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The Wilson Loop in Yang-Mills Theory in the General Axial Gauge

Brian J. Hand*
Dept. of Mathematics and Statistics, University of Guelph
Guelph, Ontario, Canada N1G 2W1
George Leibbrandt†‡
Theoretical Physics Division, C.E.R.N., CH-1211
Geneva, Switzerland

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Abstract

We test the unified-gauge formalism by computing a Wilson loop in Yang-Mills theory to one-loop order. The unified-gauge formalism is characterized by the abritrary, but fixed, four-vector N_μ, which collectively represents the light-cone gauge ($N^2 = 0$), the temporal gauge ($N^2 > 0$), the pure axial gauge ($N^2 < 0$) and the planar gauge ($N^2 < 0$). A novel feature of the calculation is the use of distinct sets of vectors, $\{n_\mu, n^*_\mu\}$ and $\{N_\mu, N^*_\mu\}$, for the path and for the gauge-fixing constraint, respectively. The answer for the Wilson loop is independent of N_μ, and agrees numerically with the result obtained in the Feymman gauge.

*bhand@sgi1.mathstat.uoguelph.ca
†gleibbra@msnet.mathstat.uoguelph.ca
‡Permanent address: Dept. of Math & Stats., U. of Guelph, Guelph, ON N1G 2W1

1 Introduction

The Wilson loop has proven to be an excellent framework for testing the consistency of axial-type gauges. In 1982, Caracciolo, Curci and Menotti [1] computed the Wilson loop to demonstrate that the principal-value prescription fails for the temporal gauge, $A_0 = 0$, in both Abelian and non-Abelian gauge theories. It was later shown [2,3] in the context of the unified-gauge formalism, that the n^*_μ-prescription [4,5] does give the correct result for the Wilson loop. The unified-gauge formalism was developed several years ago by one of the authors [6,7], and tested in detail for the two-loop Yang-Mills self-energy [8].

In 1989, Hűffel, Landshoff and Taylor carried out a successful test of the unified-gauge prescription by demonstrating that the time dependence of a typical Wilson loop exponentiates to order g^4 [9]. The path in Figure 1 has been used in several previous computations of the Wilson loop. For instance, Korchemskaya and Korchemsky [10], employing dimensional regularization, examined the Wilson loop to second order perturbation theory in the Feynman gauge. In 1992, Andrăsi and Taylor [2] evaluated the same Wilson loop in the light-cone gauge, suggesting a breakdown of the n^*_μ-prescription. However, a detailed analysis by Bassetto and his co-workers subsequently revealed the absence of any inconsistencies in the n^*_μ-prescription [3]. In fact, their light-cone gauge result for the Wilson loop turned out to be in complete agreement with the corresponding calculation in the Feynman gauge.

In axial-type gauges, the Lagrangian density for massless Yang-Mills theory is given by (notice that n_μ in the preceding paragraphs is now replaced by the letter N_μ)

$$L_{YM} = -\frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{2\alpha}(N \cdot A^a)^2, \alpha \to 0, \tag{1}$$

where $N_\mu = (N_0, \mathbf{N})$ is the gauge-fixing vector, and

$$N^\mu A^a_\mu = 0, \mu = 0, 1, 2, 3, \tag{2}$$

the gauge-fixing constraint.

The gauge-field propagator, with gauge indices omitted, reads

$$G_{\mu\nu}(q) = \frac{-i}{q^2 + i\epsilon}\left[g_{\mu\nu} - \frac{(q_\mu N\nu + q_\nu N_\mu)}{q \cdot N} + (N^2 + \alpha q^2)\frac{q_\mu q_\nu}{(q \cdot N)^2}\right], \tag{3}$$

where $\epsilon > 0$ and $\alpha \to 0$.

We shall treat the poles of $(q \cdot N)^{-1}$ and $(q \cdot N)^{-2}$ in Eq. (3) with the unified-gauge prescription [6,7], which is a generalization of the light-cone gauge prescription developed by Mandelstam [4] and one of the authors [5]:

$$\left.\frac{1}{q \cdot N}\right|_{l.c.} = \lim_{\epsilon \to 0} \frac{q \cdot N^*}{q \cdot N q \cdot N^* + i\epsilon}, \tag{4}$$

$N^*_\mu \equiv (N_0, -\mathbf{N})$ being the dual vector of N_μ.

The purpose of this article is to test the unified-gauge formalism in Yang-Mills theory by evaluating the one-loop expectation value of the Wilson loop [6,7,8,9,11,12] for the rectangular path shown in Figure 1. The path lies in Minkowski space and is charaterized in terms of the two light-cone vectors, $n_\mu \equiv (n_0, \mathbf{n})$ and $n^*_\mu \equiv (n_0, -\mathbf{n})$: $n^2 = (n^*)^2 = 0$. The four sides of the oriented path from a to d are parameterized thus:

$$\begin{aligned}
x^a_\mu &= n^*_\mu t, t \in [0,1), \\
x^b_\mu &= n^*_\mu + n_\mu s, s \in [0,1), \\
x^c_\mu &= n_\mu + n^*_\mu u, s \in [1,0), \\
x^d_\mu &= n_\mu v, v \in [1,0).
\end{aligned} \tag{5}$$

Notice the novel approach of using *distinct* sets of vectors for the path (5), $\{n_\mu, n^*_\mu\}$, and for the gauge-fixing condition (2), namely $\{N_\mu, N^*_\mu\}$.

Figure 2 shows the ten diagrams contributing to the first-order expectation value of the Wilson loop, $W^{(1)}$. These diagrams lead to the following expression:

$$\begin{aligned}
W^{(1)} = {} & (ig)^2 C_F \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} G^{\mu\nu}(q) \int_0^1 dt \int_0^1 dt' \Big[n^*_\mu n^*_\nu (e^{iq\cdot n^*(t-t')} \\
& - e^{-iq\cdot n^*(t-t')+iq\cdot n}) + n_\mu n_\nu (e^{iq\cdot n(t-t')} - e^{-iq\cdot n(t-t')-iq\cdot n^*}) \\
& + n_\mu n^*_\nu (e^{-iq\cdot n^* t + iq\cdot n t' + iq\cdot n^*} - e^{-iq\cdot n t + iq\cdot n^* t' + iq\cdot(n-n^*)} \\
& + e^{-iq\cdot n^* t + iq\cdot n t' - iq\cdot n} - e^{iq\cdot n^* t - iq\cdot n t'}) \Big] .
\end{aligned} \tag{6}$$

We now have to decide whether to perform *first* the momentum integration and *then* the path integrations, or whether to begin by integrating first over t and t'. Of course, the traditional and generally more convenient approach has been to start with the d^4q integration (see Section 3). But, as we shall demonstrate in Section 2, it is also technically feasible to begin with the t, t' integrations. As expected both approaches yield identical results.

Figure 1: Rectangular Wilson loop with light-like segments.

Figure 2: The ten first-order diagrams for the Wilson loop depicted in Figure

2 Performing the Path Integrations First

Integration over the path variables t and t' in Eq. (6) yields the following intermediate result for $W^{(1)}$:

$$\begin{aligned} W^{(1)}_{path} &= (ig)^2 C_F \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} G^{\mu\nu}(q) \left\{ \frac{n^*_\mu n^*_\nu}{(q\cdot n^*)^2}(2 - e^{iq\cdot n^*} - e^{-iq\cdot n^*}) \right. \\ &\quad \times (1 - e^{-iq\cdot n}) + \frac{n_\mu n_\nu}{(q\cdot n)^2}(2 - e^{iq\cdot n} - e^{-iq\cdot n})(1 - e^{-iq\cdot n^*}) \\ &\quad - \frac{n_\mu n^*_\nu}{q\cdot n q\cdot n^*}[(e^{iq\cdot n} - 1)(e^{iq\cdot n^*} - 1) + (e^{iq\cdot n} - 1)(e^{-iq\cdot n^*} - 1) \\ &\quad \left. + (e^{-iq\cdot n} - 1)(e^{-iq\cdot n^*} - 1) + (e^{-iq\cdot n} - 1)(e^{iq\cdot n^*} - 1)] \right\}. \end{aligned} \tag{7}$$

Notice the initial presence of the three denonimators, namely $(q\cdot n^*)^2$, $(q\cdot n)^2$, and $(q\cdot n q\cdot n^*)$. Surprisingly, all three denominators disappear upon contraction of the Lorentz indices:

$$\begin{aligned} \frac{n_\mu n^*_\nu G^{\mu\nu}(q)}{q\cdot n q\cdot n^*} &= \frac{-i}{q^2 + i\epsilon}\left[\frac{n\cdot n^*}{q\cdot n q\cdot n^*} - \frac{N\cdot n^*}{q\cdot N q\cdot n^*} - \frac{N\cdot n}{q\cdot N q\cdot n} + \frac{N^2}{(q\cdot N)^2}\right], \\ \frac{n^*_\mu n^*_\nu G^{\mu\nu}(q)}{(q\cdot n^*)^2} &= \frac{-i}{q^2 + i\epsilon}\left[\frac{-2N\cdot n^*}{q\cdot N q\cdot n^*} + \frac{N^2}{(q\cdot N)^2}\right], \\ \frac{n_\mu n_\nu G^{\mu\nu}(q)}{(q\cdot n)^2} &= \frac{-i}{q^2 + i\epsilon}\left[\frac{-2N\cdot n}{q\cdot N q\cdot n} + \frac{N^2}{(q\cdot N)^2}\right]. \end{aligned} \tag{8}$$

When Eqs. (8) are substituted into Eq. (7), we obtain:

$$\begin{aligned} W^{(1)}_{path} &= (ig)^2 C_F \mu^{4-D} 2n\cdot n^* \int \frac{d^D q}{(2\pi)^D}\left(\frac{-i}{q^2 + i\epsilon}\right)\frac{1}{q\cdot n q\cdot n^*} \\ &\quad \times [-2 + 2e^{iq\cdot n} + 2e^{iq\cdot n^*} - e^{iq\cdot(n+n^*)} - e^{iq\cdot(n-n^*)}]. \end{aligned} \tag{9}$$

The remarkable fact about Eq. (9) is that there is *no dependence* on the gauge-fixing vector N_μ. Our approach of using distinct vectors for the path of the Wilson loop (n_μ) and for the gauge-constraint (N_μ) has allowed us to exhibit unambiguously the gauge invariance of the Wilson loop.

The potential pole from the $q\cdot n^*$-term in the denominator of Eq. (9) is harmless, since it is cancelled by the numerator in the limit as $q\cdot n^* \to 0$.

There is, however a singularity from the $q \cdot n$ pole, which may be treated by the prescription given in Eq. (4).

In order to perform the momentum integration in Eq. (9), we first parameterize the denominators as follows:

$$\begin{aligned} \frac{1}{q^2 + i\epsilon} &= -i \int_0^\infty d\alpha e^{i\alpha(q^2+i\epsilon)},\ \epsilon > 0. \\ \frac{1}{q\cdot nq\cdot n^* + i\epsilon} &= -i \int_0^\infty d\beta e^{i\beta(q\cdot nq\cdot n^* + i\epsilon)}. \end{aligned} \tag{10}$$

Substitution of the above parameterizations into Eq. (9) yields the following expression for $W^{(1)}$:

$$\begin{aligned} W^{(1)}_{path} &= (ig)^2 C_F \mu^{4-D} 2n\cdot n^* \frac{i}{(2\pi)^D} \int_0^\infty d\alpha \int_0^\infty d\beta e^{-(\alpha+\beta)\epsilon} \int d^D q e^{iq_0^2(\alpha+\beta n_0^2)} \\ &\times e^{-i\alpha \mathbf{q}^2 - i\beta(\mathbf{q}\cdot\mathbf{n})^2} [-2 + 2e^{iq\cdot n} + 2e^{iq\cdot n^*} - e^{2iq_0 n_0} - e^{-2i\mathbf{q}\cdot\mathbf{n}}]. \end{aligned} \tag{11}$$

The momentum integration then gives us

$$\begin{aligned} W^{(1)}_{path} &= (ig)^2 C_F \mu^{4-D} 2n\cdot n^* \frac{i\pi^{D/2}}{(2\pi)^D} \int_0^\infty d\alpha (i\alpha)^{1-D/2} \int_0^\infty d\beta \frac{e^{-(\alpha+\beta)\epsilon}}{\alpha + \beta n_0^2} \\ &\times \left[2 - \exp\left(\frac{-in_0^2}{\alpha+\beta n_0^2}\right) - \exp\left(\frac{i\mathbf{n}^2}{\alpha+\beta n_0^2}\right)\right]. \end{aligned} \tag{12}$$

Letting $\beta' = \beta n_0^2$, and making the substitution

$$\alpha = \lambda(1-\xi),\ \ \beta' = \lambda\xi, \tag{13}$$

we find that

$$\begin{aligned} W^{(1)}_{path} &= (ig)^2 C_F \mu^{4-D} \frac{4\pi^{D/2}}{(2\pi)^D} i^{2-D/2} \int_0^1 d\xi (1-\xi)^{1-D/2} \\ &\times \int_0^\infty d\lambda \frac{e^{-\lambda(1-\xi+\xi/n_0^2)\epsilon}}{\lambda^{D/2-1}} \left[2 - \exp\left(\frac{-in_0^2}{\lambda}\right) - \exp\left(\frac{i\mathbf{n}^2}{\lambda}\right)\right]. \end{aligned} \tag{14}$$

Since $(1-\xi+\xi/n_0^2) > 0$, we may set $(1-\xi+\xi/n_0^2)\epsilon = \epsilon'$ to get for the ξ-integration,

$$\int_0^1 d\xi (1-\xi)^{1-D/2} = \frac{\Gamma(2-D/2)}{\Gamma(3-D/2)} = \frac{2}{4-D} + 0(4-D). \tag{15}$$

Hence one of the two expected poles as $D \to 4$ is provided by the ξ-integration:

$$
\begin{aligned}
W^{(1)}_{path} &= (ig)^2 C_F^{\mu^{4-D}} \frac{8}{(4\pi)^{D/2}} \frac{i^{2-D/2}}{4-D} \int_0^\infty \frac{d\lambda}{\lambda^{D/2-1}} \\
&\times \left[2 - \exp\left(\frac{-in_0^2}{\lambda}\right) - \exp\left(\frac{i\mathbf{n}^2}{\lambda}\right) \right], \\
W^{(1)}_{path} &= \frac{-g^2 C_F \mu^{4-D} i^{2-D/2}}{(2\pi)^{D/2}} \frac{4\Gamma(\frac{D}{2}-1}{(4-D)^2} \left[(\mu_0^2 + i\eta')^{2-D/2} \right. \\
&\left. + (-\mu_0^2 + i\eta')^{2-D/2} \right], \eta' > 0. \qquad (16)
\end{aligned}
$$

3 Performing the Momentum Integration First

The first step is to apply prescription (4) to the gauge-field propagator in Eq. (3), setting $\alpha = 0$:

$$
G_{\mu\nu}(q) = \frac{-i}{q^2 + i\epsilon} \left[g_{\mu\nu} - \frac{q \cdot N^*(q_\mu N_\nu + q_\nu N_\mu)}{q \cdot N q \cdot N^* + i\epsilon} + \frac{N^2 (q \cdot N^*)^2 q_\mu q_\nu}{(q \cdot N q \cdot N^* + i\epsilon)^2} \right]. \quad (17)
$$

Substitution of Eq. (17) into Eq. (6), followed by a suitable re-arrangement of terms, yields the expression

$$
W^{(1)}_{mom} = (ig)^2 C_F \frac{\mu^{4-D}}{(2\pi)^D} \sum_{i=1}^{5} I_i, \qquad (18)
$$

where

$$
\begin{aligned}
I_1 &= 4iN_0^2 n_0 \int_0^1 dt \int_0^1 dt' \frac{\partial}{\partial t} \int d^D q \frac{-iq_0}{(q \cdot N q \cdot N^* + i\epsilon)(q^2 + i\epsilon)} \\
&\times (e^{iq\cdot n(t-t')} - e^{iq\cdot n(t-t') + iq\cdot n^*} + e^{iq\cdot n^* t + iq\cdot t'} - e^{iq\cdot n^* t - iq\cdot nt'}), \qquad (19) \\
I_2 &= -4i\mathbf{n} \cdot \mathbf{N} \int_0^1 dt \int_0^1 dt' \frac{\partial}{\partial t} \int d^D q \frac{-i\mathbf{q} \cdot \mathbf{N}}{(q \cdot N q \cdot N^* + i\epsilon)(q^2 + i\epsilon)} \\
&\times (e^{iq\cdot n(t-t')} - e^{iq\cdot n(t-t') + iq\cdot n^*} + e^{iq\cdot n^* t + iq\cdot t'} - e^{iq\cdot n^* t - iq\cdot nt'}), \qquad (20) \\
I_3 &= 2N^2 \int_0^1 dt \int_0^1 dt' \frac{\partial^2}{\partial t\, \partial t'} \int d^D q \frac{-iq_0^2 N_0^2}{(q \cdot N q \cdot N^* + i\epsilon)(q^2 + i\epsilon)} \\
&\times (e^{iq\cdot n(t-t')} - e^{iq\cdot n(t-t') + iq\cdot n^*} - e^{iq\cdot n^* t + iq\cdot nt'} - e^{iq\cdot n^* t - iq\cdot nt'}), \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
I_4 &= 2N^2\int_0^1 dt\int_0^1 dt'\frac{\partial^2}{\partial t\,\partial t'}\int d^Dq\frac{-i(\mathbf{q}\cdot\mathbf{N})^2}{(q\cdot Nq\cdot N^*+i\epsilon)(q^2+i\epsilon)} \\
&\quad\times(e^{iq\cdot n(t-t')}-e^{iq\cdot n(t-t')+iq\cdot n^*}-e^{iq\cdot n^*t+iq\cdot nt'}-e^{iq\cdot n^*t-iq\cdot nt'}), \qquad (22) \\
I_5 &= 2n\cdot n^*\int_0^1 dt\int_0^1 dt'\int d^Dq(e^{iq\cdot n^*t+iq\cdot nt'}-e^{iq\cdot n^*t-iq\cdot nt'})\frac{-i}{q^2+i\epsilon}. \qquad (23)
\end{aligned}
$$

The contributions $I_1,\ldots,I_4$ vanish. Let us demonstrate the vanishing of I_2. When the d^Dq integration is performed in Eq. (20), we obtain

$$
\begin{aligned}
I_2 &= -4i\int_0^\infty dt\int_0^\infty dt'(\mathbf{n}\cdot\mathbf{N})^2\int_0^\infty d\alpha\int_0^\infty d\beta\frac{\pi^{D/2}(i\alpha)^{1-D/2}e^{-(\alpha+\beta)\epsilon}}{(\alpha+\beta\vec{N}^2)^{3/2}(\alpha+\beta N_0^2)^{1/2}} \\
&\times\frac{1}{2}\frac{\partial}{\partial t}\left[-i(t-t')e^{a(t-t')^2+b(t-t')^2}+i(t-t')e^{a(t+t')^2+b(t-t')^2}\right. \\
&\left.-i(1-t+t')e^{a(1+t-t')^2+b(1-t+t')^2}-i(t+t')e^{a(t-t')^2+b(t+t')^2}\right], \qquad (24)
\end{aligned}
$$

where

$$
a=-\frac{in_0^2}{4(\alpha+\beta N_0^2)},\quad b=\frac{i\mathbf{n}^2}{4\alpha}-\frac{i\beta(\mathbf{n}\cdot\mathbf{N})^2}{4\alpha(\alpha+\beta\mathbf{N}^2)}. \qquad (25)
$$

Setting $\beta'=\beta n_0^2$, and making the substitution

$$
\alpha=\lambda(1-\xi),\quad \beta'=\lambda\xi, \qquad (26)
$$

we see that

$$
\begin{aligned}
I_2 &= 2i\frac{(\mathbf{n}\cdot\mathbf{N})^2}{N_0^2}\pi^{D/2}i^{1-D/2}\int_0^1 d\xi\frac{(1-\xi)^{1-D/2}}{(1-\xi+\xi\mathbf{N}^2/N_0^2)^{3/2}}\int_0^1 dt\int_0^1 dt' \\
&\times\int_0^{\inf} d\lambda\frac{e^{-\lambda(1-\xi+\xi/N_0^2)\epsilon}}{\lambda^{D/2}}\frac{\partial}{\partial t}\left[i(t-t')e^{iA(t-t')^2/\lambda+iB(t-t')^2/\lambda}\right. \\
&+i(1-t+t')e^{iA(1+t-t')^2/\lambda+iB(1-t+t')^2/\lambda}-i(t-t')e^{iA(t+t')^2/\lambda+iB(t-t')^2/\lambda} \\
&\left.+i(t+t')e^{a(t-t')^2/\lambda+iB(t+t')^2/\lambda}\right]; \qquad (27)
\end{aligned}
$$

here,

$$
A=-\frac{n_0^2}{4},\quad B=\frac{\mathbf{n}^2}{4(1-\xi)}-\frac{\xi(\mathbf{n}\cdot\mathbf{N})^2}{4N_0^2(1-\xi)(1-\xi+\xi\mathbf{N}^2/N_0^2)}. \qquad (28)
$$

The λ integration yields

$$
\begin{aligned}
I_2 &= \frac{i(\mathbf{n}\cdot\mathbf{N})^2}{2N_0^2} 2^D \pi^{D/2} \int_0^1 d\xi \frac{(1-\xi)^{1-D/2}}{(1-\xi+\xi\vec{N}^2/N_0^2)^{3/2}} \int_0^1 dt \int_0^1 dt' \\
&\times \frac{\partial}{\partial t}\left\{ \frac{i(t-t')}{[-(A(t-t')^2+B(t-t')^2)^2+i\epsilon]^{D/2}} \right. \\
&- \frac{i(t-t'-1)}{[-(A(t-t'+1)^2+B(t-t'-1)^2)^2+i\epsilon]^{D/2}} \\
&- \frac{i(t-t')}{[-(A(t+t')^2+B(t-t')^2)^2+i\epsilon]^{D/2}} \\
&\left. + \frac{i(t+t')}{[-(A(t-t')^2+B(t+t')^2)^2+i\epsilon]^{D/2}} \right\}.
\end{aligned} \tag{29}
$$

The reader may convince himself that the t-integration gives the result $I_2 = 0$. Before continuing with I_5, we notice the curious fact that the four integrals $I_1, \ldots, I_4$ in Eqs. (19) - (22) all *depend* on the gauge-fixing vectors N_μ, N_μ^*, while I_5 in Eq. (23) is N_μ- independent. Since $I_1, \ldots, I_4$ are zero, however, the expression for the Wilson loop $W_{mom}^{(1)}$ in Eq. (18) is indeed gauge-*independent*. To evaluate the only non-zero contribution in Eq. (23), we proceed by first noting the formula [13]:

$$
\int \frac{d^D q\, e^{ip.m}}{p^2+i\epsilon} = \pi^{D/2}\Gamma(\frac{D}{2}-1)(4/m^2)^{\frac{D}{2}-1}. \tag{30}
$$

Accordingly, the two momentum integrals in Eq. (23) give

$$
\begin{aligned}
\int \frac{d^D q\, e^{i(q\cdot n^* t + q\cdot n t')}}{q^2+i\epsilon} &= \pi^{D/2}\Gamma(\frac{D}{2}-1)(tt'n_0^2+i\eta)^{1-\frac{D}{2}}, \epsilon>0, \eta>0; \\
\int \frac{d^D q\, e^{i(q\cdot n^* t - q\cdot n t')}}{q^2+i\epsilon} &= \pi^{D/2}\Gamma(\frac{D}{2}-1)(-tt'n_0^2+i\eta)^{1-\frac{D}{2}}, \epsilon>0, \eta>0,
\end{aligned} \tag{31}
$$

so that

$$
\begin{aligned}
I_5 &= -2in\cdot n^*(n_0^2)^{1-\frac{D}{2}}\pi^{\frac{D}{2}}\Gamma(\frac{D}{2}-1) \\
&\int_0^1 dt \int_0^1 dt' \{(tt'+i\eta')^{1-\frac{D}{2}} - (-tt'+i\eta')^{1-\frac{D}{2}}\},
\end{aligned} \tag{32}
$$

where $n \cdot n^* = 2n_0^2$ and $\eta' = \eta/\eta_0^2$. The integration over t and t' is easy and leads, in the limit as $D \to 4$, to

$$I_5 = \frac{-16\pi^{\frac{D}{2}}\Gamma(\frac{D}{2}-1)}{(4-D)^2}\left[(n_0^2+i\eta')^{2-\frac{D}{2}}+(-n_0^2+i\eta')^{2-\frac{D}{2}}\right], \eta' > 0. \qquad (33)$$

Substituting the result (33) into the expression for the Wilson loop $W^{(1)}_{mom}$, Eq. (18), we finally obtain

$$\begin{aligned} W^{(1)}_{mom} &= (ig)^2 C_F \frac{\mu^{4-D}}{(2\pi)^D} I_5, \\ W^{(1)}_{mom} &= \frac{+4ig^2 C_F \mu^{4-D}\Gamma(\frac{D}{2}-1)}{(2\pi)^{\frac{D}{2}}(4-D)^2} \\ &\quad \left[(n_0^2+i\eta')^{2-\frac{D}{2}}+(-n_0^2+i\eta')^{2-\frac{D}{2}}\right], \eta' > 0. \end{aligned} \qquad (34)$$

This answer agrees with Eq. (3.1) in ref. [3].

Comparing the result (34) with $W^{(1)}_{path}$ in Eq. (16), we see that the two distinct integration sequences give identical results (the inessential factor $i^{2-\frac{D}{2}}$ in Eq. (16) reduces to unity as $D \to 4$).

4 Discussion

In this paper we have demonstrated the gauge independence of the Wilson loop to one- loop order for a general class of axial-type gauges. Our final results are listed in Eqs. (16) and (34). Working in the unified-gauge formalism, characterized by the fixed four-vector N_μ, we were able to convince ourselves that all integrations were ambiguity-free, regardless of the nature of N_μ, and regardless of the order of integration.

To assist us in our analysis we decided to use *distinct* sets of vectors for the paths, $\{n_\mu, n^*_\mu\}$, and for the gauge-fixing constraint, $\{N_\mu, N^*_\mu\}$. With the help of this technical "fine-tuning", we showed that the correct result (Eq. (16), or Eq. (34)) could be obtained, either by integrating $first$ over the path variables t and t' and then over the momentum variables d^4q (cf. $W^{(1)}_{path}$), or by $first$ integrating over the momenta (cf. $W^{(1)}_{mom}$). Judging from the specifics of each calculation, it would appear that the procedure leading to $W^{(1)}_{path}$ is shorter and, perhaps, wrought with fewer difficulties, than the approach for $W^{(1)}_{mom}$.

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