to improve the phenomenological understanding of the intricate details of particle motion in phase space. This experiment was carried out by successive teams over a period of ten years. The techniques, results, and conclusions are summarized in this report.

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Contents

1 INTRODUCTION

All high-energy hadron accelerators currently planned or under construction need superconducting technology to reach the high fields needed to guide and focus the beams. Whereas in the classic technology the required fields were reproduced by accurately shaping the magnetic pole pieces, in superconducting magnets the field quality depends almost entirely on the position and properties of the superconducting filaments inside the coils. As a result it is more difficult to avoid unwanted multipolar errors that affect the particle dynamics [1, 2].

Since hadrons lose a negligible part of their energy through synchrotron radiation, their motion in phase space can be adequately described in the framework of the Hamiltonian formalism. In such conservative systems, the nonlinearities can make the motion of the particles chaotic in parts of the phase space through which they slowly migrate outwards until they are lost at some obstacle in the beam pipe.

It is of utmost importance for the design of a machine like the Large Hadron Collider (LHC), the 7 TeV on 7 TeV proton collider planned in the Large Electron–Positron (LEP) tunnel at CERN, to understand in detail the nonlinear dynamics of the circulating particles. The main purpose of the dynamic aperture experiment at the Super Proton Synchrotron (SPS) was therefore to simulate a nonlinear machine including tune modulation, which is known to enhance the destabilizing effects of nonlinearities [3], and to investigate particle losses under these conditions.

The dynamic aperture, corresponding to the stable phase-space area, can be defined for our purpose as the maximum betatron amplitude below which no particle loss takes place within a time interval of interest. In the case of the LHC, the injection time is the relevant period since the transverse magnetic field errors attain large values and the particles have to stay at injection for 15 min with the beam size at its maximum. Unfortunately there is at present no tool or technique available to evaluate these loss times despite a decade of intense theoretical research. Hence, we have to rely on brute-force tracking. One of the main motivations for the experiments was to use the SPS as a test bench to compare experimental results from a controlled nonlinear machine with the predictions from tracking.

Answering this important practical problem was not the only incentive for the aperture studies. Much effort has been devoted to improving our phenomenological understanding of the intricate nature of nonlinear motion in phase space. We hope that theorists will find a consistent way to describe the slow-particle-loss mechanisms observed in the SPS experiment.

The next section puts our studies into context with experiments at other accelerator centers and gives a general overview of the results of the SPS experiment up to 1991. Section 3 describes the instrumentation, including its calibration, necessary for preparing the SPS machine. The experimental conditions, the tracking model, and preparatory experiments are stated as well. In Sec. 4 the experiment with scrapers is explained together with experimental and tracking results. The experimental dynamic aperture is compared with tracking results in Sec. 5. Finally, in Sec. 6, a detailed summary is given of all issues concerning our improved insight into nonlinear particle motion in hadron storage rings.

2 HISTORICAL OVERVIEW

2.1 Experiments on transverse nonlinear dynamics

Transverse nonlinear resonance phenomena have been of interest to the accelerator community for many years (see Ref. [4]). With the planning of a new generation of large hadron colliders like the Superconducting Super Collider (SSC) and the LHC, intense experimental activity was started in the US at Fermilab, and in Europe at the CERN SPS and the DESY Hadronen Elektronen Ring Anlage (HERA) proton ring (see Refs. [5, 6, 7], respectively).

In the Fermilab E778 experiment [8] the measurements were predominantly made in the vicinity of the 2/5 resonance. Various measurements were performed: detuning with amplitude, "smear" measurements, island capture (including tune modulation), and time-dependent beam profiles. The theoretical treatment relied on the Hamiltonian formalism, the phase diagram was studied in depth and the beam losses were described by a diffusive model.

In all experiments the sextupole magnets served as the dominant source of nonlinearities with the exception of the HERA experiment where the dynamic aperture due to the multipolar errors of the superconducting magnets was measured. In this experiment a good agreement between the tracking results (all known magnetic errors considered [7]) and the experimental measurements was achieved when the experimental conditions were well understood.

Dynamic aperture experiments have also been performed at the Indiana University Cyclotron Facility (IUCF) in which low-order resonances with and without tune modulation were studied and compared with Hamiltonian models (see Refs. [9] and [10]). At the Aladdin electron ring at the Synchrotron Radiation Center in Stoughton, Wisconsin, third-order resonances were studied [11] as well. The results were similar to those found in conservative systems since experimental periods were studied which were small compared to the damping time. More recently, studies have been started at the SPEAR storage ring at Stanford to analyze the full six-dimensional phase space [12].

2.2 The SPS experiment

The purpose of this section is to summarize the SPS experiments up to the end of 1991. We will restrict ourselves here to a discussion of the results while a thorough description of the experimental conditions will be given in the next section.

The early experiments can be grouped into three periods. In the initial phase [13] in 1986 the basic machine setup was defined and first short-term results were obtained. In 1988 the short-term dynamic aperture was studied in detail and a slow loss process was found [14]. From 1989 to 1991 an attempt was made [15] to understand this loss process quantitatively. To this end the experimental setup, the instrumentation, and the measurement and simulation techniques were revised. Some progress could be reported but reliable results have only been obtained in the period from 1992–1994. They will be discussed later.

2.2.1 Experimental session in 1986

The operational conditions of the SPS were carefully chosen so as to obtain a well-tunable but also very linear machine (see next section). With these conditions as a starting point, the SPS was made nonlinear in a controlled way with eight strong sextupoles. Two different configurations were tested: one leads to a strong excitation of the third-order resonance, a low-order resonance which is to be avoided for safe operation of a machine with colliding beams; the other configuration suppresses this resonance so that the particle motion is dominated by higher order resonances. This has a greater resemblance to a machine like the LHC, which is very nonlinear due to the strong multipolar errors of its superconducting magnets. The latter configuration has been exclusively used for all following experiments. To test the dependence of nonlinear behavior on the strength of the sextupoles, two different current values were used. The higher value was taken just below the saturation level and the lower value was 1.8 times smaller. In all following experiments, except in 1988, we used the lower value because it provided a sufficiently high level of the nonlinearities at the chosen energy.

Wire scans were used to visualize a kicked beam: after one kick one finds the signature of a hollow beam filamented in phase space that shows a double-peak structure in the projection. A second kick partly restores the original one-peak structure that the beam had before the kick. Moreover, close to the third-order resonance the wire-scan profile is distorted revealing phase space deformations due to this resonance. Another important tool was the Schottky detector which allows a measurement of the tune distributions with a high resolution. With this instrument, losses due to particular resonances could be detected easily.

The experiment mainly studied the short-term dynamic aperture of various working points close to the fifth- and seventh-order resonances. This short-term dynamic aperture corresponds to a few seconds of SPS storage time or some 10^5 turns. In the tracking, with simulations over 10^2 to 10^3 turns the short-term dynamic aperture could be predicted quite well when the measured closed orbit and synchrotron oscillations were taken into account. Taking the simulations to 10^5 turns did not change the tracked dynamic aperture very much. The presence of a vertical closed orbit gave rise to skew resonances which led to particle loss when these resonances were approached in the experiment. As a consequence, in all following experiments, the closed orbit (in particular the vertical one) was carefully measured and corrected and whatever was left over after the correction was introduced in the simulations (see below). Moreover, some preliminary studies were done to understand the effect of a tune modulation ΔQ_x of some 10^{-3}.

Finally, long-term experiments (some minutes of storage time) were started without the synchrotron oscillations to avoid the additional effect due to the too strong RF noise. For the same reason we restricted all further studies to the four-dimensional phase space.

2.2.2 Experimental session in 1988

In this session two working points were studied close to the nest of coupling resonances of 5th and 7th order, respectively. The detuning with amplitude was measured and found to be in good agreement with tracking at those working points and also at the low and high sextupole excitation (see above). An example is shown in Fig. 1 which also shows that the border of chaotic motion, as determined in the tracking, fits well the amplitude where the short-term losses start to be visible in the experiment. As expected, this short-term dynamic aperture is smaller for the 5th order resonance which should have larger driving terms according to perturbation theory.

The conclusion was that the basic nonlinear parameters were well under control and that the short-term dynamic aperture can be well predicted by tracking. However, in the long-term study of this experiment a slow particle transport was found. The nature of this slow transport is still not fully understood and there are indications [16] that a simple model of diffusion is not sufficient to describe it. The experiment can be described as follows (see Fig. 2): at the beginning the lifetime is about 75 min, then a scraper is moved in till the lifetime is reduced to 40 min. After a minute the scraper is retracted by 3 mm which results in a period of roughly 1 min in which almost no losses are detected. It seems that this is the time the particles need to "diffuse" out till they reach the retracted scraper. Thereafter one finds that the lifetime stabilizes at 65 min which is close to the original value. Finally, the lifetime of 36 min is recorded (which is close to the 40 min after the first scraping) when the scraper is put back to its closest location with respect to the beam center. From this a "diffusion" rate of 3 mm per minute can be calculated. It goes without saying that the SPS was also studied without strong sextupoles: no "diffusion" could be found in that case.

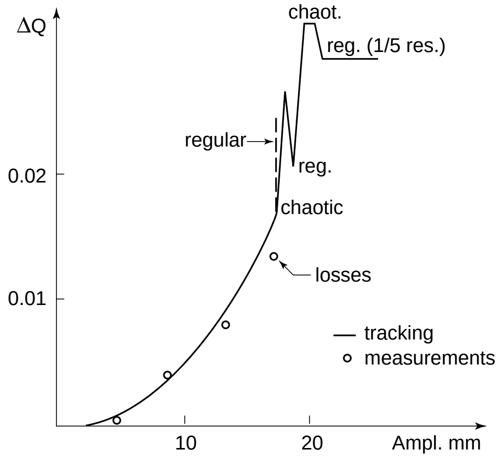


Figure 1: Comparison of tracking with experiment (close to 5th order resonance).

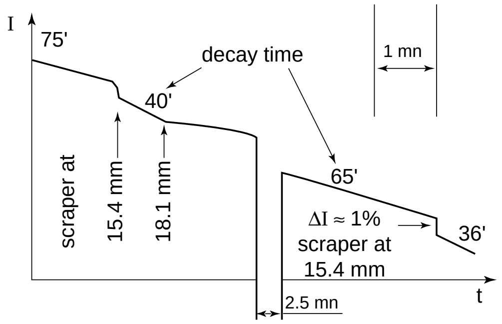


Figure 2: Detection of slow "diffusion".

One of the goals of the SPS experiment was to establish a criterion of stability for the LHC. The whole concept of such a stability criterion has been subject to controversial discussions, a summary of which is given in Appendix A. It is shown there as well that the stability criterion is violated by the measured "diffusion" rate. This reduction of the long-term dynamic aperture cannot be easily explained by tracking. In fact, in the same amplitude range, the tracking predicts regular (and therefore completely stable) motion. A rather strong effect is clearly left out. A good candidate to explain this effect is the power supply ripple that causes a tune modulation which, in conjunction with the nonlinearities, may reduce the dynamic aperture. In the tracking [17] such an effect on the dynamic aperture is found when the tune modulation depth is of the order of some 10^{-3}. The same study also finds that in the range of tune modulation frequencies between 10 Hz and 600 Hz, the smaller frequencies are much more dangerous than the larger ones. It has been demonstrated that this effect does not vanish for a different working point or another sextupole configuration. Moreover, it has been shown that the effect of a modulation due to a large number of elements can be well approximated by one single element when its strength is adjusted accordingly.

2.2.3 Experimental sessions from 1989 to 1991

The 1988 experiments and the theoretical studies led to the understanding of slow particle losses in a qualitative manner. Moreover, it was confirmed in an independent experiment in 1989 at Fermilab [18] that the combined effect of tune modulation and nonlinearities leads to a strong reduction of the dynamic aperture. In the following years we performed our experiments with a view to measuring "diffusion" rates as a function of betatron amplitude, of tune modulation depth (larger than the natural one), and of tune modulation frequency. It turned out, however, to be much more difficult than expected to get quantitative and reproducible results from the experiment.

It was realized that a much better control of the machine parameters was needed and that the measurement techniques had to be reviewed. The closed orbit, in particular the vertical one, had to be corrected to extremely low values. The linear coupling as well as the chromaticity were corrected to the best levels possible. It also became clear that the beam in the SPS had to be considered as a distribution of particles rather than a pencil beam. This remained true even with all these careful machine adjustments in place and after removing the tails of the distribution with scrapers. To sample a certain betatron amplitude it was therefore necessary to use a single kick instead of heating the beam with many small kicks. Moreover, it became mandatory to apply the same measurement procedure each time in order to arrive, at least approximately, at reproducible transverse distributions of protons. The problem of knowing the precise particle distribution also made it very difficult to draw conclusions from lifetime measurements. Finally, we found that the range of amplitudes of interest was of the order of the width of the distribution, which further complicated the interpretation of the results. This range of amplitudes is defined on the upper side by the smallest amplitude at which the particle losses are very fast and on the lower side by the largest amplitude where the "diffusion" rate becomes immeasurable.

Despite these refinements to the experiment our efforts to get quantitative results did not meet with full success. Nevertheless, some relevant intermediate results were obtained. The natural ripple spectrum was measured [3] to be $\Delta Q_x = \pm 1 \times 10^{-4}$ of which one half can be attributed to seven ripple lines between 50 Hz and 1000 Hz. One successful measurement is shown in Fig. 3: after having reproduced the detuning curve with amplitude it was possible to use octupoles of the SPS to reduce the detuning by roughly a factor of ten. Even more important is the improvement of the dynamic aperture by some 30% obtained as a result, which makes us confident that the detuning correction in the LHC will lead to some improvement as well. Tracking studies were performed up to 2.6×10^7 turns which represents 10 min of SPS storage time. It is interesting to note that a considerable reduction (10%) of the dynamic aperture still occured after one million turns were tracked. The tracking results predicted a dynamic aperture that was 20% larger than the long-term stability border found in the experiment. However, this result was considered somewhat preliminary and we concluded that a more systematic study was needed. A general, more qualitative result of the experiment was the fact that the lifetime depended strongly on the additional tune modulation. This dependence on tune modulation depth was also present in the tracking but did not agree quantitatively with the experiment. Moreover, in the experiment one could not confirm the frequency dependence that was expected from the tracking: the beam lifetime in the SPS seems to be rather insensitive to this parameter but very sensitive to the tune modulation depth. It has to be mentioned that the way the stability border was determined differs in the tracking and in the experiment: in the tracking the border of the onset of chaos was used, while in the experiment the actual particle loss was taken. To test the importance of this difference a toy model (a simple FODO cell plus a sextupole) was tracked for many millions of turns (see Fig. 4) with and without tune modulation. One found that the difference between cases with tune modulation on or off becomes apparent after some 10^5 turns, while the onset of chaotic motion shows up after only 20 000 turns. The more pronounced effect of the smaller tune modulation frequencies is only visible after more than 10^7 turns. This has two consequences: firstly it could very well be that the difference of the effect of the tune modulation frequencies is only visible after a very long time in the experiment (may be even after the storage time of interest) and secondly, in the case with tune modulation, the onset of chaotic motion is generally much too pessimistic as a criterion for the dynamic aperture which in our case is always the amplitude below which the motion is stable for a *given time interval.*

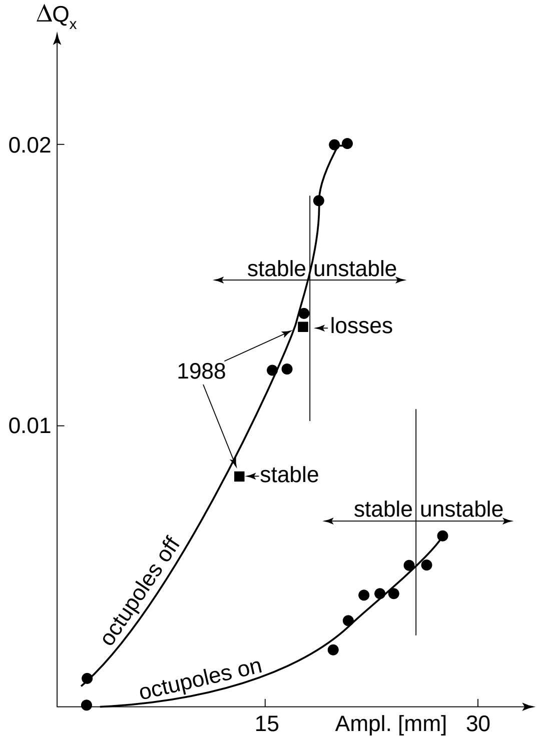


Figure 3: Detuning and stability.

An interesting side issue was the study of the effect of two simultaneous tune modulation frequencies compared with only one frequency while keeping the total tune modulation depth equal in both cases. The two tune modulation frequencies (see Fig. 5) reduced the lifetime by more than a factor of three compared with the one-frequency case. These findings led to a study which treated the case with more than one frequency in a more rigorous theoretical framework [19]. It has to be mentioned that this phenomenon may not be generally applicable but may depend on the particular parameters chosen in the SPS (see below). Another feature often found in the experiment is the appearance of the so-called "shoulder": right after a retraction of the scraper the intensity stays almost constant (infinite lifetime), and after a certain time interval, the intensity bends over rather abruptly without a smooth transition leading to a constant finite lifetime. This phenomenon was studied in Ref. [16] and found to be in contradiction with a simple diffusion mechanism. Finally we would like to mention a related study concerning slow particle losses in hadron colliders [20] which gives a phenomenological description of these losses in phase space.

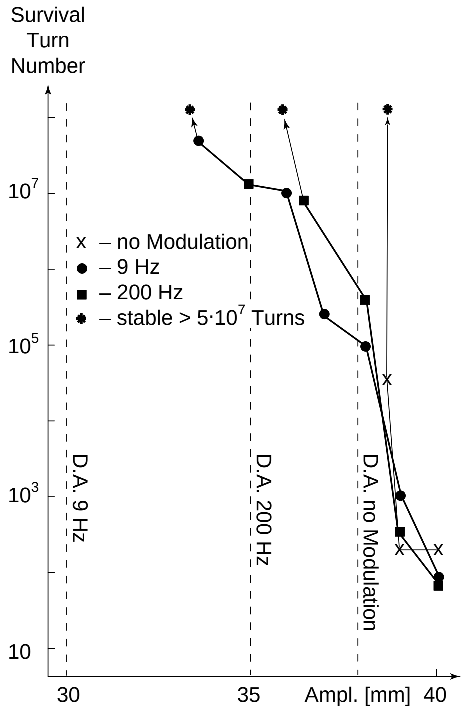


Figure 4: Long-term stability with tune modulation.

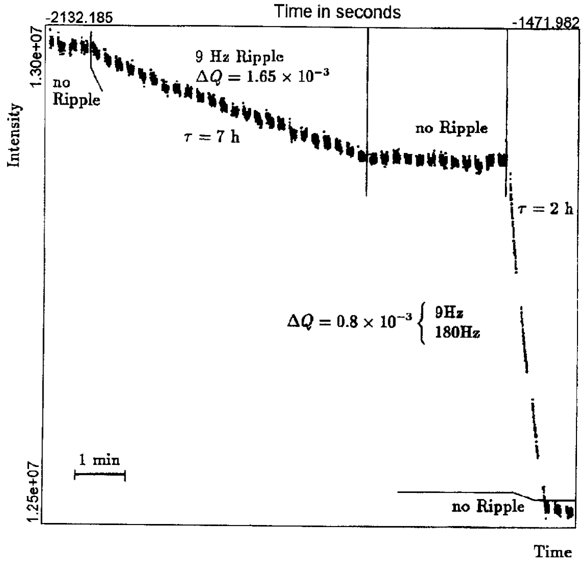


Figure 5: One and two tune-modulation frequencies.

3 PREPARATION OF THE SPS AND ITS MODEL

In this section we describe the instrumentation for the dynamic aperture experiment at the SPS. In particular our main instruments, the data acquisition system BOSC (the acronym stands for Beam OSCillations) and the wire scanner system, are covered in detail. The experimental conditions are presented in the next subsection. Then we describe the tracking model for the SPS and explain in detail all methods used in our tracking simulations. In the last subsection we show that our model reproduces the basic nonlinear behavior of the SPS. This is absolutely mandatory in order to proceed with the long-term studies. Furthermore it was also necessary to carefully calibrate our instruments.

3.1 Instrumentation for the SPS experiment

The CERN SPS used for the experiment is a synchrotron with 1100 m mean radius. It can accelerate protons from 14 GeV up to 450 GeV for fixed-target experiments. In the following we describe the general instrumentation (see Fig. 6) needed for this experiment.

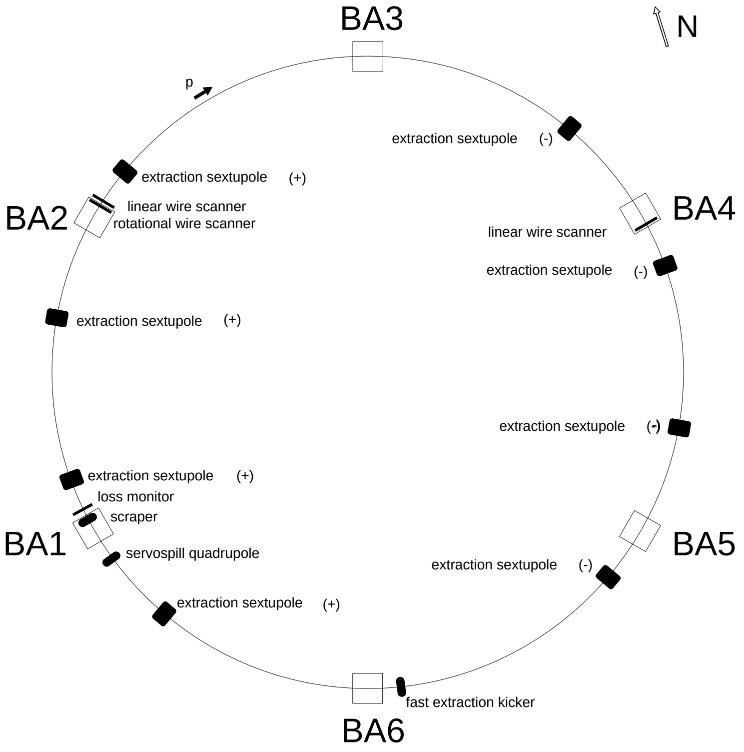


Figure 6: Instrumentation at the SPS. The eight strong sextupoles, the servospill quadrupole (the source of additional tune modulation), and the kicker magnet are shown at their locations in the SPS. The positions of the linear and rotational wire scanners, the scrapers, and the loss monitor are also given. (BA stands for access hall.)

Sizable nonlinearities are introduced by eight sextupoles (part of the slow extraction system) which are about 10 times stronger than the chromaticity sextupoles located at the focusing quadrupoles. They are grouped in two families with different polarity (Fig. 6) so as to avoid a change of the chromaticity and the excitation of the third-order resonances. Analytical calculations predict that these strong sextupoles lead to a detuning with action 10 times larger than in the normal machine.

A single quadrupole (BA1 in Fig. 6) is used to introduce additional tune modulation. This quadrupole operates linearly up to a modulation depth of $\Delta Q_x = \pm 1.5 \times 10^{-3}$ which is more than ten times the natural one. In the frequency range up to about 200 Hz the response of the magnet to the signal input is linear as well.

A kicker magnet (BA6 in Fig. 6) has been used to vary the average amplitude of the particle beam. It has been calibrated several times (see Fig. 7) and shows a very linear behavior down to small kick amplitudes. Whereas the measurement of 1993 showed a degradation in the kick strength of 10% compared with earlier measurements, the calibration of 1995 showed practically no difference from the measurement of 1993.

Pairs of horizontal and vertical scrapers (BA1 in Fig. 6) served as aperture limiters and the losses at these scrapers were detected by a scintillator placed close by (45 m downstream). Scraper positions and the loss detector signal were recorded by the BOSC system (Sec. 3.1.1).

Momentum and tune distributions were measured with a Schottky system [21]. Using the same hardware, a continuous tune measurement system was set up to measure the natural tune ripple spectrum (Fig. 8).

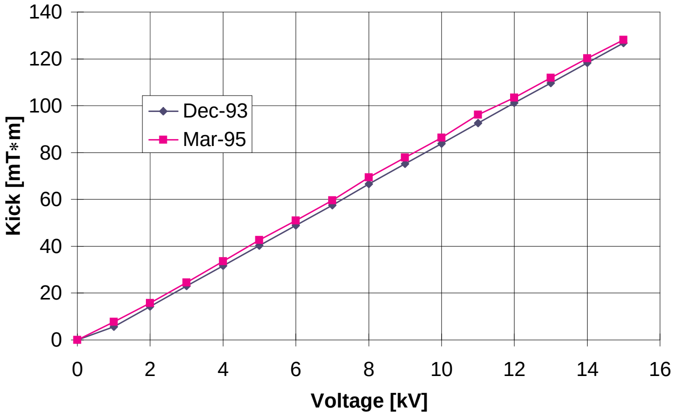


Figure 7: Calibration of the kicker in 1993 and 1995.

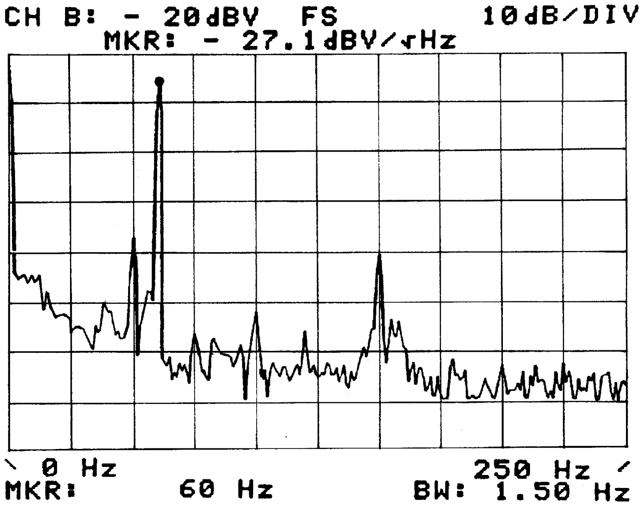


Figure 8: Natural tune ripple spectrum. Up to 250 Hz, three natural tune ripple lines are above the noise level (50, 100 and 150 Hz) as expected from the voltage power supply ripple. The large 60 Hz line has been introduced by our modulated quadrupole and is used for calibration purposes.

The turn-by-turn beam data acquisition system and the flying wire system which are most relevant for our experiment will now be described in some detail.

3.1.1 Data acquisition system BOSC

The turn-by-turn data acquisition system BOSC [22] was originally intended to be used in the SPS to measure the intensity and the position of the individual proton and antiproton bunches over a full machine cycle. Now a wide range of different signals can be recorded. The system consists of a VME crate with a 68030 CPU card, a timing module, a bunch selector card, and 12 ADC cards each with two channels to acquire and store the data from up to one million turns. Three such crates on the SPS site and one crate on the LEP site can be addressed by Apollo or HP-UX workstations via Ethernet and Token Ring. Very flexible measurement requests can be sent to the crates in data structures which are then filled with the requested data and sent back to the workstation for processing. In the crate a complex control software running under OS9 has been developed, with several application software programs now running either on Apollo or on HP-UX workstations under Xwindows/MOTIF. Apart from its use in the SPS dynamic aperture experiment, with its constantly changing requirements, BOSC also provides the operational tune measurement in the SPS.

a. Hardware description

The BOSC acquisition system in its final form is housed in a VME crate. It can handle up to 24 analog signals which are taken from homodyne receivers. They are organized in 12 dual-channel electronic cards (*dual sampler*) which contain 1 MByte memory for each channel to allow the measurement turn by turn over a period of more than 20 s in the SPS. The acquisition bandwidth of 5 MHz is well matched to the bandwidth of the receivers. A logic cell array common to both channels on the ADC card acts as a slave to the crate CPU. The system is mainly intended for the measurement of single bunches. The *bunch selector* picks a given bunch circulating in the machine. The time resolution of this selection is determined by the bandwidth of the system and is at present of the order of 200 ns. Special care has been taken to isolate the low-power analog circuits from the high-power digital circuitry. The connection between the two is made in the *VME bridge* module. The information concerning the machine cycle-time is fed into the system by a timing module.

At present three units are installed in the SPS and one in LEP:

- A first one is dedicated to turn-by-turn position measurements. It is used to derive the machine tunes. A number of channels are connected to 200 MHz receivers. They are well adapted to measure single lepton bunches. However, they are also used to measure the SPS proton beams bunched at 200 MHz. Special low-frequency FET amplifiers with a bandwidth reduced to 5 MHz to match the acquisition system are connected to a second set of channels. They allow the measurement of bunched and unbunched beams. The excitor for this measurement can either be a special fast kicker magnet or the deflector plates of the transverse feedback system. The excitation of the latter is controlled by BOSC using the *sequencer* unit.
- A second BOSC unit is devoted to single-bunch intensity measurements. The signals are generated by 20 MHz homodyne receivers.
- A third unit acquires much slower signals generated by DC current transformers, collimator movements, and scintillators.
- The LEP unit records the intensity and beam-position signals of one beam-position monitor for one bunch of electrons and positrons, respectively.

6. Control software description

The control software [23], running under OS9 on the 68030 processor, has the following tasks: setting up the communication between the crate and the workstation, setting and changing some hardware parameters, taking measurement requests from a workstation, starting the data acquisition on the crate, and sending the data to the workstation after an acquisition has been made. It is capable of handling several requests simultaneously on the same crate.

The communication is done over Ethernet and Token Ring where sockets under TCP/IP are used. For the data transfer in any direction MOPS data structures are used [24]. A schematic overview of the system consisting of the crate, the workstation, and the communication part is given in Fig. 9.

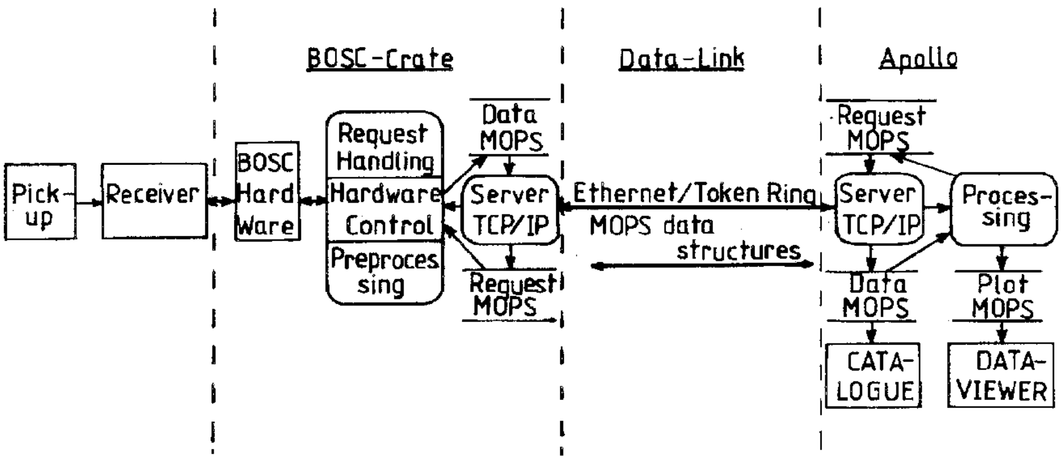


Figure 9: Setup of BOSC.

It is possible to change some of the parameters controlling the hardware such as base addresses, gains, and bunch selector settings. The use of base addresses allows the translation from physical to logical channel addresses, so as to freely choose channels without the need to swap cables. The receiver gain can be changed from 14 dB to 70 dB in 14 dB steps, each ADC channel gain can be changed from 0 dB to 24 dB in 6 dB steps.

The MOPS data structure that is sent to start the measurement on the crate in one of its objects holds a coded request (nine integer numbers) that specifies the measurement parameters: on which BOSC crate to run the measurement, the number of super cycles to be measured, the start time of the measurement in the SPS super cycle, the time between blocks of acquisitions, the number of acquisition blocks, the time between sub-blocks of acquisitions, the number of sub-blocks in one block, the number of turns per sub-block, and the channels to be used for the measurement. An example of the use of some of these parameters can be found in Fig. 10. A server program runs on both ends to receive MOPS data structures with measurement requests or acquired data, respectively. The data read from the ADC memory and hardware settings, such as timing information, are added to the request MOPS data structure that has been sent from the driving workstation.

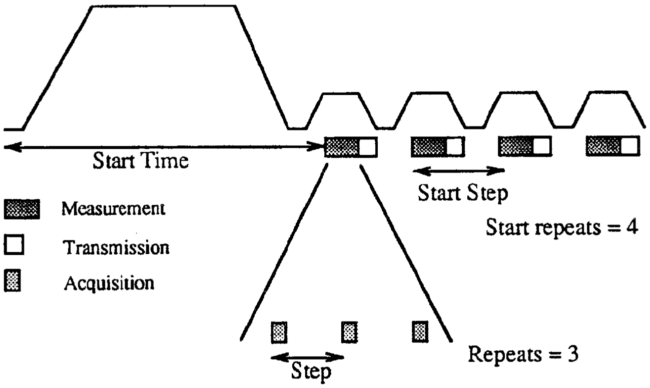


Figure 10: Time structure of a BOSC measurement [23].

c. Application software description

The application software [23] is an interface for starting a measurement, displaying the data acquired, storing data, and performing a detailed postprocessing analysis. For displaying data the *dataviewer* program is used [25], the archiving is managed by a catalog package [24]. BOSC is now used as an operational tool for tune measurement [26] as well as a tool for the dynamic aperture experiments performed on the SPS. In the following we only report on the latter application software package. There are two main types of measurements that can be performed:

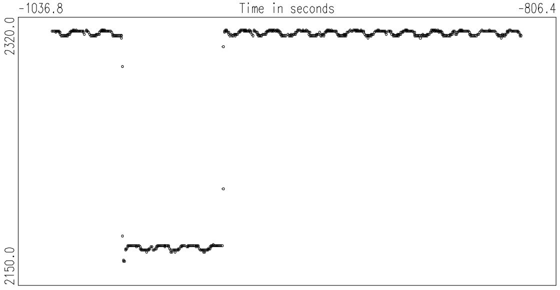
- Lifetime measurements
- Phase space measurements

In the dynamic aperture experiments we want to investigate effects that influence particle stability over long periods. It is therefore very convenient to use BOSC for simultaneous and continuous beam-intensity, scraper-position, and loss-monitor readings. Different phenomena leading to particle loss can thereby be easily distinguished (see Fig. 11).

For a phase space measurement the position and intensity signals of one or more pickups can be recorded. After having applied a kick to the beam, the Fast Fourier Transforms from the position signals give the tunes and the line spectra due to resonances. Figure 12 shows how readings of two pickups separated by a multiple of 90 degrees allow the depiction of phase space projections. Currently we take and analyze online two samples of up to 65 000 turns, the repetition rate being 30 s. This allows a very precise determination of the tune, but also linear coupling correction, chromaticity compensation, and identification of high-order resonances.

For phase space measurements there is a tool box which contains four programs. The *fft_mod* program allows one to compute the spectra for a selected range of turns. With this facility one can detect changes in the tunes, for instance due to power supply ripple. The *stroboscope* program plots only every nth point in phase space thereby visualizing resonances in the horizontal, vertical, and physical phase space projection. The *fake* program has the same functionality as *stroboscope* but uses the information of only one pickup by plotting an x-coordinate at turn number i versus the x-coordinate at turn number i + *skip_step*. The *smear* program computes the horizontal and vertical decoherence, the decoherence-corrected amplitude, and the "smear" [27].

Average DC Signal avr_h_4



Average DC Signal avr_h_7