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Heavy Flavour Physics

Matthias Neubert

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

The current status of the theory and phenomenology of weak decays of hadrons containing a heavy quark is reviewed. Exclusive semileptonic and rare decays of B mesons are discussed, as well as inclusive decay rates, the semileptonic branching ratio of the B meson, and the lifetimes of b-flavoured hadrons. Determinations of α_s from Υ spectroscopy are briefly presented.

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HEAVY FLAVOUR PHYSICS

MATTHIAS NEUBERT

Theory Division, CERN, CH-1211 Geneva 23, Switzerland
E-mail: neubert@nxth04.cern.ch

ABSTRACT

The current status of the theory and phenomenology of weak decays of hadrons containing a heavy quark is reviewed. Exclusive semileptonic and rare decays of B mesons are discussed, as well as inclusive decay rates, the semileptonic branching ratio of the B meson, and the lifetimes of b-flavoured hadrons. Determinations of α_s from Υ spectroscopy are briefly presented.

1. Introduction

Studies of weak decays of heavy flavours play a key role in testing the Standard Model and determining some of its parameters, which are related to flavour physics. In these decays information about the quark masses and the Cabibbo–Kobayashi–Maskawa (CKM) matrix can be obtained. A precise measurement of these parameters is a prerequisite for testing such intriguing phenomena as CP violation, and for exploring new physics beyond the Standard Model. On the other hand, weak decays of hadrons serve as a probe of that part of strong interaction physics which is least understood: the confining forces that bind quarks and gluons inside hadrons. In fact, the phenomenology of hadronic weak decays is characterized by an intricate interplay between the weak and strong interactions, which has to be disentangled before any information about Standard Model parameters can be extracted.

The recent experimental progress in heavy flavour physics has been summarized in the talks by T. Skwarnicki[1], J. Kroll[2] and S.L. Wu[3] at this Conference. Here I will present the theoretical framework for a description and interpretation of some of the data presented there. Since the discovery of heavy-quark symmetry[4−8] and the establishment of the heavy-quark expansion[9−25] this field has flourished. It is, therefore, unavoidable that I have to be selective and focus on few topics of particular interest. In this selection I was guided mainly by the relevance of a subject to current experiments. I apologize to all those authors whose work will thus be omitted here. In particular, I will not be able to report on theoretical progress in the areas of meson decay constants[26,27], exclusive nonleptonic decays of B mesons[28,29], and inclusive decay spectra in semileptonic and rare B decays[30,31], although there was a large activity devoted to these subjects. I will also have to leave out some formal developments, such as the study of renormalons in the heavy-quark effective theory[32−41].

This article is divided into two parts; the first covers exclusive semileptonic and rare decays, the second is devoted to inclusive decay rates and lifetimes. At the end, I will briefly discuss extractions of the strong coupling constant α_s from Υ spectroscopy.

2. Exclusive Semileptonic Decays

Semileptonic decays of B mesons have received a lot of attention in recent years. The decay channel $B \to D^* \ell \, \bar{\nu}$ has the largest branching fraction of all B-meson decay modes, and large data samples have been collected by various experimental groups. From the theoretical point of view, semileptonic decays are simple enough to allow for a reliable, quantitative description. Yet, the analysis of these decays provides much information about the strong forces that bind the quarks and gluons into hadrons. Schematically, a semileptonic decay process is shown in Fig. 1. The strength of the $b \to c$ transition is governed by the element V_{cb} of the CKM matrix. The entries of this matrix are fundamental parameters of the Standard Model. A primary goal of the study of semileptonic decays of B mesons is to extract with high precision the values of V_{cb} and V_{ub}. The problem is that the Standard Model Lagrangian is formulated in terms of quark and gluon fields, whereas the physical hadrons are bound states of these degrees of freedom. Thus, an understanding of the transition from the quark to the hadron world is necessary before the fundamental parameters can be extracted from experimental data.
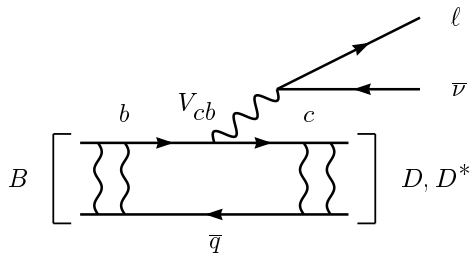

Fig. 1. Semileptonic decay of a B meson.

In the case of transitions between two heavy quarks, such as $b \to c \, \ell \, \bar{\nu}$, heavy-quark symmetry helps to eliminate (or at least reduce) the hadronic uncertainties in the theoretical description. The physical picture underlying this symmetry is the following[4−8]: In a heavy-light bound state such as a heavy meson or baryon, the typical momenta exchanged between the heavy and light constituents are of order the confinement scale Λ. The heavy quark is surrounded by a most complicated, strongly interacting cloud of light quarks, antiquarks, and gluons. However, the fact that $1/m_Q \ll 1/\Lambda$, i.e. the Compton wavelength of the heavy quark is much smaller than the size of the hadron, leads to simplifications. To resolve the quantum numbers of the heavy quark would require a hard probe; soft gluons can only resolve distances much larger than $1/m_Q$. Therefore, the light degrees of freedom are blind to the flavour (mass) and spin of the heavy quark; they only experience its colour field, which extends over large distances because of confinement. It follows that, in the $m_Q \to \infty$ limit, hadronic systems which differ only in the flavour or spin quantum

numbers of the heavy quark have the same configuration of their light degrees of freedom. Although this observation still does not allow us to calculate what this configuration is, it provides relations between the properties of such particles as the heavy mesons B, D, B^* and D^*, or the heavy baryons Λ_b and Λ_c. These relations result from new symmetries of the effective strong interactions of heavy quarks at low energies[7]. The configuration of light degrees of freedom in a hadron containing a single heavy quark with velocity v and spin s does not change if this quark is replaced by another heavy quark with different flavour or spin, but with the same velocity. For N_h heavy quark flavours, there is thus an $SU(2N_h)$ spin-flavour symmetry. Most importantly, this symmetry relates all hadronic form factors in semileptonic decays of the type $B \to D\,\ell\,\bar\nu$ and $B \to D^*\ell\,\bar\nu$ to a single universal form factor, the Isgur–Wise function, and fixes the normalization of this function at maximum q^2 (corresponding to zero recoil or equal meson velocities). Heavy-quark symmetry is an approximate symmetry, and corrections of order $\alpha_s(m_Q)$ or Λ/m_Q arise since the quark masses are not infinite. A systematic framework for analyzing them is provided by the heavy-quark effective theory (HQET)[14−16].

2.1. Determination of $|V_{cb}|$

A model-independent determination of $|V_{cb}|$ based on heavy-quark symmetry can be obtained by measuring the recoil spectrum of D^* mesons produced in $B \to D^*\ell\,\bar\nu$ decays[42]. In terms of the variable

$$w = v_B \cdot v_{D^*} = \frac{E_{D^*}}{m_{D^*}} = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}},$$ (1)

the differential decay rate reads

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3}\,(m_B - m_{D^*})^2\,m_{D^*}^3\,\sqrt{w^2 - 1}\,(w + 1)^2$$
$$\times \left[1 + \frac{4w}{w+1}\frac{m_B^2 - 2w\,m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right]|V_{cb}|^2\,\mathcal{F}^2(w).$$ (2)

The hadronic form factor $\mathcal{F}(w)$ coincides with the Isgur–Wise function up to symmetry-breaking corrections of order $\alpha_s(m_Q)$ and Λ/m_Q. The idea is to measure the product $|V_{cb}|\,\mathcal{F}(w)$ as a function of w, and to extract $|V_{cb}|$ from an extrapolation of the data to the zero-recoil point $w = 1$, where the B and the D^* mesons have a common rest frame. At this kinematic point, heavy-quark symmetry helps to calculate the normalization $\mathcal{F}(1)$ with small and controlled theoretical errors, so that the determination of $|V_{cb}|$ becomes model independent. Since the range of w values accessible in this decay is rather small ($1 < w < 1.5$), the extrapolation can be done using an expansion around $w = 1$,

$$\mathcal{F}(w) = \mathcal{F}(1)\left\{1 - \hat{\varrho}^2\,(w - 1) + \hat{c}\,(w - 1)^2 + \dots\right\}.$$ (3)

Usually a linear form of the form factor is assumed, and the slope $\widehat{\varrho}^2$ is treated as a parameter.

Measurements of the recoil spectrum have been performed first by the ARGUS[43] and CLEO[44] Collaborations in experiments operating at the $\Upsilon(4s)$ resonance, and more recently by the ALEPH[45] and DELPHI[46] Collaborations at LEP. These measurements have been discussed in detail by T. Skwarnicki[1] at this Conference. The weighted average of the results is

$$|V_{cb}|\,\mathcal{F}(1) = (35.1 \pm 1.7^{+1.4}_{-0.0}) \times 10^{-3}\,,$$
$$\widehat{\varrho}^2 = 0.87 \pm 0.16\,. \tag{4}$$

The effect of a positive curvature of the form factor has been investigated by Stone[47], who finds that the value of $|V_{cb}|\,\mathcal{F}(1)$ may change by up to +4%. This uncertainty is included by the second error quoted above.

2.1.1. Calculations of $\mathcal{F}(1)$

Heavy-quark symmetry implies that the general structure of the symmetry-breaking corrections to the form factor at zero recoil must be of the form[42]

$$\mathcal{F}(1) = \eta_A \left(1 + 0 \cdot \frac{\Lambda}{m_Q} + c_2 \frac{\Lambda^2}{m_Q^2} + \dots \right) = \eta_A \left(1 + \delta_{1/m^2} \right), \tag{5}$$

where η_A is a short-distance correction arising from a finite renormalization of the flavour-changing axial current at zero recoil, and δ_{1/m^2} parametrizes second-order (and higher) power corrections. The absence of first-order power corrections at zero recoil is a consequence of the Luke theorem[16], which is the analogue of the Ademollo–Gatto theorem[48] for heavy-quark symmetry.

The one-loop expression for η_A is known since a long time[49,6,50]:

$$\eta_A = 1 + \frac{\alpha_s(M)}{\pi} \left(\frac{m_b + m_c}{m_b - m_c} \ln \frac{m_b}{m_c} - \frac{8}{3} \right) \simeq 0.96\,. \tag{6}$$

The scale M in the running coupling constant can be fixed by adopting the prescription of Brodsky, Lepage and Mackenzie[51], where it is identified with the average virtuality of the gluon in the one-loop diagrams that contribute to η_A. If $\alpha_s(M)$ is defined in the $\overline{\rm MS}$ scheme, the result is[52] $M \simeq 0.51\sqrt{m_c m_b}$. Several estimates of higher-order corrections to η_A have been discussed. A renormalization-group resummation of mass logarithms of the type $(\alpha_s \ln m_b/m_c)^n$, $\alpha_s(\alpha_s \ln m_b/m_c)^n$ and $m_c/m_b(\alpha_s \ln m_b/m_c)^n$ leads to[53-57] $\eta_A \simeq 0.985$. On the other hand, a resummation of renormalon-chain contributions of the form $\beta_0^{n-1}\alpha_s^n$, where $\beta_0 = 11 - \frac{2}{3}n_f$ is the first coefficient of the β-function, gives[58] $\eta_A \simeq 0.945$. Using these partial resummations to estimate the uncertainty, I quote

$$\eta_A = 0.965 \pm 0.020\,. \tag{7}$$

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The accuracy of this result could be improved with an exact two-loop calculation.

An analysis of the power corrections δ_{1/m^2} is more difficult, since it cannot rely on perturbation theory. Three approaches have been discussed: in the "exclusive approach", all $1/m_Q^2$ operators in the HQET are classified and their matrix elements estimated[59,60], leading to $\delta_{1/m^2} = -(3 \pm 2)\%$; the "inclusive approach" has been used to derive the bound[61] $\delta_{1/m^2} < -3\%$, and to estimate that $\delta_{1/m^2} = -(7 \pm 3)\%$; the "hybrid approach" combines the virtues of the former two to obtain a more restrictive lower bound on δ_{1/m^2}. The result is[62]

$$\delta_{1/m^2} = -0.055 \pm 0.025 \,, \tag{8}$$

which is consistent with previous estimates. To obtain a more precise prediction, one should attempt to calculate this quantity using lattice simulations of QCD.

Combining the above results, adding the theoretical errors linearly to be conservative, gives

$$\mathcal{F}(1) = 0.91 \pm 0.04 \tag{9}$$

for the normalization of the hadronic form factor at zero recoil. This can be used to extract from the experimental result (4) the model-independent value

$$|V_{cb}| = (38.6^{+2.4}_{-1.9} \text{ exp} \pm 1.7_{\text{th}}) \times 10^{-3} \,. \tag{10}$$

After $|V_{ud}|$ and $|V_{us}|$, this is now the third-best known entry in the CKM matrix.

2.1.2. Bounds and predictions for $\hat{\varrho}^2$

The slope parameter $\hat{\varrho}^2$ in the expansion of the physical form factor in (3) differs from the slope parameter ϱ^2 of the universal Isgur–Wise function by corrections that violate the heavy-quark symmetry. The short-distance corrections have been calculated, with the result[62]

$$\hat{\varrho}^2 = \varrho^2 + (0.16 \pm 0.02) + O(1/m_Q) \,. \tag{11}$$

The slope of the Isgur–Wise function is constrained by sum rules, which relate the inclusive decay rate of the B meson to a sum over exclusive channels. At lowest order, Bjorken and Voloshin have derived two such sum rules, which imply the bounds[63−65]

$$\frac{1}{4} < \varrho^2 < \frac{1}{4} + \frac{\bar{\Lambda}}{2E_0} \simeq 0.8 \,, \tag{12}$$

where $\bar{\Lambda} = m_B - m_b$, and $E_0 = m_{B^{**}} - m_B$. Corrections to this result can be calculated in a systematic way using the Operator Product Expansion (OPE), where one introduces a momentum scale $\mu \sim \text{few} \times \Lambda$ chosen large enough so that $\alpha_s(\mu)$ and power corrections of order $(\Lambda/\mu)^n$ are small, but otherwise as small as possible to

suppress contributions from excited states[66]. The result is[67] $\varrho^2_{\min}(\mu) < \varrho^2 < \varrho^2_{\max}(\mu)$, where the boundary values are shown in Fig. 2 as a function of the scale μ. Assuming that the OPE works down to values $\mu \simeq 0.8$ GeV, one obtains rather tight bounds for the slope parameters:

$$0.5 < \varrho^2 < 0.8 \,,$$
$$0.5 < \widehat{\varrho}^2 < 1.1 \,. \tag{13}$$

The allowed region for $\widehat{\varrho}^2$ has been increased in order to account for the unknown $1/m_Q$ corrections in the relation (11). The experimental result given in (4) falls inside this region.
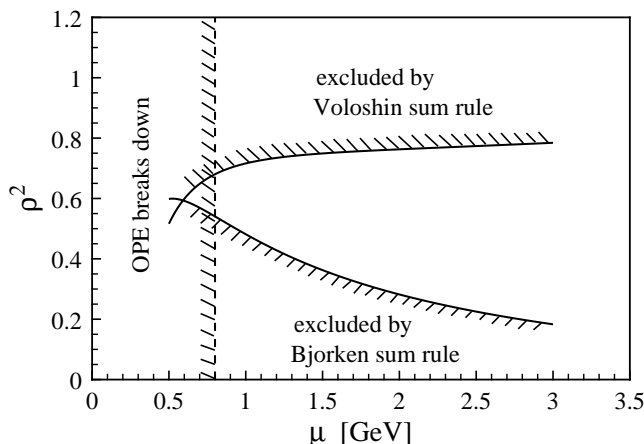

Fig. 2. Bounds for the slope parameter ϱ^2 following from the Bjorken and Voloshin sum rules.

These bounds compare well with theoretical calculations of the slope parameters. QCD sum rules have been used to calculate the slope of the Isgur–Wise function; the results obtained by various authors are $\varrho^2 = 0.84 \pm 0.02$ (Bagan *et al.*[68]), 0.7 ± 0.1 (Neubert[69]), 0.70 ± 0.25 (Blok and Shifman[70]), and 1.00 ± 0.02 (Narison[71]). The UKQCD Collaboration has presented a lattice calculation of the slope of the form factor $\mathcal{F}(w)$, yielding[72] $\widehat{\varrho}^2 = 0.9^{+0.2+0.4}_{-0.3-0.2}$. I stress that the sum rule bounds in (13) are largely model independent; model calculations in strong disagreement with these bounds should be discarded.

2.1.3. Analyticity bounds and correlations between $\widehat{\varrho}^2$ and $\widehat{c}$

A model-independent method of constraining the q^2 dependence of form factors using analyticity properties of QCD spectral functions and unitarity was proposed some time ago[73]. This method has been applied to the elastic form factor of the B meson[74−76], which is related by heavy-quark symmetry to the Isgur–Wise function. It has also been applied directly to the form factors of interest for $B \to D^{(*)}\ell\bar{\nu}$ decays[77].

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Thereby, bounds have been derived for the slope and curvature of the function $\mathcal{F}(w)$ in (3). These bounds are rather weak, however, due to the presence of B_c poles below threshold. The lack of information about the residues of these poles reduced considerably the constraining power of the method.

The problem of sub-threshold poles, which weaken the analyticity bounds, can be avoided by using heavy-quark symmetry[78]. Instead of relying on model-dependent predictions about the properties of B_c mesons, one can identify a specific $B \to D$ form factor which does not receive contributions from the ground-state B_c poles. Strong model-independent constraints on the slope and curvature of this form factor can be derived, and heavy-quark symmetry can be used to relate this form factor to the function $\mathcal{F}(w)$ describing $B \to D^* \ell \bar\nu$ decays. These relations receive symmetry-breaking corrections, which can however be estimated and turn out to weaken the bounds only softly.

For a more detailed discussion of the results of these analyses, I refer to the original literature[77,78].

2.2. Measurement of $B \to D^* \ell \bar\nu$ form factors

If the lepton mass is neglected, the differential decay distributions in $B \to D^* \ell \bar\nu$ decays can be parametrized by three helicity amplitudes, or equivalently by three independent combinations of form factors. It has been suggested that a good choice for such three quantities should be inspired by the heavy-quark expansion[8,79]. One thus defines a form factor $h_{A1}(w)$, which up to symmetry-breaking corrections coincides with the Isgur–Wise function, and two form factor ratios

$$
\begin{aligned}
R_1(w) &= \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{V(q^2)}{A_1(q^2)}, \\
R_2(w) &= \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{A_2(q^2)}{A_1(q^2)}.
\end{aligned}
\tag{14}
$$

The relation between w and q^2 has been given in (1). This definition is such that in the heavy-quark limit $R_1(w) = R_2(w) = 1$ independently of w.

To extract the functions $h_{A1}(w)$, $R_1(w)$ and $R_2(w)$ from experimental data is a complicated task. However, HQET-based calculations suggest that the w-dependence of the form factor ratios, which is induced by symmetry-breaking effects, is rather mild[79]. Moreover, the form factor $h_{A1}(w)$ is expected to have a nearly linear shape over the accessible w range. This motivates to introduce three parameters ϱ_{A1}^2, R_1 and R_2 by

$$
\begin{aligned}
h_{A1}(w) &= \mathcal{F}(1) \left\{1 - \varrho_{A1}^2 (w - 1) + O[(w - 1)^2]\right\}, \\
R_1(w) &= R_1 \left\{1 + O(w - 1)\right\}, \\
R_2(w) &= R_2 \left\{1 + O(w - 1)\right\},
\end{aligned}
\tag{15}
$$

where $\mathcal{F}(1) = 0.91 \pm 0.04$ from (9). The CLEO Collaboration has extracted these three parameters from a joint analysis of the angular distributions in $B \to D^* \ell \bar{\nu}$ decays[80]. The result is:

$$
\begin{aligned}
\varrho_{A1}^2 &= 0.91 \pm 0.15 \pm 0.06 \,, \\
R_1 &= 1.18 \pm 0.30 \pm 0.12 \,, \\
R_2 &= 0.71 \pm 0.22 \pm 0.07 \,.
\end{aligned}
\tag{16}
$$

Using the HQET, one obtains an essentially model-independent prediction for the symmetry-breaking corrections to R_1, whereas the corrections to R_2 are more model dependent. To good approximation[8]

$$
\begin{aligned}
R_1 &\simeq 1 + \frac{4\alpha_s(m_c)}{3\pi} + \frac{\bar{\Lambda}}{2m_c} \simeq 1.3 \pm 0.1 \,, \\
R_2 &\simeq 1 - \kappa \frac{\bar{\Lambda}}{2m_c} \simeq 0.8 \pm 0.2 \,,
\end{aligned}
\tag{17}
$$

with $\kappa \simeq 1$ from QCD sum rules[79]. A quark-model calculation of R_1 and R_2 gives similar results[81]: $R_1 \simeq 1.15$ and $R_2 \simeq 0.91$. The experimental data confirm the theoretical prediction that $R_1 > 1$ and $R_2 < 1$, although the errors are still large.

There is a model-independent relation between the three parameters determined from the joined angular analysis and the slope parameter $\hat{\varrho}^2$ extracted from the semileptonic spectrum. It reads[62]

$$
\varrho_{A1}^2 - \hat{\varrho}^2 = \frac{1}{6}\left(R_1^2 - 1\right) + \frac{m_B}{3(m_B - m_{D^*})}\left(1 - R_2\right).
\tag{18}
$$

The CLEO data give 0.07 ± 0.20 for the difference of the slope parameters on the left-hand side, and 0.22 ± 0.18 for the right-hand side. Both values are compatible within errors.

In my opinion the results of this analysis are very encouraging. Within errors, the experiment confirms the HQET predictions, starting to test them at the level of symmetry-breaking corrections.

2.3. Decays to charmless final states

Very recently, the CLEO Collaboration has reported a first signal for exclusive semileptonic decays of B mesons into charmless final states in the decay modes $B \to \pi \ell \bar{\nu}$ and $B \to \rho \ell \bar{\nu}$. The underlying quark process for these transitions is $b \to u \ell \bar{\nu}$. Thus, these decays provide information on the strength of the CKM matrix element V_{ub}. The observed branching fractions are[82]:

$$
\mathrm{B}(B \to \pi \ell \bar{\nu}) =
\begin{cases}
(1.34 \pm 0.45) \times 10^{-4}; & \text{ISGW}, \\
(1.63 \pm 0.57) \times 10^{-4}; & \text{BSW},
\end{cases}
$$

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$$\mathrm{B}(B \to \rho\,\ell\,\bar{\nu}) = \begin{cases} (2.28^{+0.69}_{-0.83}) \times 10^{-4}; & \text{ISGW,} \\ (3.88^{+1.15}_{-1.39}) \times 10^{-4}; & \text{BSW.} \end{cases} \tag{19}$$

There is a significant model dependence in the simulation of the reconstruction efficiencies, for which the models of Isgur, Scora, Grinstein and Wise[83] (ISGW) and Bauer, Stech and Wirbel[84] (BSW) have been used.

The theoretical description of these heavy-to-light ($b \to u$) decays is more model dependent than that for heavy-to-heavy ($b \to c$) transitions, because heavy-quark symmetry does not help to determine the relevant hadronic form factors. A variety of calculations for such form factors exists, based on QCD sum rules, lattice gauge theory, perturbative QCD, or quark models. In Table 1, I give a summary of values extracted for the ratio $|V_{ub}/V_{cb}|$ from a selection of such calculations. Clearly, some approaches are more consistent than others in that the extracted values are compatible for the two decay modes. With few exceptions, the results lie in the range

$$\left| \frac{V_{ub}}{V_{cb}} \right|_{\text{excl}} = 0.06\text{--}0.11\,, \tag{20}$$

which is in good agreement with the measurement of $|V_{ub}|$ obtained from endpoint region of the lepton spectrum in inclusive semileptonic decays[85,86]:

$$\left| \frac{V_{ub}}{V_{cb}} \right|_{\text{incl}} = 0.08 \pm 0.01_{\text{exp}} \pm 0.02_{\text{th}}\,. \tag{21}$$

Table 1. Values for $|V_{ub}/V_{cb}|$ extracted from the CLEO measurement of exclusive semileptonic B decays into charmless final states, taking $|V_{cb}| = 0.040$. An average over the experimental results in (19) is used for all except the ISGW and BSW models, where the numbers corresponding to these models are used. The first error quoted is experimental, the second (when available) is theoretical.

Method	Reference	$B \to \pi\,\ell\,\bar{\nu}$	$B \to \rho\,\ell\,\bar{\nu}$
QCD sum rules	Narison[87]	$0.159 \pm 0.019 \pm 0.001$	$0.066^{+0.007}_{-0.009} \pm 0.003$
	Ball[88]	$0.105 \pm 0.013 \pm 0.011$	$0.094^{+0.010}_{-0.012} \pm 0.016$
	Yang, Hwang[89]	$0.102 \pm 0.012^{+0.015}_{-0.013}$	$0.184^{+0.020+0.027}_{-0.024-0.015}$
lattice QCD	UKQCD[90]	$0.103 \pm 0.012^{+0.012}_{-0.010}$	—
	APE[91]	$0.084 \pm 0.010 \pm 0.021$	—
pQCD	Li, Yu[92]	0.054 ± 0.006	—
quark models	BSW[84]	0.093 ± 0.016	$0.076^{+0.011}_{-0.014}$
	KS[93]	0.088 ± 0.011	$0.056^{+0.006}_{-0.007}$
	ISGW2[94]	0.074 ± 0.012	$0.079^{+0.012}_{-0.014}$

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Clearly, this is only the first step towards a more reliable determination of $|V_{ub}|$; yet, with the discovery of exclusive $b \to u$ transitions an important milestone has been met. Efforts must now concentrate on more reliable methods to determine the form factors for heavy-to-light transitions. Some new ideas in this direction have been discussed recently. They are based on lattice calculations[95], analyticity constraints[76,96], or variants of the form-factor relations for heavy-to-heavy transitions[97].

3. Exclusive rare radiative decays

Rare decays of B mesons play an important role in testing the Standard Model, as they are sensitive probes of new physics at high energy scales. On the quark level, rare decays involve flavour-changing neutral currents such as $b \to s\gamma$ or $b \to d\,\ell^+\ell^-$. In the Standard Model they are forbidden at the tree level, but can proceed at the one-loop level through penguin or box diagrams, see Fig. 3.
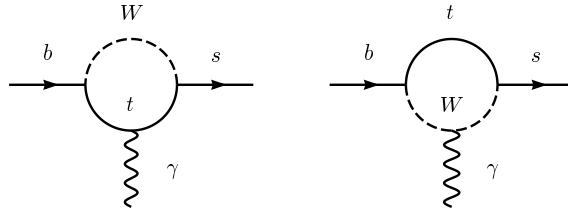

Fig. 3. Penguin diagrams for the quark transition $b \to s\gamma$.

The effective Hamiltonian describing the rare radiative decay $b \to s\gamma$ is

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb}\, c_7(\mu)\, \frac{e\, m_b}{8\pi^2}\, \bar{s}\sigma_{\mu\nu}(1+\gamma_5)b\, F^{\mu\nu} + \dots . \tag{22}$$

The Wilson coefficient $c_7(\mu)$ contains the short-distance physics of the heavy particles in the loop (t and W in the Standard Model). Its value is sensitive to new physics, such as the existence of charged Higgs bosons, which can in principle be probed by measuring the inclusive decay rates for $B \to X_{s,d}\gamma$. The uncertainty in the calculation of $c_7(\mu)$ in the Standard Model is still of order[98,99] $\pm 15\%$, however, reducing significantly the constraining power of such measurements.

The study of exclusive rare decays focuses on ratios such as

$$R_{K^*} = \frac{\Gamma(B \to K^*\gamma)}{\Gamma(B \to X_s\gamma)}, \qquad R_\rho = \frac{\Gamma(B \to \rho\,\gamma)}{\Gamma(B \to X_d\gamma)}, \tag{23}$$

which are no longer sensitive to new physics (since the coefficient $c_7(\mu)$ cancels out), but test some strong interaction matrix elements. The measurement reported by the CLEO Collaboration[100],

$$R_{K^*} = (19 \pm 7 \pm 4)\%, \tag{24}$$

can be confronted with theoretical predictions, which have however a wide spread. QCD sum-rule calculations lead to results in the range[101−104] $R_{K^*} = (17 \pm 5)\%$, while quark model predictions range between[105] 4% and 30%. Lattice simulations of the relevant form factors have been performed over a limited range in q^2 only, and the results for R_{K^*} depend rather strongly on the assumption about the q^2 dependence outside this region. Studies of the various groups[90,106−108] have led to values between 5% and 35%. More work is needed to reduce the systematic uncertainties in these calculations.

In the theoretical analyses described above, it is assumed that $B \to K^* \gamma$ decays are short-distance dominated. This assumption has been questioned recently by Atwood, Blok and Soni[109], who pointed out the possibility of large long-distance contributions in the decay $B \to K^* \gamma$, and even more so in the decay $B \to \rho \gamma$. Examples of such long-distance contributions are shown in Fig. 4. The first graph shows a "long-distance penguin" diagram, in which the c-quark in the loop is close to its mass shell. The $c\bar{c}$ pair forms a virtual vector meson state ψ^*, which then decays into a photon. The second graph shows the "weak annihilation" of the quark and antiquark in the B meson. For $B \to K^* \gamma$, this process is CKM suppressed with respect to the penguin diagram, but this suppression is not operative for $B \to \rho \gamma$.
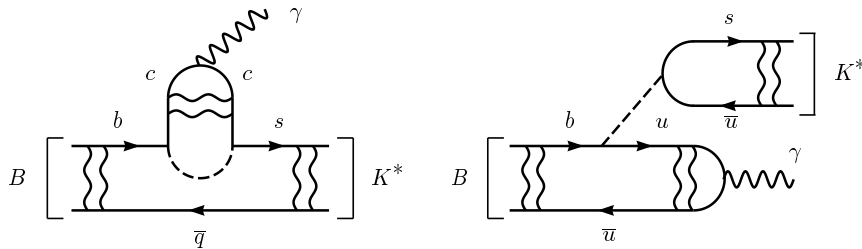

Fig. 4. Long-distance contributions to rare radiative decays.

Estimates of these long-distance contributions are difficult and currently controversial[110−116]. For $B \to K^* \gamma$, predictions for the ratio of the long- and short-distance amplitudes, $|A_{\mathrm{ld}}/A_{\mathrm{sd}}|$, range from 15–50% to < 10%. In $B^- \to \rho^- \gamma$ decays, most authors expect long-distance effects at a level of 10–30%, whereas the effects are much smaller, $\sim$ 1–10%, in the neutral channel $B^0 \to \rho^0 \gamma$. Further investigation of this important subject is necessary before a conclusion can be drawn. A clarification of this issue is also important with regard to a determination of the ratio of CKM elements $|V_{td}/V_{ts}|$ from the comparison of the decay rates for $B \to \rho \gamma$ and $B \to K^* \gamma$.

4. Inclusive decay rates

Inclusive decay rates determine the probability of the decay of a particle into the sum of all possible final states with given quantum numbers. The theoretical framework to describe inclusive decays of heavy flavours is provided by the $1/m_Q$

expansion[17−25], which is a "Minkowskian version" of the OPE. This means that the theoretical treatment of inclusive rates has a solid foundation in QCD, however with one assumption: that of quark–hadron duality. In the description of semileptonic decays (e.g. $B \to \ell\,\bar{\nu} + $ hadrons), where the integration over the lepton and neutrino phase space provides a "smearing" over the invariant hadronic mass, so-called "global" duality is needed[117], whereas the treatment of nonleptonic decays (e.g. $B \to$ hadrons), for which the total hadronic mass is fixed, requires the stronger assumption of local duality. It is important to stress that quark–hadron duality cannot be derived from QCD, although it is a common assumption in QCD phenomenology.

The main results of the $1/m_Q$ expansion for inclusive decays are that the free quark decay (i.e. the parton model) provides the first term in a systematic $1/m_Q$ expansion, and the nonperturbative corrections to it are suppressed by (at least) two powers of the heavy quark mass, i.e. they are of relative order $(\Lambda/m_Q)^2$. The generic expression of any inclusive decay rate of a hadron H_Q containing a heavy quark into some final state with quantum numbers f is of the form[18−20,118,119]

$$\Gamma(H_Q \to X_f) = \frac{G_F^2 m_Q^5}{192\pi^3}\,|\mathrm{KM}|^2 \left\{ c_3^f \left(1 - \frac{\langle \bar{Q}(i\vec{D})^2 Q\rangle_H}{2m_Q^2}\right) \right.$$
$$\left. + c_5^f\,\frac{\langle \bar{Q}g_s\sigma_{\mu\nu}G^{\mu\nu}Q\rangle_H}{m_Q^2} + \sum_i c_{6,i}^f\,\frac{\langle \bar{Q}\Gamma_i q\bar{q}\Gamma_i Q\rangle_H}{m_Q^3} + \ldots \right\}, \quad (25)$$

where $|\mathrm{KM}|$ is a combination of CKM matrix elements, c_n^f are calculable coefficient functions, and $\langle O_n\rangle_H$ are the (normalized) forward matrix elements of local operators between H_Q states. The matrix elements of the dimension-five operators are[59]

$$\langle \bar{Q}(i\vec{D})^2 Q\rangle_H = -\lambda_1 = \mu_\pi^2\,,$$
$$\langle \bar{Q}g_s\sigma_{\mu\nu}G^{\mu\nu}Q\rangle_H = 2d_H\lambda_2\,, \quad (26)$$

where $d_P = 3$, $d_V = -1$ and $\lambda_2 = \frac{1}{4}(m_{B^*}^2 - m_B^2) \simeq 0.12$ GeV^2 for the ground state pseudoscalar (P_Q) and vector (V_Q) mesons, and $d_\Lambda = 0$ for the Λ_Q baryon. The matrix element of the "kinetic energy operator", $\mu_\pi^2 = -\lambda_1$, has been estimated by several authors[120−122]; below I shall use the value $-\lambda_1 = (0.4 \pm 0.2)$ GeV^2 with a conservative error. Meson matrix elements of the dimension-six operator in (25) can be related, in the vacuum saturation approximation[123], to the decay constant f_B of the B meson. I shall now discuss the most important applications of this general formalism to inclusive decays of b-flavoured mesons and baryons.

4.1. Determination of $|V_{cb}|$ from inclusive semileptonic decays

The extraction of $|V_{cb}|$ from the inclusive semileptonic decay rate of the B meson is based on the expression[18−20]

$$\Gamma(B \to X_c \ell\,\bar{\nu}) = \frac{G_F^2 m_b^5}{192\pi^3}\,|V_{cb}|^2 \left\{\left(1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2}\right) f(m_c/m_b)\right.$$

$$-\frac{6\lambda_2}{m_b^2}\left(1-\frac{m_c^2}{m_b^2}\right)^4 + \frac{\alpha_s(M)}{\pi} g(m_c/m_b) + \ldots \Bigg\}, \qquad (27)$$

where m_b is the pole mass of the b quark (defined to a given order in perturbation theory), and $f(x)$ and $g(x)$ are phase space functions given elsewhere[124]. The theoretical uncertainties in this determination of $|V_{cb}|$ are quite different from the ones entering the analysis of exclusive decays. In inclusive decays there appear the quark masses rather than the meson masses. Moreover, the theoretical description relies on the assumption of global quark–hadron duality, which is not necessary for exclusive decays. I will now discuss the theoretical uncertainties in detail.

4.1.1. Nonperturbative corrections

The nonperturbative corrections are very small; with $-\lambda_1 = (0.4\pm0.2)$ GeV^2 and $\lambda_2 = 0.12$ GeV^2, one finds a reduction of the parton model rate by $-(4.2\pm0.5)\%$. The uncertainty in this number is below 1% and thus completely negligible.

4.1.2. Dependence on quark masses

Although $\Gamma \sim m_b^5\, f(m_c/m_b)$, the dependence on m_b becomes milder if one chooses m_b and $\Delta m = m_b - m_c$ as independent variables. This is apparent from Fig. 5, which shows that $\Gamma \sim m_b^{2.3}\,\Delta m^{2.7}$. Moreover, these variables have essentially uncorrelated theoretical uncertainties. Whereas $m_b = m_B - \bar\Lambda + \ldots$ is mainly determined by the $\bar\Lambda$ parameter of the HQET[125], the mass difference Δm obeys the expansion[59]

$$\Delta m = (\bar m_B - \bar m_D)\left\{1 + \frac{(-\lambda_1)}{2\bar m_B\bar m_D} + \ldots\right\} = (3.40\pm0.03\pm0.03)\ \text{GeV}, \qquad (28)$$

i.e. it is sensitive to the kinetic energy parameter λ_1. Here $\bar m_B = 5.31$ GeV and $\bar m_D = 1.97$ GeV denote the "spin-averaged" meson masses, e.g. $\bar m_B = \frac{1}{4}(m_B + 3m_{B^*})$.

I think that theoretical uncertainties of 60 MeV on Δm and 200 MeV on m_b are reasonable; values much smaller than this are probably too optimistic. This leads to

$$\left(\frac{\delta\Gamma}{\Gamma}\right)_{\text{masses}} = \sqrt{\left(0.10\,\frac{\delta m_b}{200\ \text{MeV}}\right)^2 + \left(0.05\,\frac{\delta\Delta m}{60\ \text{MeV}}\right)^2} \simeq 11\%. \qquad (29)$$

4.1.3. Perturbative corrections

This is the most subtle part of the analysis. The semileptonic rate is known exactly only to order[124] α_s, though a partial calculation of the coefficient of order α_s^2 exists[126]. The result is

$$\frac{\Gamma}{\Gamma_{\text{tree}}} = 1 - 1.67\,\frac{\alpha_s(m_b)}{\pi} - (1.68\beta_0 + \ldots)\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 + \ldots. \qquad (30)$$
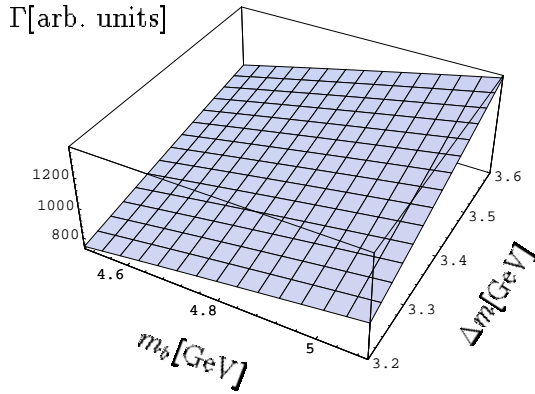
Fig. 5. Dependence of the inclusive semileptonic decay rate on m_b and $\Delta m = m_b - m_c$.

The one-loop correction is moderate; it amounts to about -11%. Of the two-loop coefficient, only the part proportional to the β-function coefficient $\beta_0 = 11 - \frac{2}{3}n_f$ is known. For $n_f = 3$ light quark flavours, this term is $1.68\beta_0 \simeq 15.1$ and gives rise to a rather large correction of about -6%. One may take this as an estimate of the perturbative uncertainty. The dependence of the result on the choice of the renormalization scale and scheme has been investigated, too, and found to be of order[127] 6%.

Yet, the actual perturbative uncertainty may be larger than that. A subset of higher-order corrections, the so-called renormalon contributions of the form $\beta_0^{n-1}\alpha_s^n$, can be summed to all orders in perturbation theory, leading to[41] $\Gamma/\Gamma_{\text{tree}} = 0.77 \pm 0.05$, which is equivalent to choosing the rather low scale $M \simeq 1$ GeV in (27). This estimate is 12% lower than the one-loop result.

These considerations show that there are substantial perturbative uncertainties in the calculation of the semileptonic decay rate. They could only be reduced with a complete two-loop calculation, which is however quite a formidable task. At present, I consider $(\delta\Gamma/\Gamma)_{\text{pert}} \simeq 10\%$ a reasonable estimate.

4.1.4. Result for $|V_{cb}|$

Adding, as previously, the theoretical errors linearly and taking the square root, I find

$$\frac{\delta|V_{cb}|}{|V_{cb}|} \simeq 10\% \tag{31}$$

for the theoretical uncertainty in the determination of $|V_{cb}|$ from inclusive decays, keeping in mind that in addition this method relies on the assumption of global duality. Taking the result of Ball et $al.$[41] for the central value, I quote

$$|V_{cb}| = (0.0398 \pm 0.0040) \left(\frac{B_{\text{SL}}}{10.77\%}\right)^{1/2} \left(\frac{\tau_B}{1.6 \text{ ps}}\right)^{-1/2}. \tag{32}$$

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Using the new world averages for the semileptonic branching ratio[1], $B_{\mathrm{SL}} = (10.77 \pm 0.43)\%$, and for the average B meson lifetime[2], $\tau_B = (1.60 \pm 0.03)$ ps, I obtain

$$|V_{cb}| = (39.8 \pm 0.9_{\mathrm{exp}} \pm 4.0_{\mathrm{th}}) \times 10^{-3}\,, \tag{33}$$

which is is excellent agreement with the measurement in exclusive decays reported in (10). This agreement is gratifying given the differences of the methods used, and it provides an indirect test of global quark–hadron duality. Combining the two measurements gives the final result

$$|V_{cb}| = 0.039 \pm 0.002\,. \tag{34}$$

4.2. Semileptonic branching ratio and charm counting

The semileptonic branching ratio of the B meson is defined as

$$B_{\mathrm{SL}} = \frac{\Gamma(B \to X\, e\, \bar{\nu})}{\sum_\ell \Gamma(B \to X\, \ell\, \bar{\nu}) + \Gamma_{\mathrm{NL}} + \Gamma_{\mathrm{rare}}}\,, \tag{35}$$

where Γ_{NL} and Γ_{rare} are the inclusive rates for nonleptonic and rare decays, respectively, the latter being negligible. The main difficulty in calculating B_{SL} is not in the semileptonic width, but in the nonleptonic one. As mentioned above, the calculation of nonleptonic decays in the $1/m_Q$ expansion relies on the strong assumption of local quark–hadron duality.

Measurements of the semileptonic branching ratio have been performed in various experiments, using both model-dependent and model-independent analyses. The situation has been summarized by T. Skwarnicki[1] at this Conference. The new world average is

$$B_{\mathrm{SL}} = (10.77 \pm 0.43)\%\,. \tag{36}$$

An important aspect in understanding this result is charm counting, i.e. the measurement of the average number n_c of charm hadrons produced per B decay. The CLEO Collaboration has presented a new result for n_c, which is[1,128]

$$n_c = 1.16 \pm 0.05\,. \tag{37}$$

In the naive parton model, one finds[129] $B_{\mathrm{SL}} \sim 15$–16% and $n_c \simeq 1.15$–1.20. Whereas n_c is in agreement with experiment, the semileptonic branching ratio is predicted too large. With the establishment of the $1/m_Q$ expansion, the nonperturbative corrections to the parton model could be computed and turned out too small to improve the prediction. This lead Bigi *et al.* to conclude that values $B_{\mathrm{SL}} < 12.5\%$ cannot be accommodated by theory, thus giving rise to a puzzle referred to as the "baffling semileptonic branching ratio"[130]. The situation has changed recently, however. Bagan *et al.* found indications that higher-order perturbative corrections lower

the value of $B_{\rm SL}$ significantly[131]. The exact order-α_s corrections to the nonleptonic width have been computed for $m_c \neq 0$, and an analysis of the renormalization scale and scheme dependence has been performed. In particular, it turns out that radiative corrections increase the partial width $\Gamma(b \to c\bar{c}s)$. This has two effects: it lowers the semileptonic branching ratio, but at the price of a higher value of n_c. The results in two popular renormalization schemes are[132]

$$
B_{\rm SL} = \begin{cases} 12.1 \pm 0.7^{+0.9}_{-1.2}\%; & \text{on-shell scheme}, \\ 11.7 \pm 0.7^{+0.9}_{-1.3}\%; & \overline{\rm MS} \text{ scheme}, \end{cases}
$$

$$(38)$$

$$
n_c = 1.21 \mp 0.04 \mp 0.01; \quad \text{both schemes}.
$$

The errors in the two quantities are anti-correlated. The first error reflects the uncertainties in the input parameters, whereas the second one shows the dependence on the renormalization scale, which is varied in the range $m_b/2 < \mu < 2m_b$. Lowering μ decreases the value of $B_{\rm SL}$ and vice versa. Note that using a low renormalization scale is not unnatural; Luke et $al.$ have estimated that $\mu \simeq 0.3 m_b$ is an appropriate scale in this case[126]. Values $B_{\rm SL} < 12\%$ can thus easily be accommodated. Only a complete order-α_s^2 calculation could reduce the perturbative uncertainties.
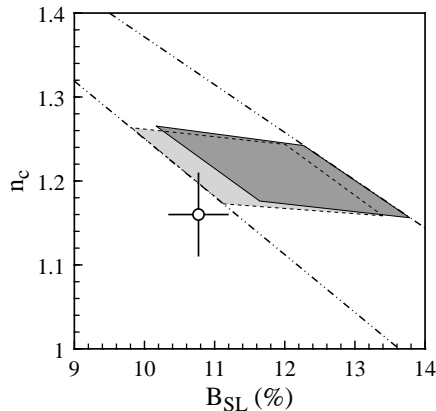

Fig. 6. Correlation between the semileptonic branching ratio and charm counting. The dark area is the theoretically allowed region in the on-shell scheme, whereas the light area refers to the $\overline{\rm MS}$ scheme[132]. The dash-dotted lines indicate the allowed region if the calculation of $\Gamma(b \to c\bar{c}s)$ is ignored and this partial rate is treated as a free parameter[133]. The data point shows the world average for $B_{\rm SL}$ and the new CLEO result for n_c presented at this Conference.

The above discussion shows that it is the combination of a low semileptonic branching ratio and a low value of n_c that constitutes a potential problem. This is illustrated in Fig. 6, which is an updated version of a figure shown in a recent work of Buchalla et $al.$[133]. With the new experimental and theoretical results for $B_{\rm SL}$ and n_c, only a small discrepancy remains between theory and experiment. It has been argued that the current experimental value of n_c may depend on model assumptions about

the production of charm hadrons, which are sometimes questionable[133,134]. Another possibility, which has been pointed out by Palmer and Stech[135] and others[136−138], is that local quark–hadron duality could be violated in nonleptonic B decays. If so, this will most likely happen in the channel $b \to c\bar{c}s$, where the energy release, $E = m_B - m_{X(c\bar{c}s)}$, is of order 1.5 GeV or smaller. If the discrepancy between theory and experiment persists, this possibility should be taken seriously before a "new physics" explanation[139,140] is advocated.

For completeness, I briefly discuss the semileptonic branching ratio for B decays into a τ lepton, which is suppressed by phase space. The ratio of the semileptonic rates for decays into τ leptons and into electrons can be calculated reliably. The result is[141]

$$B(B \to X\,\tau\,\bar{\nu}_\tau) = (2.32 \pm 0.23)\% \times \frac{B_{\mathrm{SL}}}{10.77\%} = (2.32 \pm 0.25)\%\,. \qquad (39)$$

It is in good agreement with the new world average[1]

$$B(B \to X\,\tau\,\bar{\nu}_\tau) = (2.60 \pm 0.32)\%\,. \qquad (40)$$

4.3. Lifetime ratios of b-hadrons

The $1/m_Q$ expansion predicts that the lifetimes of all b-flavoured hadrons agree up to nonperturbative corrections suppressed by at least two powers of $1/m_b$. This prediction can be tested with new high precision data, which have been summarized by J. Kroll[2] at this Conference.

4.3.1. Lifetime ratio for B^- and B^0

The lifetimes of the charged and neutral B mesons differ at order $1/m_b^3$ in the heavy quark expansion. The corresponding corrections arise from effects referred to as interference and weak annihilation[142,143]. They are illustrated in Fig. 7. In the operator language, these spectator effects are represented by hadronic matrix elements of local four-quark operators of the type

$$\langle \bar{b}\Gamma q\bar{q}\Gamma b\rangle_B \sim f_B^2\, m_B \sim \Lambda^3\,, \qquad (41)$$

where the vacuum insertion approximation[123] has been used. It turns out that interference gives rise to the dominant corrections (weak annihilation is strongly CKM suppressed), which decrease the decay rate for B^-, i.e. enhance its lifetime. The result is[119]

$$\Delta\Gamma_{\mathrm{int}}(B^-) = \frac{G_F^2 m_b^5}{192\pi^3}\,|V_{cb}|^2\,16\pi^2\,\frac{f_B^2\,m_B}{m_b^3}\,\zeta_{\mathrm{QCD}}\,, \qquad (42)$$

where
$$\zeta_{\mathrm{QCD}} = 2c_+^2(m_b) - c_-^2(m_b) = \begin{cases} 1; & \text{at tree level,} \\ -0.6; & \text{with QCD corrections.} \end{cases} \tag{43}$$

After including short-distance corrections to the four-fermion interactions the interference becomes destructive. The numerical result is

$$\frac{\tau(B^-)}{\tau(B^0)} \simeq 1 + 0.04 \left(\frac{f_B}{180 \text{ MeV}} \right)^2, \tag{44}$$

consistent with the experimental value[2]

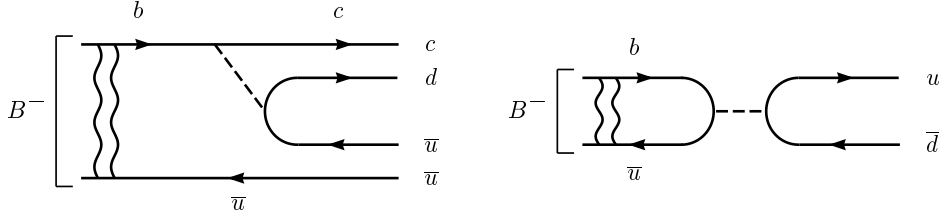$$\frac{\tau(B^-)}{\tau(B^0)} = 1.02 \pm 0.04. \tag{45}$$

Fig. 7. Interference and weak-annihilation contributions to the lifetime of the B^- meson. The interference effect in the first diagram arises from the presence of two identical $\bar{u}$ quarks in the final state.

The theoretical prediction depends on the vacuum saturation assumption[123], which has been criticized. Chernyak has estimated that nonfactorizable contributions can be as large as 50% of the factorizable ones[144]. Another important observation is the following one: in nonleptonic decays, spectator effects appearing at order $1/m_b^3$ are enhanced by a factor $16\pi^2$ resulting from the two-body versus three-body phase space. In fact, the scale of the correction in (42) is unexpectedly large:

$$16\pi^2 \frac{f_B^2\, m_B}{m_b^3} \simeq \left(\frac{4\pi f_B}{m_b} \right)^2 \simeq 0.2. \tag{46}$$

The presence of this phase-space enhancement factor leads to a peculiar structure of the $1/m_Q$ expansion for nonleptonic rates, which can be displayed as follows:

$$\Gamma \sim \Gamma_0 \left\{ 1 + \left(\frac{\Lambda}{m_Q} \right)^2 + \left(\frac{\Lambda}{m_Q} \right)^3 + \ldots + 16\pi^2 \left[\left(\frac{\Lambda}{m_Q} \right)^3 + \left(\frac{\Lambda}{m_Q} \right)^4 + \ldots \right] \right\}. \tag{47}$$

Numerically, the terms of order $16\pi^2 \, (\Lambda/m_Q)^3$ are more important than the ones of order $(\Lambda/m_Q)^2$. I draw two conclusion from this observation: it is important to include this type of $1/m_b^3$ corrections to all predictions for nonleptonic rates; there is a challenge to calculate the hadronic matrix elements of four-quark operators with

high accuracy. Lattice calculations could help to improve the existing estimates of such matrix elements.

4.3.2. Lifetime ratio for B_s and B_d

The lifetimes of the two neutral mesons B_s and B_d differ by corrections that are due to spectator effects referred to as W exchange. They are smaller than the interference effects discussed above. The theoretical prediction is[119]

$$\frac{\tau(B_s)}{\tau(B_d)} = 1 \pm O(1\%) \,, \tag{48}$$

consistent with the experimental value[2]

$$\frac{\tau(B_s)}{\tau(B_d)} = 1.01 \pm 0.07 \,. \tag{49}$$

Note that $\tau(B_s)$ denotes the average lifetime of the two B_s states.

4.3.3. Lifetime ratio for Λ_b and B^0

Although differences between the lifetimes of heavy mesons and baryons start at order $1/m_b^2$, the main effects come again at order $1/m_b^3$. However, here one encounters the problem that the matrix elements of four-quark operators are needed not only between meson states (where the vacuum saturation approximation may be used), but also between baryon states. Very little is known about such matrix elements. Bigi *et al.* have adopted a simple nonrelativistic quark model and conclude that[119]

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = 0.90\text{–}0.95 \,. \tag{50}$$

The experimental result for this ratio is significantly lower[2]:

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = 0.76 \pm 0.05 \,. \tag{51}$$

In my opinion, a dedicated theoretical effort to understand this result is desirable. In view of the above discussion, one should first question (and improve) the calculation of baryonic matrix elements of four-quark operators, then question the vacuum saturation approximation, and finally question the validity of local quark–hadron duality.

5. Determinations of α_s from Υ spectroscopy

Before summarizing, I want to touch upon a topic not related to B decays. The large mass of the b quark makes it possible to describe the spectrum and properties

of $(b\bar{b})$ bound states with high accuracy, using the heavy-quark expansion. From a comparison with experiment, it is then possible to extract the strong coupling constant α_s. Analyses of this type have been performed based on lattice calculations and QCD sum rules. The current status of the determination of α_s from calculations of the Υ spectrum in lattice gauge theory has been summarized by C. Michael[145] at this Conference. When translated into a value of $\alpha_s(m_Z)$ in the $\overline{\text{MS}}$ scheme, a conservative result is[146,147]

$$\alpha_s(m_Z) = 0.112 \pm 0.007 \,. \tag{52}$$

A more precise value, $\alpha_s(m_Z) = 0.115 \pm 0.002$, has been reported by the NRQCD Collaboration[148]; the small error has been criticized, however[146,147].

Voloshin has performed an analysis of the Υ spectrum using QCD sum rules, including a resummation of large Coulomb corrections to all orders in perturbation theory[149]. He quotes $\alpha_s(1\text{ GeV}) = 0.336 \pm 0.011$, which translates into

$$\alpha_s(m_Z) = 0.109 \pm 0.001 \,. \tag{53}$$

The very small error may have been underestimated. It is important to understand better the sources of theoretical uncertainty before this result can be trusted.

Despite such reservations, it looks promising that ultimately the Υ system may provide one of the best ways to measure α_s with high precision at low energies.

6. Summary and conclusions

I have reviewed the status of the theory of weak decays of heavy flavours, concentrating on topics relevant to current experiments. Weak decays play a unique role in testing the Standard Model at low energies. Ultimately, a precise determination of the parameters of the flavour sector (elements of the Cabibbo–Kobayashi–Maskawa matrix and quark masses) will help to explore such intricate phenomena as CP violation, and is crucial in searches for new physics beyond the Standard Model.

Exclusive semileptonic decays mediated by the heavy-quark transition $b \to c\,\ell\,\bar{\nu}$ are of particular importance, as they allow for a model-independent description provided by heavy-quark symmetry and the heavy-quark effective theory. These concepts can now be tested with detailed measurements of the form factors in the decay $B \to D^*\ell\,\bar{\nu}$. The most striking result of the analysis of this decay is a very precise determination of the strength of $b \to c$ transitions: $|V_{cb}| = (38.6 \pm 2.5) \times 10^{-3}$. In the future, studies of the related decays $B \to D\,\ell\,\bar{\nu}$, $B_s \to D_s^{(*)}\ell\,\bar{\nu}$ and $\Lambda_b \to \Lambda_c\ell\,\bar{\nu}$ may provide further tests of heavy-quark symmetry and teach us about the dependence of the Isgur–Wise function on the flavour and spin quantum numbers of the light degrees of freedom in a heavy hadron.

The discovery of exclusive semileptonic decays into charmless final states is a milestone on the way towards a precise determination of $|V_{ub}|$. Currently, new theoretical

ideas are being discussed, and existing approaches are being refined, which may help to get a better handle on the calculation of heavy-to-light transition form factors. These form factors also appear in the description of rare radiative decays, which are mediated by flavour-changing neutral currents and in the Standard Model proceed through loop (penguin) diagrams.

The second part of my talk was devoted to inclusive decays of b-flavoured hadrons. The theoretical description of inclusive rates is based on the $1/m_Q$ expansion, which is a "Minkowskian" version of the operator product expansion. It can be derived from QCD if one accepts the hypothesis of quark–hadron duality. Duality is an important concept in QCD phenomenology, which however cannot be derived yet from first principles, so it needs to be tested. The measurement of the inclusive semileptonic decay rate of the B meson provides an alternative way to determine $|V_{cb}|$, which leads to $|V_{cb}| = (39.8 \pm 4.1) \times 10^{-3}$, in excellent agreement with the value obtained from exclusive decays. This agreement provides an indirect test of global quark–hadron duality. The theoretical calculation of the semileptonic branching ratio, B_{SL}, suffers from a sizable renormalization-scheme dependence, which does not allow to state a discrepancy between the data and theory for B_{SL} alone. However, the combination of a low value of B_{SL} and a low value of n_c, the number of charm hadrons per B decay, remains a problem that deserves further investigation.

A particularly clean test of the heavy-quark expansion is provided by the study of lifetime ratios of b-flavoured hadrons. This tests the assumption of local quark–hadron duality, as well as our capability to evaluate the hadronic matrix elements of the operators appearing in the $1/m_b$ expansion. For the lifetime ratios $\tau(B_d)/\tau(B_s)$ and $\tau(B^-)/\tau(B^0)$ there is good agreement between theory and experiment, although the data have not yet reached the precision required to perform a stringent test of the theoretical predictions. However, in the case of the lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ there are indications for a significant discrepancy; the data indicate much larger power corrections than anticipated by theory. A better understanding of this discrepancy, if it persists, is most desirable.

Given the limitations in space and time, the material covered in this talk is only a selection of current topics in heavy flavour physics. With continuous advances on the theoretical front, and with ongoing experimental efforts and the construction of new facilities (B factories), this field will remain of great interest and will continue to provide us with new exciting results in the future.

References:

1. T. Skwarnicki, these Proceedings.
2. I.J. Kroll, these Proceedings.
3. S.L. Wu, these Proceedings.
4. E.V. Shuryak, Phys. Lett. B **93**, 134 (1980); Nucl. Phys. B **198**, 83 (1982).
5. S. Nussinov and W. Wetzel, Phys. Rev. D **36**, 130 (1987).
6. M.B. Voloshin and M.A. Shifman, Yad. Fiz. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 292 (1987)]; **47**, 801 (1988) [**47**, 511 (1988)].
7. N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
8. M. Neubert, Phys. Rep. **245**, 259 (1994).
9. E. Eichten and F. Feinberg, Phys. Rev. D **23**, 2724 (1981); E. Eichten, in: Field Theory on the Lattice, edited by A. Billoire *et al.*, Nucl. Phys. B (Proc. Suppl.) **4**, 170 (1988).
10. W.E. Caswell and G.P. Lepage, Phys. Lett. B **167**, 437 (1986); G.P. Lepage and B.A. Thacker, in: Field Theory on the Lattice, edited by A. Billoire *et al.*, Nucl. Phys. B (Proc. Suppl.) **4**, 199 (1988).
11. H.D. Politzer and M.B. Wise, Phys. Lett. B **206**, 681 (1988); **208**, 504 (1988).
12. E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990); **243**, 427 (1990).
13. B. Grinstein, Nucl. Phys. B **339**, 253 (1990).
14. H. Georgi, Phys. Lett. B **240**, 447 (1990).
15. A.F. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nucl. Phys. B **343**, 1 (1990); A.F. Falk, B. Grinstein and M.E. Luke, *ibid.* **357**, 185 (1991).
16. M.E. Luke, Phys. Lett. B **252**, 447 (1990).
17. J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B **247**, 399 (1990).
18. I.I. Bigi, N.G. Uraltsev and A.I. Vainshtein, Phys. Lett. B **293**, 430 (1992) [E: **297**, 477 (1993)]; I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993); I.I. Bigi *et al.*, in: Proceedings of the Annual Meeting of the Division of Particles and Fields of the APS, Batavia, Illinois, 1992, edited by C. Albright *et al.* (World Scientific, Singapore, 1993), p. 610.
19. B. Blok, L. Koyrakh, M.A. Shifman and A.I. Vainshtein, Phys. Rev. D **49**, 3356 (1994) [E: **50**, 3572 (1994)].
20. A.V. Manohar and M.B. Wise, Phys. Rev. D **49**, 1310 (1994).
21. M. Luke and M.J. Savage, Phys. Lett. B **321**, 88 (1994); A.F. Falk, M. Luke and M.J. Savage, Phys. Rev. D **49**, 3367 (1994).
22. T. Mannel, Nucl. Phys. B **413**, 396 (1994).
23. A.F. Falk, Z. Ligeti, M. Neubert and Y. Nir, Phys. Lett. B **326**, 145 (1994).
24. M. Neubert, Phys. Rev. D **49**, 3392 and 4623 (1994).
25. I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Int. J. Mod. Phys. A **9**, 2467 (1994).

26. New experimental upper limits on leptonic decays of B mesons have been reported in: M. Artuso *et al.* (CLEO Collaboration), Phys. Rev. Lett. **75**, 785 (1995).

27. A recent QCD sum rule analysis of heavy-meson decay constants can be found in: T. Huang and C.-W. Luo, preprint BIHEP-TH-94-39 (1994).

28. A recent phenomenological work is: F. Buccella *et al.*, Phys. Rev. D **51**, 3478 (1995).

29. Recent theoretical work on two-body nonleptonic B decays has been presented in: M. Gronau, O.F. Hernández, D. London and J.L. Rosner, Phys. Rev. D **50**, 4529 (1994); preprints TECHNION-PH-95-10 [hep-ph/9504326] and TECHNION-PH-95-11 (1995) [hep-ph/9504327]; A.J. Buras and R. Fleischer, Phys. Lett. B **341**, 379 (1995); **360**, 138 (1995); preprint TUM-T31-95-95 (1995) [hep-ph/9507303]; A. Khodzhamirian and R. Rückl, preprint MPI-PHT-95-97 (1995) [hep-ph/9510294], to appear in: Beauty 95, Proceedings of the 3rd International Conference on B-Physics at Hadron Machines, Oxford, England, July 1995.

30. The theoretical description of inclusive decay spectra has been discussed recently in: A. Kapustin, Z. Ligeti and H.D. Politzer, Phys. Lett. B **357**, 653 (1995); R.D. Dikeman, M. Shifman and N.G. Uraltsev, preprint TPI-MINN-95/9-T (1995) [hep-ph/9505397]; A.F. Falk and A. Kyatkin, Phys. Rev. D **52**, 5049 (1995); A. Kyatkin, Phys. Lett. B **361**, 105 (1995); H. Li, preprint CCUTH-95-02 (1995) [hep-ph/9510415]; H. Li and Hoi-Lai Yu, preprint CCUTH-95-04 (1995) [hep-ph/9509408].

31. The calculation of moments of inclusive decay spectra has been investigated in: M.B. Voloshin, Phys. Rev. D **51**, 4934 (1995); A. Kapustin and Z. Ligeti, Phys. Lett. B **355**, 318 (1995); A.F. Falk, M. Luke and M.J. Savage, preprint UTPT-95-11 (1995) [hep-ph/9507284].

32. M. Beneke and V.M. Braun, Nucl. Phys. B **426**, 301 (1994).

33. I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. D **50**, 2234 (1994).

34. M. Beneke, V.M. Braun and V.I. Zakharov, Phys. Rev. Lett. **73**, 3058 (1994).

35. M. Neubert and C.T. Sachrajda, Nucl. Phys. B **438**, 235 (1995).

36. M. Luke, A.V. Manohar and M.J. Savage, Phys. Rev. D **51**, 4924 (1995).

37. G. Martinelli and C.T. Sachrajda, Phys. Lett. B **354**, 423 (1995).

38. G. Martinelli, M. Neubert and C.T. Sachrajda, preprint CERN-TH.7540/94 (1995) [hep-ph/9504217].

39. M. Neubert, Phys. Rev. D **51**, 5924 (1995).

40. M. Beneke and V.M. Braun, Phys. Lett. B **348**, 513 (1995).

41. P. Ball, M. Beneke and V.M. Braun, Nucl. Phys. B **452**, 563 (1995); Phys. Rev. D **52**, 3929 (1995).

42. M. Neubert, Phys. Lett. B **264**, 455 (1991).

43. H. Albrecht *et al.* (ARGUS Collaboration), Z. Phys. C **57**, 533 (1993).

44. B. Barish *et al.* (CLEO Collaboration), Phys. Rev. D **51**, 1041 (1995).

45. D. Buskulic *et al.* (ALEPH Collaboration), Phys. Lett. B **359**, 236 (1995).

46. D. Bloch *et al.* (DELPHI Collaboration), conference contribution DELPHI 95-110 PHYS 545 (1995) [eps0569].

47. S. Stone, in: Proceedings of the 8th Annual Meeting of the Division of Particles and Fields of the APS, Albuquerque, New Mexico, 1994, edited by S. Seidel (World Scientific, Singapore, 1995), p. 871.

48. M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).

49. J.E. Paschalis and G.J. Gounaris, Nucl. Phys. B **222**, 473 (1983); F.E. Close, G.J. Gounaris and J.E. Paschalis, Phys. Lett. B **149**, 209 (1984).

50. M. Neubert, Nucl. Phys. B **371**, 149 (1992).

51. S.J. Brodsky, G.P. Lepage and P.B. Mackenzie, Phys. Rev. D **28**, 228 (1983); G.P. Lepage and P.B. Mackenzie, Phys. Rev. D **48**, 2250 (1993).

52. M. Neubert, Phys. Lett. B **341**, 367 (1995).

53. H.D. Politzer and M.B. Wise, Phys. Lett. B **206**, 681 (1988); *ibid.* **208**, 504 (1988).

54. X. Ji and M.J. Musolf, Phys. Lett. B **257**, 409 (1991).

55. D.J. Broadhurst and A.G. Grozin, Phys. Lett. B **267**, 105 (1991).

56. A.F. Falk and B. Grinstein, Phys. Lett. B **247**, 406 (1990).

57. M. Neubert, Phys. Rev. D **46**, 2212 (1992).

58. M. Neubert, Phys. Rev. D **51**, 5924 (1995).

59. A.F. Falk and M. Neubert, Phys. Rev. D **47**, 2965 and 2982 (1993).

60. T. Mannel, Phys. Rev. D **50**, 428 (1994).

61. M. Shifman, N.G. Uraltsev and A. Vainshtein, Phys. Rev. D **51**, 2217 (1995).

62. M. Neubert, Phys. Lett. B **338**, 84 (1994).

63. J.D. Bjorken, in: Results and Perspectives in Particle Physics, Proceedings of the 4th Rencontres de Physique de la Vallé d'Aoste, La Thuile, Italy, 1990, ed. M. Greco (Editions Frontières, Gif-sur-Yvette, 1990), p. 583.

64. N. Isgur and M.B. Wise, Phys. Rev. D **43**, 819 (1991).

65. M.B. Voloshin, Phys. Rev. D **46**, 3062 (1992).

66. A.G. Grozin and G.P. Korchemsky, preprint OUT-4102-53 (1994) [hep-ph/9411323].

67. M. Neubert, preprint CERN-TH/95-107 (1995) [hep-ph/9505238], to appear in: Electroweak Interactions and Unified Theories, Proceedings of the 30th Rencontres de Moriond, Les Arcs, France, March 1995; G.P. Korchemsky and M. Neubert, preprint CERN-TH/95-108, in preparation.

68. E. Bagan, P. Ball and P. Gosdzinsky, Phys. Lett. B **301**, 249 (1993).

69. M. Neubert, Phys. Rev. D **47**, 4063 (1993).

70. B. Blok and M. Shifman, Phys. Rev. D **47**, 2949 (1993).

71. S. Narison, Phys. Lett. B **325**, 197 (1994).

72. K.C. Bowler *et al.* (UKQCD Collaboration), Phys. Rev. D **52**, 5067 (1995).

73. C. Bourrely, B. Machet and E. de Rafael, Nucl. Phys. B **189**, 157 (1981).

74. E. de Rafael and J. Taron, Phys. Lett. B **282**, 215 (1992); Phys. Rev. D **50**, 373 (1994).

75. I. Caprini, Z. Phys. C **61**, 651 (1994); Phys. Lett B **339**, 187 (1994).

76. C.G. Boyd, B. Grinstein and R.F. Lebed, Phys. Rev. Lett. **74**, 4603 (1995); Phys. Lett. B **353**, 306 (1995).

77. C.G. Boyd, B. Grinstein and R.F. Lebed, preprint UCSD/PTH 95-11 (1995) [hep-ph/9508211].

78. I. Caprini and M. Neubert, preprint CERN-TH/95-255 (1995), in preparation.

79. M. Neubert, Phys. Rev. D **46**, 3914 (1992); Z. Ligeti, Y. Nir and M. Neubert, *ibid.* **49**, 1302 (1994).

80. J. Gronberg *et al.* (CLEO Collaboration), conference contribution CLEO CONF 95-27 (1995) [eps0164].

81. F.E. Close and A. Wambach, Phys. Lett. B **348**, 207 (1995).

82. R. Ammar *et al.* (CLEO Collaboration), conference contribution CLEO CONF 95-09 (1995) [eps0165].

83. N. Isgur, D. Scora, B. Grinstein and M.B. Wise, Phys. Rev. D **39**, 799 (1989).

84. M. Wirbel, B. Stech and M. Bauer, Z. Phys. C **29**, 637 (1985); M. Bauer, B. Stech and M. Wirbel, *ibid.* **34**, 103 (1987).

85. H. Albrecht *et al.* (ARGUS Collaboration), Phys. Lett. B **255**, 297 (1991).

86. J. Bartelt *et al.* (CLEO Collaboration), Phys. Rev. Lett. **71**, 4111 (1993).

87. S. Narison, Phys. Lett. B **283**, 384 (1992).

88. P. Ball, Phys. Rev. D **48**, 3190 (1993).

89. K.-C. Yang and W.-Y.P. Hwang, preprint NTUTH-94-17 (1995).

90. D.R. Burford *et al.* (UKQCD Collaboration), Nucl. Phys. B **447**, 425 (1995).

91. C.R. Allton *et al.* (APE Collaboration), Phys. Lett. B **345**, 513 (1995).

92. H-n. Li and H.-L. Yu, Phys. Rev. Lett. **74**, 4388 (1995).

93. J.G. Körner and G.A. Schuler, Z. Phys. C **38**, 511 (1988).

94. D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1995).

95. J.M. Flynn *et al.* (UKQCD Collaboration), preprint SHEP-95-18 (1995) [hep-ph/9506398].

96. L. Lellouch, preprint CPT-95-P-3236 (1995) [hep-ph/9509358].

97. B. Stech, Phys. Lett. B **354**, 447 (1995).

98. A. Ali and C. Greub, Z. Phys. C **60**, 433 (1993).

99. A.J. Buras, M. Misiak, M. Münz and S. Pokorski, Nucl. Phys. B **424**, 374 (1994).

100. R. Ammar *et al.* (CLEO Collaboration), Phys. Rev. Lett. **71**, 674 (1993); K.W. Edwards *et al.* (CLEO Collaboration), conference contribution CLEO CONF 95-6 (1995) [eps0160].

101. P. Colangelo, C.A. Dominguez, G. Nardulli and N. Paver, Phys. Lett. B **317**,

183 (1993).

102. P. Ball, preprint TUM-T31-43-93 (1993) [hep-ph/9308244], unpublished.

103. A. Ali, V.M. Braun and H. Simma, Z. Phys. C **63**, 437 (1994).

104. S. Narison, Phys. Lett. B **327**, 354 (1994).

105. A compilation of quark-model results on R_{K^*} can be found in Ref.[100].

106. C. Bernard, P. Hsieh and A. Soni, Phys. Rev. Lett. **72**, 1402 (1994).

107. A. Abada *et al.* (APE Collaboration), preprint CERN-TH/95-59 (1995) [hep-lat/9503020].

108. T. Bhattacharya and R. Gupta, in: Lattice 94, edited by F. Karsch *et al.*, Nucl. Phys. B (Proc. Suppl.) **42**, 935 (1995).

109. D. Atwood, B. Blok and A. Soni, preprint SLAC-PUB-95-6635 (1994) [hep-ph/9408373].

110. E. Golowich and S. Pakvasa, Phys. Rev. D **51**, 1215 (1995).

111. H.-Y. Cheng, preprint IP-ASTP-23-94 (1994) [hep-ph/9411330].

112. G. Ricciardi, Phys. Lett. B **355**, 313 (1995).

113. G. Eilam, A. Ioannisian and R.R. Mendel, preprint TECHNION-PH-95-4 (1995) [hep-ph/9505222].

114. J. Milana, preprint UMD-PP-95-110 (1995) [hep-ph/9503376].

115. A. Ali and V.M. Braun, Phys. Lett. B **359**, 223 (1995).

116. A. Khodjamirian, G. Stoll and D. Wyler, Phys. Lett. B **358**, 129 (1995).

117. E.C. Poggio, H.R. Quinn and S. Weinberg, Phys. Rev. D **13**, 1958 (1976).

118. B. Blok and M. Shifman, Nucl. Phys. B **399**, 441 and 459 (1993); in: Proceedings of the 3rd Workshop on the Tau–Charm Factory, Marbella, Spain, June 1993, edited by J. and R. Kirkby (Editions Frontières, Gif-sur-Yvette, 1994), p. 247.

119. I.I. Bigi *et al.*, in: B Decays, edited by S. Stone, Second Edition (World Scientific, Singapore, 1994), p. 134; for a recent review, see: I.I. Bigi, preprint UND-HEP-95-BIG02 (1995) [hep-ph/9508408], and references therein.

120. P. Ball and V.M. Braun, Phys. Rev. D **49**, 2472 (1994).

121. V. Eletsky and E. Shuryak, Phys. Lett. B **276**, 191 (1992).

122. M. Neubert, Phys. Lett. B **322**, 419 (1994).

123. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B **147**, 385 and 448 (1979).

124. Y. Nir, Phys. Lett. B **221**, 184 (1989).

125. A.F. Falk, M. Neubert and M.E. Luke, Nucl. Phys. B **388**, 363 (1992).

126. M. Luke, M.J. Savage and M.B. Wise, Phys. Lett. B **343**, 329 (1995); **345**, 301 (1995).

127. P. Ball and U. Nierste, Phys. Rev. D **50**, 5841 (1994).

128. T. Browder, preprint UH 511-836-95 (1995), to appear in: Proceedings of the International Europhysics Conference on High Energy Physics, Brussels, Belgium, September 1995.

129. G. Altarelli and S. Petrarca, Phys. Lett. B **261**, 303 (1991).

130. I. Bigi, B. Blok, M.A. Shifman and A. Vainshtein, Phys. Lett. B **323**, 408 (1994).

131. E. Bagan, P. Ball, V.M. Braun and P. Gosdzinsky, Nucl. Phys. B **432**, 3 (1994); Phys. Lett. B **342**, 362 (1995); E. Bagan, P. Ball, B. Fiol and P. Gosdzinsky, Phys. Lett. B **351**, 546 (1995).

132. These are updated and corrected numbers obtained using the formulae of Ref.[131] (P. Ball, private communication).

133. G. Buchalla, I. Dunietz and H. Yamamoto, preprint FERMILAB-PUB-95-167-T (1995) [hep-ph/9507437].

134. I. Dunietz, P.S. Cooper, A.F. Falk and M.B. Wise, Phys. Rev. Lett. **73**, 1075 (1994).

135. W.F. Palmer and B. Stech, Phys. Rev. D **48**, 4174 (1993).

136. A.F. Falk, M.B. Wise and I. Dunietz, Phys. Rev. D **51**, 1183 (1995).

137. B. Blok and T. Mannel, preprint TTP-95-22 (1995).

138. C.G. Boyd, B. Grinstein and A.V. Manohar, preprint UCSD-PTH-95-19 (1995) [hep-ph/9511233].

139. A.L. Kagan, Phys. Rev. D **51**, 6196 (1995).

140. L. Roszkowski and M. Shifman, preprint TPI-MINN-95-04-T (1995) [hep-ph/9503358].

141. A.F. Falk, Z. Ligeti, M. Neubert and Y. Nir, Phys. Lett. B **326**, 145 (1994).

142. B. Guberina, S. Nussinov, R. Peccei and R. Rückl, Phys. Lett. B **89**, 111 (1979); N. Bilic, B. Guberina and J. Trampetic, Nucl. Phys. B **248**, 261 (1984); B. Guberina, R. Rückl and J. Trampetic, Z. Phys. C **33**, 297 (1986).

143. M. Shifman and M. Voloshin, Sov. J. Nucl. Phys. **41**, 120 (1985); JETP **64**, 698 (1986).

144. V. Chernyak, preprints BUDKERINP-94-69 (1994) [hep-ph/9407353] and BUDKERINP-95-18 (1995) [hep-ph/9503208].

145. C. Michael, these Proceedings.

146. C. Michael, in: Lattice 94, edited by F. Karsch *et al.*, Nucl. Phys. B (Proc. Suppl.) **42**, 147 (1995).

147. P. Weisz, preprint [hep-lat/9511017], to appear in: Lattice 95, Proceedings of the International Symposium on Lattice Field Theory, Melbourne, Australia, July 1995.

148. C.T.H. Davies *et al.* (NRQCD Collaboration), Phys. Lett. B **345**, 42 (1995).

149. M.B. Voloshin, Int. J. Mod. Phys. A **10**, 2865 (1995).