Superstring Extension of the Standard Model and Gravitation

Costas Kounnas[*]

Theory Division, CERN,
CH-1211, Geneva 23, SWITZERLAND [†]

ABSTRACT

Some of the four-dimensional Superstring solutions provide a consistent framework for a Supersymmetric Unification of all interactions including gravity. A class of them extends successfully the validity of the standard model up to the string scale $\mathcal{O}(10^{17})$ GeV. We stress the importance of string corrections which are relevant for low energy $\mathcal{O}(1)$ TeV predictions of gauge and Yukawa couplings as well as the spectrum of the supersymmetric particles after supersymmetry breaking.

Talk given at the

SUSY-95 International Workshop
"Supersymmetry and Unification of Fundamental Interactions"
Palaiseau, 15-19 May 1995
and at the
Four Seas Conference
Trieste, June 25-July 1, 1995.

arXiv:hep-th/9512034v1 6 Dec 1995

[*]On leave from Ecole Normale Supérieure, 24 rue Lhomond, F-75231, Paris, Cedex 05, FRANCE.
[†]e-mail address: KOUNNAS@NXTH04.CERN.CH

1 Introduction and Motivations

If one tries to extend the validity of an effective field theory to energy scales much higher than its characteristic mass scale, and quantum corrections appear carrying positive powers of the cut-off scale Λ, one is faced with a scale hierarchy problem. The typical example is the scale hierarchy problem [1] of the Standard Model (SM) of strong and electroweak interactions, seen as a low-energy effective field theory. When the SM is extrapolated to cut-off scales $\Lambda \gg 1$ TeV, there is no symmetry protecting the mass of the elementary Higgs field, and therefore the masses of the weak gauge bosons, from large quantum corrections proportional to Λ. The most popular solution to the scale hierarchy problem of the SM is [2] to extend the latter to a model with global $N = 1$ supersymmetry, effectively broken at a scale $M_{\mathrm{SUSY}} \lesssim 1$ TeV. These extensions of the SM, for instance [3] the Minimal Supersymmetric Standard Model (MSSM), can be safely extrapolated up to cut-off scales much higher than the electroweak scale, such as the supersymmetric unification scale $M_{\mathrm{U}} \sim 10^{16}$ GeV, the string scale $M_S \sim 10^{17}$ GeV, or the Planck scale $M_{\mathrm{P}} \equiv G_N^{-1/2}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV.

To see in a more quantitative way the properties of supersymmetry that guarantee the stability of the hierarchy of scales against quantum corrections in the MSSM and its variants, it is useful take a closer look to the one-loop effective potential of a generic theory using a momentum cut-off Λ, [4]

$$V_1 = V_0 + \frac{1}{64\pi^2} \mathrm{Str}\, \mathcal{M}^0 \cdot \Lambda^4 \log \frac{\Lambda^2}{\mu^2} + \frac{1}{32\pi^2} \mathrm{Str}\, \mathcal{M}^2 \cdot \Lambda^2 +$$

$$\frac{1}{64\pi^2} \mathrm{Str}\, \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} + \dots, \tag{1.1}$$

where the dots stand for Λ- independent contributions, μ is the scale parameter, and

$$\mathrm{Str}\, \mathcal{M}^n \equiv \sum_i (-1)^{2J_i} (2J_i + 1) m_i^n \tag{1.2}$$

is a sum over the n-th power of the various field- dependent mass eigenvalues m_i, with weights accounting for the number of degrees of freedom and the statistics of particles of different spin J_i. In eq. (1.1), V_0 is the classical potential, which in the case of the SM (and of the MSSM) should contain mass terms at most of the order of the electroweak scale.

The quantum correction to the vacuum energy with the highest degree of ultraviolet divergence is the Λ^4 term, whose coefficient $\mathrm{Str}\, \mathcal{M}^0$ is always field- independent, and equal to the number of bosonic minus fermionic degrees of freedom. Being field- independent, this term can affect the discussion of the cosmological constant problem (when the theory is coupled to gravity), but does not affect the discussion of the hierarchy problem. Anyway, this term is always absent in supersymmetric theories, which possess equal numbers of bosonic and fermionic degrees of freedom.

The second most divergent term in eq. (1.1) is the quadratically divergent contribution which is proportional to $\mathrm{Str}\, \mathcal{M}^2$. In the SM, $\mathrm{Str}\, \mathcal{M}^2$ depends on the Higgs field, and induces a quadratically divergent contribution to the Higgs squared mass, the well-known

source of the hierarchy problem. An early attempt to get rid of the quadratically divergent one-loop contributions to the SM Higgs squared mass consisted [5] in imposing the mass relation $[(\partial^2/\partial\varphi^2)\mathrm{Str}\,\mathcal{M}^2(\varphi)]_{\varphi=v} = 0$; neglecting the light fermion masses, this amounts to requiring $3m_H^2 + 6m_W^2 + 3m_Z^2 - 12m_t^2 = 0$. It is clear (for recent discussions, see e.g. [6] and references therein) that such a requirement is modified at higher orders in perturbation theory, since it amounts to a relation among the dimensionless couplings of the SM that is not stable under the renormalization group. A more satisfactory solution of the problem is provided by $N = 1$ global supersymmetry. For unbroken $N = 1$ global supersymmetry, $\mathrm{Str}\,\mathcal{M}^n$ is identically vanishing for any n, due to the fermion-boson degeneracy within supersymmetric multiplets. The vanishing of $\mathrm{Str}\,\mathcal{M}^2$ persists, as a field identity, if global supersymmetry is spontaneously broken in the absence of anomalous $U(1)$ factors [7]. Indeed, to keep the scale hierarchy stable it is sufficient that supersymmetry breaking does not reintroduce field- dependent quadratically divergent contributions to the vacuum energy. This still allows for a harmless, field- independent quadratically divergent contribution to the effective potential, and is actually used to classify the so-called soft supersymmetry- breaking terms [8]. In the case of softly broken supersymmetry, the Λ^2 term of eq. (1.1) only contributes to the cosmological constant. With a typical mass splitting M_{SUSY} within the MSSM supermultiplets, the logarithmic term in eq. (1.1) induces corrections to the Higgs mass terms (before minimization), which are at most $O(M_{\mathrm{SUSY}}^2)$: the hierarchy is then stable if $M_{\mathrm{SUSY}} \lesssim \mathcal{O}(1)$ TeV.

Without any further assumption, the predictability of the supersymmetric standard model is rather weak, since one must include all soft- breaking terms that are consistent with the SU(3)×SU(2)×U(1) gauge symmetry. This means that the mass spectra of the squarks and sleptons are unknown parameters, and that the only restriction on them is just to be smaller than or equal to $M_{\mathrm{SUSY}} \leq \mathcal{O}(1)$ TeV.

In order to go further and give a more restricted mass spectrum for the squarks, sleptons, etc., more assumptions must be introduced in the would-be fundamental theory. One interesting possibility is the unification of the strong and electroweak gauge couplings, at high energy scales ($E \geq 10^{16}$ GeV). It is very interesting that the SUSY Grand Unification idea works perfectly [9] and predicts correctly the present experimental value for $sin^2\theta_W(m_Z) = 0.2334 \pm 0.0008$, once we set M_{SUSY} around the TeV scale and use the actual values for $\alpha_s(m_Z) = 0.118 \pm 0.008$ and $\alpha_{em}^{-1}(m_Z) = 127.9 \pm 0.2$. The bottom to tau mass ratio ($m_b/m_\tau \simeq 2.8$) is also predicted correctly; the proton lifetime is predicted to be above the present experimental limits ($\tau_p^{SUSY} \simeq 6 \times 10^{34}$ years, $\tau_p^{exp} \geq 6.8 \times 10^{32}$ years), due to the relatively large grand-unification scale $M_U \simeq 1.1 \times 10^{16}$ GeV.

Within the SUSY grand-unification framework, the number of independent supersymmetry soft-breaking mass parameters is drastically reduced at the unification scale to the well known by now supersymmetry-breaking parameters parameters.

$$m_0, \; m_{1/2}, \; A, \; B \text{ and } \mu, \text{ at } M_U,$$

Due to the large radiative corrections going from the M_U to the m_Z scale, the simple tree-level relations get modified [10,11]. The gauge radiative corrections give positive contribution to the (mass)^2 parameters of the squarks sleptons and Higgses. On the other

hand, the large Yukawa couplings of the third generation, give large negative radiative corrections to the (mass)^2 of the scalars and especially to the one of the two Higgses. When the third generation Yukawa couplings are sufficiently large, m_H^2 can become negative at m_Z scale; the SU(2)×U(1) gauge symmetry is then spontaneously broken by the gauge and Yukawa radiative corrections [10,11]. The precise value of the h_t and h_b couplings, which are necessary to break the SU(2)×U(1) at the correct scale, depend on the soft supersymmetry- breaking parameters. Obviously, the radiative breaking mechanism requests a special relation between the gauge couplings and the Yukawa couplings, and thus it demands the top quark to be heavy enough to make the m_H^2 mass parameter negative. A detailed analysis, made twelve years ago [10, 11] and more recently [12] shows that the radiative breaking holds in certain regions of the soft breaking parameters space provided that the top Yukawa coupling is sufficiently large. Furthermore the radiative breaking mechanism implies definite relations among gauge and Yukawa couplings. It is impossible to understand such relations in the framework of non-supersymmetric or $N = 1$ supersymmetric field theory. The only thing we may say at this point is that they are necessary to explain the SU(2)× U(1) breaking at the experimentally known scale. In more fundamental theories like superstring theories, there are super- unification relations among gauge and Yukawa couplings which are compatible with the radiative breaking mechanism (see later) and the present experimental data.

2 Soft Breaking Terms from N=1 Supergravity

In order to go beyond the *technical solution* of the hierarchy problem in renormalizable softly broken supersymmetry, we must move in to a more fundamental theory where the soft breaking terms are generated via a spontaneous breaking of supersymmetry. *The only possible candidate* for such a theory is $N = 1$ supergravity coupled to gauge and matter fields [13]. In contrast with the case of global supersymmetry the spontaneous breaking of local supersymmetry is not incompatible with vanishing vacuum energy. In $N = 1$ supergravity, the spin 2 graviton has for superpartner the spin 3/2 gravitino, and the only consistent way of breaking supersymmetry is *spontaneously*, via the super-Higgs mechanism. One is then bound to interpret the MSSM as an effective low-energy theory derived from a spontaneously broken supergravity. The scale of soft supersymmetry breaking in the MSSM, M_{SUSY}, is related (in a model-dependent way) to the gravitino mass $m_{3/2}$, which sets the scale of the spontaneous breaking of local supersymmetry. One might naively think that, whatever mechanism breaks local supersymmetry and generates the hierarchy $m_{3/2} \ll M_{\mathrm{P}}$, the condition $M_{\mathrm{SUSY}} \sim m_{3/2} \lesssim 1\,\mathrm{TeV} \ll M_{\mathrm{P}}$ remains sufficient to guarantee the stability of such a hierarchy against quantum corrections. To explain why this expectation is generically incorrect [14], we need first to review some general facts about spontaneously broken $N = 1$ supergravity.

Even barring higher-derivative terms, the general structure of $N = 1$ supergravity still allows for a large amount of arbitrariness [13]. First, one is free to choose the field content. Besides the gravitational supermultiplet, containing as physical degrees of freedom the

graviton and the gravitino, one has a number of vector supermultiplets, whose physical degrees of freedom are the spin 1 gauge bosons A_μ^a and the spin 1/2 Majorana gauginos λ^a, transforming in the adjoint representation of the chosen gauge group. One is also free to choose the number of chiral supermultiplets, whose physical degrees of freedom are spin 1/2 Weyl fermions χ^I and complex spin 0 scalars z^I, and their transformation properties under the gauge group. Furthermore, one has the freedom to choose a real gauge-invariant Kähler function

$$\mathcal{G}(z,\overline{z}) = K(z,\overline{z}) + \log |w(z)|^2 \,, \tag{2.3}$$

where K is the Kähler potential whose second derivatives determine the kinetic terms for the fields in the chiral supermultiplets, and w is the (analytic) superpotential. One can also choose a second (analytic) function $f_{ab}(z)$, transforming as a symmetric product of adjoint representations of the gauge group, which determines the kinetic terms for the fields in the vector supermultiplets, and in particular the gauge coupling constants and axionic couplings,

$$g_{ab}^{-2} = \operatorname{Re} f_{ab} \,, \qquad \theta_{ab} = \operatorname{Im} f_{ab} \,. \tag{2.4}$$

Once the functions $\mathcal{G}$ and f are given, the full supergravity Lagrangian is specified. In particular (using here and in the following the standard supergravity mass units in which $M_{\rm P} = 1$), the classical scalar potential reads

$$V = V_F + V_D$$

$$V_F = e^{\mathcal{G}} \left(\mathcal{G}^I \mathcal{G}_I - 3 \right)$$

$$V_D = \frac{[(\operatorname{Re} f)^{-1}]^{ab}}{2} \left(\mathcal{G}_I T_a^I {}_{\overline{J}} \overline{z}^{\overline{J}} \right) \left(z^L T_{b\,L} {}^{\overline{K}} \mathcal{G}_{\overline{K}} \right) \,. \tag{2.5}$$

In our notation, repeated indices are summed, unless otherwise stated; we use Hermitian generators, $[(T_a)^I {}_{\overline{J}}]^\dagger = T_a^J {}_{\overline{I}}$; derivatives of the Kähler function are denoted by $\partial \mathcal{G}/\partial z^I \equiv \partial_I \mathcal{G} \equiv \mathcal{G}_I$ and $\partial \mathcal{G}/\partial \overline{z}^{\overline{I}} \equiv \partial_{\overline{I}} \mathcal{G} \equiv \mathcal{G}_{\overline{I}}$; and the Kähler metric is $\mathcal{G}_{I\overline{J}} = \mathcal{G}_{\overline{J}I} = K_{I\overline{J}} = K_{\overline{J}I}$. The inverse Kähler metric $\mathcal{G}^{I\overline{J}}$, such that $\mathcal{G}^{I\overline{J}} \mathcal{G}_{\overline{J}K} = \delta_K^I$, can be used to define

$$\mathcal{G}^I \equiv \mathcal{G}^{I\overline{J}} \mathcal{G}_{\overline{J}} \,, \qquad \mathcal{G}^{\overline{I}} \equiv \mathcal{G}_J \mathcal{G}^{J\overline{I}} \,. \tag{2.6}$$

Notice that the D-term part of the scalar potential is always positive semi-definite, $V_D \geq 0$, as in global supersymmetry. However, in contrast with global supersymmetry, the F-term part of the scalar potential is not semi- positive definite in general. On the one hand, this allows for spontaneous supersymmetry breaking with vanishing classical vacuum energy, as required by consistency with a flat background. On the other hand, the requirement of vanishing vacuum energy imposes a non-trivial constraint on the structure of the theory,

$$\langle \mathcal{G}^I \mathcal{G}_I \rangle = 3 \quad \text{if} \quad \langle V_D \rangle = 0 \,. \tag{2.7}$$

The order parameter of local supersymmetry breaking in flat space is the gravitino mass,

$$m_{3/2}^2(z,\overline{z}) = |w(z)|^2 e^{K(z,\overline{z})} \,, \tag{2.8}$$

which depends on the vacuum expectation values of the scalar fields of the theory, determined in turn by the condition of minimum vacuum energy. In the following, we shall assume that the D breaking is absent at tree level, as is the case in all interesting situations. For convenience, we shall also classify the fields as $z^I \equiv (z^\alpha, z^i)$, where the fields with Greek indices have non-vanishing auxiliary fields $\mathcal{G}^\alpha \neq 0$ and thus participate to the supersymmetry breaking; the fields with small Latin indices have vanishing $\mathcal{G}^i = 0$. With these conventions, eq. (2.7) can be split as

$$\langle \mathcal{G}^\alpha \mathcal{G}_\alpha \rangle = 3\,, \qquad \langle \mathcal{G}^i \mathcal{G}_i \rangle = 0\,, \tag{2.9}$$

The above F- breaking defines the goldstino direction in terms of the chiral fermions χ^I,

$$\tilde{\eta} = e^{\frac{\mathcal{G}}{2}} \mathcal{G}_\alpha \chi^\alpha\,. \tag{2.10}$$

and the $N = 1$ local supersymmetry is spontaneously broken on a flat background. The coefficient of the one-loop quadratically divergent contributions to the vacuum energy, associated to this breaking, is given by [15]

$$\mathrm{Str}\,\mathcal{M}^2(z, \overline{z}) = 2\,Q(z, \overline{z})\,m_{3/2}^2(z, \overline{z})\,, \tag{2.11}$$

where

$$Q(z, \overline{z}) = N_{TOT} - 1 - \mathcal{G}^I(z, \overline{z}) H_{I\overline{J}}(z, \overline{z}) \mathcal{G}^{\overline{J}}(z, \overline{z})\,, \tag{2.12}$$

$$H_{I\overline{J}}(z, \overline{z}) = R_{I\overline{J}}(z, \overline{z}) + F_{I\overline{J}}(z, \overline{z})$$

$$R_{I\overline{J}}(z, \overline{z}) \equiv \partial_I \partial_{\overline{J}} \log \det \mathcal{G}_{M\overline{N}}(z, \overline{z})\,,$$

$$F_{I\overline{J}}(z, \overline{z}) \equiv \partial_I \partial_{\overline{J}} \log \det \mathrm{Re}\,[f_{ab}(z)]\,. \tag{2.13}$$

Clearly, the only non-vanishing contributions to $\mathrm{Str}\,\mathcal{M}^2$ come from the field directions z^α for which $\langle \mathcal{G}^\alpha \mathcal{G}_\alpha \rangle \neq 0$ (not summed). In eq. (2.13), $R_{I\overline{J}}$ is the Ricci tensor of the Kähler manifold for the chiral multiplets, whose total number is denoted by N_{TOT}; $F_{I\overline{J}}$ has also a geometrical interpretation, since the way it is constructed from the gauge field metric is very similar to the way $R_{I\overline{J}}$ is constructed from the Kähler metric. It is important to observe that both $R_{I\overline{J}}$ and $F_{I\overline{J}}$ *do not depend at all* on the superpotential of the theory, but only depend on the metrics for the chiral and gauge superfields. This very fact allows for the possibility that, for special geometrical properties of these two metrics, the dimensionless quantity $Q(z, \overline{z})$ may turn out to be field-independent and hopefully vanishing.

In a general spontaneously broken $N = 1$ supergravity, the non-vanishing of $Q(z, \overline{z})$ induces, at the one-loop level, a contribution to the vacuum energy quadratic in the cut-off Λ. This leads to a very uncomfortable situation, not only in relation with the cosmological constant problem (a vacuum energy of order $m_{3/2}^2 \Lambda^2$ cannot be cancelled by any physics at lower energy scales) but also in relation with the gauge hierarchy problem, which asks for a gravitino mass not much larger than the electroweak scale. Since $m_{3/2}(z, \overline{z})$ is a field-dependent object, and its expectation value must arise from minimizing the vacuum energy, quadratically divergent loop corrections to the latter may generically destabilize [14, 16] the desired hierarchy $m_{3/2} \ll \Lambda$, attracting the gravitino mass either to $m_{3/2} = 0$ (unbroken

supersymmetry) or to $m_{3/2} \sim \Lambda$ (no hierarchy). This destabilization problem cannot be solved just by moving from the cut-off regulated supergravity to the quantum supergravity defined by four- dimensional superstrings [17–22], since the only practical difference will be to replace the cut-off scale Λ by an effective scale of order M_S. In a generic supergravity theory, we still have the freedom to evade this problem, by postulating the existence of an extra sector of the theory, which gives an opposite contribution to Q, so that $Q + \Delta Q = 0$. Such a request, however, is very unnatural, and implies a severe fine- tuning among the parameters of the old theory and of the extra sector. In string- derived supergravities, the possibility of such a cheap way out is lost, since all the degrees of freedom of the theory are known and the total contribution to Q is well defined. We no longer have the freedom to compensate a non-zero Q by modifying the theory!

From the previous discussion, it is clear that a satisfactory solution of the hierarchy problem ($m_{3/2} \ll M_{\mathrm{P}}$), and the perturbative stability of the flat background, at least up to $\mathcal{O}(m_{3/2}^4)$ corrections, require the vanishing of $Q(z, \overline{z})$ [14]. It is also clear that, if such a solution exists, this will put strong constraints on the scalar and gauge metrics, see eqs. (2.11)–(2.13). Even if one arrange such relations by hand they will not be preserved in the quantum (gravitational) level. In my opinion *there is no any hope* to find such relations in the framework of supergravity field theories. In order to have any hope to find any solution to the above problem, it is necessary to have a framework in which the quantum corrections including the gravitational ones are well defined. The only candidate theory we have so far, that contains a consistent theory of quantum gravity is *superstring* theory.

Before moving to the superstring effective supergavities, I would like to present at this point a special class of $N = 1$ supergravity models, the so called *no-scale models* [23, 14]. The main property of no-scale models is that the classical vacuum energy is identically equal to zero for all scalar field directions z^a which participate to the breaking of supersymmetry with $\mathcal{G}^a \neq 0$.

$$V(z^a; z^i) \geq 0, \quad \text{and} \quad V_{min}(z^a; z^i) = 0 \quad \text{for any } z^a. \tag{2.14}$$

The vacuum is classically degenerate even after the supersymmetry breaking. In these models the supersymmetry breaking scale $m_{3/2}(z, \overline{z})$ is undetermined at the classical level. At the quantum level, however, the z^a-effective potential is deformed, and in general one finds preferred values for the vev's of z^a and thus for $m_{3/2}(z, \overline{z})$. There are two generic possibilities after the quantum corrections; either,

$$i) \ m_{3/2}(z, \overline{z}) \simeq O(M_{\mathrm{P}}) \ ,$$

or

$$ii) \ m_{3/2}(z, \overline{z}) \simeq O(Q_0) \ , \tag{2.15}$$

where Q_0 is a dimensional transmutation scale, at which the mass^2 of the SM–Higgs become negative, $m_H^2(Q_0) \sim 0$ and which is created by the gauge and Yukawa radiative corrections. The first possibility may take place if the quantum gravity corrections are large and do not respect the flatness properties of the z^a-effective potential. This possibility cannot be

examined in any theory in which the quantum gravity is not consistently quantized. Only in the string framework can one check this possibility.

For the same reasons the second possibility, $m_{3/2}(z, \overline{z}) \simeq \mathcal{O}(Q_0)$ can exist only under the assumption that the quantum gravitational effects do not change drastically the z^a-flatness properties of the effective potential. This requirement strongly restricts the fundamental theory at energy scales of order $E \simeq M_P$. Even in the string framework, there is no definite answer yet, even though there are some indications that this is the case in a certain class of string- induced no-scale models. *Assuming* this property of the quantum gravitational corrections and that $Q(z, \overline{z}) =)$, we can examine in detail the effects of the gauge and Yukawa interactions at all energy scales $E < M_P$. It turns out that $m_{3/2}(z, \overline{z})$ is always attracted to an infrared critical scale near Q_0, which is hierarchically smaller than M_P; $m_{3/2}(z, \overline{z}) = \mathcal{O}(10^{-15}) M_P$ [23, 14]. Here I will just sketch this mechanism, since details of the no-scale radiative breaking mechanism can be found in the literature [23, 14]. It is important that Q_0 exists for any value of $m_{3/2}(z, \overline{z}) \leq Q_0$ and is almost z^a-independent. For all z^a such that $m_{3/2}(z, \overline{z}) \gg Q_0$, m_H^2 is positive, which implies the non-existence of SU(2) × U(1) breaking minima. On the other hand for $m_{3/2}(z, \overline{z}) \leq Q_0$, $m_H^2 < 0$, and so an SU(2) × U(1) breaking minimum is developed with $m_{3/2}(z, \overline{z})$ near the transmutation scale Q_0. These results are obtained by a minimization of the one-loop effective potential in the H_i and $m_{3/2}(z, \overline{z})$ field directions [23, 14]. One finds

$$m_Z \simeq m_{3/2}(z, \overline{z}) \leq O(Q_0) . \tag{2.16}$$

The no-scale SU(2) × U(1) radiative breaking provides the *only known mechanism* that explains the important hierarchy between the electroweak scales $m_Z \sim m_{3/2} \leq Q_0$ and the fundamental gravitational scale M_P. This scale hierarchy is a consequence of the very slow variation (logarithmic) of the renormalized soft-breaking parameters:

$$Q_0 = M_P \exp -\frac{O(1)}{\alpha_t} \simeq O(10^{-15}) M_P . \tag{2.17}$$

where $\alpha_t = h_t^2/4\pi$ is the top Yukawa coupling at M_z. The $O(1)$ coefficient is determined in any specific no-scale model and depends only on dimensionless parameters, gauge and Yukawa coupling constants and on the soft- breaking terms mass ratios, $m_{3/2}/M_{1/2}$, $\mu/M_{1/2}$, $A/M_{1/2}$, $B/M_{1/2}$.

In the framework of no-scale models, the SUSY spectrum at low energies is more constrained than in a general $N = 1$ supergravity model. It depends on at least one parameter less than the general $N = 1$ models, since a linear combination of $m_{3/2}$ and $M_{1/2}$, is determined by the minimization of the effective potential. In practice, the no-scale models are even more restrictive, because of the flatness requirement in the z^a-direction of the scalar potential. This requirement drastically restricts the choice of interesting $N = 1$ supergravity models. As we already stressed, the main assumption of no-scale models is about the quantum gravitational corrections. In order to check this assumption, it is necessary to go even beyond the $N = 1$ supergravity framework, and try to find theories in which the quantum gravitational corrections make sense and behave like $N = 1$ no-scale supergravities for energies $E < M_P$.

It is remarkable that such an extension exists in the framework of $N = 1$ four-dimensional superstrings [17–22]. In that framework, the quantum gravitational corrections are under control, and all interactions (gauge, Yukawa and gravitational) are unified at the string scale $M_S \simeq mpl$. At the very beginning of the string revolution, it was noticed that there existed a connection between the superstring effective theories [24–28] and the $N = 1$ no-scale supergravity models. Since then, many candidates were found and confirmed their no-scale structure. However, for technical reasons, which are related to the complexity of string solutions, as well as to the ambiguities related to the supersymmetry-breaking mechanism on strings (or in string- effective theories), we do not have up to now precise quantitative predictions; we rather have qualitative predictions, which are not, in fact, more restrictive than the no-scale effective theories.

3   4d-Superstrings and their Effective Theories

Superstring theory extents the validity of quantum field theory to very short distances and defines consistently the quantization of all interactions. It is thus appropriate to try to investigate the behavior of string dynamics at high energies and in regions of spacetime where the gravitational field is strong. There are several problems in gravity where the classical, and even worse the semiclassical treatment have perplexed physicists for decades. We are referring here to questions concerning the behavior in regions of strong (or infinite) curvature with both astrophysical (black holes) and cosmological (big-bang, wormholes) interest. It is only appropriate to try to elucidate such questions in the context of stringy gravity. There has been progress towards this direction, and by now we have at least some ideas on how different string gravity can be from general relativity in regions of space- time with strong curvatures [29]. We need however exact classical solutions of string theory in order to have more quantitative control on phenomena that are characteristic of stringy gravity.

On the other hand the special characteristics of superstrings do not only reflect in the gravitational sector of the theory. There are also important implications for particle physics, namely, concerning the string low energy predictions at M_Z and at the accessible by the future, energy scale of $\mathcal{O}(1)\text{TeV}$. The first main property of superstrings is that they are ultraviolet finite theories (at least perturbatively). The second important property is that they unify gravity with all other interactions. There are several ways to construct four- dimensional superstrings with $N = 1$ space time supersymmetry. It is interesting however, that all N=1 superstring constructions shows some *universality* properties and thus they define a *special* class of $N = 1$ supergravity theories. The bosonic part of their effective $N = 1$ supergravity action, restricted up to two space-time derivatives, reads

$$S_{\text{bos}}^{\text{eff}} = \int d^4x \sqrt{-g} \{ \frac{1}{2}\mathcal{R} - \frac{\nabla_\mu S \nabla^\mu \bar{S}}{(S + \bar{S})} - \frac{\delta\hat{c}}{2}$$

$$-K_{I\bar{J}}(z^I)\nabla_\mu z^I \nabla^\mu \bar{z}^{\bar{J}} - \frac{V(z^I)}{(S + \bar{S})}$$

8

$$- \frac{k^a}{4}(S + \bar{S})F^a_{\mu\nu} \ F^{\mu\nu}_a + \frac{ik^a}{4}(S - \bar{S})F^a_{\mu\nu} \ \tilde{F}^{\mu\nu}_a)\} \tag{3.18}$$

where $S + \bar{S}$ is the dilaton field and $S - \bar{S}$ the pseudoscalar axion (dual to the two-index antisymmetric tensor). $F^a_{\mu\nu}$ are the field strengths of the gauge bosons, and $\tilde{F}^a_{\mu\nu}$ are their duals. The dilaton potential which is proportional to the central charge deficit $\delta\hat{c}$ is zero for the critical strings and is different from zero for the non-critical string solutions which are defined in curved space-time. For the $N = 1$ superstring solutions $\delta\hat{c} = 0$ and the scalar manifold metric $K_{I\bar{J}}$ is given in terms of derivatives of a Kälher potential $K(z^I, \bar{z}^{\bar{I}})$, $K_{I\bar{J}} \equiv \partial_I\partial_{\bar{J}}K(z^I, \bar{z}^{\bar{I}})$. For a given string solution, the gauge group and the multiplet content are uniquely specified, and so are the Kähler and the gauge kinetic functions, which do indeed exhibit the remarkable geometrical properties. Moreover, as an effect of the string unification of all interactions, these theories do not contain any explicit mass parameter besides the string mass scale M_S, in the sense that all couplings and masses of the low-energy effective theory are associated with the VEVs of some moduli fields. There is a vast literature concerning the effective supergravities corresponding to four-dimensional superstring models with unbroken $N = 1$ local supersymmetry, both at the classical [24–28] and at the quantum [30] level. The typical structure which emerges is the following. The vector multiplets are fixed by the four-dimensional gauge group characterizing the given class of string solutions. As for the chiral multiplets, there is always a universal 'dilaton-axion' multiplet, S, singlet under the gauge group, which at the classical level entirely determines the gauge kinetic function,

$$f_{ab} = k^a \delta_{ab} S \,. \tag{3.19}$$

. The above form of the string gauge kinetic function implies a tree-level unification relation at the string scale. This unification does not include only the gauge interactions but also the Yukawa ones as well as the interactions among the scalars.

$$\frac{1}{\alpha_i} = \frac{k^i}{\alpha_{str}} = 2\pi k^i(S + \bar{S}) \quad \text{at} \quad M_S \tag{3.20}$$

This unification of couplings happens at large energy scales $E_t = \mathcal{O}(M_S) = 5 \times 10^{17}$ GeV. At low scales due to the quantum corrections, the above string unification relations become

$$\frac{1}{\alpha_i(\mu)} = \frac{k^i}{\alpha_{str}} + b_0^i \ log\frac{M_S^2}{\mu^2} + \Delta_i(T^a) \tag{3.21}$$

The string unification looks very similar to the well-known unification condition in super-symmetric Grand Unified Theories (susy-GUTs) with a unification scale $M_U \sim M_S$ and $\Delta_i(T^a) = 0$ in the $\bar{DR}$ renormalization scheme; in susy-GUTs the normalization constants k_i are fixed *only* for the gauge couplings ($k_1 = k_2 = k_3 = 1$, $k_{em} = \frac{8}{3}$), but there are no relations among gauge and Yukawa couplings at all. In string effective theories, however, the normalization constants (k_i) are known for both *gauge and Yukawa interactions*. Furthermore, $\Delta_i(T^a)$ are calculable *finite* quantities for any particular string solution without any Ultra-Violet ambiguities since strings are UV finite. Thus, the predictability of a given string solution is extended for all low energy coupling constants $\alpha_i(M_Z)$ once the string-

9

induced corrections $\Delta_i(T^a)$ are determined. This determination however, requests string computations which we did not know, up to now, how to perform in full generality. It turns out that $\Delta_i(T^a)$ are non-trivial functions of the vacuum expectation values of some gauge singlet fields the so-called moduli as well as standard Higgs fields [31, 32, 33, 34], (the moduli fields are flat directions at the string classical level and they remain flat in string perturbation theory, in the exact supersymmetric limit). The $\Delta_i(T^a)$ are target space duality invariant functions, which depend on the particular string solution. Partial results for Δ_i exist [31, 32, 33, 34] in the exact supersymmetric limit in many string solutions based on orbifold [19] and fermionic constructions [20] and they are, in principle, well defined calculable quantities once we perform our calculations at the string level where all interactions including gravity are consistently defined. The full string corrections to the coupling constant unification, $\Delta_i(T^A)$, as well as the string corrections associated to the soft supersymmetry-breaking parameters

$$m_0,\ m_{1/2},\ A,\ B \text{ and } \mu, \text{ at } M_U,$$

are of main importance, since they fix the strength of the gauge and Yukawa interactions, the full spectrum of the supersymmetric particles as well as the SM Higgs and the top-quark masses at the low energy range $M_Z \leq E_t \leq \mathcal{O}(1)$ TeV.

In addition to S, there are in general other singlet chiral superfields, called 'moduli', which do not appear in the superpotential of the string effective theories and thus correspond to classically flat directions of the scalar potential. They parametrize the size and the shape of the internal compactification manifold, and will be denoted here by the generic symbols T and U. Finally, there are other chiral superfields which are in general charged under the gauge group, or at least have a potential induced by some superpotential coupling: for the moment, we shall denote them with the generic symbol C, understanding that in realistic models this class of fields should contain the matter and Higgs fields of the MSSM. ($z^I = [T, U, C]$).

The remarkable fact is that in the known four- dimensional string models, in the limit where the T and/or U moduli are large with respect to the string scale M_S, the Kähler manifold for the chiral superfields obeys well- defined scaling properties with respect to the real combinations of moduli fields $s \equiv (S + \overline{S})$, $t_i \equiv (T_i + \overline{T}_i)$ and $u_i \equiv (U_i + \overline{U}_i)$. These scaling properties are due to the discrete target- space duality symmetries [35] of four-dimensional superstrings, and the scaling weights are nothing but the modular weights with respect to the moduli fields which participate in the supersymmetry-breaking mechanism: in the limit of large moduli, non-trivial topological effects on the world sheet are exponentially suppressed and can be neglected; in this limit the discrete duality symmetries are promoted to accidental scaling symmetries of the kinetic terms in the effective supergravity theory. More precisely, the Kähler potential can be written as

$$K = -\log Y(s, t, u) + K^{(C)}(C, \overline{C}; T, \overline{T}; U, \overline{U}). \tag{3.22}$$

The function Y factorizes into three terms,

$$Y = Y^{(S)}(s) \cdot Y^{(T)}(t) \cdot Y^{(U)}(u), \tag{3.23}$$

where
$$Y^{(S)} = s \,, \tag{3.24}$$

so that
$$s\partial_s Y = Y \,. \tag{3.25}$$

Another general feature involves the moduli T_i, corresponding to harmonic $(1,1)$ forms, associated with deformations of the Kähler class of the internal compactified space. Even if their number is model-dependent, the fact that three of them are related to the three complex coordinates of the internal compactification manifold implies, in the limit of large T moduli,

$$t_i\partial_{t_i} Y = 3\, Y \,. \tag{3.26}$$

The moduli U_i are associated with harmonic $(1,2)$ forms, correspond to deformations of the complex structure of the internal compactified space, and their existence, number and properties are more model-dependent. In general, in the limit of large U moduli one can write a relation of the form

$$u_i\partial_{u_i} Y = p_U\, Y \,, \tag{3.27}$$

where $p_U = 0, 1, 2, 3$ depends on the superstring model under consideration. Finally, keeping only quadratic fluctuations of the C fields (sufficient to evaluate the Kähler metric and the mass terms around $C = \overline{C} = 0$), one can in general write

$$K^{(C)} = \sum_A K^A_{i_A \bar{j}_A}(t,u) C^{i_A} \overline{C}^{\bar{j}_A}$$

$$+ \frac{1}{2}\sum_{A,B} \left[P_{i_A j_B}(t,u) C^{i_A} C^{j_B} + \text{h.c.} \right] + \dots \,, \tag{3.28}$$

with generic scaling properties of the form

$$t_i\partial_{t_i} K^A_{i_A \bar{j}_A} = \lambda^A_t K^A_{i_A \bar{j}_A} \,, \tag{3.29}$$

$$u_i\partial_{u_i} K^A_{i_A \bar{j}_A} = \lambda^A_u K^A_{i_A \bar{j}_A} \,, \tag{3.30}$$

$$t_i\partial_{t_i} P_{i_A j_B} = \frac{\lambda^A_t + \lambda^B_t}{2} P_{i_A j_B} \,, \tag{3.31}$$

$$u_i\partial_{u_i} P_{i_A j_B} = \frac{\lambda^A_u + \lambda^B_u}{2} P_{i_A j_B} \,. \tag{3.32}$$

These remarkable scaling properties for the Kähler metric follow from the discrete target-space dualities, which are symmetries of the full Kähler function $\mathcal{G}$. Under a generic duality transformation, of the form

$$z^\alpha \longrightarrow f(z^\alpha) \,,$$

the Kähler potential transform as

$$K \longrightarrow K + \phi + \overline{\phi} \,,$$

where ϕ is an analytic function of the moduli fields z^α, and in particular it must be that

$$Y \longrightarrow Y e^{\phi + \overline{\phi}} \,. \tag{3.33}$$

Also, it is not restrictive for our purposes to consider the case in which the fields C_A transform with a specific modular weight λ_A,

$$C_A \longrightarrow e^{-\lambda_A \phi} C_A \,. \tag{3.34}$$

The fact that target-space duality is a symmetry implies then a definite transformation property for the superpotential, $w \longrightarrow e^{-\phi} w$ which in turn puts very strong restrictions on the superpotential couplings, for example the cubic Yukawa couplings of the form $h_{ABD} C^A C^B C^D$. If h_{ABD} is such that, in the large moduli limit, it goes to a non-vanishing constant (or, more generally, to a modular form of weight zero), then it must be

$$\lambda_A + \lambda_B + \lambda_D = 1 \,. \tag{3.35}$$

For example, in $Z_2 \times Z_2$ orbifolds the h_{ABD} are constants, whereas in Calabi-Yau manifolds they are modular forms of weight zero, which approach a constant in the large volume limit for the associated moduli.

In the case of unbroken supersymmetry, and in the large moduli limit, the classical superpotential w is independent of the (S, T, U) moduli fields, and at least quadratic in the C fields. From the previous scaling properties, it also follows that around $C = 0$ one can write

$$K^s K_s = 1 \,, \quad K^{t_i} K_{t_i} = 3 \,, \quad K^{u_i} K_{u_i} = p_U \,. \tag{3.36}$$

Armed with this result, we are ready to discuss spontaneous supersymmetry breaking in the superstring effective supergravities. As already explained, to have broken supersymmetry and vanishing vacuum energy one needs $w \neq 0$ and $\mathcal{G}^I \mathcal{G}_I = 3$ at the minima of the tree-level potential. If one takes the effective supergravities derived from the four-dimensional superstring model with unbroken supersymmetry, one consistently obtains a semi-positive definite scalar potential, admitting $C = 0$ minima with unbroken supersymmetry and vanishing vacuum energy, and flat directions along the S, T and U moduli fields. To obtain supersymmetry breaking minima with unbroken gauge symmetries, one has to introduce a superpotential modification which generates minima with $C = 0$, $w \neq 0$, $\mathcal{G}^I \mathcal{G}_I = 3$ when the summation index I runs over the (S, T, U) moduli, $\mathcal{G}^I \mathcal{G}_I = 0$ when I runs over the C fields. This means, however, that the superpotential modification must depend on at least some of the (S, T, U) moduli, since otherwise we would get, when summing over the moduli indices, $\mathcal{G}^I \mathcal{G}_I = 4 + p_U$, which would make the scalar potential strictly positive definite and thus not allow for the desired minima. As for the origin of possible superpotential modifications, we must refer to the two types of mechanisms for supersymmetry breaking considered so far in the framework of four-dimensional string models. The first one corresponds to exact tree-level string solutions [36, 37], in which supersymmetry is broken via orbifold compactification. The second one is based on the assumption that supersymmetry breaking is induced by non-perturbative phenomena [38], such as gaugino condensation or something else, at the level of the string effective field theory. These will be the two possibilities considered in the following.

In the case of non-perturbative supersymmetry breaking [38], in the absence of a second-quantized string formalism one can assume that, at the level of the effective supergravity,

the super- Higgs mechanism is induced by a superpotential modification which preserves target-space duality [39]. The relevant transformations are those acting non-trivially on the moduli fields z^α associated with supersymmetry breaking. If, for example, the modified superpotential has the form

$$w = w_{SUSY} + A(z^\alpha) + B_{AB}(z^\alpha)C^A C^B + \dots , \qquad (3.37)$$

target-space duality requires then the following transformation properties:

$$A(z^\alpha) \longrightarrow A(z^\alpha)e^{-\phi} , \qquad\qquad B_{AB}(z^\alpha) \longrightarrow B_{AB}(z^\alpha)e^{-(1-\lambda_A-\lambda_B)\phi} . \qquad (3.38)$$

Unfortunately, the form of the function $A(z^\alpha)$ cannot be uniquely fixed by the requirement that it is a modular form of weight (-1). However, another important constraint comes from the physical requirement that the potential must break supersymmetry and generate a vacuum energy at most $\mathcal{O}(m_{3/2}^4)$ in the large moduli limit. This implies that $A(z^\alpha) \longrightarrow$ constant $\neq 0$ for $z^\alpha \to \infty$. This is not the case for the models of supersymmetry breaking with minima of the effective potential at small values of T, which make use of the Dedekind function $\eta(T)$ in the superpotential modification [40]: either they do not break supersymmetry or they do so with a large cosmological constant, in contradiction with the assumption of a constant flat background. In the case of the function $B_{AB}(z^\alpha)$, it is sufficient to assume that, in the large moduli limit, $B_{AB}(z^\alpha) \longrightarrow$ constant. For the moduli fields that are not involved in the breaking of supersymmetry, these asymptotic conditions are not necessary and can be relaxed. The requirement that $A(z^\alpha) \longrightarrow$ constant $\neq 0$ for $z^\alpha \to \infty$ defines an approximate no-scale model, with minima of the effective potential corresponding to field configurations that are far away from possible $z^\alpha \simeq \mathcal{O}(1)$ self-dual minima with unbroken supersymmetry ($\mathcal{G}_\alpha = 0$) and negative vacuum energy $\mathcal{O}(M_P^4)$. Between these two classes of extrema, there may exist other extrema of the effective potential with $\mathcal{G}_\alpha \neq 0$, but those are generically unstable and/or have non-vanishing vacuum energy [40]. As for the VEVs of the moduli fields that do not contribute to supersymmetry breaking (those with $\mathcal{G}^i \mathcal{G}_i = 0$), they are generically fixed to some extended symmetry points (e.g. the self-dual points).

In the string models with tree-level supersymmetry-breaking [36, 37], the superpotential modifications in the large-moduli limit are fully under control, since in that case the explicit form of the one-loop string partition function is known, and one can derive the low-energy effective theory without making any assumption. In this class of models, the large-moduli limit is a necessity, since for small values of the moduli (close to their self-dual points) there exist Hagedorn-type instabilities, induced by some winding modes that become tachyonic in flat space-time [37]. At the self-dual point there is a new stable minimum with unbroken supersymmetry and negative cosmological constant, as expected. We should stress here that the prescription for a consistent effective field theory in the region of small moduli requires the addition of extra degrees freedom, corresponding to the winding modes which can become massless or tachyonic for some values of the T and/or U moduli close to the self-dual points [37]. In the large-moduli limit, however, we can disregard the effects of these extra states and not include them in the effective field theory. In this limit, the superpotential modification associated with supersymmetry breaking seems to violate

target-space duality. On the other hand, w_{SUSY} and the Kähler potential maintain the same expressions as in the case of exact supersymmetry, with the desired scaling properties that can produce a supergravity model with $Q(z, \overline{z}) = 0$ [14]. The main features of the effective theories with tree- level supersymmetry breaking are the following:

1. The Kähler potential and the gauge kinetic function of the effective theory are the same as those obtained in the limit of unbroken supersymmetry, so that, up to analytic field redefinitions, supersymmetry breaking is indeed induced only by a superpotential modification.

2. For $C = 0$, the Kähler manifold for the T and U moduli can be decomposed into the product of two factor manifolds. The first one, described by a Kähler potential K', involves one T and one U field, to be called here T' and U'

$$K' = -\log[(T' + \overline{T}')(U' + \overline{U}')], \quad (C = 0), \tag{3.39}$$

and the second one, described by a Kähler potential K'', involves all the remaining T and U moduli.

3. The superpotential modification associated with supersymmetry breaking does not involve the fields S, T' and U', so that $\mathcal{G}_S = K_S$, $\mathcal{G}_{T'} = K_{T'}$, $\mathcal{G}_{U'} = K_{U'}$. The condition $\mathcal{G}^\alpha \mathcal{G}_\alpha = 3$, which must be satisfied at the minima, is identically saturated by the fact that for $C = 0$ it is $\mathcal{G}^S \mathcal{G}_S = \mathcal{G}^{T'} \mathcal{G}_{T'} = \mathcal{G}^{U'} \mathcal{G}_{U'} = 1$. The goldstino direction is then along some linear combination of the (S, T', U') fields.

4. The superpotential modification associated with supersymmetry breaking involves the fields appearing in K'', so that, restricting the sum over I to these fields, the condition $\mathcal{G}^I \mathcal{G}_I = 0$ can be satisfied at all minima.

We can now discuss the other proposed mechanism for spontaneous supersymmetry breaking in string- derived supergravity models, i.e. the possibility of non- perturbative phenomena, which at the level of the effective supergravity theory can again be described by a modification of the superpotential [38]. In order to obtain a consistent model, with broken supersymmetry and classically vanishing vacuum energy, the superpotential modification must be such that $\mathcal{G}^I \mathcal{G}_I = 3$ around $C = 0$. The superpotential modification must then contain some dependence on the (S, T, U) moduli fields in order to avoid a strictly positive potential. For example, the simplest choice $w = k + O(C^2)$, with w independent of the (S, T, U) moduli, would give $\mathcal{G}^I \mathcal{G}_I = K^{\hat{I}} K_{\hat{I}} = 4 + p_U$, where the index $\hat{I}$ runs over the (S, T, U) moduli, and therefore a potential around $C = 0$ of the form $V = (1 + p_U)|k|^2 e^K = (1 + p_U)|k|^2/Y$. Since the quantity at the numerator is field-independent and strictly positive-definite, there is no stationary point for the potential, with the exception of the boundaries of moduli space, $Y \to \infty$, for example the decompactification limit $Y^{(T)} Y^{(U)} \to \infty$ or the zero-coupling limit $Y^{(S)} \to \infty$. It is then clear that, to avoid this problem, the superpotential must depend on some of the moduli, in order to fix some of the VEVs associated with the moduli directions.

14

The simplest superpotential modification follows from the conjecture of gaugino condensation [38], and includes a non-trivial dependence on the S modulus. Such an assumption is made plausible by the fact that the gauge coupling constant of the theory is determined by the VEV of the S field. An S-dependent superpotential modification can allow for minima with $\mathcal{G}_S = 0$, and fix the VEV of the S modulus at the minima. Irrespectively of the details of the S dependence of the superpotential, as long as there is a field configuration of S such that $\mathcal{G}_S = 0$, this is sufficient to create a well-behaved positive-semi-definite potential in the absence of U-type moduli ($p_U = 0$). When such moduli are present, one must make the further assumption that the superpotential contains also a non-trivial U-dependence, so that minima with $\mathcal{G}_U = 0$ can be allowed: otherwise, the scalar potential would still remain strictly positive-definite. Notice that the stabilization of the VEVs of the U-type moduli can be performed either at the string level, by moving to the points of extended symmetry associated with the U moduli, or at the level of the effective theory, by extending the assumption made for the S field.

A superpotential modification with non-trivial S and U dependence has the name of 'T-breaking', since in that case the condition of vanishing vacuum energy, $\mathcal{G}^I \mathcal{G}_I = 3$, is saturated by the T fields only. For the models in which $p_U = 3$, we may alternatively assume a superpotential modification, which depends only on S and T, so that $\mathcal{G}_S = \mathcal{G}_{T_i} = 0$ at the minima, and the condition $\mathcal{G}^I \mathcal{G}_I = 3$ is entirely saturated by the U moduli: we shall call this scenario 'U-breaking'. In the cases in which the Kähler manifolds for the T and U moduli are factorized, one may also consider intermediate scenarios of $S/T/U$-breaking, in which the superpotential modification is such that, at the minima of the potential with $C = 0$, it is identically $\mathcal{G}^S \mathcal{G}_S + \mathcal{G}^{T_i} \mathcal{G}_{T_i} + \mathcal{G}^{U_i} \mathcal{G}_{U_i} = 3$, with non-vanishing contributions from more than one sector. In this language the string tree level breaking [36, 37] is an $S/T'/U'$- breaking. I should stress here that in all these scenarios the resulting value of Q, the coefficient appearing in $\mathrm{Str}\,\mathcal{M}^2$, does not depend on the details of the superpotential modification [14], but only on the scaling weights λ^I of the different fields z^I, with respect to the moduli z^α for which $\mathcal{G}^\alpha \mathcal{G}_\alpha \neq 0$ (not summed) at the minima.

$$z^\alpha \partial_\alpha [K_{I,\bar{J}}]^C = \lambda^C \tag{3.40}$$

What is interesting is that under the above assumptions for the supersymmetry breaking, the coefficient Q and the soft breaking parameters are given *uniquely* in terms of the scaling weights λ^I and in terms of the gravitino mass scale $m_{3/2}$; there is no any further dependence coming from the superpotential [14].

- Quadratic divergences:

$$Q = \sum_A (1 + \lambda^A) + d^a \lambda^a - 1$$

- Scalar mass terms:

$$(m_0^2)_A = (1 + \lambda^A) m_{3/2}^2$$

- Gaugino mass terms:

$$(M_{1/2})_a = -\lambda^a m_{3/2}$$

- Cubic Scalar Couplings:

$$A_{A,B,C} = (3 + \lambda^A + \lambda^B + \lambda^C) m_{3/2}$$

15

- $\mu_{A,B} \; H_A \; H_B$ terms:

$$\mu_{A,B} = \frac{1}{2}(2 + \lambda^A + \lambda^B)m_{3/2}$$

- Quadratic Scalar Couplings:

$$B_{A,B} = \frac{1}{2}(4 + \lambda^A + \lambda^B)m_{3/2}$$

In the above equations λ^a is the scaling weight of the gauge kinetic function and d^a is the dimension of the gauge group.

4 Superstrings Corrections to the Unification Relations

The main obstruction in determining the exact form of the string radiative corrections $\Delta_i(T^a)$ is strongly related to the infrared divergences of the $\langle [F^a_{\mu\nu}]^2 \rangle$ two-point correlation function in superstring theory. In field theory, we can avoid this problem using off-shell calculations. In first quantized string theory we cannot do so since we do not know in general how to go off-shell. Even in field theory there are problems in defining an infrared regulator for chiral fermions especially in the presence of space- time supersymmetry.

In [41] it was suggested to use a specific space- time with negative curvature in order to achieve consistent regularization in the infrared. The proposed curved space however is not useful for string applications since it does not correspond to an exact super-string solution.

Recently, exact superstring solutions have been constructed using special four-dimensional spaces as superconformal building blocks with $\hat{c} = 4$ and $N = 4$ superconformal symmetry [42, 43, 34]. The full spectrum of string excitations for the superstring solutions based on those four-dimensional subspaces, can be derived using the techniques developed in [43]. The main characteristic property of these solutions is the existence of a mass gap μ, which is proportional to the curvature of the non-trivial four-dimensional space- time. Comparing the spectrum in a flat background with that in curved space we observe a shifting of all massless states by an amount proportional to the space- time curvature, $\Delta m^2 = Q^2/4 = \mu^2/2$, where Q is the Liouville background charge and μ is the IR cutoff. What is also interesting is that the shifted spectrum in curved space is equal for bosons and fermions due to the existence of a new space-time supersymmetry defined in curved space-time [42, 43, 34]. Therefore, in some curved space- time solutions the induced infrared regularization is consistent with supersymmetry and can be used either in field theory or string theory.

In order to calculate the renormalization of the effective couplings we need to turn on backgrounds for gauge and gravitational fields. Thus our aim is to define the deformation of the two-dimensional superconformal theory which corresponds to a non-zero field strength $F^a_{\mu\nu}$ and $R_{\mu\nu\rho\sigma}$ background and find the integrated one-loop partition function $Z(\mu, F, \mathcal{R})$,

where F is by the magnitude of the field strength, $F^2 \equiv \langle F^a_{\mu\nu} F^{\mu\nu}_a \rangle$ and $\mathcal{R}$ is that of the curvature, $\langle R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \rangle = \mathcal{R}^2$.

$$Z[\mu, F_i, \mathcal{R}] = \frac{1}{V(\mu)} \int_{\mathcal{F}} \frac{d\tau d\bar{\tau}}{(\mathrm{Im}\tau)^2} Z[\mu, F_i, \mathcal{R}; \tau, \bar{\tau}] \qquad (4.41)$$

The index i labels different simple or $U(1)$ factors of the gauge group and $V(\mu) = (k + 2)^{3/2}/8\pi$ is the volume of the three dimensional sphere. Expanding the partition function in a power series in $F, \mathcal{R}$

$$Z[\mu, F_i, \mathcal{R}] = \sum_{m,n=0}^{\infty} F^m \mathcal{R}^n \qquad (4.42)$$

we can extract the integrated correlators $\langle F^m \mathcal{R}^n \rangle = Z_{m,n}$. The one loop correction to the gauge coupling constants is given by [34]

$$\frac{4\pi}{\alpha_A^2(\mu)} = \frac{4\pi k^a}{\alpha_a^2(M_S)} + Z_{2,0}^a(\mu) \qquad (4.43)$$

In flat space, a small non-zero $F^a_{\mu\nu}$ background gives rise to an infinitesimal deformation of the 2-d σ-model action given by,

$$\Delta S^{2d}(F^{(4)}) = \int dz d\bar{z} \, F^a_{\mu\nu} [x^\mu \partial_z x^\nu + \psi^\mu \psi^\nu] \bar{J}_a \qquad (4.44)$$

Observe that for $F^a_{\mu\nu}$ constant (constant magnetic field), the left moving operator $[x^\mu \partial_z x^\nu + \psi^\mu \psi^\nu]$ is not a well-defined $(1,0)$ operator on the world sheet. Even though the right moving Kac-Moody current $\bar{J}_a$ is a well-defined $(0,1)$ operator, the total deformation is not integrable in flat space. Indeed, the 2-d σ-model β-functions are not satisfied in the presence of a constant magnetic field. This follows from the fact that there is a *non-trivial back-reaction* on the gravitational background due the non-zero magnetic field.

The important property of a non-trivial space-time background in which the 4d flat background R^4 is replaced by $W_k^{(4)} = S_k^3 \times R$, a three dimensional sphere with curvatue $1/(k+2)$ plus a non compact coordinate with a background $Q = \sqrt{2/(k+2)}$, is that we can solve exactly for the gravitational back-reaction. First observe that the deformation that corresponds to a constant magnetic field $B_i^a = \epsilon_{oijk} F_a^{ik}$ is a well-defined (1,1) integrable deformation, preserving the world-sheet supersymmetry:

$$\Delta S^{2d}(W_k^{(4)}) = \int dz d\bar{z} \, B_i^a [I^i + \frac{1}{2} \epsilon^{ijk} \psi_j \psi_k] \bar{J}_a \qquad (4.45)$$

where I^i is anyone of the $SU(2)_k \sim S_k^3$ currents. The deformed partition function $Z[\mu, F_i, \mathcal{R}]$ is IR finite due to the infrared regulator μ induced by the weakly curved space-time. In the string framework it is also UV finite, where the M_S acts as UV cutoff.

In general the exact determination of the IR regularized partition function in terms of the expectation values of the moduli fields T^I, the Higgs fields H^I and the supersymmetry auxiliary fields $\mathcal{G}^{\mathcal{I}}$, $Z[\mu, F_i, \mathcal{R}, T^I, H^I, \mathcal{G}^{\mathcal{I}}]$, defines at one loop all string quantum corrections for the gauge, Yukawa and gravitational couplings as well as the corrections to the soft supersymmetry breaking parameters.

5   Conclusions

In realistic models of spontaneously broken supergravity, the desired hierarchy $m_Z, m_{3/2} \ll M_{\mathrm{P}}$ can be stable, and eventually find a natural dynamical explanation, when quantum loop corrections to the effective potential do not contain terms quadratic in the cut-off scale M_{P}. Requiring broken supersymmetry with vacuum energy at most $\mathcal{O}(m_{3/2}^4)$ at one loop level, defines a highly non-trivial constraint on the Kähler potential K and the gauge kinetic function f_{ab}, including both the observable and the hidden sectors of the theory, as well as on the mechanism for spontaneous supersymmetry breaking. In the presence of some approximate scaling properties of the gauge and Kähler metrics the contributions to the coefficient of the quadratic divergences Q and to the soft supersymmetry breaking parameters depend only on the scaling weights of the fields λ_i, and not on the VEVs of the sliding singlet fields in the hidden sector. The expressions for Q and for the mass parameters of the MSSM in terms of λ^i find a deeper justification in the effective theories of four-dimensional superstrings, where supersymmetry breaking is described either at the string tree-level or, by assuming some non-perturbative phenomena, in the effective field theory. In these theories, the full particle content and the approximate scaling weights are completely fixed. The origin of the approximate scaling properties of the superstring effective theories is due to target-space modular invariance, and the scaling weights λ^i are nothing but the target-space duality weights with respect to the moduli fields, which participate in the supersymmetry- breaking mechanism. Indeed, in the limit of large moduli the discrete target-space duality symmetries are promoted to some accidental scaling symmetries of the gauge and matter kinetic terms in the effective supergravity theory. Thus we have identified in $Q = 0$ another criterion for a consistent choice of the supersymmetry breaking directions $\mathcal{G}^I \neq 0$ in a given 4d- superstring model. Once the breaking direction in the auxiliary field space is identified $\mathcal{G}^I \neq 0$, we can construct the IR regularized partition function at one loop in the presence of non zero magnetic field F^a, non zero curvature $\mathcal{R}$ and in terms of the expectation values of the moduli fields T^I and the Higgs fields H^I

$$Z(\mu, F^a, \mathcal{R}, T^I, H^I, \mathcal{G}^I).$$

The knowledge of the above partition function defines at one loop all string quantum corrections for the gauge, Yukawa and gravitational couplings as well as the corrections to the soft supersymmetry breaking parameters.

References

[1] E. Gildener and S. Weinberg, Phys. Rev. **D13** (1976) 3333;
S. Weinberg, Phys. Lett. **82B** (1979) 387;
G. 't Hooft, in *Recent developments in gauge theories*, G. 't Hooft et al., eds., Plenum Press, New York, 1980, p. 135.

[2] L. Maiani, Proc. Summer School on Particle Physics, Gif-sur-Yvette, 1979 (IN2P3, Paris, 1980), p. 1;
M. Veltman, Acta Phys. Polon. **B12** (1981) 437;
E. Witten, Nucl. Phys. **B188** (1981) 513;
See also: S. Weinberg, as in ref. [1].

[3] P. Fayet, Phys. Lett. **B69** (1977) 489 and **B84** (1979) 416;
S. Dimopoulos and H. Georgi, Nucl. Phys. **B193** (1981) 150.
For reviews and further references, see, e.g.:
H.-P. Nilles, Phys. Rep. **110** (1984) 1;
S. Ferrara, ed., 'Supersymmetry' (North-Holland, Amsterdam, 1987).

[4] S. Coleman and E. Weinberg, Phys. Rev. **D7** (1973) 1888;
S. Weinberg, Phys. Rev. **D7** (1973) 2887;
J. Iliopoulos, C. Itzykson and A. Martin, Rev. Mod. Phys. **47** (1975) 165.

[5] M. Veltman, as in ref. [2].

[6] M.B. Einhorn and D.R.T. Jones, Phys. Rev. **D46** (1992) 5206.

[7] S. Ferrara, L. Girardello and F. Palumbo, Phys. Rev. **D20** (1979) 403.

[8] L. Girardello and M.T. Grisaru, Nucl. Phys. **B194** (1982) 65.

[9] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. **D24** (1981)1681;
L.E. Ibáñez and G.G. Ross, Phys. Lett. **105B** (1982) 439;
J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett.**249B** (1990) 44; Nucl. Phys. **B373** (1992) 55;

U. Amaldi, W. De Boer and H. Fürstenau, Phys. Lett. **B260** (1991) 447;
P. Langacker and M. Luo, Phys. Rev. **D44** (1991) 817.

[10] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. **68** (1982) 927;
L.E. Ibáñez and C. Lopez, Phys. Lett. **126B** (1983) 94; J. Ellis, J.S. Hagelin, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. **125B** (1983) 275;
L. Alvarez-Gaumé, J. Polchinski and M.B. Wise, Nucl. Phys. **B221** (1983) 499.

[11] L.E. Ibáñez and C. Lopez, Phys. Lett. **126B** (1983) 94;
C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and M. Quirós, Phys. Lett. **132B** (1983) 95; Nucl. Phys. **B236** (1984) 438.

[12] G. Gamberini, G. Ridolfi and F. Zwirner, Nucl. Phys. **B331** (1990) 331.

[13] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Nucl. Phys. **B212** (1983) 413;
J. Bagger, Nucl. Phys. **B211** (1983) 302.

[14] S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. **B429** (1994) 589; ERRATUM, ibid. **B433** (1995) 255;
C. Kounnas, I. Pavel and F. Zwirner, Phys. Lett. **B335** (1994) 403.

[15] M.T. Grisaru, M. Rocek and A. Karlhede, Phys. Lett. **B120** (1983) 110.

[16] J. Polchinski and L. Susskind, Phys. Rev. **D26** (1982) 3661;
H.-P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. **B124** (1983) 337;
A.B. Lahanas, Phys. Lett. **B124** (1983) 341;
U. Ellwanger, Phys. Lett. **B133** (1983) 187;
J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. **B241** (1984) 406;
J. Polchinski, Proceedings of the 6th Workshop on Grand Unification, Minneapolis, April 18–20, 1985;
J. Bagger and E. Poppitz, Phys. Rev. Lett. **71** (1993) 2380.

[17] P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. **B258** (1985) 46.

[18] K.S. Narain, Phys. Lett. **B169** (1986) 41;
K.S. Narain, M.H. Sarmadi and E. Witten, Nucl. Phys. **B279** (1987) 369.

[19] L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys. **B261** (1985) 678;
K.S. Narain, M.H. Sarmadi and C. Vafa, Nucl. Phys. **B288** (1987) 551.

[20] H. Kawai, D.C. Lewellen and S.-H.H. Tye, Nucl. Phys. **B288** (1987) 1;
I. Antoniadis, C. Bachas and C. Kounnas, Nucl. Phys. **B289** (1987) 87.

[21] W. Lerche, D. Lüst and A.N. Schellekens, Nucl. Phys. **B287** (1987) 477.

[22] D. Gepner, Phys. Lett. **B199** (1987) 370; Nucl. Phys. **B296** (1988) 757.

[23] E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, Phys. Lett. **B133** (1983) 61;
J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. **B241** (1984) 406 and **B247** (1984) 373;
J. Ellis, A.B. Lahanas, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. **B134** (1984) 429.

[24] E. Witten, Phys. Lett. **B155** (1985) 151;
A. Strominger, Phys. Rev. Lett. **55** (1985) 2547;
A. Strominger and E. Witten, Comm. Math. Phys. **101** (1985) 341;
S. Ferrara, C. Kounnas and M. Porrati, Phys. Lett. **B181** (1986) 263;
C. Burgess, A. Font and F. Quevedo, Nucl. Phys. **B272** (1986) 661;
N. Seiberg, Nucl. Phys. **B303** (1988) 286;
M. Cvetic, J. Louis and B. Ovrut, Phys. Lett. **B206** (1988) 227;
S. Ferrara and M. Porrati, Phys. Lett. **B216** (1989) 1140;
M. Cvetic, J. Molera and B. Ovrut, Phys. Rev. **D40** (1989) 1140;
L. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. **B329** (1990) 27.

[25] S. Ferrara, L. Girardello, C. Kounnas and M. Porrati, Phys. Lett. **B192** (1987) 368;
I. Antoniadis, J. Ellis, E. Floratos, D.V. Nanopoulos and T. Tomaras, Phys. Lett. **B191** (1987) 96;
S. Ferrara, L. Girardello, C. Kounnas and M. Porrati, Phys. Lett. **B194** (1987) 358.

[26] S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, Phys. Lett. **B194** (1987) 366.

[27] C. Kounnas and M. Porrati, Nucl. Phys. **B310** (1988) 355.

[28] S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, Nucl. Phys. **B318** (1989) 75.

[29] See for instance: E. Kiritsis and C. Kounnas, Procedings of the Four Seas Conference, Trieste, June 1995; CERN-TH/95-224, gr-qc/9509017, and references therein.

[30] L.E. Ibàñez and H.P. Nilles, Phys. Lett. **B169** (1986) 354;
H. Itoyama and J. Leon, Phys. Rev. Lett. **56** (1986) 2352;
E. Martinec, Phys. Lett. **B171** (1986) 2352;
M. Dine and N. Seiberg, Phys. Rev. Lett. **57** (1986) 2625;
H.P. Nilles, Phys. Lett. **B180** (1986) 240;
M. Dine, N. Seiberg and E. Witten, Nucl. Phys. **B289** (1987) 589;
V.S. Kaplunovsky, Nucl. Phys. **B307** (1988) 145;
L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. **B355** (1991) 649;
J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. **B372** (1992) 145 and Phys. Lett. **B271** (1991) 307;
G. Lopez Cardoso and B.A. Ovrut, Nucl. Phys. **B369** (1992) 351; I. Antoniadis, K.S. Narain and T. Taylor, Phys. Lett. **B267** (1991) 37 and Nucl. Phys. **B383** (1992) 93.

[31] V.S. Kaplunovsky, Nucl. Phys. **B307** (1988) 145;
L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. **B355** (1991) 649.

[32] J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. **B372** (1992) 145 and Phys. Lett. **B271** (1991) 307;
G. Lopez Cardoso and B.A. Ovrut, Nucl. Phys. **B369** (1992)351;
S. Ferrara, C. Kounnas, D. Lüst and F. Zwirner, Nucl. Phys. **B365** (1991) 431.

[33] I. Antoniadis, K. Narain and T. Taylor, Phys. Lett. **B276** (1991) 37;
I. Antoniadis, E. Gava and K. Narain Phys. Lett. **B283** (1992) 209; Nucl. Phys. **B383** (1992) 93;
I. Antoniadis, E. Gava, K.S. Narain and T. Taylor, Nucl. Phys. **B407** (1993) 706; ibid. **B413** (1994) 162.

[34] E. Kiritsis and C. Kounnas, Nucl. Phys. **B41** [Proc. Suppl.] (1995) 331, (hep-th/9410212); Nucl. Phys. **B442** (1995) 472;
E. Kiritsis and C. Kounnas, hep-th/9508078.

[35] K. Kikkawa and M. Yamasaki, Phys. Lett. **B149** (1984) 357;
N. Sakai and I. Senda, Progr. Theor. Phys. **75** (1986) 692;
V.P. Nair, A. Shapere, A. Strominger and F. Wilczek, Nucl. Phys. **B287** (1987) 402;
B. Sathiapalan, Phys. Rev. Lett. **58** (1987) 1597;
R. Dijkgraaf, E. Verlinde and H. Verlinde, Commun. Math. Phys. **115** (1988) 649;
A. Giveon, E. Rabinovici and G. Veneziano, Nucl. Phys. **B322** (1989) 167;
A. Shapere and F. Wilczek, Nucl. Phys. **B320** (1989) 301;
M. Dine, P. Huet and N. Seiberg, Nucl. Phys. **B322** (1989) 301.

[36] C. Kounnas and M. Porrati, Nucl. Phys. **B310** (1988) 355;
S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, Nucl. Phys. **B318** (1989) 75;
I. Antoniadis, C. Bachas, D. Lewellen and T. Tomaras, Phys.Lett. **B207** (1988) 441;
I. Antoniadis, Phys. Lett. **B246** (1990) 377.

[37] C. Kounnas and B. Rostand, Nucl. Phys. **B341** (1990) 641;
I. Antoniadis and C. Kounnas, Phys. Lett. **B261** (1991) 369.

[38] J.-P. Derendinger, L.E. Ibàñez and H.P. Nilles, Phys. Lett. **B155** (1985) 65;
M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. **B156** (1985) 55;
C. Kounnas and M. Porrati, Phys. Lett. **B191** (1987) 91.

[39] S. Ferrara, D. Lüst, A. Shapere and S. Theisen, Phys. Lett. **B225** (1989) 363;
S. Ferrara, D. Lüst and S. Theisen, Phys. Lett. **B233** (1989) 147.

[40] A. Font, L.E. Ibàñez, D. Lüst and F. Quevedo, Phys. Lett. **B245** (1990) 401;
S. Ferrara, N. Magnoli, T.R. Taylor and G. Veneziano, Phys. Lett. **B245** (1990) 409;
H.-P. Nilles and M. Olechowski, Phys. Lett. **B248** (1990) 268;
P. Binétruy and M.K. Gaillard, Phys. Lett. **B253** (1991) 119;
V. Kaplunovsky and J. Louis, preprint UTTG-94-1, LMU-TPPW-94-1 (1994).

[41] C. Callan and F. Wilczek, Nucl. Phys. **B340** (1990) 366.

[42] C. Kounnas, Phys. Lett. **B321** (1994) 26.

[43] I. Antoniadis, S. Ferrara and C. Kounnas Nucl. Phys. **B421** (1994) 343.