OVERVIEW OF KAON DECAY PHYSICS

R. D. Peccei*

*Department of Physics, University of California at Los Angeles,
Los Angeles, CA 90095-1547*

Abstract

After a brief history of the insights gained from Kaon physics, the potential of Kaon decays for probing lepton number violation is discussed. Present tests of CTP and of Quantum Mechanics in the neutral Kaon sector are then reviewed and the potential of the Frascati Φ factory for doing incisive tests in this area is emphasized. The rest of this overview focuses on CP violating effects in the Kaon system. Although present observations of CP violation are perfectly consistent with the CKM model, we emphasize the theoretical and experimental difficulties which must be faced to establish this conclusively. In so doing, theoretical predictions and experimental prospects for detecting $\Delta S = 1$ CP violation through measurements of ϵ'/ϵ and of rare K decays are reviewed. The importance of looking for evidence for non-CKM CP-violating phases, through a search for a non-vanishing transverse muon polarization in $K_{\mu 3}$ decays, is also stressed.

1. Introduction

Ever since their discovery (1) nearly 50 years ago, Kaons have played an important part in the development of particle physics. The suggestion of Pais (2) and Gell-Mann (3) that Kaons possessed a new quantum number— strangeness—and so could only be produced in association with particles with the opposite quantum number was soon confirmed experimentally (4) and marked the beginning of the study of flavor physics. At the same time, the $\tau - \theta$ puzzle (5) provided the impetus for Lee and Yang (6) to suggest that the weak interactions did not conserve parity. With parity violation the identification of the τ, which decayed into three pions, with the θ, which decayed into two pions, was natural and Kaons were born.

In the 1960's Kaons played an important role in elucidating some of the underlying symmetries of the strong interactions, well before the advent of QCD where these symmetries are more manifest. The approximate invariance of the strong interactions under flavor SU(3) (7) led to the Gell-Mann-Okubo formula (8) interrelating the Kaon

HEP-PH-9504392

---

* Invited talk given at the 23rd INS International Symposium on Nuclear and Particle Physics with Meson Beams with 1 GeV/c Region, Tokyo, Japan, March 1995. To be published in the Symposium Proceedings.

mass with that of the pion and the η. The extension of this symmetry to a, spontaneously broken, approximate $SU(3)_V \times SU(3)_A$ invariance of the strong interactions (9) underscored the special dynamical role of the pseudoscalar meson octet (π, K, η) as near Nambu Goldstone bosons (10). It also provided important connections between weak decay amplitudes involving Kaons, like the Callan-Trieman relation (11). Almost simultaneously to these theoretical developments, the discovery of the decay $K_L \to 2\pi$ by Christianson, Cronin, Fitch and Turlay (12) provided the first indication that CP, like parity, was also not a good symmetry of nature.

Kaon physics also provided important insights into the flavor structure of the weak interactions. The weaker strength of Kaon weak decays relative to that of the pions lead to the introduction of the Cabibbo angle (13) and to the notion of flavor mixing for charged current weak interactions. The very suppressed nature of the neutral current decay $K_L \to \mu^+ \mu^-$, relative to the charged current decay $K^+ \to \mu^+ \nu_e$, found its natural explanation through the GIM mechanism (14) and lead to the prediction of a further flavor—charm—which was subsequently found (15).

Although perhaps the halcyon decays of Kaon physics are past, Kaons can be counted on, even today, to provide important future physics insights at the research frontier. In this talk I would like to focus on three such areas, where experiments with Kaon beams can substantially further our understanding:

i) Tests of flavor violation, using the intense Kaon beams now available, to probe for lepton number violation to an accuracy of one part in a trillion.

ii) Tests of CPT and of Quantum Mechanics to unprecedented accuracy, using to advantage the tiny mass difference between the K_L and the K_S states to amplify these effects and make them experimentally more accessible.

iii) Tests of CP violation in the only system where this phenomena has been observed, particularly to look for evidence for direct $(\Delta S = 1)$ CP violation and for CP violation induced by new scalar interactions.

2. Testing for Flavor Violation

Both lepton number (L) and baryon number (B) are classical global symmetries of the Standard Model. However, there are no good reasons why these symmetries should be exact in nature. In fact, it is known that quantum effects arising from the existence of chiral anomalies (16) lead to a breakdown of (B+L)-symmetry (17). Also, if the Standard Model is embedded into some Grand Unified Theory (GUT), then generally these theories have lepton and quarks in the same representation, leading to a breakdown of both L and B (18).

The violations of B and L alluded to above are highly suppressed, leading to phenomena like proton decay which have extremely long lifetimes (19). However, these may well not be the only sources of flavor violation in nature. For instance, new physics may involve interactions which are mediated by leptoquarks—objects having both quark and lepton quantum numbers. Leptoquark exchanges, as those typified by the diagram in Fig. 1, will give rise to flavor changing decays, like $K_L \to \mu^+ e^-$.
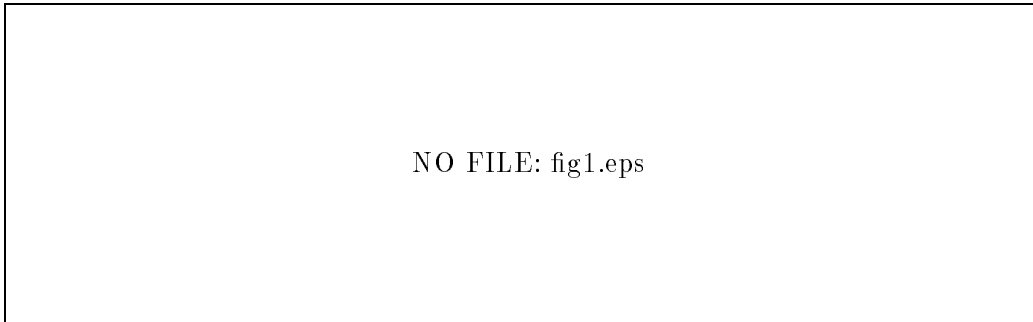
NO FILE: fig1.eps

Fig. 1. Leptoquark exchange leading to flavor violation

Assuming comparable couplings of the leptoquarks to quarks and leptons to those of the W's to these excitations, one predicts a branching ratio for this process of order

$$BR(K_\mathrm{L} \to \mu^+ e^-) \sim \left(\frac{M_\mathrm{W}}{M_\mathrm{LQ}}\right)^4$$

One sees that if one probes flavor violating processes to the level of BR $\sim 10^{-12}$ one is probing leptoquark masses (and therefore new physics) to the level $M_\mathrm{LQ} \geq 100$ TeV.

Experimentally, searches for lepton flavor violating interactions have been carried out to great accuracy. The present best bounds for the process $K_\mathrm{L} \to \mu^+ e^-$ come from experiments at Brookhaven (E791) and KEK (E137). These experiments have established 90% CL bounds of $O(10^{-10})$ for this branching ratio:

$$BR(K_\mathrm{L} \to \mu^+ e^-) \leq 3.3 \times 10^{-11} \quad (20)$$
$$BR(K_\mathrm{L} \to \mu^+ e^-) \leq 9.4 \times 10^{-11} \quad (21)$$

A new experiment (E871) has started running at Brookhaven which should be able to push down the limit for this decay to a $BR \sim 2 \times 10^{-12}$.

For the lepton violating process $K \to \pi \mu e$ the best bound to date comes from a Brookhaven experiment, E777:

$$BR(K^+ \to \pi^+ \mu^- e^+) \leq 2.1 \times 10^{-10} \quad (22)$$

A new experiment at Brookhaven (E865) has started running and hopes to push this BR also down to the level of $O(10^{-12})$.

The large improvement in precision expected from BNL E865 compared to the present bound, as well as the sharpening expected from BNL E871 to the present limit on $K_\mathrm{L} \to \mu^+ e^-$, if no effects are found will produce only a small extension of the mass limits for particles which could mediate these decays. Typically, an improvement in BR limits of a factor of 10 will only lead to an improvement in the mass limit by a factor of 2 or so, since the BR scales as M^{-4}.

3. CPT and Quantum Mechanics Tests

The CPT theorem (23) is a fundamental consequence of being able to describe elementary particle interactions by a relativistic local quantum field theory. Thus

violation of CPT invariance would signal the breakdown of some sacred principles, like locality or even Quantum Mechanics! Nevertheless, it has been suggested that some small violation of CPT invariance may possibly arise in connection to string theory (24) or may result from gravitational effects (25). In both cases a concomitant breakdown of Quantum Mechanics may also occur. The neutral Kaon system is ideal for probing these speculations since the very small $K_{\rm L} - K_{\rm S}$ mass difference allows one to probe $K^o - \bar{K}^o$ mass differences of $O(10^{-18})$ the Kaon mass. This is the right range to begin seeing possible inverse Planck mass effects:

$$\frac{M_{K^o} - M_{\bar{K}^o}}{M_{K^o}} \sim \frac{M_{K^o}}{M_{\rm Planck}} \sim 10^{-19}$$

Present day data are consistent with CPT conservation. However, more incisive tests would be welcome. These are likely to be carried out in the near future, particularly at the Frascati Φ Factory.

There have been two kinds of theoretical analyses of CPT violating phenomena in the neutral Kaon complex which differ in that in one case (26) Quantum Mechanics is assumed to hold, while in the other both CPT and Quantum Mechanics violations are included (28), (27). If CPT is violated, the phenomenology of the $K^o - \bar{K}^o$ system is modified in two ways (26):

i) The $K_{\rm L}$ and $K_{\rm S}$ states are now different superpositions of K^o and $\bar{K}^o$, characterized by separate mixing parameters $\epsilon_{\rm L}$ and $\epsilon_{\rm S}$:

$$|K_{\rm L}> \simeq \frac{1}{\sqrt{2}}\{(1+\epsilon_{\rm L})|K^o> -(1-\epsilon_{\rm L})|\bar{K}^o>\}$$

$$|K_{\rm S}> \simeq \frac{1}{\sqrt{2}}\{(1+\epsilon_{\rm S})|K^o> +(1-\epsilon_{\rm S})|\bar{K}^o>\} \ ,$$

with

$$\epsilon_{\rm L} = \epsilon_{\rm K} - \delta_{\rm K} \ ; \ \epsilon_{\rm S} = \epsilon_{\rm K} + \delta_{\rm K}$$

where the parameter $\delta_{\rm K}$ typifies mixing CPT violation.

ii) Particle and antiparticle decay amplitudes are no longer simply related by complex conjugation. Instead one has, for example(29):

$$A(K^o \rightarrow \pi^- \ell^+ \nu_e) = a + b \quad ; \quad A(\bar{K}^o \rightarrow \pi^+ \ell^- \bar{\nu}_e) = a^* - b^*$$

$$A(K^o \rightarrow 2\pi; I) = (A_I + B_I)e^{i\delta_I} \quad ; \quad A(\bar{K}^o \rightarrow 2\pi; I) = (A_i^* - B_I^*)e^{i\delta_I}$$

In the above, the b and B_I amplitudes violate CPT.

If, in addition, also Quantum Mechanics is violated then, besides the above modifications due to CPT non-invariance, the time evolution of the $K^o - \bar{K}^o$ complex is different from the usual Schrödinger evolution. This is most easily described in terms of the evolution of the density matrix ρ of the $K^o - \bar{K}^o$ system. Quantum Mechanics violation is introduced (25) through the appearance of an extra term in the Schrödinger equation for ρ [1]

$$i\frac{\partial}{\partial t}\rho = H\rho - \rho H^\dagger + \delta h \rho \ .$$

---

1 For the $K^o - \bar{K}^o$ system the 2×2 Hamiltonian H is not Hermitian, since it also describes the decay of these states: $H = M - \frac{i}{2}\Gamma$.

Because of the presence of the δh term above, the evolution of ρ with time has no longer the Schrödinger form. Given δh this evolution can be straightforwardly computed (28),(27). Ellis *et al.* (30) show that the simplest δh, which is consistent with some general principles like probability conservation, can be parametrized by three CPT and Quantum Mechanics violating parametes: α, β, γ, with

$$\alpha, \gamma > 0 \; ; \; \alpha\gamma > \beta^2 \; .$$

Present day data is not sufficient to determine all these CPT violating parameters. In addition, for the case where one assumes that there is also a violation of Quantum Mechanics, one should really do a fit of the experimental data with the modified evolution equation. Without a violation of Quantum Mechanics, there are basically two independent tests of CPT: one where the K_L semileptonic decay asymmetry A_{K_L} is compared to the real part of η_{+-}, and the other where the phase of ϵ is compared to the superweak phase ϕ_{SW} (26).[2] The first test is sensitive to amplitude CPT violation and one has

$$\frac{1}{2}A_{K_L} - Re \; \eta_{+-} = \frac{Re \; b}{Re \; a} - \frac{Re \; B_o}{Re \; A_o} = (1.3 \pm 6.6) \times 10^{-5} \; ,$$

where the right-hand side uses the PDG values (31) for the experimental quantities. For the second test one uses the fact that one can decompose ϵ into a CP violating and a CPT violating piece, with these terms being 90° out of phase (32):

$$\epsilon = \epsilon_{C\!\!\!/P}e^{i\phi_{SW}} + \epsilon_{C\!\!\!/PT}e^{i\left(\phi_{SW}+\frac{\pi}{2}\right)}$$

with

$$\phi_{SW} = \tan^{-1}\frac{2\Delta m}{\Gamma_S - \Gamma_L} = (43.64 \pm 0.14)° \; .$$

One finds, again using PDG values, that

$$\epsilon_{C\!\!\!/PT} \simeq \sqrt{2} \; Im \; \delta_K \simeq \sqrt{2}\left(\frac{Re \; B_o}{Re \; A_o} - Re \; \delta_K\right) = (2.6 \pm 3.2) \times 10^{-5} \; ,$$

so that $\epsilon_{C\!\!\!/PT}$ is at most of order 1% of ϵ. Hence the decay $K_L \to 2\pi$ is either wholly, or predominantly, a result of CP violation, not CPT violation. Nevertheless, because what one measures are essentially differences of CPT violating parameters, one cannot exclude an accidental cancellation and thus the possibility of having a hidden large CPT violation (26). If amplitude CPT violation is neglected, then this cancellation is excluded and a measurement of $\epsilon_{C\!\!\!/PT}$ at the level indicated above gives a strong bound on the $K^o - \bar{K}^o$ mass difference:

$$M_{K^o} - M_{\bar{K}^o} \simeq 2\sqrt{2} \; \Delta m\epsilon_{C\!\!\!/PT} = (2.57 \pm 3.18) \times 10^{-19} \; \text{GeV} \; .$$

Huet and Peskin (27) have recently performed an analysis of the time evolution of the decay of an initial K^o into $\pi^+\pi^-$, under the assumption that Quantum Mechanics is violated by the δh perturbation discussed above. Such decays are studied in the CP

---

2   For these tests, given the present accuracy and the smallness of ϵ', one can neglect ϵ' altogether. Thus one has $\eta_{+-} \simeq \eta_{oo} \simeq \epsilon$.

Lear experiment (34), since in $p\bar{p}$ annihilation one can tag the produced K^o or $\bar{K}^o$ states with the sign of the accompanying produced charged Kaon. Neglecting amplitude CPT violation, the decay of an initial K^o into $\pi^+\pi^-$ can be written as

$$R_{+-}(t) = e^{-\Gamma_{\rm S}t} + R_{\rm L}e^{-\Gamma_{\rm L}t} + 2|\epsilon_{\rm L}^-|\cos(\Delta mt + \phi_{+-})\exp\left[-\frac{(\Gamma_{\rm L} + \Gamma_{\rm S})t}{2}\right] \ .$$

If there is no violation of Quantum Mechanics, then

$$|\epsilon_{\rm L}^-| = |\eta_{+-}| \ ; \ \ R_{\rm L} = |\eta_{+-}|^2 \ .$$

If Quantum Mechanics is violated, however, $R_{\rm L}$ and $\epsilon_{\rm L}^-$ are no longer simply related and they depend on the CPT violating parameters β and γ[3]. One finds:

$$\epsilon_{\rm L}^- = \epsilon_{\rm L} - \frac{\beta}{d} \ ; \ \ R_{\rm L} = |\epsilon_{\rm L}^-|^2 + \frac{\gamma}{\Delta\Gamma} + \frac{4\beta}{\Delta\Gamma}\,{\rm Im}\,\frac{\epsilon_{\rm L}^- d}{d^*} \ ,$$

where the kinematical parameter d is

$$d = \Delta m + \frac{i}{2}(\Gamma_{\rm S} - \Gamma_{\rm L}) \equiv \Delta m + \frac{i}{2}\Delta\Gamma \simeq (5 \times 10^{-15}{\rm GeV})e^{i\phi_{\rm SW}} \ .$$

By comparing the time evolution $R_{+-}(t)$ observed by CP Lear (34) with their expression, Huet and Peskin (27) are able to extract values for the parameters β and γ. Interestingly, even assuming that there are Quantum Mechanics violations, one can attribute at most only 10% of ϵ to CPT violation. So, even in this more extreme scenario, the measurement of a nonvanishing value for ϵ is principally, or exclusively, a signal of CP violation. I quote below the results obtained by Huet and Peskin (27) when also amplitude CPT violation is included. They find [4]:

$$\beta + \frac{|d|}{2\sin\phi_{\rm SW}}\left[\frac{{\rm Re}\ b}{{\rm Re}\ a} - \frac{{\rm Re}\ B_o}{{\rm Re}\ A_o}\right] = (1.2 \pm 4.4) \times 10^{-19}\ {\rm GeV}$$

$$\gamma - 2|\eta_{+-}|d\left[\frac{{\rm Re}\ b}{{\rm Re}\ a} - \frac{{\rm Re}\ B_o}{{\rm Re}\ A_o}\right] = (-1.1 \pm 3.6) \times 10^{-21}\ {\rm GeV} \ .$$

The parameter β also gives a contribution to ϵ at 90° to $\phi_{\rm SW}$. So, if Quantum Mechanics is violated, the phase difference of the phase of $\epsilon \simeq \eta_{+-}$ from $\phi_{\rm SW}$ now not only measures $\epsilon_{\rm C\!\!\!/PT}$ but $\epsilon_{\rm C\!\!\!/PT} - \frac{\sqrt{2}\beta}{|d|}$. Using the PDG values for the difference between ϕ_{+-} and $\phi_{\rm SW}$ one finds the additional constraint:

$$\beta - \frac{|d|}{\sqrt{2}}\epsilon_{\rm C\!\!\!/PT} = (-0.9 \pm 1.1) \times 10^{-19}\ {\rm GeV} \ .$$

One must do more than just study $K_{\rm L}$ semileptonic decays and $K_{\rm S}$ and $K_{\rm L}$ decays into 2 pions to distinguish all of the parameters connected with possible CPT

---

3   The parameter α affects the precise exponential decrease in the above equation. However, this change can be neglected in the analysis(27).

4   These results if $\beta = \gamma = 0$ give looser bounds on the CPT violating amplitude combination $\frac{{\rm Re}\ b}{{\rm Re}\ a} - \frac{{\rm Re}\ B_o}{{\rm Re}\ A_o}$ than what was quoted above, since only the CP Lear data was used. From the rate determination one has a value of $(3.3 \pm 10.3) \times 10^{-5}$ for the CPT violating amplitude combination, while this becomes $(4.8 \pm 15.9) \times 10^{-5}$ from the interference determination.

and Quantum Mechanics violations. The Φ factory presently under construction at Frascati is ideal for this task, although already some important new information should emerge from CP Lear. Indeed, we learned at this meeting (35) that CP Lear has a preliminary measurement of the K_S semileptonic symmetry A_{K_S} which agrees with A_{K_L} within 10%. If Quantum Mechanics is OK, one expects

$$A_{K_S} - A_{K_L} = -4\mathrm{Re}\ \delta_K$$

and such a measurement isolates Re δ_K directly.

At a Φ factory one can perform CPT and Quantum Mechanics tests principally by using the accelerator as a $K^o - \bar{K}^o$ interferometer. Additionally, one can use K_L decays as a tag to study K_S decays and perform tests of the type described above. The initial state produced at a Φ factory, when the Φ decays, is a coherent superposition of K_S and K_L states:

$$|\Phi> = \frac{1}{\sqrt{2}}\{|K_S(\vec{p})\rangle|K_L(-\vec{p})\rangle - |K_S(-\vec{p})\rangle|K_L(\vec{p})\rangle\}\ .$$

As a result, when the K_S and K_L states eventually decay into final states f_1 and f_2, the relative time decay probability will show a characteristic interference pattern reflecting the initial coherent superposition. This interference pattern is sensitive to CP and CPT violation parameters (36). For instance, if the final states f_1 and f_2 are $\pi^+\pi^-$ and $\pi^o\pi^o$, for large time differences between the times t_1 and t_2 where the $\pi^+\pi^-$ and $\pi^o\pi^o$ are produced, the decay probability will fall as $e^{-\Gamma_L|t_1-t_2|}$. However, the coefficient of this exponential is different depending on whether $t_1 \gg t_2$ or $t_2 \gg t_1$, with this difference being related to Re ϵ'/ϵ (37).

In the case Quantum Mechanics is violated, these interference patterns will be altered. By studying in detail the time evolution of the system one should then be able to separate out pure effects of CPT violation from effects in which both CPT and Quantum Mechanics are violated. A nice example to study (27) is the pattern of the time evolution for identical final states ($f_1 = f_2$). Because of the antisymmetry in the initial K_L, K_S state, it is easy to see that quantum mechanically the decay probability vanishes if the decays into f_1 and f_2 occur at precisely the same time ($t_1 = t_2$). This is no longer the case when one admits possible Quantum Mechanics violations. For example, if f_1 and f_2 are both semileptonic states, one has (27):

$$
\begin{aligned}
I(\ell^\pm\pi^\mp\nu_\ell(t_1);\ \ell^\pm\pi^\mp\nu_\ell(t_2)) = &\left\{(1 \pm 4\ \mathrm{Re}\ \epsilon_K)\left[e^{-\Gamma_S t_1 - \Gamma_L t_2}\right.\right. \\
+ &\ e^{-\Gamma_S t_2 - \Gamma_L t_1} - 2\cos\Delta m(t_1 - t_2)\exp-(\frac{\Gamma_S + \Gamma_L}{2} + \alpha - \gamma)(t_1 + t_2)\Big] \\
\pm &\ \frac{4\beta}{|d|}\left[\left[\sin(\Delta m t_1 - \phi_{SW})\exp-[\frac{\Gamma_S + \Gamma_L}{2} + \alpha - \gamma]t_1\ e^{-\Gamma_S t_2} + (t_1 \leftrightarrow t_2)\right]\right. \\
+ &\ [\Gamma_S \leftrightarrow \Gamma_L;\ \phi_{SW} \leftrightarrow -\phi_{SW}]\Big] \\
+ &\ \frac{2\alpha}{\Delta m}\sin\Delta m(t_1 + t_2)\exp-(\frac{\Gamma_S + \Gamma_L}{2} + \alpha - \gamma)(t_1 + t_2) \\
+ &\ \frac{2\gamma}{\Gamma_S - \Gamma_L}\left[e^{-\Gamma_L(t_1+t_2)} - e^{-\Gamma_S(t_1+t_2)}\right]\Big\}\ .
\end{aligned}
$$

The first term in the curly bracket, if $\alpha - \gamma = 0$, is the usual quantum mechanical expression which vanishes when $t_1 = t_2$. The others three terms are proportional to the additional parameters α, β and γ connected with Quantum Mechanics violation. Because the time dependence of all these four terms is different, in principle a careful study of this quantity would allow a separate determination of α, β, γ and Re ϵ_K.

4. CP Violation

To date the neutral Kaon system is the only place where a violation of CP has been observed.[5] In the modern gauge theory paradigm this phenomena can have one of two possible origins. Either

i) the full Lagrangian of the theory is CP invariant, but this symmetry is not preserved by the vacuum state: CP $|0\rangle \neq |0\rangle$. In this case CP is a spontaneously broken symmetry (39).

or

ii) there are terms in the Lagrangian of the theory which are not invariant under CP transformations. CP is explicitly broken by these terms and is no longer a symmetry of the theory.

The first possibility, unfortunately, runs into a potential cosmological problem(40). As the universe cools below a temperature T^* where spontaneous CP violation occurs, one expects that domains of different CP should form. These domains are separated by walls having a typical surface energy density $\sigma \sim T^{*3}$. The energy density associated with these walls dissipates slowly as the universe cools further and eventually contributes an energy density to the universe at temperature T of order $\rho_{\text{Wall}} \sim T^{*3}T$. Such an energy density today would typically exceed the universe closure density by many orders of magnitude:

$$\rho_{\text{Wall}} \sim 10^{-7} \left(\frac{T^*}{\text{TeV}}\right)^3 \text{GeV}^{-4} \gg \rho_{\text{closure}} \sim 10^{-46} \text{ GeV}^{-4} \ .$$

One can avoid this difficulty by imaging that the scale where CP is spontaneously violated is very high, so that T^* is above the temperature where inflation occurs. In this case the problem disappears, since the domains get inflated anyway. Nevertheless, there are still problems, since it proves difficult to connect this high energy spontaneous breaking of CP with the observed phenomenon at low energies. What emerges, in general, are models which are quite complex(41), with CP violation being associated with new interactions much as in the original superweak model of Wolfenstein(42).

If, on the other hand, CP is explicitly broken the phenomenology of neutral Kaon CP violation is a quite natural result of the standard model of the electroweak interactions. There is, however, a requirement emerging from the demand of renormalizability which bears mentioning. Namely, if CP is explicitly broken then renormalizability requires that all the parameters in the Lagrangian which can be complex

---

5   This is not quite correct, since to obtain a non-trivial asymmetry between matter and antimatter in the universe, it is necessary that there should be processes that violate CP(38).

must be so. A corollary of this is that the number of possible CP violating phases in the theory increases with the complexity of the theory, as there are then more terms which can have imaginary coefficients.

In this respect, the three generation ($N_g = 3$) standard model with only one Higgs doublet is the simplest possible model, since it has only one phase. With just one Higgs doublet, the Hermiticity of the scalar potential allows no complex parameters to appear. If CP is not a symmetry, complex Yukawa couplings are, however, allowed. After the breakdown of the $SU(2) \times U(1)$ symmetry, these couplings produce complex mass matrices. Going to a physical basis with real diagonal masses introduces a complex mixing matrix in the charged currents of the theory. For the quark sector, this is the famous Cabibbo-Kobayashi Maskawa (CKM) matrix(43).[6] This $N_g \times N_g$ unitary matrix contains $N_g(N_g - 1)/2$ real angles and $N_g(N_g + 1)/2$ phases. However, $2N_g - 1$ of these phases can be rotated away by redefinitions of the quark fields leaving only $(N_g - 1)(N_g - 2)/2$ phases. Thus for $N_g = 3$ the standard model has only one physical complex phase to describe all CP violating phenomena.[7]

If CP is broken explicitly, it follows by the renormalizability corollary that any extensions of the SM will involve further CP violating phases. For instance, if one has two Higgs doublets, Φ_1 and Φ_2, then the Hermiticity of the scalar potential no longer forbids the appearance of complex terms like

$$V = \ldots \mu_{12} \Phi_1^\dagger \Phi_2 + \mu_{12}^* \Phi_2^\dagger \Phi_1 \ .$$

Indeed, if one did not include such terms, the presence of complex Yukawa couplings would induce such terms at one loop.[8]

Testing the CKM paradigm

One does not really know if the complex phase present in the CKM matrix is responsible for the CP violating phenomena observed in the neutral Kaon system. Indeed, one does not know whether there are further phases besides the CKM phase. Nevertheless, it is remarkable that, as a result of the hierarchial structure of the CKM matrix and of other dynamical circumstances, one can **qualitatively** explain all we know experimentally about CP violation today on the basis of the CKM picture. In what follows, I make use of the CKM matrix in the parametrization adopted by the PDG(31) and expand the three real angles in the manner suggested by Wolfenstein(45)

---

6   If the neutrinos are massless, there is no corresponding matrix in the lepton sector since it can be reabsorbed by redefining the neutrino fields.

7   This is not quite true. In the SM there is also another phase related to the QCD vacuum angle which leads to a CP violating interaction involving the gluonic field strength and its dual:

$$\mathcal{L}_{\text{CP viol.}} = \bar{\theta} \frac{\alpha_{\text{S}}}{8\pi} F_a^{\mu\nu} \tilde{F}_{a\mu\nu} \ .$$

The phase angle $\bar{\theta}$ contributes to the neutron electric dipole moment and, to respect the existing bound on d_n (31) must be extremely small: $\bar{\theta} \leq 10^{-9} - 10^{10}$. Why this should be so is unknown and constitutes the strong CP problem (44).

8   More precisely, one needs complex counterterms to absorb the complex quadratic divergences induced through the Yukawa couplings.

in powers of the sine of the Cabibbo angle λ. To order λ^3 one has then

$$V = \begin{vmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{vmatrix}$$

with A, ρ and η being parameters one needs to fix from experiments—with $\eta \neq 0$ signalling CP violation.[9]

Three pieces of experimental data provide today independent dynamical information on CP violation. These are:

i) The value of the K_L to K_S amplitude ratios, $n_{+-} = \epsilon + \epsilon'$; $\eta_{oo} = \epsilon - 2\epsilon'$, with

$$|\eta_{+-}| \simeq |\eta_{oo}| \sim 2 \times 10^{-3} .$$

ii) The small value of the $\Delta S = 1$ CP violating parameter ϵ', with the ratio

$$\epsilon'/\epsilon \lesssim 10^{-3} .$$

iii) The very strong bounds on the electric dipole moments of the neutron and the electron

$$d_e, d_n \lesssim 10^{-25} \text{ ecm} .$$

Other information at hand is either too insensitive, like the corresponding CP violating parameters for $K \rightarrow 3\pi$ decays η_{+-o} and η_{ooo}, or is dynamically fixed, like $A_{K_L} = 2 \text{ Re } \eta_{+-}$ or $\phi_{+-} = \phi_{SW}$ which follows as a result of CPT invariance.

One can "understand" the above three facts quite simply in the CKM paradigm. In the model η_{+-}, or the parameter ϵ, is determined by the ratio of the imaginary to the real part of the box graph of Fig. 2a. It is easy to check that this ratio is of order

$$\epsilon \sim \lambda^4 \sin \delta \sim 10^{-3} \sin \delta .$$

That is, ϵ is naturally small because of the suppression of interfamily mixing without requiring the CKM phase δ to be small. Similarly, one can qualitatively understand why ϵ'/ϵ is small. This ratio is suppressed by the $\Delta I = 1/2$ rule and it is induced only by the Penguin diagrams of Fig. 2b involving the emission of virtual gluons (or photons), which are Zweig suppressed(46). Typically this gives

$$\frac{\epsilon'}{\epsilon} \sim \frac{1}{20} \cdot \left[\frac{\alpha_S}{12\pi} \ln \frac{m_t^2}{m_c^2} \right] \sim 10^{-3} .$$

Finally, in the CKM model the electric dipole moments are small since the first non-vanishing contributions(47) occur at three loops, as shown in Fig. 2c, leading to the estimate(48)

$$d_{q,e} \sim em_{q,e} \left[\frac{\alpha^2 \alpha_S}{\pi^3} \right] \left[\frac{m_t^2 m_b^2}{M_W^6} \right] \lambda^6 \sin \delta \sim 10^{-32} \text{ ecm} .$$

---

9   It is often convenient instead of using $\rho - i\eta$ to write this in terms of a magnitude and phase: $\rho - i\eta = \sigma e^{-i\delta}$, with δ being the CP violating CKM phase.
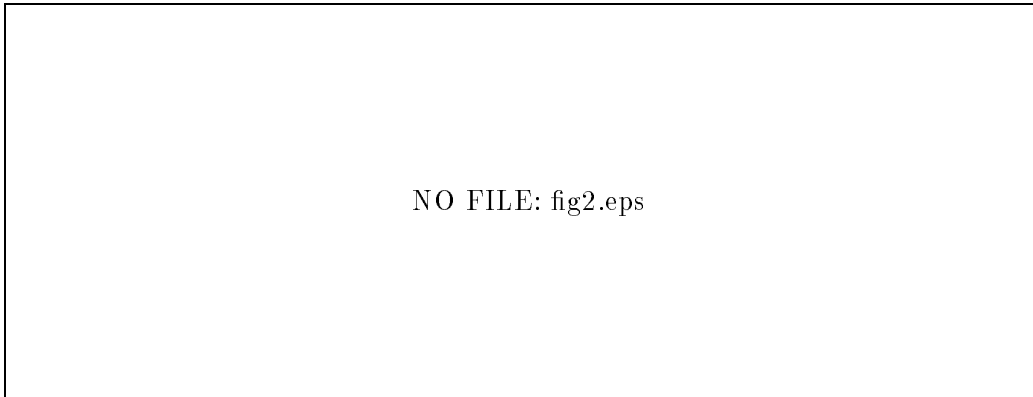
NO FILE: fig2.eps

Fig. 2. Graphs contributing to ϵ, ϵ' and $d_{q,e}$

One can, of course, use the precise value of ϵ measured experimentally to determine an allowed region for the parameters entering in the CKM matrix. Because of theoretical uncertainties in evaluating the hadronic matrix element of the $\Delta S = 2$ operator associated with the box graph of Fig. 2a this parameter space region is rather large. Further restrictions on the allowed values of CKM parameters come from semileptonic B decays and from $B_d - B_{\bar{d}}$ mixing. Because the parameter A, related to V_{cb}, is better known, it has become traditional to present the result of these analysis as a plot in the $\rho - \eta$ plane. Fig. 3 shows the results of a recent analysis, done in collaboration with my student, K. Wang(49). The input parameters used, as well as the range allowed for certain hadronic amplitudes and other CKM matrix elements is detailed in Table. 1.

Table 1. Parameters used in the $\rho - \eta$ analysis of (49)

$$
\begin{array}{rcll}
|\epsilon| &=& (2.26 \pm 0.02) \times 10^{-3} & (31) \\
\Delta m_d &=& (0.496 \pm 0.032) ps^{-1} & (50) \\
m_t &=& (174 \pm 10^{+13}_{-12}) \text{ GeV} & (51) \\
|V_{cb}| &=& 0.0378 \pm 0.0026 & (52) \\
|V_{ub}|/|V_{cb}| &=& 0.08 \pm 0.02 & (52) \\
B_{\mathrm{K}} &=& 0.825 \pm 0.035 & (53) \\
\sqrt{B_d}\, f_{B_d} &=& (180 \pm 30) \text{ MeV} & (54)
\end{array}
$$

The resulting 1σ allowed contour emerging from the overlap of the three constraints coming from ϵ, $B_{\mathrm{d}} - B_{\bar{\mathrm{d}}}$ mixing and the ratio of $|V_{ub}|/|V_{cb}|$, shown in Fig. 4, gives a roughly symmetric region around $\rho = 0$ within the ranges

$$0.2 \leq \eta \leq 0.5 \; ; \;\; -0.4 \leq \rho \leq 0.4 \; .$$

As anticipated by our qualitative discussion this region implies that the CKM phase δ is large ($\rho = 0$ corresponds to $\delta = \pi/2$). One should note, however, that this analysis does not establish the CKM paradigm. Using only the B physics constraints one sees

NO FILE: fig3.eps
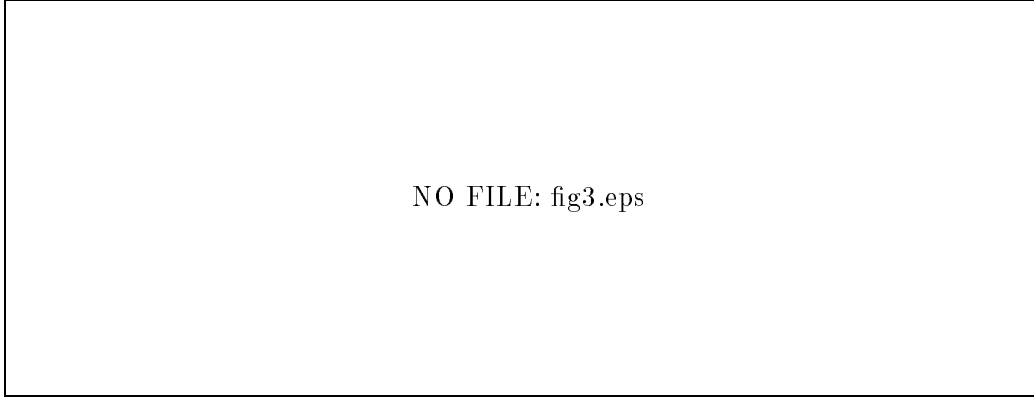
Fig. 3. Constraints on the (ρ, η) plot

NO FILE: fig4.eps
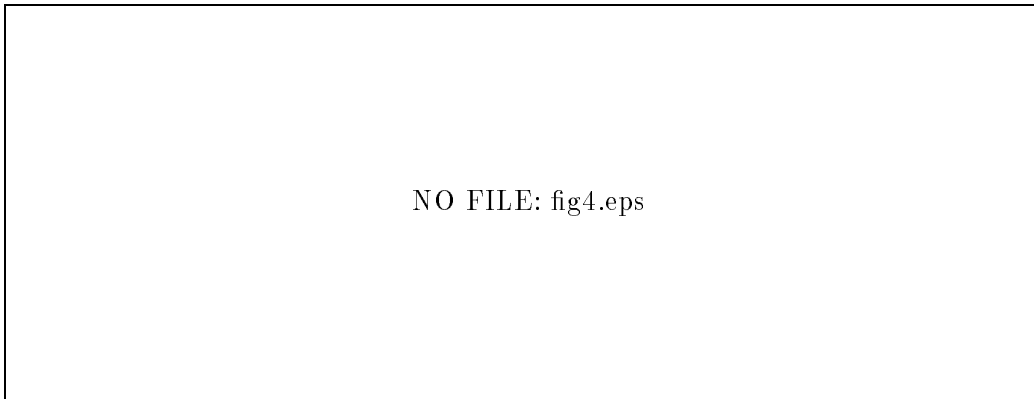
Fig. 4. Allowed region in the $\rho - \eta$ plane

that in Fig. 3 there is also an overlap region for $\eta = 0$, which gives $\rho = -0.33 \pm 0.08$ (49). So one can still imagine that ϵ is due to some other CP violating interaction, as in the superweak model (42), with the CKM phase η being very small. Obviously, it is important to exclude such a possibility, but this is not going to be easy. Wang and I (49) discussed how this could perhaps happen as a result of improving the bounds on $B_{\mathrm{s}} - B_{\bar{\mathrm{s}}}$ mixing. Here I would like to concentrate on what can the Kaon system tell us on this issue.

In principle, one can obtain quantitative tests of the CKM model with Kaon experiments. However, the needed experiments are very challenging, either due to the high precision required or due to the rarity of the processes to be studied. Furthermore, the analysis of these results is also theoretically very challenging, since it will require better estimates of hadronic matrix elements than what we have at present.

A good example of both of these challenges is provided by ϵ'/ϵ. The present data on this ratio is inconclusive, with the result obtained at Fermilab (55).

$$\mathrm{Re}\ \frac{\epsilon'}{\epsilon} = (7.4 \pm 5.2 \pm 2.9) \times 10^{-4} \quad [\mathrm{E731}]$$

being consistent with zero within the error, while the result of the NA31 experiment

at CERN (56) giving a non-zero value to 3σ:

$$\mathrm{Re}\ \frac{\epsilon'}{\epsilon} = (23.0 \pm 3.6 \pm 5.4) \times 10^{-4} \quad [\mathrm{NA31}]$$

Theoretically, the predictions for ϵ'/ϵ are dependent both on the value of the CKM matrix elements and, more importantly, on an estimate of certain hadronic matrix elements. Buras and Lautenbacher (57) give for this ratio the approximate formula

$$\mathrm{Re}\ \frac{\epsilon'}{\epsilon} \simeq 3.6 \times 10^{-3} A^2 \eta \left[B_6 - 0.175 \left(\frac{m_t^2}{M_{\mathrm{W}}^2} \right)^{0.93} B_8 \right] \ .$$

Here B_6 and B_8 are quantities related to the matrix elements of the dominant gluonic and electroweak Penguin operators, respectively. The electroweak Penguin contribution is suppressed relative to the gluonic Penguin contribution by a factor of $\alpha/\alpha_{\mathrm{S}}$. However, it does not suffer from the $\Delta I = 3/2$ suppression and so one gains back a factor of 20. Furthermore, as Flynn and Randall (58) first noted, the contribution of these terms can become significant for large top mass because it grows approximately as m_t^2. The result of the CKM analysis presented earlier suggested that

$$0.12 \leq A^2 \eta \leq 0.31 \ ,$$

while for $m_t = 175$ GeV the square bracket above reduces to $[B_6 - 0.75 B_8]$. Hence one can write the expectation from theory for ϵ'/ϵ as

$$4.3 \times 10^{-4} [B_6 - 0.75 B_8] \leq \mathrm{Re}\ \frac{\epsilon'}{\epsilon} \leq 11.2 \times 10^{-4} [B_6 - 0.75 B_8] \ .$$

Because the top mass is so large, the predicted value for ϵ'/ϵ depends rather crucially on **both** B_6 and B_8. These (normalized) matrix elements have been estimated by both lattice (59) and $1/N$ (60) calculations to be equal to each other, with an individual error of $\pm 20\%$:

$$B_6 = B_8 = 1 \pm 0.20 \ .$$

Thus, unfortunately, the combination entering in ϵ'/ϵ is poorly known. It appears that the best one can say theoretically is that $\mathrm{Re}\ \epsilon'/\epsilon$ should be a "few" times 10^{-4}, with a "few" being difficult to pin down more precisely. Theory, at any rate, seems to favor the E731 experimental result over that of NA31.

Fortunately, we may learn something more in this area in the next five years or so. There are 3rd generation experiments in preparation both at Fermilab (KTeV) and CERN (NA48). These experiments should begin taking data in a year or so and are designed to reach statistical and systematic accuracy for ϵ'/ϵ at the level of 10^{-4}. The Frascati Φ factory which should begin operations in 1997, in principle, can also provide interesting information for ϵ'/ϵ. At the Φ factory one will need an integrated luminosity of $\int \mathcal{L}\, dt = 10\ fb^{-1}$ to arrive at a statistical sensitivity for ϵ'/ϵ at the level of 10^{-4}. However, if this statistical sensitivity is reached, the systematic uncertainties will be quite different than those at KTeV and NA48, providing a very useful cross check. It is important to remark that, irrespective of detailed theoretical prediction, the observation of a non-zero value for ϵ'/ϵ at a significant level is very important, for it would provide direct evidence for $\Delta S = 1$ CP violation and would rule out a superweak explanation for the observed CP violation in the neutral K sector.

Rare Kaon Decays

There are alternatives to the ϵ'/ϵ measurement which could reveal $\Delta S = 1$ (direct) CP violation. However, these alternatives involve daunting experiments(61), which are probably out of reach in the near term. Whether these experiments can (or will?) eventually be carried out is an open question which I will return to later.

K_S decays

CP Lear already and the Frascati Φ factory soon will enable a more thorough study of K_S decays by more efficient tagging. The decay $K_\mathrm{S} - 3\pi^o$ is CP-violating, while the $K_\mathrm{S} \to \pi^+\pi^-\pi^o$ mode has both CP-conserving and CP-violating pieces. However, even in this case the CP conserving piece is small and vanishing in the center of the Dalitz plot. Hence one can extract information about CP violation from $K_\mathrm{S} \to 3\pi$ decays. The analogue $K_\mathrm{S}/K_\mathrm{L}$ amplitude ratios to η_{+-} and η_{oo} for $K \to 3\pi$ have both $\Delta S = 1$ and $\Delta S = 2$ pieces:

$$\eta_{ooo} = \epsilon + \epsilon'_{ooo} \; ; \; \eta_{+-o} = \epsilon + \epsilon'_{+-o} \; .$$

However, in contrast to what obtains in the $K \to 2\pi$ case, here the $\Delta S = 1$ pieces can be larger. Cheng (62) gives estimates for ϵ'_{+-o}/ϵ and ϵ'_{ooo}/ϵ of $O(10^{-2})$, while others are more pessimistic (63). Even so, there does not appear to be any realistic prospects in the near future to probe for $\Delta S = 1$ CP-violating amplitudes in $K_\mathrm{S} \to 3\pi$. For instance, at a Φ-factory even with an integrated luminosity of 10 fb^{-1} one can only reach an accuracy for η_{+-o} and η_{ooo} of order 3×10^{-3}, which is at the level of ϵ not ϵ'.

Asymmetries in charged K decays

CP violating effects involving charged Kaons can only be due to $\Delta S = 1$ transitions, since $K^+ \leftrightarrow K^-$ $\Delta S = 2$ mixing is forbidden by charge conservation. A typical CP-violating effect in charged Kaon decays necessitates a comparison between K^+ and K^- processes. However, a CP-violating asymmetry between these processes can occur only if there are at least two decay amplitudes involved and these amplitudes have both a relative weak CP-violating phase and a relative strong rescattering phase between each other. Thus the resulting asymmetry necessarily depends on strong dynamics. To appreciate this fact, imagine writing the decay amplitude for K^+ decay to a final state f^+ as

$$A(K^+ \to f^+) = A_1 \, e^{i\delta_\mathrm{W_1}} e^{i\delta_\mathrm{S_1}} + A_2 \, e^{i\delta_\mathrm{W_2}} e^{i\delta_\mathrm{S_2}} \; .$$

Then the corresponding amplitude for the decay $K^- \to f^-$ is

$$A(K^- \to f^-) = A_1 \, e^{-i\delta_\mathrm{W_1}} e^{i\delta_\mathrm{S_1}} + A_2 \, e^{-i\delta_\mathrm{W_2}} e^{i\delta_\mathrm{S_2}} \; .$$

That is, the strong rescattering phases are the same but one complex conjugates the weak amplitudes. From the above one sees that the rate asymmetry between these processes is

$$\begin{aligned} \mathcal{A}(f^+; f^-) &= \frac{\Gamma(K^+ \to f^+) - \Gamma(K^+ \to f^-)}{\Gamma(K^+ \to f^+) + \Gamma(K^- \to f^-)} \\ &= \frac{2A_1 A_2 \sin(\delta_\mathrm{W_2} - \delta_\mathrm{W_1}) \sin(\delta_\mathrm{S_2} - \delta_\mathrm{S_1})}{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\delta_\mathrm{W_2} - \delta_\mathrm{W_1}) \cos(\delta_\mathrm{S_2} - \delta_\mathrm{S_1})} \; . \end{aligned}$$

Table 2. Predictions for Asymmetries in $K^{\pm}$ Decays

Asymmetry	Prediction		Φ Factory Reach
$\mathcal{A}(\pi^+\pi^+\pi^-;\pi^-\pi^-\pi^+)$	5×10^{-8}	(66)	3×10^{-5}
$\mathcal{A}(\pi^+\pi^o\pi^o;\pi^-\pi^o\pi^o)$	2×10^{-7}	(66)	5×10^{-5}
$\mathcal{A}_{\mathrm{Dalitz}}(\pi^+\pi^+\pi^-;\pi^-\pi^+\pi^+)$	2×10^{-6}	(66)	3×10^{-4}
$\mathcal{A}_{\mathrm{Dalitz}}(\pi^+\pi^o\pi^o;\pi^-\pi^o\pi^o)$	1×10^{-6}	(66)	2×10^{-4}
$\mathcal{A}(\pi^+\pi^o\gamma;\pi^-\pi^o\gamma)$	10^{-5}	(67)	2×10^{-3}

Unfortunately, detailed calculations in the standard CKM paradigm for rate asymmetries and asymmetries in Dalitz plot parameters for various charged Kaon decays give quite tiny predictions. This can be qualitatively understood as follows. The ratio $A_2 \sin(\delta_{W_2} - \delta_{W_1})/A_1$ is typically that of a Penguin amplitude to a weak decay amplitude and so is of order ϵ'/ϵ. Furthermore, because of the small phase space for $K \rightarrow 3\pi$ decays or because one is dealing with electromagnetic rescattering in $K \rightarrow \pi\pi\gamma$, the rescattering contribution suppress these asymmetries even more. Table 2 gives typical predictions, contrasting them to the expected reach of the Frascati Φ factory with $\int \mathcal{L} \, dt = 10$ fb^{-1}. For the $K \rightarrow 3\pi$ decays, Belkov *et al.* (64) give numbers at least a factor of 10 above those given in Table 2. However, these numbers are predicated on having very large rescattering phases which do not appear to be realistic(65). One is lead to conclude that, if the CKM paradigm is correct, it is unlikely that one will see a CP-violating signal in charged Kaon decays.

$K_{\mathrm{L}} \rightarrow \pi^o \ell^+ \ell^-$; $K_{\mathrm{L}} \rightarrow \pi^o \nu\bar{\nu}$

Perhaps more promising are decays of the K_{L} to π^o plus lepton pairs. If the lepton pair is charged, then the process has a CP conserving piece in which the decay proceeds via a 2γ intermediate state. Although there was some initial controversy (68), the rate for the process $K_{\mathrm{L}} \rightarrow \pi^o \ell^+ \ell^-$ arising from the CP-conserving 2γ transitiion is probably at, or below, the 10^{-12} level (69):

$$B(K_{\mathrm{L}} \rightarrow \pi^o \ell^+ \ell^-)_{\mathrm{CP \; cons.}} = (0.3 - 1.2) \times 10^{-12}$$

and is just a small correction to the dominant CP violating contribution going through an effective spin 1 virtual state, $K_{\mathrm{L}} \rightarrow \pi^o J^*$. Since $\pi^o J^*$ is CP even, this part of the amplitude is CP violating and has two distinct pieces (70): an indirect contribution from the CP even piece (ϵK_1) in the K_{L} state and a direct $\Delta S = 1$ CP-violating piece coming from the K_2 part of K_L:

$$A(K_{\mathrm{L}} \rightarrow \pi^o J^*) = \epsilon A(K_1 \rightarrow \pi^o J^*) + A(K_2 \rightarrow \pi^o J^*) \; .$$

To isolate the interesting direct CP contribution in this process requires understanding first the size of the indirect contribution. The amplitude $A(K_1 \rightarrow \pi^o J^*)$ could be determined absolutely if one had a measurement of the process $K_{\mathrm{S}} \rightarrow \pi^o \ell^+ \ell^-$. Since this is not at hand, at the moment one has to rely on various guesstimates. These

give the following range for the indirect CP-violating branching ratio(71)

$$B(K_{\mathrm{L}} \to \pi^o \ell^+ \ell^-)^{\mathrm{indirect}}_{\mathrm{CP\ violating}} = (1.6 - 6) \times 10^{-12} \ ,$$

where the smaller number is the estimate coming from chiral perturbation theory, which the other comes from relating $A(K_1 \to \pi^o J^*)$ to the analogous amplitude for charged K decays.

The calculation of the direct CP-violating contribution to the process $K_{\mathrm{L}} \to \pi^o \ell^+ \ell^-$, as a result of electroweak Penguin and box contributions and their gluonic corrections, is perhaps the one that is most reliably known. The branching ratio obtained by Buras, Lautenbacher, Misiak and Münz in their next to leading order calculation of the process(72) is

$$B(K_{\mathrm{L}} \to \pi^o \ell^+ \ell^-)^{\mathrm{direct}}_{\mathrm{CP\ violating}} = (5 \pm 2) \times 10^{-12}$$

where the error arises mostly from the imperfect knowledge of the CKM matrix.

Experimentally one has the following 90% C.L. for the two $K_{\mathrm{L}} \to \pi^o \ell^+ \ell^-$ processes:

$$
\begin{aligned}
B(K_{\mathrm{L}} \to \pi^o \mu^+ \mu^-) &< 5.1 \times 10^{-9} \\
B(K_{\mathrm{L}} \to \pi^o e^+ e^-) &< 1.8 \times 10^{-9}
\end{aligned}
$$

The first limit comes from the E799 experiment at Fermilab(73), while the second limit combines the bounds obtained by the E845 experiment at Brookhaven(74) and the E799 Fermilab experiment(75). Forthcoming experiments at KEK and Fermilab should be able to improve these limits by at least an order of magnitude[10], if they can control the dangerous background arising from the decay $K_{\mathrm{L}} \to \gamma\gamma e^+ e^-$(76). Even more distant future experiment may actually reach the level expected theoretically for the $K_{\mathrm{L}} \to \pi^o e^+ e^-$ rate (77). However, it will be difficult to unravel the direct CP-violating contribution from the indirect CP-violating contribution, unless the $K_{\mathrm{S}} \to \pi^o e^+ e^-$ rate is also measured simultaneously.

In this respect, the process $K_{\mathrm{L}} \to \pi^o \nu\bar{\nu}$ is very much cleaner. This process is purely CP-violating, since it has no 2γ contribution. Furthermore, it has a tiny indirect CP contribution, since this is of order ϵ times the already small $K^+ \to \pi^+ \nu\bar{\nu}$ amplitude(78). Next to leading QCD calculations for the direct rate have been carried out by Buchalla and Buras(79) who give the following approximate formula for the branching ratio for this process

$$B(K_{\mathrm{L}} \to \pi^o \nu\bar{\nu}) = 8.2 \times 10^{-11} A^4 \eta^2 \left(\frac{m_t}{M_{\mathrm{W}}} \right)^{2.3} \ .$$

This value is very far below the present 90% C.L. obtained by the E799 experiment at Fermilab(80)

$$B(K_{\mathrm{L}} \to \pi^o \nu\bar{\nu}) < 5.8 \times 10^{-5} \ .$$

KTeV should be able to lower this bound substantially, perhaps to the level of 10^{-8} but this still leaves a long way to go!

---

10   KEK 162 goal is to get to a BR of $O(10^{-10})$ while KTeV hopes to push this BR down to 5×10^{-11}.

$K^+ \to \pi^+ \nu \bar\nu$

The last process I would like to consider is the charged Kaon analogue to the process just discussed. Although the decay $K^+ \to \pi^+ \nu \bar\nu$ is not CP violating, it is sensitive to $|V_{\rm td}|^2 \simeq A^2 \lambda^6 [(1 - \rho)^2 + \eta^2]$ and so, indirectly, it is sensitive to the CP violating CKM parameter η. For the CP violating decay $K_{\rm L} \to \pi^o \nu \bar\nu$ the electroweak Penguin and box contributions are dominated by loops containing top quarks. Here, because one is not looking at the imaginary part one cannot neglect altogether the contribution from charm quarks. If one could do so, the branching ratio formula for $K^+ \to \pi^+ \nu \bar\nu$ would be given by an analogous formula to that for $K_{\rm L} \to \pi^o \nu \bar\nu$ but with $\eta^2 \to \eta^2 + (1 - \rho)^2$.

Because m_t is large, the $K^+ \to \pi^+ \nu \bar\nu$ branching ratio is not extremely sensitive to the contribution of the charm-quark loops (81). Furthermore, when next to leading QCD corrections are computed the sensitivity of the result to the precise value of the charm-quark mass is reduced considerably(82). Buras $et~al.$(83) give the following approximate formula for the $K^+ \to \pi^+ \nu \bar\nu$ branching ratio

$$B(K^+ \to \pi^+ \nu \bar\nu) = 2 \times 10^{-11} A^4 \left[\eta^2 + \frac{2}{3}(\rho^e - \rho)^2 + \frac{1}{3}(\rho^\tau - \rho)^2 \right] \left(\frac{m_t}{M_{\rm W}} \right)^{2.3} .$$

In the above the parameters ρ^e and ρ^τ differ from unity because of the presence of the charm-quark contributions. Taking $m_t = 175$ GeV and $m_c(m_c) = 1.30 \pm 0.05$ GeV (84), Buras $et~al.$(83) find that ρ^e and ρ^τ lie in the ranges

$$1.42 \le \rho^e \le 1.55 ; \ \ 1.27 \le \rho^\tau \le 1.38 .$$

Using the range of η and ρ determined by the CKM analysis discussed here gives about a 40% uncertainty for the $K^+ \to \pi^+ \nu \bar\nu$ branching ratio, leading to the expectation

$$B(K^+ \to \pi^+ \nu \bar\nu) = (1 \pm 0.4) \times 10^{-10} .$$

This number is to be compared to the best present limit coming from the E787 experiment at Brookhaven. Careful cuts must be made in the accepted π^+ range and π^+ momentum to avoid potentially dangerous backgrounds, like $K^+ \to \pi^+ \pi^o$ and $K^+ \to \mu^+ \pi^o \nu$. Littenberg(85) at this meeting has given a new preliminary result for this branching ratio

$$B(K^+ \to \pi^+ \nu \bar\nu) < 3 \times 10^{-9} \ \ \ (90\% \ {\rm C.L.})$$

which updates the previously published result from the E787 collaboration(86). This value is still about a factor of 30 from the interesting CKM model range, but there are hopes that one can get close to this sensitivity in the present run of this experiment.

Looking for new CP-violating phases

Positive signals for $\epsilon'/\epsilon \neq 0$ will indicate the general validity of the CKM picture. However, given the large theoretical uncertainty, it is clear that values of ϵ'/ϵ consistent with zero at the 10^{-4} level cannot disprove this picture. In my view, it is more likely that B-physics experiments (particularly the detection of the expected large asymmetry in $B_d \to \psi K_{\rm S}$ decays(87)) will provide the crucial smoking gun for

the CKM paradigm, with rare Kaon decays filling in the detailed picture. However, whether the CKM picture is (essentially) correct or not, experiments in the Kaon sector may provide the first glimpse at **other** CP-violating phases.

There are good theoretical arguments for having further CP-violating phases, besides the CKM phase δ. For instance, to establish a matter-antimatter asymmetry in the Universe one needs to have processes which involve CP violation(38). If the origin of this asymmetry comes from processes at the GUT scale, then, in general, the GUT interactions contain further CP-violating phases besides the CKM phase δ (88). If this asymmetry is established at the electroweak scale(89), then most likely one again needs further phases, both because intrafamily suppression gives not enough CP violation in the CKM case to generate the asymmetry and because one needs to have more than one Higgs doublet(90). Indeed this last point gives the fundamental reason why one should expect to have further CP-violating phases, besides the CKM phase δ. It is likely that the standard model is part of a larger theory. For instance, supersymmetric extensions of the SM have been much in vogue lately. Any such extensions will introduce further particles and couplings and thus, by the simple corollary mentioned at the beginning of this section, they will introduce new CP-violating phases.

The best place to look for non-CKM phases is in processes where CP violation within the CKM paradigm is either vanishing or very suppressed. One such example is provided by experiments aimed at measuring the electric dipole moments of the neutron or the electron, since electric dipole moments are predicted to be extremely small in the CKM model. Another example concerns measurements of the transverse muon polarization $\langle P_\perp^\mu \rangle$ in $K_{\mu 3}$ decays, which vanishes in the CKM paradigm(91). The transverse muon polarization measures a T-violating triple correlation(92)

$$\langle P_\perp^\mu \rangle \sim \langle \vec{s}_\mu \cdot (\vec{p}_\mu \times \vec{p}_\pi) \rangle \;.$$

In as much as one can produce such an effect also as a result of final state interactions (FSI) this is not a totally clear test for new CP-violating phases. With two charged particles in the final state, like for the decay $K_L \to \pi^- \mu^+ \nu_\mu$, one expects FSI to give typically $\langle P_\perp^\mu \rangle_{\rm FSI} \sim \alpha/\pi \sim 10^{-3}$ (93). However, for the process $K^+ \to \pi^o \mu^+ \nu_\mu$ with only one charged particle in the final state, the FSI effects should be much smaller. Indeed, Zhitnitski(94) estimates for this proces that $\langle P_\perp^\mu \rangle_{\rm FSI} \sim 10^{-6}$. So a $\langle P_\perp^\mu \rangle$ measurement in the $K^+ \to \pi^o \mu^+ \nu_\mu$ decay is a good place to test for additional CP-violating phases.

The transverse muon polarization $\langle P_\perp^\mu \rangle$ is particularly sensitive to scalar interactions and thus is a good probe of CP-violating phases arising from the Higgs sector(95). One can write the effective $K_{\mu 3}$ amplitude(96) as

$$A = G_{\rm F} \sin \theta_c f_+(q^2) \left\{ p_K^\mu \bar{\mu} \gamma_\mu (1 - \gamma_5) \nu_\mu + f_S(q^2) m_\mu \bar{\mu} (1 - \gamma_5)^\nu_\mu \right\} \;.$$

Then

$$\langle P_\perp^\mu \rangle = \frac{m_\mu}{M_{\rm K}} {\rm Im}\, f_{\rm S} \left[\frac{|\vec{p}_\mu|}{E_\mu + |\vec{p}_\mu| n_\mu \cdot n_\nu - m_\mu^2/M_{\rm K}} \right] \simeq 0.2\, {\rm Im}\, f_{\rm S} \;.$$

Here $n_\nu (n_\nu)$ are unit vectors along the muon (neutrino) directions and the numerical value represents the expectation after doing an average over phase space(97).

Contributions to Im f_S can arise in multi-Higgs models (like the Weinberg 3-Higgs model(98)) from the charged Higgs exchange shown in Fig. 5, leading to (99)

$$\mathrm{Im}\ f_S \simeq \mathrm{Im}(\alpha^*\gamma)\frac{M_K^2}{M_{H^-}^2}\ .$$

Here $\alpha(\gamma)$ are constants associated with the charged Higgs coupling to quarks (leptons). Because a leptonic vertex is involved, one in general does not have a strong constraint on $\mathrm{Im}(\alpha^*\gamma)$. By examining possible non-standard contributions to the B semileptonic decay $B \to X\tau\nu_\tau$, Grossman(100) obtains

$$\mathrm{Im}(\alpha^*\gamma) < \frac{0.23\ M_{H^-}^2}{[\mathrm{GeV}]^2}$$

which yields a bound for $\langle P_\perp^\mu \rangle$ of $\langle P_\perp^\mu \rangle < 10^{-2}$. Amusingly, this is the same bound one infers from the most accurate measurement of $\langle P_\perp^\mu \rangle$ done at Brookhaven about a decade ago (101), which yielded

$$\langle P_\perp^\mu \rangle = (-3.1 \pm 5.3) \times 10^{-3}\ .$$
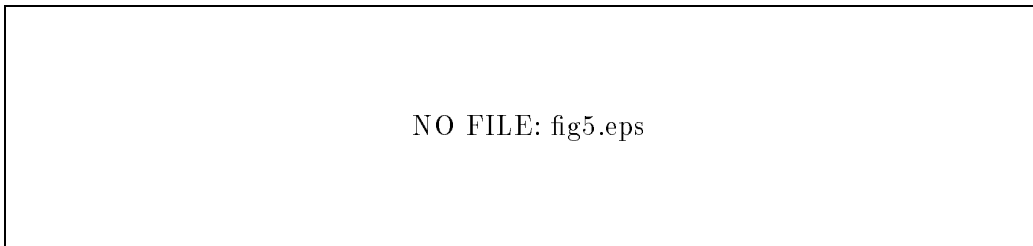
NO FILE: fig5.eps

Fig. 5. Graphs contributing to $\langle P_\perp^\mu \rangle$

In specific models, however, the leptonic phases associated with charged Higgs couplings are correlated with the hadronic phases. In this case, one can obtain more specific restrictions on $\langle P_\perp^\mu \rangle$ due to the strong bounds on the neutron electric dipole moment. For instance, for the Weinberg 3 Higgs model, one relates $\mathrm{Im}(\alpha^*\gamma)$ to a similar product of couplings of the charged Higgs to quarks(99):

$$\mathrm{Im}(\alpha^*\gamma) = \left(\frac{v_u}{v_e}\right)^2\ \mathrm{Im}(\alpha^*\beta)\ ,$$

where v_u (v_e) are the VEV of the Higgs doublets which couples to up-like quarks (leptons). The strong bound on the neutron electric dipole moment(31) then gives the constraint

$$\mathrm{Im}(\alpha^*\beta) \leq \frac{4 \times 10^{-3}\ M_{H^-}^2}{[\mathrm{GeV}]^2}\ .$$

If one assumes that $v_u \sim v_e$, this latter bound gives a strong constraint on $\langle P_\perp^\mu \rangle$ $[\langle P_\perp^\mu \rangle < 10^{-4}]$. However, this constraint is removed if $v_u/v_e \sim m_t/m_\tau$.

Similar results are obtained in the simplest supersymmetric extension of the SM. In this case, Im f_S arises from a complex phase associated with the gluino mass. Assuming all supersymmetric masses are of the same order, Christova and Fabbrichesi(102) arrive at the estimate

$$\text{Im } f_S \simeq \frac{M_K^2}{m_{\tilde{g}}^2} \frac{\alpha_s}{12\pi} \sin\phi_{\text{susy}} \ ,$$

where ϕ_{susy} is the gluino mass CP-violating phase. This phase, however, is strongly restricted by the neutron electric dipole moment. Typically, one finds(103)

$$\sin\phi_{\text{susy}} \leq \frac{10^{-7} \ m_{\tilde{g}}^2}{[\text{GeV}]^2}$$

leading to a negligible contribution for $\langle P_\perp^\mu \rangle$, below the level of $\langle P_\perp^\mu \rangle_{\text{FSI}}$.

An experiment (E246) is presently underway at KEK aimed at improving the bound on $\langle P_\perp^\mu \rangle$ obtained earlier at Brookhaven. The sensitivity of E246 is such that one should be able to achieve an error $\delta\langle P_\perp^\mu \rangle \sim 5 \times 10^{-4}(97)$. This level of precisiion is very interesting and, in some ways, it is comparable or better to d_n measurements for probing CP-violating phases from the scalar sector. This is the case, for instance, in the Weinberg model if v_u/v_e is large. At any rate, if a positive signal were to be found, it would be a clear indication for a non-CKM CP-violating phase. Furthermore, as Garisto(104) has pointed out, a positive signal at the level aimed by the E246 experiment would imply very large effects in the corresponding decays in the B system involving τ-leptons (processes like $B^+ \to D^o \tau^+ \nu_\tau$), since one expects, roughly,

$$\langle P_\perp^\tau \rangle_B \sim \frac{M_B}{M_K} \frac{m_\tau}{m_\mu} \langle P_\perp^\mu \rangle_K \ .$$

Thus, in principle, a very interesting experimental cross-check could be done.

5. Concluding Remarks

In the past we have learned profound lessons by doing experiments with Kaon beams. It is my impression that in the future we will continue to learn from Kaons important information, as the planned experiments have an increasing level of precision and sophistication. Indeed, in the next five years, there are a number of experiments which could produce big surprises [flavor violation; CPT violation; evidence for non-CKM phases; decay rates above the SM expectations] and others which could further strengthen our present paradigm for CP violation, through a non-zero measurement of ϵ'/ϵ.

This said, it is a fact that all the experiments presently under construction or taking data are extraordinarily hard and require tremendous sophistication. Thus it seems almost inconceivable (impossible?) to go beyond them. For this reason, it would seem sensible to me to adopt a "plan now, decide later" attitude for new Kaon experiments, beyond those now on the books. That is, it would seem prudent before deciding to go the next step to await the results of the data which will be forthcoming in the next half decade.

Acknowledgments

References

[1] R. H. Brown *et al.*, Nature **163** (1949) 82.

[2] A. Pais, Phys. Rev. **86**(1952) 663.

[3] M. Gell-Mann, Phys. Rev. **92**(1953) 833.

[4] W. B. Fowler *et al.*, Phys. Rev. **93** (1954) 861.

[5] R. H. Dalitz, Phys. Rev. **94**(1954) 1046.

[6] T. D. Lee, and C. N. Yang, Phys. Rev. **104** (1956) 254.

[7] M. Gell-Mann, Caltech Report CTSL-20 (1961); Phys. Rev. **125** (1962) 1067; Y. Neeman, Nucl. Phys. **26** (1961) 222.

[8] M. Gell-Mann (7); S. Okubo, Prog. Theor. Phys. **27**(1962) 949.

[9] M. Gell-Mann, Physics **1**(1964) 63.

[10] Y. Nambu, Phys. Rev. Lett. **4**(1960) 380; J. Goldstone, Nuovo Cimento **19**(1961) 15.

[11] C. G. Callan and S. B. Treman, Phys. Rev. Lett. **16**(1966) 153.

[12] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. **13**(1964) 138.

[13] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531.

[14] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D**2**(1970) 1285.

[15] J. J. Aubert *et al.*, Phys. Rev. Lett **33**(1974) 1404; J. E. Augustin *et al.*, Phys. Rev. Lett. **33**(1974) 1406.

[16] S. Adler, Phys. Rev. **177**(1969) 2426; J. S. Bell and R. Jackiw, Nuovo Cimento **60A**(1969) 47.

[17] G. 't Hooft, Phys. Rev. Lett. **37**(1976) 8.

[18] J. C. Pati and A. Salam, Phys. Rev. D**8**(1973) 1240; H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**(1974) 1438.

[19] See for example, Ch. Berger *et al.*, Zeit. f. Phys. C**50**(1991) 385.

[20] E791 Collaboration: K. Arisaka *et al.*, Phys. Rev. Lett. **70**(1993) 1049.

[21] E137 Collaboration: T. Akagi *et al.*, Phys. Rev. Lett. **67** (1991) 2614.

[22] E777 Collaboration: A. M. Lee *et al.*, Phys. Rev. Lett. **64**(1990) 165.

[23] J. Schwinger, Phys. Rev. **82** (1951) 914; G. Lüders, Dansk. Mat. Fys. Midd. **28**(1954) 17; W. Pauli in **Niels Bohr and the Development of Physics**(Pergamon Press, New York, 1955).

[24] V. A. Kostelecky and R. Potting, Nucl. Phys. **B359** (1991) 545; for a proof of CPT in the context of world-sheet strings, see however, H. Sonoda, Nucl. Phys. **B326**(1989) 135.

[25] S. W. Hawking, Phys. Rev. **D14**(1975) 2460; Commun. Math. Phys. **87**(1982) 395; D. N. Page, Gen. Rel. Grav. **14**(1982) 299.

[26] C. O. Dib and R. D. Peccei, Phys. Rev. **D46**(1992) 2265.

[27] P. Huet and M. Peskin, SLAC-PUB 6454, Nucl. Phys. B (to be published).

[28] J. Ellis, N. L. Mavromatos and D. V. Nanopoulos, Phys. Lett. **B293**(1992) 142; CERN TH6755/92.

[29] V. V. Barmin *et al.*, Nucl. Phys. **B247** (1984) 293; (Erratum) Nucl. Phys. **B254**(1985) 747. See also, N. W. Tanner and R. H. Dalitz, Ann. Phys. **171**(1986) 463.

[30] J. Ellis, J. S. Hagelin, D. V. Nanopoulos and M. Srednicki, Nucl. Phys. **B241**(1984) 381.

[31] Particle Data Group: L. Montanet *et al.*, Phys. Rev. **D50**(1994) 1173.

[32] J. W. Cronin, Rev. Mod. Phys. **53**(1981) 373.

[33] C. D. Buchanan *et al.*, Phys. Rev. **D45**(1992) 4088.

[34] CP Lear Collaboration: R. Adler *et al.*, Phys. Lett. **B286**(1992) 180.

[35] B. Pagels, these Proceedings.

[36] H. J. Lipkin, Phys. Rev. **176**(1987) 1785; I. Dunietz, J. Hauser and J. Rosner, Phys. Rev. **D35**(1987) 2166; J. Bernabeu, F. J. Botella, and J. Roldán, Phys. Lett. **211B** (1988) 226.

[37] See, for example, **The DAΦNE Physics Handbook** L. Maiani, G. Pancheri, and N. Paver, eds. (INFN, Frascati 1992).

[38] A. Sakharov, JETP Lett. **5**(1967) 24.

[39] T. D. Lee, Phys. Reports **9C**(1974) 148.

[40] I. Kobzarev, L. Okun, and Y. Zeldovich, Phys. Lett. **50B**(1974) 340.

[41] S. Barr, Phys. Rev. **D30**(1984) 1805; A. Nelson, Phys. Lett. **143B**(1934) 165. See also, R. D. Peccei, in **25 Years of CP Violation**, ed. J. Tran Than Van (Editions Frontiers, Gif-sur-Yvette, 1990).

[42] L. Wolfenstein, Phys. Rev. Lett. **13**(1964) 562.

[43] N. Cabibbo, Phys. Rev. Lett. **10**(1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**(1973) 652.

[44] For a review, see, for example, R. D. Peccei in **CP Violation** ed. C. Jarlskog (World Scientific, Singapore, 1989).

[45] L. Wolfenstein, Phys. Rev. Lett. **51**(1983) 1945.

[46] F. Gilman and M. Wise, Phys. Lett. B**83**(1979) 83; Phys. Rev. D**20**(1979) 83; B. Guberina and R. D. Peccei, Nucl. Phys. B**163**(1980) 289.

[47] E. P. Shabalin, Sov. J. Nucl. Phys. **28**(1978) 75.

[48] See, for example, Y. Nir in Proceedings of the 1992 SLAC Summer School (SLAC, Stanford, California, 1993).

[49] R. D. Peccei and K. Wang, UCLA/95/TEP/2, Phys. Lett. B (to be published).

[50] R. Forty, to appear in the Proceedings of the International Conference on High Energy Physics (ICHEP 94), Glasgow, Scotland, July 1994.

[51] CDF Collaboration: F. Abe *et al.*, Phys. Rev. D**50** (1994) 2966.

[52] S. Stone, to appear in the Proceedings of the 1994 DPF Conference, Albuquerque, New Mexico, August 1994.

[53] S. Sharpe, Nucl. Phys. B (Proc. Suppl.)**34**(1994) 403.

[54] This is a guesstimate of the value of this parameter coming from Lattice QCD calculations. For a compilation of the most recent results, see J. Shigemitsu, to appear in the Proceedings of the International Conference on High Energy Physics (ICHEP 94), Glasgow, Scotland, July 1994.

[55] E731 Collaboration: L. K. Gibbons *et al.*, Phys. Rev. Lett. **70**(1993) 1203.

[56] NA31 Collaboration: G. D. Barr *et al.*, Phys. Lett. B**317** (1993) 1233.

[57] A. J. Buras and M. E. Lautenbacher, Phys. Lett. B**318** (1993) 212. See also, A. J. Buras, in **Phenomenology of Unification from Present to Future**, ed. G. Diambrini-Palazzi *et al.* (World-Scientific, Singapore, 1994).

[58] J. Flynn and L. Randall, Phys. Lett. B**216**(1989) 221; *ibid.* B**224**(1989) 221; Nucl. Phys. B**326**(1989) 3.

[59] M. Ciuchini, E. Franco, G. Martinelli, and L. Reina, Phys. Lett. B**301**(1993) 263.

[60] A. J. Buras, M. Jamin, and M. T. Lautenbacher, Nucl. Phys. B**408**(1993) 209.

[61] For a review see, J. C. Ritchie and S. G. Wojcicki, Rev. Mod. Phys. **65**(1993) 113.

[62] H. Y. Cheng, Phys. Rev. D**43**(1991) 1579.

[63] J. F. Donoghue, B. R. Holstein, and G. Valencia, Phys. Rev. D**36**(1987) 798.

[64] A. A. Belkov *et al.*, Phys. Lett. B**232**(1989) 118.

[65] G. Isidori, L. Maiani and A. Pugliese, Nuc. Phys. B**381** (1992) 522.

[66] These results come from the Ph.D. Thesis of F. Pettit (UCLA 1995). They are quite similar to those obtained by G. Isidori, L. Maiani, and A. Pugliese, Nucl. Phys. B**381**(1992) 522.

[67] H. Y. Cheng, Phys. Lett. B**315**(1993) 170; Phys. Rev. D**49**(1994) 3771; N. Paver, Riazzudin, and F. Simeoni, Phys. Lett. B**316**(1993) 397. These papers supersede an earlier, more optimistic, estimate of C. O. Dib and R. D. Peccei, Phys. Lett. B**249** (1990) 325.

[68] L. M. Seghal, Phys. Rev. D**38**(1988) 808; D**41**(1991) 161; T. Morozumi and H. Iwasaki, Prog. Theor. Phys. **82**(1989) 371.

[69] A. G. Cohen, G. Ecker, and A. Pich, Phys. Lett. B**304** (1993) 347; see also G. D'Ambrosio *et al.*, CERN TH-7503, 1994.

[70] C. O. Dib, I. Dunietz, and F. Gilman, Phys. Lett. B**218** (1989) 487; Phys. Rev. D**39**(1989) 2639.

[71] See for example, B. Winstein and L. Wolfenstein, Rev. Mod. Phys. **65**(1993) 1113.

[72] A. J. Buras, M. E. Lautenbacher, M. Misiak, and M. Münz, TUM-T31-60/94.

[73] E779 Collaboration: D. H. Harris *et al*, Phys. Rev. Lett. **71**(1993) 3914.

[74] E845 Collaboration: K. E. Ohl *et al.*, Phys. Rev. Lett. **64** (1990) 2755.

[75] E799 Collaboration: D. H. Harris *et al*, Phys. Rev. Lett. **71**(1993) 3918.

[76] B. H. Greenlee, Phys. Rev. D**42**(1990) 3724.

[77] B. Winstein, these Proceedings.

[78] L. S. Littenberg, Phys. Rev. D**39**(1989) 3222.

[79] G. Buchalla and A. J. Buras, Nucl. Phys. B**400**(1993) 225.

[80] M. Weaver, Phys. Rev. Lett. **72**(1994) 3758.

[81] C. O. Dib, Phys. Lett. B**282**(1992) 201.

[82] G. Buchalla and A. J. Buras, Nucl. Phys. B**412**(1994) 106.

[83] A. J. Buras, M. E. Lautenbacher, and G. Ostermaier, Phys. Rev. D**50** (1994) 3433.

[84] J. Gasser and H. Leutwyler, Phys. Reports **87C**(1982) 77.

[85] L. S. Littenberg, these Proceedings.

[86] E787 Collaboration: M. S. Atiya, Phys. Rev. Lett. **70**(1993) 2521; Phys. Rev. D**48**(1993) R1.

[87] For a discussion, see for example, Y. Nir and H. Quinn, in **B Decays**, ed. S. Stone (World Scientific, Singapore, 1992).

[88] See for example, R. D. Peccei, in **25 Years of CP Violation**, ed. J. Tran Than Van (Editions Frontiers, Gif-sur-Yvette, 1990).

[89] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposnikov, Phys. Lett. **155B**(1985) 36; For a review, see for example, M. E. Shaposnikov, Physica Scripta, **T36**(1991) 183.

[90] For a discussion, see for example, M. Dine, Nucl. Phys. B (Proc. Suppl.) **37A**(1994) 127.

[91] M. Leurer, Phys. Rev. Lett. **62** (1989) 1967.

[92] J. J. Sakurai, Phys. Rev. **109**(1958) 980.

[93] G. S. Adkins, Phys. Rev. D**28**(1983) 2285.

[94] A. R. Zhitnitski, Sov. J. Nucl. Phys. **31**(1980) 529.

[95] P. Castoldi, J. M. Frere, and G. L. Kane, Phys. Rev. D**39**(1989) 2633.

[96] G. Belanger and C. O. Geng, Phys. Rev. D**44**(1991) 2789.

[97] Y. Kuno, Nucl. Phys. B (Proc. Suppl.) **37A**(1994) 87.

[98] S. Weinberg, Phys. Rev. Lett. **37**(1976) 657.

[99] R. Garisto and G. Kane, Phys. Rev. D**44**(1991) 2038; G. Belanger and C. O. Geng, Phys. Rev. D**44** (1991) 2789.

[100] Y. Grossman, Nucl. Phys. B**426**(1994) 355.

[101] S. R. Blatt *et al.*, Phys. Rev. D**27**(1983) 1056.

[102] E. Christova and M. Fabbrichesi, Phys. Lett. B**315** (1993) 113.

[103] M. Dugan, B. Grinstein, and L. Hall, Nucl. Phys. B**255** (1985) 413.

[104] R. Garisto, Phys. Rev. D**51**(1995) 1107.