

Perturbative and non-perturbative monodromies in $N = 2$ heterotic string vacua

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In this talk we summarise our recent results on perturbative and non-perturbative monodromies in four-dimensional heterotic strings with $N = 2$ space-time supersymmetry, and we compare our results with the rigid $SU(2)$ $N = 2$ Yang-Mills monodromies.

1. Classical results, enhanced gauge symmetries and Weyl reflections

We will, in the following, consider four-dimensional $N = 2$ heterotic string vacua which are based on compactifications of six-dimensional $N = 1$ heterotic vacua on a two-torus T_2 . The moduli of T_2 are commonly denoted by T and U where U describes the deformations of the complex structure, $U = (\sqrt{G} - iG_{12})/G_{11}$ (G_{ij} is the metric of T_2), while T parametrizes the deformations of the area and of the antisymmetric tensor, $T = 2(\sqrt{G} + iB)$. (Other possibly existing vector fields will not be considered in the following.) The scalar fields T and U are the spin-zero component fields of two $U(1)$ $N = 2$ vector supermultiplets. Classically, the physical properties of two-torus compactifications are invariant under the group $SO(2, 2, \mathbf{Z})$ of discrete target space duality transformations. It contains the $T \leftrightarrow U$ exchange symmetry, with group element denoted by σ , as well as the $PSL(2, \mathbf{Z})_T \times PSL(2, \mathbf{Z})_U$ dualities, which act on T and U as

$$(T, U) \longrightarrow \left(\frac{aT - ib}{icT + d}, \frac{a'U - ib'}{ic'U + d'} \right) \quad (1)$$

where $ad - bc = a'd' - b'c' = 1$. The classical monodromy group, which is a true symmetry of the classical effective Lagrangian, is generated [1] by the elements σ , g_1 , $g_1: T \rightarrow 1/T$ and g_2 , $g_2: T \rightarrow 1/(T - i)$. The transformation

$t: T \rightarrow T + i$, which is of infinite order, corresponds to $t = g_2^{-1}g_1$. $PSL(2, \mathbf{Z})_T$ is generated by g_1 and g_2 , whereas the corresponding elements in $PSL(2, \mathbf{Z})_U$ are obtained by conjugation [1] with σ , i.e. $g'_i = \sigma^{-1}g_i\sigma$.

Any $N = 2$ heterotic string vacuum contains two further $U(1)$ vector fields, namely the graviphoton field, which has no physical scalar partner, and the dilaton-axion field, denoted by S . Thus the Abelian gauge symmetry we will be considering in the following is given by $U(1)_L^2 \times U(1)_R^2$. At special lines in the (T, U) moduli space, additional vector fields become massless and the $U(1)_L^2$ becomes enlarged to a non-Abelian gauge symmetry. Specifically, there are four inequivalent lines in the moduli space where two charged gauge bosons become massless, namely at $U = T$, $U = \frac{1}{T}$, $U = T - i$ and $U = \frac{T}{iT+1}$. We will, in this paper, only consider the $T = U \neq 1, \rho$ line for definiteness. The other 3 lines can be discussed in a similar way [2].

At the critical line $T = U \neq 1, \rho$, the $U(1)_L^2$ is extended to $SU(2)_{(1)} \times U(1)$. The Weyl reflection w_1 of the enhanced gauge group $SU(2)_{(1)}$ acts as follows on T and U , $w_1: T \leftrightarrow U$. Thus, $w_1 = \sigma$ and the Weyl reflection w_1 is a target space duality transformation. Similarly, the Weyl transformations associated with the other 3 critical lines are also target space modular transformations and therefore also elements of the monodromy group. All Weyl reflections can be expressed in terms of the generators g_1, g_2 and σ .

*Talk presented by G. L. Cardoso

The four critical lines are fixed under the corresponding Weyl transformation. The number of additional massless states at a given critical line/point agrees with the order of the fixed point transformation at that critical line/point [3].

The moduli fields T and U can be expressed in terms of the field theory Higgs fields whose non-vanishing vacuum expectation values spontaneously break the enlarged gauge symmetry down to $U(1)_L^2$. The Higgs field of $SU(2)_{(1)}$, for instance, is given by $a_1 \propto (T - U)$.

The classical vector couplings are determined by the holomorphic prepotential which is a homogeneous function of degree two of the fields X^I ($I = 0, \dots, 3$). It is given by [4,5,1]

$$F = i \frac{X^1 X^2 X^3}{X^0} = -STU \quad (2)$$

where the physical vector fields are defined as $S = i \frac{X^1}{X^0}$, $T = -i \frac{X^2}{X^0}$, $U = -i \frac{X^3}{X^0}$ and the graviphoton corresponds to X^0 . As explained in [4,5], the period vector (X^I, iF_I) ($F_I = \frac{\partial F}{\partial X^I}$), that follows from the prepotential (2), does not lead to classical gauge couplings which all become small in the limit of large S . In order to arrive at a ‘physical’ period vector [4,5] one has to perform the following symplectic transformation $(X^I, iF_I) \rightarrow (P^I, iQ_I)$, where $P^1 = iF_1$, $Q_1 = iX^1$, and $P^i = X^i$, $Q_i = F_i$ for $i = 0, 2, 3$. In this new basis the classical period vector takes the form

$$\Omega^T = (1, TU, iT, iU, iSTU, iS, -SU, -ST) \quad (3)$$

where $X^0 = 1$. One sees that all electric vector fields P^I depend only on T and U , whereas the magnetic fields Q_I are all proportional to S .

The basis Ω is also well adapted to discuss the action of the target space duality transformations and, as particular elements of the target space duality group, of the four inequivalent Weyl reflections associated with the four critical lines of gauge symmetry enhancement. In general, the field equations of the $N = 2$ supergravity action are invariant under the following symplectic $Sp(8, \mathbf{Z})$ transformations, which act on the period vector Ω as [4]

$$\begin{pmatrix} P^I \\ iQ_I \end{pmatrix} \rightarrow \Gamma \begin{pmatrix} P^I \\ iQ_I \end{pmatrix} = \begin{pmatrix} U & Z \\ W & V \end{pmatrix} \begin{pmatrix} P^I \\ iQ_I \end{pmatrix} \quad (4)$$

where the 4×4 sub-matrices U, V, W, Z have to satisfy the symplectic constraints $U^T V - W^T Z = V^T U - Z^T W = 1$, $U^T W = W^T U$, $Z^T V = V^T Z$. Invariance of the lagrangian implies that $W = Z = 0$, $VU^T = 1$. In case that $Z = 0$, $W \neq 0$ and hence $VU^T = 1$ the action is invariant up to shifts in the θ -angles. The non-vanishing matrix W corresponds to a non-trivial one-loop monodromy due to logarithmic singularities in the prepotential. (see section 3.) Finally, if $Z \neq 0$ then the electric fields transform into magnetic fields; these transformations are the non-perturbative monodromies due to logarithmic singularities induced by monopoles, dyons or other non-perturbative excitations (see section 4).

As mentioned before, the classical action is completely invariant under the target space duality transformations. Thus the classical monodromies have $W, Z = 0$. The classical monodromy matrix U ($V = U^T,^{-1} = U^*$) associated with the Weyl reflection $w_1 = \sigma$ is given by

$$U_\sigma = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad (5)$$

At the classical level the S -field is invariant under target space duality transformations.

2. Perturbative results

Loop corrections to the holomorphic prepotential only occur at one-loop. Simple power counting arguments imply that the one-loop correction to the prepotential must be independent of the dilaton field S . Thus the perturbative prepotential takes the form [5,1]

$$\begin{aligned} F &= F^{(Tree)}(X) + F^{(1-loop)}(X) \\ &= i \frac{X^1 X^2 X^3}{X^0} + (X^0)^2 f(T, U) \\ &= -STU + f(T, U) \end{aligned} \quad (6)$$

Since the target space duality transformations are known to be a symmetry in each order of perturbation theory, the tree level plus one-loop effective action must be invariant under these transformations up to discrete shifts in the various θ angles due to monodromies around semi-classical

singularities in the moduli space where massive string modes become massless. The period vector $\Omega^T = (P^I, iQ_I)$ transforms perturbatively as follows

$$P^I \rightarrow U^I_J P^J, \quad iQ_I \rightarrow V_I^J iQ_J + W_{IJ} P^J \quad (7)$$

where

$$V = (U^T)^{-1}, \quad W = V\Lambda, \quad \Lambda = \Lambda^T \quad (8)$$

and U belongs to $SO(2, 2, \mathbf{Z})$. Classically, $\Lambda = 0$, but in the quantum theory, Λ is a real symmetric matrix, which should be integer valued in some basis.

In addition, the effective action is also invariant, up to discrete shifts in the θ -angles, under discrete shifts in the S -field, $D: S \rightarrow S - i$. Thus, the full perturbative monodromy group contains the following $Sp(8, \mathbf{Z})$ transformation

$$V_S = U_S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$W_S = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad Z_S = 0 \quad (9)$$

As explained in [5,1] the dilaton is not any longer invariant under the target space duality transformations at the one-loop level. Indeed, it can be shown [5] that (7) implies that

$$S \longrightarrow S + \frac{V_1^J (F_J^{(1-loop)} - i\Lambda_{JK} P^K)}{U_I^I P^I} \quad (10)$$

Near the critical lines of classical gauge symmetry enhancement the one-loop prepotential exhibits logarithmic singularities and is therefore not a singlevalued function when transporting the moduli fields around these singular lines. For example around the singular $SU(2)_{(1)}$ line $T = U \neq 1, \rho$ the function f must have the following form [5,1,2]

$$f(T, U) = \frac{1}{\pi} (T - U)^2 \log(T - U) + \Delta(T, U) \quad (11)$$

where $\Delta(T, U)$ is finite and single valued at $T = U \neq 1, \rho$. At the remaining three critical lines $f(T, U)$ takes an analogous form.

2.1. Perturbative $SU(2)_{(1)}$ monodromies

Let us now consider the element σ which corresponds to the Weyl reflection in the enhanced $SU(2)_{(1)}$. Under the mirror transformation σ , $T \leftrightarrow U, T - U \rightarrow e^{-i\pi}(T - U)$, and the P transform classically and perturbatively as

$$P^0 \rightarrow P^0, \quad P^1 \rightarrow P^1, \quad P^2 \rightarrow P^3, \quad P^3 \rightarrow P^2 \quad (12)$$

Using (11), the 1-loop corrected Q_2 and Q_3 are computed to be

$$Q_2 = iSU - \frac{2i}{\pi}(T - U) \log(T - U) - \frac{i}{\pi}(T - U) - i\Delta_T$$

$$Q_3 = iST + \frac{2i}{\pi}(T - U) \log(T - U) + \frac{i}{\pi}(T - U) - i\Delta_U \quad (13)$$

It follows from (10) that, under mirror symmetry $T \leftrightarrow U$, the dilaton S transforms as

$$S \rightarrow S + i \quad (14)$$

Then, it follows that perturbatively

$$\begin{pmatrix} Q_2 \\ Q_3 \end{pmatrix} \rightarrow \begin{pmatrix} Q_3 \\ Q_2 \end{pmatrix} + \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} T \\ U \end{pmatrix} \quad (15)$$

Thus, the section Ω transforms perturbatively as $\Omega \rightarrow \Gamma_\infty^{w_1} \Omega$, where [1,2]

$$\Gamma_\infty^{w_1} = \begin{pmatrix} U_\sigma & 0 \\ U_\sigma \Lambda & U_\sigma \end{pmatrix}, \quad U_\sigma = \begin{pmatrix} I & 0 \\ 0 & \eta \end{pmatrix}$$

$$\Lambda = -\begin{pmatrix} \eta & 0 \\ 0 & \mathcal{C} \end{pmatrix}, \quad \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathcal{C} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \quad (16)$$

2.2. Truncation to the rigid case of Seiberg/Witten

In order to truncate [2] the perturbative $SU(2)_{(1)}$ monodromy matrix $\Gamma_\infty^{w_1}$ to the rigid one of Seiberg/Witten [6], we will take the limit $\kappa^2 = \frac{8\pi}{M_{pl}^2} \rightarrow 0$ and expand $T = T_0 + \kappa\delta T$, $U = T_0 + \kappa\delta U$. Here we have expanded the moduli fields T and U around the same vev $T_0 \neq 1, \rho$. Both δT and δU denote fluctuating fields of mass

dimension one. We will also freeze in the dilaton field to a large vev, $S = \langle S \rangle$. Then, the Q_2 and Q_3 given in (13) can be expanded as

$$\begin{aligned} Q_2 &= i\langle S \rangle T_0 + \kappa \tilde{Q}_2 \quad , \quad Q_3 = i\langle S \rangle T_0 + \kappa \tilde{Q}_3 \\ \tilde{Q}_2 &= i\langle S \rangle \delta U - \frac{2i}{\pi} (\delta T - \delta U) \log \kappa (\delta T - \delta U) \\ &\quad - \frac{i}{\pi} (\delta T - \delta U) - i\Delta_T (\delta T, \delta U) \\ \tilde{Q}_3 &= i\langle S \rangle \delta T + \frac{2i}{\pi} (\delta T - \delta U) \log \kappa (\delta T - \delta U) \\ &\quad + \frac{i}{\pi} (\delta T - \delta U) - i\Delta_U (\delta T, \delta U) \end{aligned} \quad (17)$$

Next, one has to specify how mirror symmetry is to act on the vev's T_0 and $\langle S \rangle$ as well as on δT and δU . We will take that under mirror symmetry

$$T_0 \rightarrow T_0 \quad , \quad \delta T \leftrightarrow \delta U \quad , \quad \langle S \rangle \rightarrow \langle S \rangle \quad (18)$$

Note that we have taken $\langle S \rangle$ to be invariant under mirror symmetry. This is an important difference to (14). Using (18) and that $\delta T - \delta U \rightarrow e^{-i\pi} (\delta T - \delta U)$, it follows that the truncated quantities \tilde{Q}_2 and \tilde{Q}_3 transform as follows under mirror symmetry

$$\begin{pmatrix} \tilde{Q}_2 \\ \tilde{Q}_3 \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{Q}_3 \\ \tilde{Q}_2 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} \delta T \\ \delta U \end{pmatrix} \quad (19)$$

Defining a truncated section $\tilde{\Omega}^T = (\tilde{P}^2, \tilde{P}^3, i\tilde{Q}_2, i\tilde{Q}_3) = (i\delta T, i\delta U, i\tilde{Q}_2, i\tilde{Q}_3)$, it follows that $\tilde{\Omega}$ transforms as $\tilde{\Omega} \rightarrow \tilde{\Gamma}_\infty^{w_1} \tilde{\Omega}$ under mirror symmetry (18), where

$$\begin{aligned} \tilde{\Gamma}_\infty^{w_1} &= \begin{pmatrix} \tilde{U} & 0 \\ \tilde{U}\tilde{\Lambda} & \tilde{U} \end{pmatrix} \quad , \quad \tilde{U} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \tilde{\Lambda} &= \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix} \end{aligned} \quad (20)$$

Note that, because of the invariance of $\langle S \rangle$ under mirror symmetry, $\tilde{\Lambda} \neq -\mathcal{C}$, contrary to what one would have gotten by performing a naive truncation of (16) consisting in keeping only rows and columns associated with (P^2, P^3, iQ_2, iQ_3) .

Finally, in order to compare the truncated $SU(2)$ monodromy (20) with the perturbative $SU(2)$ monodromy of Seiberg/Witten [6], one has to perform a change of basis from moduli fields

to Higgs fields, as follows

$$\begin{aligned} \begin{pmatrix} a \\ a_D \end{pmatrix} &= M\tilde{\Omega} \quad , \quad M = \begin{pmatrix} m & \\ & m^* \end{pmatrix} \\ m &= \frac{\gamma}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \end{aligned} \quad (21)$$

where γ denotes a constant to be fixed below. Then, the perturbative $SU(2)$ monodromy in the Higgs basis is given by

$$\begin{aligned} \tilde{\Gamma}_\infty^{Higgs} &= M\tilde{\Gamma}_\infty^{w_1} M^{-1} \\ &= \begin{pmatrix} m\tilde{U}m^{-1} & 0 \\ m^*\tilde{U}\tilde{\Lambda}m^{-1} & m^*\tilde{U}m^T \end{pmatrix} \end{aligned} \quad (22)$$

which is computed to be

$$\tilde{\Gamma}_\infty^{Higgs, w_1} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ \frac{4}{\gamma^2} & 0 & -1 & 0 \\ & & & 1 \end{pmatrix} \quad (23)$$

The monodromy matrix (23) correctly reproduces the perturbative $SU(2)$ monodromy of Seiberg/Witten [6] for $\gamma^2 = 2$, whereas comparison with the perturbative $SU(2)$ monodromy of Klemm et al [7] gives that $\gamma^2 = 1$.

3. Non perturbative monodromies

In order to obtain some information about non-perturbative monodromies in $N = 2$ heterotic string compactifications, we will follow Seiberg/Witten's strategy in the rigid case [6] and we will first try to decompose the perturbative monodromy matrix $\Gamma_\infty^{w_1}$ into $\Gamma_\infty^{w_1} = \Gamma_M^{w_1} \Gamma_D^{w_1}$ with $\Gamma_M^{w_1}$ ($\Gamma_D^{w_1}$) possessing monopole (dyonic) like fixed points. Thus, the critical line $T = U$ of classical gauge symmetry enhancement will split into two non-perturbative singular lines where magnetic monopoles or dyons respectively become massless. Similar considerations can be made for any of the other singular lines [2].

3.1. Non perturbative monodromies for $SU(2)_{(1)}$

We will assume that

1. $\Gamma_\infty^{w_1}$ is to be decomposed into precisely two factors, namely

$$\Gamma_\infty^{w_1} = \Gamma_M^{w_1} \Gamma_D^{w_1} \quad (24)$$

2. $\Gamma_M^{w_1}$ and therefore $\Gamma_D^{w_1}$ must be symplectic.
3. $\Gamma_M^{w_1}$ ($\Gamma_D^{w_1}$) must have a monopole (dyon) like fixed point.
4. $\Gamma_M^{w_1}$ and $\Gamma_D^{w_1}$ should be conjugated, as it is the case in the rigid theory.
5. $\Gamma_\infty^{w_1}$ has a peculiar block structure in that the indices $j = 0, 1$ of the section (P_j, iQ_j) are never mixed with the indices $j = 2, 3$. We will assume that $\Gamma_M^{w_1}$ and $\Gamma_D^{w_1}$ also have this structure.
6. We will take $\Gamma_M^{w_1}$ to be the identity matrix on its diagonal. The existence of a basis where the non-perturbative monodromies have this special form will be a posteriori justified by the fact that it leads to a consistent truncation to the rigid case.

Putting all these things together yields [2] the following 8×8 non-perturbative monodromy matrix $\Gamma_M^{w_1}$ that depends on four parameters x, y, v and p and that consistently describes the splitting of the $T = U$ line

$$\Gamma_M^{w_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -2/3 & 2/3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2/3 & -2/3 \\ x & y & 0 & 0 & 1 & 0 & 0 & 0 \\ y & v & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & p & p & 0 & 0 & 1 & 0 \\ 0 & 0 & p & p & 0 & 0 & 0 & 1 \end{bmatrix} \quad (25)$$

$\Gamma_D^{w_1}$ then immediately follows from (24).

The associated fixed points have the form

$$(N, -M) = (0, 0, N^2, -N^2, 0, 0, 0, 0) \quad (26)$$

for the monopole and

$$(N, -M) = \left(0, 0, N^2, -N^2, 0, 0, \frac{3}{2}N^2, -\frac{3}{2}N^2\right) \quad (27)$$

for the dyon.

Note that demanding the monopole matrix to be conjugated to the dyonic monodromy matrix leads to the requirement $p \neq 0$ [2].

3.2. Truncating the $SU(2)_{(1)}$ monopole monodromy to the rigid case

The monopole monodromy matrix $\Gamma_M^{w_1}$ depends on four yet undetermined parameters, namely x, v, y and $p \neq 0$. One way to determine their values is to demand that, upon truncation of (25) to the rigid case, one recovers the rigid non-perturbative monodromies of Seiberg/Witten.

Consider the 4×4 monopole subblock which acts on (P^2, P^3, iQ_2, iQ_3)

$$\Gamma_{M23}^{w_1} = \begin{bmatrix} 1 & 0 & -2\alpha & 2\alpha \\ 0 & 1 & 2\alpha & -2\alpha \\ p & p & 1 & 0 \\ p & p & 0 & 1 \end{bmatrix} \quad (28)$$

where $\alpha = \frac{1}{3}, p \neq 0$. Rotating it into the Higgs basis gives that

$$\begin{aligned} \Gamma_M^{Higgs, w_1} &= M \Gamma_{M23}^{w_1} M^{-1} \\ &= \begin{bmatrix} 1 & 0 & -4\alpha\gamma^2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{2p}{\gamma^2} & 0 & 1 \end{bmatrix} \end{aligned} \quad (29)$$

where M is given in equation (21). In the rigid case, on the other hand, one expects to find for the rigid monopole monodromy matrix in the Higgs basis that

$$\tilde{\Gamma}_M^{Higgs, w_1} = \begin{bmatrix} 1 & 0 & -4\tilde{\alpha}\gamma^2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{2\tilde{p}}{\gamma^2} & 0 & 1 \end{bmatrix} \quad (30)$$

where $\tilde{\alpha} = \frac{1}{4}, \tilde{p} = 0$. The first and third lines of (30) are nothing but the monodromy matrix for one $SU(2)$ monopole ($\gamma^2 = 2$ in the conventions of Seiberg/Witten [6], and $\gamma^2 = 1$ in the conventions of Klemm et al [7]). Thus, truncating the monopole monodromy matrix (25) to the rigid case appears to produce jumps in the parameters $p \rightarrow \tilde{p} = 0$ and $\alpha \rightarrow \tilde{\alpha}$ as given above. These jumps are due to the freezing in of the dilaton (see eq.(18)) when taking the flat limit. In [2]

we presented a field theoretical explanation for the jumps occurring in the parameters p and α when taking the rigid limit described in section 2.2. This explanation also determines, as a bonus, the values of the parameters v, y and p , namely $y = \frac{8}{3}$, $p = \frac{4}{3}$ and $v = 0$. Moreover, one can show that, in order to decouple the four $U(1)$'s at the non-perturbative level, one has to have $x = v$ and consequently $x = 0$.

4. Comparison with type II

The monopole monodromy matrix $\Gamma_M^{w_1}$ given in (25), which was obtained by decomposing $\Gamma_\infty^{w_1}$, is not an integer valued matrix. Recall that $\Gamma_\infty^{w_1}$ doesn't leave the dilaton S invariant, but rather induces the shift $S \rightarrow S + i$. Thus, consider [8] combining $\Gamma_\infty^{w_1}$ with a compensating shift $S \rightarrow S - i$ for the dilaton, which is generated by Γ_S given in (9). Then, under $\hat{\Gamma}_\infty^{w_1} = \Gamma_S \Gamma_\infty^{w_1}$, $T \leftrightarrow U$ whereas $S \rightarrow S$. $\hat{\Gamma}_\infty^{w_1}$ is given by

$$\hat{\Gamma}_\infty^{w_1} = \begin{pmatrix} U_\sigma & 0 \\ X & U_\sigma \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 0 \\ 0 & \tilde{U}\tilde{\Lambda} \end{pmatrix} \quad (31)$$

Comparing with (20) shows that the non-trivial submatrix of $\hat{\Gamma}_\infty^{w_1}$, which acts only on the rows and columns associated with (P^2, P^3, iQ_2, iQ_3) , turns in the Higgs basis (21) precisely into the rigid perturbative $SU(2)$ monodromy matrix of Seiberg/Witten. Thus, $\hat{\Gamma}_\infty^{w_1}$ can be straightforwardly decomposed into $\hat{\Gamma}_\infty^{w_1} = \hat{\Gamma}_M^{w_1} \hat{\Gamma}_D^{w_1}$. The integer valued monopole matrix $\hat{\Gamma}_M^{w_1}$, for instance, reads

$$\hat{\Gamma}_M^{w_1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (32)$$

The decomposition of $\hat{\Gamma}_\infty^{w_1}$ leads to integer valued non perturbative monodromy matrices $\hat{\Gamma}_M^{w_1}$ and

$\hat{\Gamma}_D^{w_1}$, which are in agreement with the ones computed on the type II Calabi-Yau side [8,9].

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