

# Spontaneous Breaking of $N=2$ to $N=1$ in Rigid and Local Supersymmetric Theories

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## ABSTRACT

We analyze the relation between rigid and local supersymmetric  $N=2$  field theories, when half of the supersymmetries are spontaneously broken. In particular, we show that the recently found partial supersymmetry breaking induced by electric and magnetic Fayet-Iliopoulos terms in rigid theories can be obtained by a suitable flat limit of previously constructed  $N=2$  supergravity models with partial super-Higgs in the observable sector.

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# Introduction

Recent work on D-brane physics has renewed interest in the analysis of supersymmetry breaking in  $N=2$  supergravity models that may describe the low-energy effective actions of string theories that take into account non-perturbative phenomena, such as the role of R-R charged states in conifold transitions, R-R Fayet-Iliopoulos (F-I) terms, and p-form condensation [1].

The generation of a non-perturbative scalar potential, which could stabilize flat directions of supersymmetric vacua, opens the way to studying dynamically generated mechanisms of supersymmetry breaking, and to further explore the possibility of breaking  $N=2$  supersymmetry to  $N=1$ .

In a previous note we focused our attention on a minimal model in which the simultaneous occurrence of the Higgs and super-Higgs mechanisms was shown to be a necessary condition for triggering the breaking of  $N=2$  to  $N=1$ , and for the lifting of two of the original six flat directions of the theory.

The model considered there was a spontaneously broken phase of  $N=2$  supergravity coupled to a  $U(1)$  vector multiplet and a hypermultiplet, charged with respect to the  $U(1)^2$  gauge group of the theory. Prior to gauging, the moduli space of the model was the six-dimensional manifold

$$\frac{SO(4,1)}{SO(4)} \times \frac{SU(1,1)}{U(1)}. \quad (1)$$

After gauging, the theory consisted of an  $N=1$  supergravity theory, with a massive spin-3/2 multiplet together with two massless chiral multiplets, whose scalars provided the residual (two complex) flat directions of the theory. In  $N=2$  supergravity, the presence of a charged hypermultiplet is necessary to Higgs the  $U(1)^2$ . This Higgsing is needed to give a mass to the two spin-1 components of the spin-3/2 massive multiplet [2].

On the other hand, very recently, it was observed in ref. [3] that spontaneous breaking of  $N=2$  rigid supersymmetry to  $N=1$  can already occur in a self-interacting (non-renormalizable)  $U(1)$  abelian theory, if F-I terms with simultaneous electric and magnetic components are introduced.

In the minimal theory of such kind, the photino is the goldstino and the broken phase consists of just a massless spin  $(1,1/2)$  multiplet together with a massive chiral multiplet.

This theory is in apparent violation of “common wisdom,” asserting that it is impossible to break spontaneously  $N=2$  to  $N=1$  in rigid theories. However, Hughes and Polchinski pointed out in ref. [4] that a generalization of the supersymmetry current algebra can contradict this wisdom. In fact, two- and four-dimensional counterexamples were provided within string [4] and membrane [5] theory. Aim of the present paper is, first, to re-analyze more carefully the general principles of supersymmetry and to show that, in agreement with ref. [4], it is indeed possible to have partial breaking of  $N=2$  in rigid theories; then, to show that such possibility is in fact encompassed by the previously studied partial breaking in  $N=2$  supergravity [6, 7], if a suitable flat limit, which preserves  $N=2$  rigid supersymmetry, is taken.

Section 1 will re-examine rigid extended supersymmetry, while Section 2 will deal with local  $N=2$  supergravity and its flat limit.

# 1 Partial SUSY Breaking in the Rigid Theory

Let us recall the argument that forbids partial braking in rigid supersymmetry. It is a simple consequence of the current algebra of extended supersymmetry, that implies, among other things, the following equation:

$$\int d^3\vec{y}\{J_{B0\dot{\alpha}}(y^0, \vec{y}), J_{\mu\alpha}^A(x)\} = 2\sigma_{\alpha\dot{\alpha}}^\nu \delta_B^A T_{\mu\nu}(x), \quad A, B = 1, \dots, N. \quad (2)$$

This equation means that the variation under the  $B$ -supersymmetry of the  $A$ -th supercurrent is diagonal in the extension indices, and equal to the gamma-trace of the stress-energy tensor. This equation, which makes sense even when supersymmetry is broken, due to the fact that commutators of fields are local, immediately implies that if a supersymmetry is broken, then all of them are broken with the same strength. To see this, one takes the VEV of eq. (2) and inserts a complete basis of states in the commutator. Standard manipulations imply the existence of  $N$  spin-1/2 zero-mass particles (the goldstini), which all couple to the vacuum with equal strength,  $F_{goldstino}^2 \sim \langle 0|T_\mu^\mu|0\rangle$ .

The way out to this situation is that eq. (2) is not the most general current algebra consistent with supersymmetry [4]. Indeed, the Jacobi identities of supersymmetry [8] allow for an additional field-independent, constant term to be added to eq. (2):  $\sigma_{\mu\dot{\alpha}\alpha} C_A^B$ . This term does not modify the supersymmetry algebra *on the fields* [9], since its commutators with any quantity in the theory is obviously zero. In particular, the commutator of two supersymmetry transformations on any field *is* a translation (up to eventual gauge transformations).

The presence of this extra constant term in the supersymmetric current algebra [10] allows for the breaking of only some of the  $N$  supersymmetries. As we are going to see in a moment, the model of ref. [3] (APT model) precisely realizes this situation. The  $N=2$  supersymmetry is realized *manifestly* and *linearly* on the fields (indeed, the model can be written in terms of  $N=2$  superfields [3]), but the current algebra of the  $N=2$  supersymmetry is not the standard one, but the one with the additional constant term.

The APT model consists of one (or more)  $N=2$  vector multiplets,  $A^\Sigma$ . They can be written as constrained  $N=2$  chiral multiplets, obeying the constraint:

$$(\epsilon_{ij} D^i \sigma_{\mu\nu} D^j)^2 A^\Sigma = -96 \square A^{*\Sigma}. \quad (3)$$

The crucial observation in ref. [3] is that this constraint does not imply that the auxiliary fields of the vector multiplet,  $\vec{Y}^\Sigma$  – which transform as vectors of the extension algebra  $SU(2)$  – are real. A *constant* imaginary term is also allowed, so that  $\vec{Y}^{*\Sigma} \neq \vec{Y}^\Sigma$ . This term is a magnetic F-I term, while the real part of  $\vec{Y}^\Sigma$  is a standard  $N=2$  electric F-I term. The transformation law of the “gaugino” (i.e. the spin-1/2 field of the vector multiplet) is

$$\delta\lambda_A^\Sigma = \frac{i}{\sqrt{2}} X_{AB}^\Sigma \eta^B \equiv \frac{i}{\sqrt{2}} Y_I^\Sigma \epsilon_{AC} \sigma_B^{IC} \eta^B + \dots \quad (4)$$

Here  $I = 1, 2, 3$  and the supersymmetry parameter is  $\eta^A$ ,  $A = 1, 2$ . The ellipsis denote terms which vanish on translationally invariant backgrounds. Since  $X_{AC}^\Sigma X_{CB}^\Sigma = \vec{Y}^\Sigma \cdot \vec{Y}^\Sigma \delta_{AB}$  (no sum

on  $\Sigma$ ), we see that when  $\vec{Y}^\Sigma$  is real, one unbroken supersymmetry implies  $\vec{Y}^\Sigma = 0$ , i.e. that the two supersymmetries are both unbroken. On the other hand, a complex  $\vec{Y}^\Sigma$  may have square equal to zero without vanishing. In ref. [3], in particular, there is a single vector multiplet, and at the minimum  $\vec{Y} = 2m\Lambda^2(0, i, -1)$ ,  $m \neq 0$ . Here we have explicitly shown the dependence of  $\vec{Y}$  on the supersymmetry breaking scale  $\Lambda \sim F_{goldstino}^{1/2}$ . With this choice of  $\vec{Y}$ , the matrix  $X_{AB}$  has exactly one zero eigenvalue, i.e. N=2 is broken to N=1.

The lagrangian of the APT model, extended to contain an arbitrary number of vector multiplets, reads, in N=2 superfield notation [11]:

$$\mathcal{L} = \frac{i}{4} \int d^2\theta_1 d^2\theta_2 [\mathcal{F}(A^\Sigma) - A_\Sigma^D A^\Sigma] + \frac{1}{2} (\vec{E}_\Sigma \cdot \vec{Y}^\Sigma + \vec{M}^\Sigma \cdot \vec{Y}_\Sigma^D) + c.c., \quad (5)$$

where  $A^\Sigma$  is an *unconstrained* chiral N=2 multiplet,  $A_\Sigma^D$  is a constrained chiral multiplet playing the role of Lagrange multiplier,  $\vec{E}$ ,  $\vec{M}$  are *constant* vectors, and  $\vec{M}^\Sigma$  is real. The equations of motion for the auxiliary fields derived from lagrangian (5) read:

$$\begin{aligned} \vec{Y}^\Sigma &= -2\tau_2^{\Sigma\Delta} (\text{Re } \vec{E}_\Delta + \tau_{1\Delta\Gamma} \vec{M}^\Gamma) + 2i\vec{M}^\Sigma, \\ \tau_{1\sigma\Delta} &= \text{Re } \tau_{\Sigma\Delta}, \quad \tau_{2\Sigma\Delta} = \text{Im } \tau_{\Sigma\Delta}, \quad \tau_{\Sigma\Delta} = \mathcal{F}_{\Sigma\Delta}, \quad \tau_2^{\Sigma\Gamma} \tau_{2\Gamma\Delta} = \delta_\Delta^\Sigma. \end{aligned} \quad (6)$$

In computing the supersymmetric variation of the supercurrent, the only difference with respect to the standard case ( $\vec{M}^\Sigma = 0$ ) arises in terms involving  $\vec{Y}^\Sigma$ . In other words, all fermionic and derivative terms in the variation of the supercurrent are as in the standard case, so that the variation of the supercurrent reads

$$\int d^3\vec{y} \{J_{B0\dot{\alpha}}(y^0, \vec{y}), J_{\mu\alpha}^A(x)\} = \frac{1}{2} \sigma_{\mu\alpha\dot{\alpha}} (X_{AC}^\Sigma)^* \tau_{2\Sigma\Delta} X_{CB}^\Delta + \dots = 2\sigma_{\mu\alpha\dot{\alpha}} M_B^A + \dots, \quad (7)$$

where  $X_{AB}^\Sigma$  is as in eq. (4), and the ellipsis denote terms identical with the standard case. Using eqs. (4,6) one finds

$$\begin{aligned} M_B^A &= \frac{1}{4} \tau_{2\Sigma\Delta} \vec{Y}^{*\Sigma} \cdot \vec{Y}^\Delta \delta_B^A + \frac{i}{4} \tau_{2\Sigma\Delta} \vec{\sigma}_B^A \cdot (\vec{Y}^{*\Sigma} \times \vec{Y}^\Delta) = \\ &= \tau_2^{\Sigma\Delta} (\text{Re } \vec{E}_\Sigma + \tau_{\Sigma\Gamma} \vec{M}^\Gamma) (\text{Re } \vec{E}_\Delta + \tau_{\Delta\Pi}^* \vec{M}^\Pi) \delta_B^A + 2\vec{\sigma}_B^A \cdot (\text{Re } \vec{E}_\Sigma \times \vec{M}^\Sigma). \end{aligned} \quad (8)$$

The term proportional to  $\delta_B^A$  is the scalar potential, as in the standard case. Since  $M_B^A$  is given by the square of the fermionic shifts in eq. (4), it is positive semidefinite. Its eigenvalues are

$$\lambda_\pm = \frac{1}{4} \tau_{2\Sigma\Delta} \vec{Y}^\Sigma \cdot \vec{Y}^{*\Delta} \pm \frac{1}{4} \|i\tau_{2\Sigma\Delta} \vec{Y}^\Sigma \times \vec{Y}^{*\Delta}\|, \quad \|\vec{V}\| \equiv (\vec{V} \cdot \vec{V})^{1/2}. \quad (9)$$

Partial breaking is possible whenever  $\lambda_- = 0$ .

By substituting eq. (8) into the variation of the supercurrent we find that the supersymmetry current algebra receives a *field-independent* modification, as expected:

$$\int d^3\vec{y} \{J_{B0\dot{\alpha}}(y^0, \vec{y}), J_{\mu\alpha}^A(x)\} = 2\sigma_{\alpha\dot{\alpha}}^\nu \delta_B^A T_{\mu\nu}(x) + 4\sigma_{\mu\alpha\dot{\alpha}} \vec{\sigma}_B^A \cdot (\text{Re } \vec{E}_\Sigma \times \vec{M}^\Sigma). \quad (10)$$

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<sup>3</sup>Notice that the off-shell algebra still closes, since an imaginary constant does not contribute to  $\delta\vec{Y}^\Sigma$ .

As we noticed before, this additional constant term in the current algebra does not affect the form of the supersymmetry transformations on fields. Indeed, in the presence of a nonzero  $\vec{M}$ , the commutator of two supersymmetry transformations still has the standard form on all fields except on the spin-1 field, where it gets an extra term

$$[\delta_1, \delta_2]A_\mu^\Sigma = 2i(\eta_A^1 \sigma_\mu \bar{\eta}^{2B} - \eta_A^2 \sigma_\mu \bar{\eta}^{1B}) \vec{\sigma}_B^A \cdot \vec{M}^\Sigma. \quad (11)$$

This extra term is a harmless gauge transformation when the supersymmetry parameters are constant. This is no longer the case when they are given a dependence on space-time coordinate; in other words, the APT model cannot be coupled naively to supergravity. We shall see in a moment that in spite of this, it can be recovered as an appropriate flat limit of N=2 supergravity.

## 2 N=2 Supergravity with Partial Breaking and its Flat Limit

In this section we show that the APT model arises as a flat limit of a N=2 supergravity. The limit is  $M_{Pl} \rightarrow \infty$ ,  $\Lambda = \text{constant}$ , where, as before,  $\Lambda \sim F_{goldstino}^{1/2}$  is the scale of supersymmetry breaking. For sake of simplicity we will study the original APT theory, which contains a single self-interacting abelian vector multiplet.

As we have just seen, the current algebra (i.e. the Ward identity) of rigid supersymmetry can be modified by adding a constant term to it. In local supersymmetry, this freedom no longer exists, instead, the algebra is modified because of the presence of the gravitini. When restricted to translationally invariant backgrounds, the Ward identity becomes to so-called ‘‘T-identity’’ [12, 13]

$$\delta_A \psi_L^i \delta^B \psi_R^j \mathcal{Z}_{ij} - 3M_{Pl}^2 \mathcal{M}_{AC} \mathcal{M}^{*CB} = V \delta_A^B, \quad (12)$$

where  $\delta_A \psi^i$  denote the shift, under the A-th supersymmetry, of the spin one-half fermions, while  $\mathcal{M}_{AB}$  is the gravitino mass matrix and  $\mathcal{Z}_{ij}$  is the kinetic term of the fermions. We have kept the dependence on  $M_{Pl}$  in this formula because we will be interested in studying an appropriate flat limit  $M_{Pl} \rightarrow \infty$ . These identities show that even when  $V = 0$ , one may still have, say

$$\delta_1 \psi_L^i \delta^1 \psi_R^j \mathcal{Z}_{ij} = 3\mathcal{M}_{1C} \mathcal{M}^{C1} = 0, \quad (13)$$

but instead

$$\delta_2 \psi_L^i \delta^2 \psi_R^j \mathcal{Z}_{ij} = 3\mathcal{M}_{2C} \mathcal{M}^{C2} \neq 0. \quad (14)$$

In N=2, this corresponds to breaking half of the supersymmetries (N=1 unbroken), at zero cosmological constant.

We must emphasize that a field configuration for which at least one supersymmetry is unbroken gives automatically an absolutely stable local minimum of the potential [12].

A supergravity model with partial breaking of N=2 supersymmetry can be constructed along the lines of ref. [6]. The matter content of the model is a charged hypermultiplet, whose scalars parametrize the quaternionic manifold  $SO(4, 1)/SO(4)$ , coupled to an abelian vector multiplet.

By denoting the quaternionic coordinates of the hypermultiplet manifold with  $b^u$ ,  $u = 0, 1, 2, 3$ , we can write its symplectic vielbein [14] – which determines the coupling to fermions – as [6]

$$\mathcal{U}^{\alpha A} = \frac{1}{2b^0} \epsilon^{\alpha\beta} (db^0 - i\vec{\sigma}d\vec{b})_{\beta}^A. \quad (15)$$

The special geometry of the vector-multiplet manifold is specified by four holomorphic sections  $X^{\Sigma}(z)$ ,  $F_{\sigma}(z)$ ,  $\Sigma = 0, 1$  [15], in terms of which the Kähler potential reads

$$K = -\log i(X^{*\Lambda}F_{\Lambda} - X^{\Lambda}F_{\Lambda}^*). \quad (16)$$

Our choice, which slightly generalizes that of ref. [6], is

$$X^0(z) = \frac{1}{\sqrt{2}}, \quad X^1(z) = \frac{i}{\sqrt{2}}f'(z), \quad F_0(z) = -\frac{i}{\sqrt{2}}[2f(z) - zf'(z)], \quad F_1(z) = \frac{z}{\sqrt{2}}. \quad (17)$$

This choice of sections is such that no holomorphic prepotential exists. It has been obtained by performing the symplectic transformation  $X^1 \rightarrow -F_1$ ,  $F_1 \rightarrow X^1$  on the sections obtained from the prepotential  $F(X^0, X^1) = -i(X^0)^2 f(X^1/X^0)$  [6, 15].

The gauge group in our case is  $U(1)^2$ , with one of the  $U(1)$  factors coming from the matter vector multiplet and the other from the graviphoton. The coupling of  $U(1)^2$  to the hypermultiplet is specified by the covariant derivative  $D_{\mu}b^u = \partial_{\mu}b^u + A_{\mu}^{\Sigma}k_{\Sigma}^u$ . The Killing vectors  $k_{\Sigma}^u$  in our case are a simple generalization of those of ref. [6]:

$$k_0^u = g_1\delta^{u3} + g_2\delta^{u2}, \quad k_1^u = g_3\delta^{u2}. \quad (18)$$

$g_1$ ,  $g_2$  and  $g_3$  are arbitrary constants. These Killing vectors are derived through standard N=2 formulae from the ‘‘D-term’’ prepotentials [14, 6]

$$\mathcal{P}_0^I = \frac{1}{b^0}(g_1\delta^{I3} + g_2\delta^{I2}), \quad \mathcal{P}_1^I = g_3\frac{1}{b^0}\delta^{I2}, \quad I = 1, 2, 3, \quad (19)$$

i.e.  $\vec{\mathcal{P}}_{\Sigma} = b_0^{-1}\vec{k}_{\Sigma}$ .

The formulae for the shifts of the (antichiral) gaugino  $\lambda_A^{z*}$ , hyperini  $\zeta^{\alpha}$ , and (chiral) gravitini  $\psi_{A\mu}$  are

$$\begin{aligned} \delta\lambda_A^{z*} &= -ig^{zz*}\epsilon_{BC}\vec{\sigma}_A^C \cdot \vec{\mathcal{P}}_{\Sigma}e^{K/2}(\partial_z + \partial_z K)X^{\Sigma}(z)\eta^B = W_{AB}^{z*}\eta^B, \\ \delta\zeta^{\alpha} &= -2\epsilon_{AB}\mathcal{U}_u^{\alpha B}k_{\Sigma}^ue^{K/2}X^{\Sigma}(z)\eta^A \equiv \mathcal{N}_A^{\alpha}\eta^A, \\ \delta\psi_{A\mu} &= \frac{i}{2}\epsilon_{BC}\vec{\sigma}_A^C\vec{\mathcal{P}}_{\Sigma}e^{K/2}X^{\Sigma}(z)\gamma_{\mu}\eta^B \equiv iS_{AB}\gamma_{\mu}\eta^B. \end{aligned} \quad (20)$$

To recover the APT model, we must expand around an appropriate vacuum, with zero cosmological constant at the Planck scale, and perform a suitable flat limit  $M_{Pl} \rightarrow \infty$ . It is evident that the coupling constants  $g_i$ , as well as the gravitino mass, must vanish in that limit. The reason is that the APT model does not contain any hypermultiplet, thus, to reconcile its spectrum with the one of our supergravity theory, the hypermultiplet must decouple from the vector multiplet in the flat limit. Moreover, a finite, nonzero gravitino mass would imply that the lagrangian

contains *explicit* supersymmetry-breaking terms in the  $M_{Pl} \rightarrow \infty$  limit. The crucial property that allows for a non-trivial result in the flat limit is that the gravitino shifts are proportional to the supersymmetry breaking scale  $\Lambda$ , while the latter is related to the gravitino mass by  $m_{3/2} \sim \Lambda^2/M_{Pl}$ . It is therefore possible to find a limit in which the gravitino shifts contribute to the Ward identity of rigid supersymmetry, while the gravitino mass goes to zero (together with all explicit supersymmetry-breaking terms).

The limit reproducing the APT model is specified as follows. First of all we set

$$f(z) = \frac{1}{2} + \frac{\Lambda}{M_{Pl}}z + \frac{\Lambda^2}{M_{Pl}^2}\phi(z) + O(\Lambda^3/M_{Pl}^3). \quad (21)$$

Then we choose

$$g_1 = \frac{\Lambda^2}{M_{Pl}^2}\xi, \quad g_2 = \frac{\Lambda^2}{M_{Pl}^2}e, \quad g_3 = 2\frac{\Lambda}{M_{Pl}}m. \quad (22)$$

In the limit  $M_{Pl} \rightarrow \infty$ ,  $\Lambda = \text{constant}$ , we find

$$\begin{aligned} g_{zz^*} &= \partial_z \partial_{z^*} K(z, z^*) = \frac{\Lambda^2}{M_{Pl}^2} \left\{ 1 - \frac{1}{2}\phi''(z) - \frac{1}{2}[\phi''(z)]^* \right\} + O(\Lambda^3/M_{Pl}^3), \\ K_z &= -\frac{\Lambda}{M_{Pl}} + O(\Lambda^2/M_{Pl}^2). \end{aligned} \quad (23)$$

To recover the APT model we set

$$\mathcal{F}(z) \equiv z^2 - i2\phi(z), \quad (24)$$

and we rescale the fermions so that they have a canonically normalized kinetic term:

$$\lambda_A^{z^*} \rightarrow (M_{Pl}\Lambda^2)^{-1/2}\lambda_A^{z^*}, \quad \zeta^\alpha \rightarrow M_{Pl}^{-3/2}\zeta^\alpha, \quad \psi_{A\mu} \rightarrow M_{Pl}^{-3/2}\psi_{A\mu}. \quad (25)$$

By restoring the proper mass dimension ( $-1/2$ ) in the supersymmetry breaking parameter, i.e. rescaling  $\eta^A \rightarrow M_{Pl}^{1/2}\eta^A$ , and by defining  $\tau(z) = \tau_1(z) + i\tau_2(z) = \mathcal{F}''(z)$ , we find that  $g_{zz^*} = (\Lambda^2/M_{Pl}^2)\tau_2(z)/2$ , and that in the flat limit the shifts of the fermions read

$$\begin{aligned} \delta\lambda_A^{z^*} &= i\Lambda^2 \frac{2}{\tau_2(z)} \epsilon_{BC} \left[ \frac{1}{\sqrt{2}b^0} (\xi\sigma_A^{3C} + e\sigma_A^{2C}) + \frac{1}{\sqrt{2}b^0} m\sigma_A^{2C} \tau(z) \right] \eta^B + O(\Lambda^3/M_{Pl}), \\ \delta\zeta^\alpha &= i\Lambda^2 \epsilon_{BC} \frac{1}{\sqrt{2}b^0} \left[ (\xi\sigma_A^{3C} + e\sigma_A^{2C}) + 2im\sigma_A^{2C} \right] \eta^B + O(\Lambda^3/M_{Pl}), \\ \delta\psi_{A\mu} &= \frac{i}{2}\Lambda^2 \gamma_\mu \epsilon_{BC} \frac{1}{\sqrt{2}b^0} \left[ (\xi\sigma_A^{3C} + e\sigma_A^{2C}) + 2im\sigma_A^{2C} \right] \eta^B + O(\Lambda^3/M_{Pl}). \end{aligned} \quad (26)$$

Notice that in the flat limit,  $b^0$  becomes a coupling constant, since the fluctuations in  $\partial_\mu b^0$  are  $O(M_{Pl}^{-1})$ . Therefore  $b^0$  plays a role analogous to the dilaton in the low-energy limit of string theory. Since the manifold of the hypermultiplet is homogeneous, all values of  $b^0 > 0$  give the same physics. In the flat limit the only effect of a change in  $b^0$  is to rescale the coupling constants  $\xi$ ,  $e$  and  $m$ . In particular, at  $b^0 = 1$ , we recover the APT model with a single vector multiplet and

$$\text{Re } \vec{E} = \Lambda^2(0, e, \xi), \quad \vec{M} = \Lambda^2(0, m, 0). \quad (27)$$

Eq. (26) shows that in the flat limit the shift of the gaugino is identical with the one of the APT model, whereas the gravitino and hyperino shifts are nonzero, *field independent constants*. We are thus guaranteed that the potential of our model agrees with the APT one, up to field independent terms. This statement can be easily verified by using the T-identities eq. (12). In the normalizations of this section these identities read

$$-12(S_{AC})^* S_{CB} + \frac{\tau_2(z)}{2}(W_{AC}^{z*})^* W_{CB}^{z*} + 2(\mathcal{N}_A^\alpha)^* \mathcal{N}_B^\alpha = \delta_B^A V(z), \quad (28)$$

where we have used the re-scaled fermion shifts throughout. As expected, the (field independent) terms that are off-diagonal in the extension indices  $A, B$ , and that arise from the square of the gaugino shifts, are exactly cancelled by the combined shifts of the hyperini and gravitini. The same shifts also contribute an additional *field-independent* term to the scalar potential, equal to  $-(1/2)\Lambda^4(\xi^2 + e^2 + 4m^2)$ . The complete potential reads

$$V(z) = \frac{1}{b^0 2} \left[ \frac{1}{\tau_2(z)} |\text{Re } \vec{E} + \tau(z) \vec{M}|^2 - \frac{1}{2} \Lambda^4 (\xi^2 + e^2 + 4m^2) \right]. \quad (29)$$

Some comments are now in order.

1. The potential in eq. (28) differs from the one in [3] or in eq. (8). Since the difference depends only on  $b^0$ , it becomes irrelevant in the flat limit, when gravitational interactions decouple and  $b^0$  does not fluctuate. The stationary point  $\tau_1(z) = -e/m$ ,  $\tau_2(z) = |\xi/m|$  is a stable minimum [3] at  $M_{Pl} = \infty$ . At this minimum,

$$V_{min} = \frac{1}{2b^0 2} \Lambda^4 (4|\xi m| - \xi^2 - e^2 - 4m^2). \quad (30)$$

2. Since the hyperino and gravitino shifts depend only on  $b^0$ , for a generic choice of  $\xi, e, m$ , they break both supersymmetries. On the other hand, since the gaugino shift depends also on the field  $z$ , whenever the “prepotential”  $\mathcal{F}(z)$  gives rise to non-renormalizable interactions, one may be able to find a VEV  $z$  such that the gaugino-shift matrix has exactly one zero eigenvalue. In this case N=2 supersymmetry is broken to N=1 in both the “observable” gaugino sector and the “hidden” hyperino + gravitino sector, while the residual N=1 is only broken in the hidden sector.
3. The cosmological constant is effectively zero at the Planck scale, whenever  $\Lambda \ll M_{Pl}$ . Nevertheless, for  $M_{Pl} < \infty$ , and for generic values of the parameters  $\xi, e, m$ , the potential in eq. (29) has no minimum. Indeed, in the full supergravity theory described above,  $b^0$  is a dynamical field. Minimization in  $b^0$  and  $z$  result in a constraint on the parameters of the theory, which reads, up to terms  $O(\Lambda/M_{Pl})$ :

$$4|\xi m| - \xi^2 - e^2 - 4m^2 = 0. \quad (31)$$

This equation is solved by  $e = 0, \xi = 2m$ . These parameters define a supergravity model undergoing partial super-Higgs from N=2 to N=1: N=1 is unbroken in both the “observable” sector and the “hidden” sector. A general theorem [12] guarantees that in this case the stationary point in  $z$  and  $b^0$  exists and is a stable minimum.



4. For any other choice of the parameters, only “cosmological” solutions with a time-dependent runaway  $b^0$  VEV exist. The characteristic time of evolution of these solutions,  $db^0/dt$ , is  $\kappa M_{Pl}/\Lambda^2$ , where  $\kappa$  is  $O(1)$  for a generic choice of parameters. If we set  $\Lambda \sim 100 \text{ GeV}$ , this time is approximately  $10^{-10} \text{ sec}$ . Needless to say, this means that a “realistic” supersymmetry breaking scale does not give rise to a realistic – i.e. almost stationary – perturbative vacuum.
5. The model of ref. [6] has an accidental flat direction, giving rise to an extra massless  $N=1$  scalar multiplet, due to a special choice of the vector-multiplet metric. In ref. [6], indeed, the vector multiplet manifold was the homogeneous space  $SU(1,1)/U(1)$ , corresponding to choosing  $f(z) = (1/2) + (\Lambda/M_{Pl})z$  in eq. (21).

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