

CERN-TH/95-308

hep-th/9511131

Gaugino Condensation, Duality and Supersymmetry Breaking

Fernando Quevedo *

Theory Division CERN

CH-1211 Geneva 23

Switzerland.

e-mail:quevedo@nxth04.cern.ch

Abstract

The status of gaugino condensation in low-energy string theory is reviewed. Emphasis is given to the determination of the effective action below condensation scale in terms of the 2PI and Wilson actions. We illustrate how the different perturbative duality symmetries survive this simple nonperturbative phenomenon, providing evidence for the belief that these are exact nonperturbative symmetries of string theory. Consistency with T duality lifts the moduli degeneracy. The $B_{\mu\nu}$ - *axion* duality also survives in a nontrivial way in which the degree of freedom corresponding to $B_{\mu\nu}$ is replaced by a massive $H_{\mu\nu\rho}$ field but duality is preserved. S duality may also be implemented in this process. Some general problems of this mechanism are mentioned and the possible nonperturbative scenarios for supersymmetry breaking in string theory are discussed.

CERN-TH/95-308

November 1995

*Contribution to the Conference on S-duality and Mirror Symmetry, Trieste, June 1995

1 Introduction

In the efforts to extract a relation between string theory and physics, we find two main problems, namely how the large vacuum degeneracy is lifted and how supersymmetry is broken at low energies. These problems, when present at string tree level, cannot be solved at any order in string perturbation theory. The reason is the following: It is known that at tree-level, setting all the matter fields to zero forces the superpotential to vanish, for *any* value of the moduli and dilaton fields. The corresponding scalar potential vanishes implying flat directions for the moduli and dilaton. Also, the F and D auxiliary fields, which are the order parameters for supersymmetry breaking, vanish in this situation, implying unbroken supersymmetry. Since the superpotential does not get renormalized in perturbation theory, if it vanishes at tree level it will also vanish at all orders of string perturbation theory. Then the F-term part of the potential also vanishes perturbatively. The only perturbative correction that could alter this situation is the generation of a Fayet-Iliopoulos D-term by an ‘anomalous’ $U(1)$, usually present in 4D strings. However, in all the cases considered so far there are charged fields getting nonvanishing vev’s which cancel the D -term, breaking gauge symmetries instead of supersymmetry.

Therefore these problems are exact in perturbation theory and the only hope to solve them is nonperturbative physics. This has a good and a bad side. The good side is that nonperturbative effects represent the most natural way to generate large hierarchies due to their exponential suppression, this is precisely what is needed to obtain the Weinberg-Salam scale from the fundamental string or Planck scale. The bad side is that despite many efforts, we do not yet have a nonperturbative formulation of string theory. At the moment, the only concrete nonperturbative information we can extract is from the purely *field theoretical* nonperturbative effects inside string theory. Probably the simplest and certainly the most studied of those effects is gaugino condensation in a hidden sector of the gauge group, since it has the potential of breaking supersymmetry as well as lifting some of the flat directions, as we will presently discuss.

2 Gaugino Condensation

The idea of breaking supersymmetry in a dynamical way was first presented in refs. [1]. In those articles a general topological argument was developed in terms of the Witten index $Tr(-)^F$, showing that dynamical supersymmetry breaking *cannot*

be achieved unless there is chiral matter or we include supergravity effects for which the index argument does not apply. This was subsequently verified by explicitly studying gaugino condensation in pure supersymmetric Yang-Mills, a vector-like theory, for which gauginos condense but do not break global supersymmetry [2] (for a review see [3]). Breaking global supersymmetry with chiral matter was an open possibility in principle, but this approach ran into many problems when tried to be realized in practice.

The situation improved very much with the coupling to supergravity. The reason was that simple gaugino condensation was argued to be sufficient to break supersymmetry once the coupling to gravity was included. This works in a hidden sector mechanism where gravity is the messenger of supersymmetry breaking to the observable sector [4]. Furthermore, string theory provided a natural realization of this mechanism [7, 6] by having naturally a hidden sector especially in the $E_8 \times E_8$ versions. Also, it gave another direction to the mechanism by the fact that gauge couplings are field dependent (as anticipated for supergravity models in ref. [5]). This same fact raised the hope that gaugino condensation could lift the moduli and dilaton flat directions, but soon it was recognized that it only changed flat to runaway potentials, thus destabilizing those fields in the ‘wrong’ direction (zero gauge coupling and infinite radius)¹.

A simple way to see this is by setting the gaugino condensate $\langle \lambda^\alpha \lambda_\alpha \rangle \sim \Lambda^3$ with $\Lambda \sim M \exp(-1/(bg^2))$, the renormalization group invariant scale. Here $M \sim 10^{19}$ Gev is the compactification scale, b the coefficient of the one-loop beta function of the hidden sector group and g the corresponding gauge coupling. In string theory we have that $4\pi g^{-2} \sim \langle S + S^* \rangle$ where S is the chiral dilaton field (including also the axion and fermionic partner). Also, $M^{-1} \sim \langle T + T^* \rangle$ with T being one of the moduli fields. Substituting naively $\langle \lambda^\alpha \lambda_\alpha \rangle$ into the lagrangian induces a scalar potential for the real parts of S and T (S_R and T_R respectively), namely $V(S_R, T_R) \sim \frac{1}{S_R T_R^3} \exp(-3S_R/4\pi b)$. This potential has a runaway behaviour for both S_R and T_R , as advertized.

The T dependence of the potential was completely changed after the consideration of target space or T duality. In its simplest form, this symmetry acts on the field T as an $SL(2, \mathbf{Z})$ symmetry:

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad ad - bc = 1. \quad (1)$$

¹The possibility of a nonvanishing $\langle H_{ijk} \rangle$ stabilizing the potential with vanishing cosmological constant [6], was discarded after it was realized that this field was always quantized, breaking supersymmetry at the Planck scale, also its incorporation does not seem consistent with T -duality.

It was shown [8], that imposing this symmetry changes the structure of the scalar potential for the moduli fields in such a way that it develops a minimum at $T \sim 1.2$ (in string units), whereas the potential blows-up at the decompactification limit ($T_R \rightarrow \infty$), as desired. The modifications due to imposing T duality can be traced to the fact that the gauge couplings get moduli dependent threshold corrections from loops of heavy string states [9]. This in turn generates a moduli dependence on the superpotential induced by gaugino condensation of the form $W(S, T) \sim \eta(T)^{-6} \exp(-3S/8\pi b)$ with $\eta(T)$ the Dedekind function.

This mechanism however did not help in changing the runaway behaviour of the potential in the direction of S . For stabilizing S , the only proposal was to consider gaugino condensation of a nonsemisimple gauge group, inducing a sum of exponentials in the superpotential $W(S) \sim \sum_i \alpha_i \exp(-3S/8\pi b_i)$ which conspire to generate a local minimum for S [10]. These have been named ‘racetrack’ models in the recent literature.

It was later found that combining the previous ideas together with the addition of matter fields in the hidden sector (natural in many string models)[11, 12], was sufficient to find a minimum with almost all the right properties, namely, S and T fixed at the desired value, $S_R \sim 25, T_R \sim 1$, supersymmetry broken at a small scale ($\sim 10^{2-4}$ GeV) in the observable sector, etc. This lead to studies of the induced soft breaking terms at low energies.

Besides that relative succes, there are at least five problems that assures us that we are far from a satisfactory treatment of these issues.

- (i) Unlike the case for T , fixing the *vev* of the dilaton field S , at the phenomenologically interesting value, is not achieved in a satisfactory way. The conspiracy of several condensates with hidden matter to generate a local minimum at a good value, requires certain amount of fine tuning and cannot be called natural.
- (ii) The cosmological constant turns out to be always negative, which looks like an unsourmountable problem at present. This also makes the analysis of soft breaking terms less reliable, because in order to talk about them, a constant piece has to be added to the lagrangian that cancels the cosmological constant. It is then hard to believe that the unknown mechanism generating this term would leave the results on soft breaking terms (such as small gaugino masses) untouched.
- (iii) The derivation of the effective theory below condensation is not completely understood. There are several approaches to this and the exact relation among

them is not completely clear.

- (iv) There is an inherently stringy problem which is due to the fact that the S field is not stringy. S is only the dual of another field, L which is the one created by string vertex operators, having the dilaton and the antisymmetric tensor field $B_{\mu\nu}$ (instead of the axion) as the bosonic components. The problem resides in the fact that, if there is not a Peccei-Quinn (PQ) symmetry $S \rightarrow S + i \text{constant}$, as in the many condensates scenario, it is not clear if the theory in terms of S is any longer dual to the L theory. This sets serious doubts on whether the S approach mentioned above is valid at all. Another way to express this problem is to ask if it is possible to formulate directly gaugino condensation in terms of the stringy field L .
- (v) Finally, even if the previous problems were solved, there are at least two serious cosmological problems for the gaugino condensation scenario. First, it was found under very general grounds, that it was not possible to get inflation with the type of dilaton potentials obtained from gaugino condensation [13]. Second is the so-called ‘cosmological moduli problem’ which applies to any (nonrenormalizable) hidden sector scenario including gaugino condensation [14]. In this case, it can be shown that the moduli and dilaton fields acquire masses of the electroweak scale ($\sim 10^2$ GeV) after supersymmetry breaking. Therefore if stable, they overclose the universe, if unstable, they destroy nucleosynthesis by their late decay, since they only have gravitational strength interactions.

In the next section, I will present a general description of the effective theory below condensation scale, addressing the issue of problem (iii) above. Section 4 will show the solution of problem (iv) whereas in section 5, I will discuss ideas towards solving problems (i) and (v). The resolution of problem (ii) is left to the reader.

3 Wilson vs 2PI Actions

To study the effects of gaugino condensation we should be able to answer the following questions: Do gauginos condense? If so, is supersymmetry broken by this effect? What is the effective theory below the scale of condensation? In order to answer these questions, several ideas have been put forward [2, 5, 6, 15]. Let me revise briefly the different approaches.

In ref. [2], a chiral superfield U was introduced representing the condensate $W^\alpha W_\alpha$. The effective supersymmetric theory in terms of U was found by matching

the anomaly of an original R -symmetry of the underlying supersymmetric Yang-Mills action.

In refs. [6], [16], the same anomalous symmetry was used to reproduce the effective action below condensation scale, without the need of introducing U . That gave rise to the superpotential $W(S) \sim \exp(-3S/8\pi b)$ mentioned before. The earlier approach of ref. [5] was based on the direct substitution of $\lambda^\alpha \lambda_\alpha$ in the original supergravity lagrangian. A more recent analysis of ref. [15], uses a Nambu-Jona-Laisinio approach to describe the condensation mechanism.

Even though some of these approaches gave similar results, there are important differences among them. In particular, following ref. [5], since they substitute $\lambda^\alpha \lambda_\alpha$ directly into the supersymmetric action in components, the effective lagrangian is not explicitly supersymmetric unlike for instance the results of ref. [6].² Also, the approach of [15], even though it reproduces the results in [2] at tree-level, by including quantum corrections, they find very different results, for instance, the dilaton could be stabilized with a single condensing group. Finally the formalisms of [2] and [6] have been compared in [11, 17]. They eliminate the field U by *assuming* it does not break global supersymmetry, *ie* by using $\partial W/\partial U = 0$ and find agreement between the two methods. However this condition should not be imposed beforehand and it is not well justified in the supergravity case.

We can see there is no satisfactory understanding of the effective theory below condensation. Furthermore, the anomalous symmetry argument which is the most solid description of the single condensing case, cannot be used for the interesting case of several condensing groups.

We will now present a self contained discussion which will at the end identify the main approaches with known field theory quantities, *ie* the 2PI and Wilsonian effective actions [18], and mention how these two approaches are actually related in a consistent manner.

3.1 Supergravity Basics

Since the fields S and T are expected to have very large vev's, it is more convenient to work with local supersymmetry without taking the Planck scale to ∞ . The most general action for chiral matter supermultiplets Σ coupled to supergravity can be

²These two approaches were shown to be equivalent in ref. [17], once the superconformal structure of the original supergravity action is considered in detail, giving rise an explicit supersymmetric action as in [6]

written as [19]:

$$\mathcal{I} = \int d^4x \left\{ -\frac{3}{4} [S_0 S_0^* e^{-K(\Sigma, \Sigma^*)/3}]_D + [S_0^3 W(\Sigma)]_F + \left[\frac{1}{4} f_{ab}(\Sigma) W^{\alpha a} W_{\alpha}^b \right]_F + \text{cc} \right\} \quad (2)$$

where the Kähler potential $K(\Sigma, \Sigma^*)$, the superpotential $W(\Sigma)$ and the gauge kinetic function $f_{ab}(\Sigma)$ define a particular theory. The field S_0 is an extra chiral superfield called ‘the compensator’. Its existence is due to the fact that action (2) is not only invariant under super Poincaré symmetries but under the full superconformal symmetry. This simplifies the treatment of the theory in particular the calculation of the action in components. Super Poincaré supergravity is easily obtained by explicitly fixing the field S_0 to a particular value, it is usually chosen in such a way that the coefficient of the Einstein term in the action is just Newton’s constant.

Two symmetries of the superconformal algebra have a particular importance for us: Weyl and chiral $U(1)$ transformations. These two symmetries do not commute with supersymmetry. The chiral $U(1)$ group is at the origin of the R-symmetry of Poincaré theories. Weyl and chiral transformations with parameters λ and θ respectively, act on component fields with a factor $e^{w_j \lambda + i n_j \theta/2}$, w_j and n_j being the Weyl and chiral weights of the component field. For a left-handed chiral multiplet (z, ψ, f) , one finds the following weights:

$$\begin{aligned} z : & \quad w, & n = w, \\ \psi : & \quad w + \frac{1}{2}, & n = \frac{3}{2}, \\ f : & \quad w + 1, & n = 3. \end{aligned} \quad (3)$$

Chiral matter multiplets Σ have $w = n = 0$, except for S_0 which has $w = n = 1$. The chiral multiplet of gauge field strength W^a has $w = n = 3/2$. The $U(1)$ transformations of (left-handed) gauginos and chiral fermions are therefore:

$$\lambda^a \longrightarrow e^{3i\theta/4} \lambda^a, \quad \psi \longrightarrow e^{-3i\theta/4} \psi. \quad (4)$$

These transformations generate a gauge-chiral $U(1)$ mixed anomaly. This anomaly can be cancelled by the ‘Green-Schwarz’ counterterm [20, 17] :

$$\Delta \mathcal{I} = -c \left\{ \int d^4x \left[\frac{1}{4} \text{Tr} W^{\alpha} W_{\alpha} \log S_0 \right]_F + \text{cc} \right\}. \quad (5)$$

where $c = \frac{3}{2\pi} [C(G) - \sum_I C(R_I)]$, (C here represents the Casimir of the representation, for the case without matter we have that $c = 8\pi b$). This counterterm is

claimed to cancel the anomaly to all orders in perturbation theory [17] and plays an important role in what follows.

The action (12) has also a symmetry under Kähler transformations: $K \rightarrow K + \varphi(\Sigma) + \varphi^*(\Sigma^*)$, $W \rightarrow e^{-\varphi(\Sigma)} W$ since any such a transformation can be absorbed by redefining S_0 : $S_0 \rightarrow e^{\varphi/3} S_0$.

3.2 The Wilson Effective Action

Let us now restrict to a simple case that has all the properties we need to discuss gaugino condensation, *ie* a single chiral multiplet S coupled to supergravity and a nonabelian gauge group with $K = K_p(S + S^*)$ arbitrary, $W(S) = 0$ and $f(S) = S$. This is the case for the dilaton in string theory at the perturbative level. This defines the effective (Wilson) action at scales $M \geq E \geq \Lambda$. We are interested in the Wilson action at scales $\Lambda \geq E \geq 10^2$ GeV in which we expect that gauginos have condensed and S is the only degree of freedom, that means we want to integrate out the full gauge supermultiplet to obtain the effective action for S at low energies. This is precisely the approach of ref. [6] mentioned above. We need to compute:

$$e^{i\Gamma(S, S_0)} \equiv \int DV \exp i \int d^4x \{[(S - c \log S_0) \text{Tr } W^\alpha W_\alpha]_F + \text{cc}\} \quad (6)$$

First of all we can observe that $\Gamma(S, S_0)$ depends on its arguments only through the combination $S_0 \exp(-S/c)$. Second, since the result of the integration has to be superconformal invariant (because the anomaly is cancelled), we know that $\Gamma[S_0 \exp(-S/c)]$ has to be written in the form of equation (2) (plus higher derivative terms) with $f = 0$ since there are no gauge fields. Since the powers of S_0 are exactly given by (2) and S_0 only appears multiplying $\exp(-S/c)$ we can just read the super and Kähler potentials to be:

$$\begin{aligned} W(S) &= w e^{-3S/c} \\ e^{-K/3} &= e^{-K_p/3} - k e^{-(S+S^*)/c} \end{aligned} \quad (7)$$

where w and k are arbitrary constants ($k > 0$ to assure positive kinetic energy). The superpotential is just the one found in [6]. The correction to the Kähler potential is new [18]. Notice that both are corrections of order $\exp -1/g^2$ as expected. A word of caution is in order. Unlike the superpotential which has no corrections in perturbation theory, the Kähler potential can be corrected order by order in perturbation theory, therefore in practice the perturbative part of the Kähler potential K_p

is simply unknown and for weak coupling those corrections are bigger than the non-perturbative correction found here. Our result could be useful, only after the exact perturbative Kähler potential is known. It is still interesting to realize that such a simple symmetry argument can give us the exact expressions for the *nonperturbative* super and Kähler potentials, without the need of holomorphy!

3.3 The 2PI Effective Action

To answer the questions posed at the beginning of this chapter, *ie* whether gauginos condense and break supersymmetry, it is convenient to think about the case of spontaneous breaking of gauge symmetries. In that case we minimize the effective potential for a Higgs field, obtained from the 1PI effective action and see if the minimum breaks or not the corresponding gauge symmetry. In our case, we are interested in the expectation value of a composite field, namely $\lambda^\alpha \lambda_\alpha$ or its supersymmetric expression $W^\alpha W_\alpha$. Therefore we need the so-called two particle irreducible effective action.

We start then with the generating functional in the presence of an external current J coupled to the operator that we want the expectation value of, namely, $W^\alpha W_\alpha$:

$$e^{i\mathcal{W}[S, S_0, J]} \equiv \int DV \exp i \int d^4x \{[(S - c \log S_0 + J) \text{Tr} W^\alpha W_\alpha]_F + \text{cc}\} \quad (8)$$

From this we have

$$\frac{\delta \mathcal{W}}{\delta J} = \langle W^\alpha W_\alpha \rangle \equiv U \quad (9)$$

and define the 2PI action as

$$\Gamma[S, S_0, \hat{U}] \equiv \mathcal{W} - \int d^4x (\hat{U} J) \quad (10)$$

To find the explicit form of Γ we use the fact that \mathcal{W} depends on its three arguments only through the combination $S + J - c \log S_0$, therefore, we can see that $\delta \Gamma / \delta (S - c \log S_0) = \delta \Gamma / \delta J = \hat{U}$. Integrating this equation determines the dependence of Γ in S and S_0 :

$$\Gamma[S, S_0, \hat{U}] = \hat{U} (S - c \log S_0) + \Xi(\hat{U}) \quad (11)$$

where $\Xi(\hat{U})$ can be determined using symmetry arguments as follows. First we define a chiral superfield U by $\hat{U} \equiv S_0^3 U$. Therefore U is a standard chiral superfield with vanishing chiral and conformal weight ($w = n = 0$). Then $\Gamma[S, S_0, U S_0^3]$ can be written in the form (2) with chiral fields S and U . Again the fact that the S_0

dependence of (2) is very restricted, allows us to just read again the corresponding Kähler and superpotential. We find:

$$\begin{aligned} W[S, U] &= U[S + \frac{c}{3} \log U + \xi] \\ e^{-K/3} &= e^{-K_p/3} - a (UU^*)^{1/3} \end{aligned} \tag{12}$$

Here ξ is an arbitrary constant. We can see that the superpotential corresponds to the one found in [2]. The Kähler potential is new, in [2] it was found for the global case, to which this reduces in the global limit.

Notice that we have identified the two main approaches to gaugino condensation with the two relevant actions in field theory, namely the Wilson and 2PI effective actions. Our approach to the 2PI action is a reinterpretation of the one in [2]. We have to stress that in our treatment U is only a *classical* field, not to be integrated out in any path integral. It also does not make sense to consider loop corrections to its potential, this solves the question raised in [15] where loop corrections to the U potential could change the tree level results. Furthermore, since U is classical we can eliminate it by just solving its field equations: $\partial\Gamma/\partial U = 0$. (Since this implies $J = 0$, it makes equations (11) and (9) reduce to (7).) These equations cannot be solved explicitly but we find the solution in an $1/\Lambda$ expansion. We find that the solution of these equations reproduce the Wilson action derived in the previous subsection (obtaining both $W(S)$ and $K(S+S^*)$ as in equation (9)) plus extra terms suppressed by inverse powers of the condensation scale. This shows explicitly the relation between the two approaches.

We can also consider the case of several condensates. This case shows the power of the techniques used previously. Following the original discussions of [6] it was needed to use the PQ symmetry of S to cancel the $U(1)_R$ anomaly, however when there are several condensing groups we would have needed several S fields to cancel the anomaly (see [18]) but there is only one S field in string theory. In our approach however, we use the counterterm (5) which in the case of several groups is a sum of terms [17]. Therefore we have one counterterm for each group and so the path integrals just factorize into products for each of the *many* condensates, implying that the total superpotential (W) and $e^{-K/3}$ functions are the sum of the ones for one single condensate. This is the first real *derivation* of this well used result!

By studying the effective potential for U we recover the previously known results. For one condensate and field independent gauge couplings (no field S) the gauginos condense ($U \neq 0$) but supersymmetry is unbroken. For field dependent gauge coupling, the minimum is for $U = 0$ ($S \rightarrow \infty$) so gauginos do not condense (this

is reflected in the runaway behaviour of the Wilsonian action for S). For several condensing groups we find $U \neq 0$ and supersymmetry broken or not, depending on the case [12].

4 Linear vs Chiral Formalisms

Here we report on the resolution of question (iv) of section 2 [21] : perturbative $4D$ string theory has in its spectrum a two-index antisymmetric tensor field $B_{\mu\nu}$. Because it only has derivative couplings, $B_{\mu\nu}$ is dual to a pseudoscalar field, the axion a . We can transform back and forth from the $B_{\mu\nu}$ and a formulations as long as the corresponding shift symmetries are preserved. It is known that nonperturbative effects break the PQ symmetry of a giving it a mass, then the puzzle is: what happens to the stringy $B_{\mu\nu}$ field in the presence of non-perturbative effects? Is the duality symmetry also broken by those effects? Is it then correct to forget about the $B_{\mu\nu}$ field, as it is usually done, and work only with a ? (Since, unlike the axion, $B_{\mu\nu}$ is the field created by string vertex operators). The answer to these questions is very interesting: duality symmetry is *not* broken by the nonperturbative effects but the $B_{\mu\nu}$ field disappears from the propagating spectrum! Its place is taken by a massive 3-index antisymmetric tensor field $H_{\mu\nu\rho}$ dual to the massive axion.

Here I will just sketch the main steps of the derivation and refer the reader to [21] for further details. In $4D$ strings, the antisymmetric tensor belongs to a linear superfield L ($\overline{D}D L = 0$), together with the dilaton and the dilatino. For simplicity we only consider the couplings of this field to gauge superfields in global supersymmetry (the supergravity extension is straightforward), the most general action is then the D -term of an arbitrary function Φ , $\mathcal{L}_L = [\Phi(\hat{L})]_D$, with $\hat{L} \equiv L - \Omega$ and Ω the Chern Simons superfield, satisfying $\overline{D}D\Omega = W^\alpha W_\alpha$.

Since the gauginos appear in the lagrangian through the arbitrary function Φ , the analysis of gaugino condensation is far more complicated in the linear case than in the chiral case. Furthermore, the Wilson action is not well defined in this case, because the field L is not gauge invariant, we cannot just integrate the gauge fields out leaving an effective action for L alone as we did for S . Therefore we have to consider the 2PI action, and to find it, we have to work in the first order formalism where the gauge fields appear only through $\text{Tr } W^\alpha W_\alpha$ as in the S case. This will also allow us to perform a duality transformation and show that the L and S approaches are equivalent.

The duality transformation is obtained by starting with the first order system

coupled to the external current J :

$$e^{i\mathcal{W}(J)} = \int DV DS DY \exp i \int d^4x (\mathcal{L}(Y, S) + 2 \Re[J \text{Tr} W^\alpha W_\alpha]_F) \quad (13)$$

Where V is the gauge superfield, Y an arbitrary vector superfield with the lagrangian $\mathcal{L}(Y, S) = \{\Phi(Y)\}_D + \{S\overline{\mathcal{D}\mathcal{D}}(Y + \Omega)\}_F$, and S (the same S of of the previous section!) starting life as a Lagrange multiplier chiral superfield.

Integrating out S , implies $\overline{\mathcal{D}\mathcal{D}}(Y + \Omega) = 0$ or $Y = L - \Omega \equiv \hat{L}$, giving back the original theory. On the other hand integrating first Y gives the dual theory in terms of S and V . This is the situation above the condensation scale. Below condensation, however, we have to integrate first the gauge fields, after that we have the same two options for getting the two dual theories, the difference now is that the integration over V breaks the PQ symmetry (if there are at least two condensing gauge groups) and we are left with a duality without global symmetries.

To see this, we will concentrate on the $2PI$ effective action $\Gamma(U, Y, S)$ obtained in the standard way for $U \equiv \langle \text{Tr} W^\alpha W_\alpha \rangle$ [18]. The important result is that since \mathcal{W} depends on S and J only through the combination $S + J$, we can see as in eq. (12) that $\Gamma(U, S, Y) = US + \Xi(U, Y)$, where $\Xi(U, Y)$ is arbitrary, therefore S appears only linearly in the path integral and its integration gives again a δ -function, but imposing now $\overline{\mathcal{D}\mathcal{D}}Y = -U$ instead of the constraint $\overline{\mathcal{D}\mathcal{D}}(Y + \Omega) = 0$ above condensation scale. We can then see that there is no linear multiplet implied by this new constraint. This is an indication that the $B_{\mu\nu}$ field is no longer in the spectrum.

The new propagating bosonic degrees of freedom in Y are, a scalar component, the dilaton, becoming massive after gaugino condensation and a vector field v^μ dual to a , the pseudoscalar component of S . Instead of showing the details of this duality in components, I will describe the following slightly simplified toy model which has all the relevant properties:

$$\mathcal{L}_{v^\mu, a} = -\frac{1}{2}v^\mu v_\mu - a\partial_\mu v^\mu - m^2 a^2$$

If we solve for v^μ we obtain $v_\mu = -\partial_\mu a$, substituting back we find

$$\mathcal{L}_a = \frac{1}{2}\partial^\mu a\partial_\mu a - m^2 a^2$$

describing the massive scalar a . On the other hand, solving for a we get $a = -\frac{1}{2m^2}(\partial_\mu v^\mu)$ which gives

$$\mathcal{L}'_{v^\mu} = -\frac{1}{2}v^\mu v_\mu + \frac{1}{4m^2}(\partial_\mu v^\mu)^2.$$

The lagrangian \mathcal{L}'_{v^μ} also describes a massive scalar given by the longitudinal, spin zero, component of v^μ . We can see that the only component that has time derivatives is v^0 , so the other three are auxiliary fields. Notice that for $m = 0$, we recover the standard duality among a massless axion and $B_{\mu\nu}$ field. Therefore, after the gaugino condensation process, the original $B_{\mu\nu}$ field of the linear multiplet is projected out of the spectrum in favour of a massive scalar field corresponding to the longitudinal component of v^μ or to the transverse component of the antisymmetric tensor $H_{\mu\nu\rho} \equiv \epsilon_{\mu\nu\rho\sigma} v^\sigma$. Thus solving the puzzle of the axion mass in the two dual formulations. Other interesting discussions of gaugino condensation in the linear formalism can be found in [22].

5 Scenarios for SUSY Breaking

The results of the previous sections have shown us that the general results extracted in the past years about gaugino condensation in string models, in terms of the field S , are robust. We have seen how gaugino condensation can in principle lift the string vacuum degeneracy and break supersymmetry at low energies (modulo de problems mentioned before). But this is a very particular field theoretical mechanism and it would be surprising that other nonperturbative effects at the Planck scale could be completely irrelevant for these issues. In general we should always consider the two types of nonperturbative effects: stringy (at the Planck scale) and field theoretical (like gaugino condensation). Four different scenarios can be considered depending on which class of mechanism solves each of the two problems: lifting the vacuum degeneracy and breaking supersymmetry.

For breaking supersymmetry at low energies, we expect that a field theoretical effect should be dominant in order to generate the hierarchy of scales. We are then left with two preferred scenarios: either the dominant nonperturbative effects are field theoretical, solving both problems simultaneously, or there is a ‘two steps’ scenario in which stringy effects dominate to lift vacuum degeneracy and field theory effects dominate to break supersymmetry. The first scenario has been the only one considered so far, the main reason is that we can control field theoretical nonperturbative effects but not the stringy. In this scenario, independent of the particular mechanism, we have to face the cosmological moduli problem.

In the two steps scenario the dilaton and moduli fields are fixed at high energies with a mass $\sim M_{Planck}$ thus avoiding the cosmological moduli problem. It is also reasonable to expect that Planck scale effects can generate a potential for S and T .

The problem resides in the implementation of this scenario [23], mainly due to our ignorance of nonperturbative string effects.

5.1 S Duality

To approach nonperturbative string effects we may use the conjectured $SL(2, Z)$ S -duality in $N = 1$ effective lagrangians [24] :

$$S \rightarrow \frac{aS - ib}{i c S + d}, \quad ad - bc = 1. \quad (14)$$

Even though there is mounting evidence for this symmetry in $N = 4, 2$ string backgrounds, it is not yet clear how it will be extended to $N = 1$ and if so most probably the lagrangian is not invariant under this symmetry since it usually exchanges ‘electric’ and ‘magnetic’ degrees of freedom. However, similar to the case of T duality, if we restrict to the part of the action that depends only on S , (which is the relevant part when looking for vacuum configurations) this is expected to be invariant under S duality. Recall that if we do the same for the classical action, the continuous $SL(2, R)$ transformation is a symmetry of the truncated action, so the argument that quantum effects break the continuous to the discrete S duality could actually make sense in this case. As found in ref. [24], the superpotential should be a modular form of weight -1 and can be written as:

$$W(S) = \eta(S)^{-2} Q[j(S)] \quad (15)$$

where Q is an arbitrary rational function of the absolute modular invariant function $j(S)$. Its arbitrariness forbids us to extract concrete conclusions, but there are several general issues worth mentioning. Since the weight of $W(S)$ is negative, it necessarily has poles [25]. If we further impose that the scalar potential has to vanish at $S_R \rightarrow \infty$ (zero string coupling)[27] there should be poles at finite values of S which may need interpretation. The functions $\eta(S)$ and $j(S)$ can be expressed as infinite sums of $q \equiv e^{-2\pi S}$, thus encompassing the expected nonperturbative instanton-like expansion. The selfdual points $S = 1, \exp i\pi/6$ are always extrema of the potential and very often are minima. For those points supersymmetry is unbroken, thus making the two steps scenario very plausible at least for the S field.

This way of fixing the vev of S is much more elegant than the racetrack scenario with several condensing gauge groups. It is similar to the way we understood the fixing of T . A general question to be addressed to this scenario is that usually the vev of S is very close to $S_R \sim 1$ because the nontrivial structure of the potentials is always close to the selfdual points. This is far from the phenomenologically required

value where we want $4\pi/g^2 \sim 25$. However, as emphasized in [26] the gauge coupling is S only at tree level, it is expected to get nonperturbative corrections and we may have a situation with $S_R = 1$ but with a larger value of $f(S)$ at the minimum leading to the desired gauge coupling at the string scale.

Let us mention as an aside that the gaugino condensation process can be made consistent with S -duality [27, 26, 28]. A way to do it is to write the gaugino condensation superpotential $W \sim \exp -\frac{3S}{c}$ as the first term in an infinite expansion of the form (15). Another approach is to try to *derive* the effective superpotential from nonperturbative corrections to the gauge kinetic function $f(S)$. The problem with this approach is that we do not know how $f(S)$ should transform under S duality (we cannot forget the gauge fields as we did for finding $W(S)$). In ref. [26], it was assumed that f is invariant, but then the gaugino condensation-induced superpotential $W \sim \exp -\frac{3f}{c}$ would also be invariant instead of a weight -1 form as required by S -duality. An extra factor $\eta(S)^{-2}Q[j(S)]$ has to be put in by hand without justification, losing the connection with the condensation process.

A probably better way to derive an S duality invariant effective theory after gaugino condensation, may be to assume a noninvariant $f(S)$ [29], after all that is precisely what happens in T duality for which $f(T) \sim \log \eta(T)$. If for instance we take,

$$f(S) = \frac{C}{\pi} \log \left\{ \eta(S) (j(S) - 744)^{(C-12)/24C} \right\} \quad (16)$$

nonperturbatively (here C is the Casimir of the corresponding gauge group, see discussion below equation (5)), we can see that it has the right limit for large S (*ie* $f \rightarrow S$) and induces a gaugino condensation superpotential $W(S) \sim \eta(S)^{-2}(j(S) - 744)^{(12-C)/12C}$ which has the right transformation properties under S duality and reduces to the gaugino condensation superpotential in the large S limit. The non-invariance of $f(S)$ may probably be related with S -duality anomalies [29] as it happened in the T duality case. A problem with this approach is that if we are considering nonperturbative corrections to the f function, we should also include those corrections for W and K . This may diminish the importance of the gaugino condensation-induced superpotential above, because it would be just an extra contribution to the original nonperturbative superpotential which we do not know. There may still be situations, as argued in [30], for which gaugino condensation superpotentials could nevertheless be dominant.

5.2 Two Steps Scenario

In the two steps scenario, after we have fixed the vev of the moduli by stringy effects, it remains the question of how supersymmetry is broken at low energies. Notice that we would be left with the situation present before the advent of string theory in which the gauge coupling is field *independent*. In that case we know from Witten's index that gaugino condensation cannot break global supersymmetry. Since there are no 'moduli' fields with large vev's, the supergravity correction should be negligible because we are working at energies much smaller than M_{Planck} .

In fact we can perform a calculation by setting S to a constant in eq. (12), it is straightforward to show that supersymmetry is still unbroken in that case [23], as expected. A more general way to see this is computing explicitly the $1/M_{Planck}$ correction to a global supersymmetric solution $W_\phi = 0$, and see that it coincides with the solution of $W_\phi + W K_\phi / M_p^2 = 0$ which is always a supersymmetric extremum of the supergravity scalar potential.

As mentioned in section 2, there seems to be however a counterexample in the literature. In ref. [4] a modification of the Kähler potential (12) was considered:

$$e^{-K/3} = 1 - a (UU^*)^{1/3} - b (UU^*) \quad (17)$$

with the same superpotential. For $a = -9b$ supersymmetry was found to be broken with vanishing cosmological constant. But also for this choice of parameters the global limit is such that K_{UU^*} vanishes, and so the kinetic energy for U . This makes the corresponding minimum in the global case ill defined, since there may be other nonconstant field configurations with vanishing energy. This is then not a counterexample, because the global theory is not well defined in the minimum. In any case, in our general analysis, there are no such extra corrections to the Kähler potential for U .

We are then left with a situation that if global supersymmetry is unbroken, we cannot break local supersymmetry, unless there are moduli like fields. This can bring us further back to the past and reconsider models with dynamical breaking of global supersymmetry (for a recent discussion with new insights see [31]).

6 Conclusions

- (i) Gaugino condensation provides a simple example of how supersymmetry can be broken dynamically with partial succes. Some of the problems may be

solved after having better control of the supergravity lagrangian. In particular, in the single hidden sector group case we have seen that the gauginos do not condense, but this situation may be changed after perturbative and nonperturbative corrections to the Kähler potential are considered [30]. The cosmological problems may be more generic, however.

- (ii) The gaugino condensation process is also an interesting laboratory to test non-perturbative properties of string and field theories. In particular duality symmetries survive this simple, but nontrivial, nonperturbative test.
- (iii) The different approaches to describe the effective theory underlying the condensation process correspond simply to the use of the Wilson or 2PI effective actions, therefore there is a well defined relation among them. Even though the Wilson action is usually simpler to work with, the 2PI action is more suitable to follow the condensation process, it also is the only one that could be used to describe the condensation of gauginos in the ‘linear formalism’. The Wilson action cannot be used without previously identifying the low energy degrees of freedom. We needed the 2PI action to find out that the axion degree of freedom is represented by a massive $H_{\mu\nu\rho}$ tensor.
- (iv) The linear and chiral descriptions are equivalent, even in the absence of PQ symmetries. Which formulation is more convenient depends on the situation. In the linear description, the stringy $B_{\mu\nu}$ field is replaced by the massive $H_{\mu\nu\rho}$ field. We believe, this will also be the case in more general nonperturbative effects. We may conjecture that this result could be related with the claims that ‘stringy’ nonperturbative effects are not well described by strings but better by membranes, which couple naturally to $H_{\mu\nu\rho}$ or five-branes, which provide the 10D origin of the field S . A (massless) field $H_{\mu\nu\rho}$ also appears naturally in 11D supergravity.
- (v) There is not a compelling scenario for supersymmetry breaking and the field remains open, but we have a much better perspective on the relevant issues now. The nonrenormalizable hidden sector models of which the gaugino condensation is a particular case, may need a convincing solution of the cosmological moduli problem to still be considered viable. Hopefully, this will lead to interesting feedback between cosmology and string theory [32]. Furthermore, the recent progress in understanding supersymmetric gauge theories can be of much use for reconsidering gaugino condensation with hidden matter, the

discussion in the string literature is far from complete. The understanding of models with chiral matter could also provide new insights to global supersymmetry breaking, relevant to the two steps scenario mentioned above. In any case the techniques found to be useful in the simplest gaugino condensation approach discussed here, will certainly help in understanding those more complicated models.

I thank the organizers for the invitation to participate in such an exciting conference.

References

- [1] E. Witten, Nucl. Phys. B188 (1981) 513; Nucl. Phys. B202 (1982) 253
- [2] G. Veneziano, S. Yankielowicz, Phys. Lett. 113B (1982) 231.
- [3] D. Amati, K. Konishi, Y.Meurice, G.C. Rossi, G. Veneziano, Phys. Rep. 162 (1988) 169.
- [4] H.P. Nilles, Phys. Lett. 115B (1982) 193.
- [5] S. Ferrara, L. Girardello, H.P. Nilles, Phys. Lett. 125B (1983) 457.
- [6] M. Dine, R. Rohm, N. Seiberg, E. Witten, Phys. Lett. 156B (1985) 55.
- [7] J.-P. Derendinger, L.E. Ibáñez, H.P. Nilles, Phys. Lett. 155B (1985) 65.
- [8] A. Font, L.E. Ibáñez, D. Lüst, F. Quevedo, Phys. Lett. 245B (1990) 401; S. Ferrara, N. Magnoli, T. Taylor, G. Veneziano, Phys. Lett. 245B (1990) 409; H.P. Nilles, M. Olechowski, Phys. Lett. 248B (1990) 268; P. Binetruy, M.K. Gaillard, Phys. Lett. 253B (1991) 119.
- [9] L. Dixon, V. Kaplunovsky, J. Louis, Nucl. Phys. B329 (1990) 27.
- [10] N. Krasnikov, Phys. Lett. 193B (1987) 37; L. Dixon, in *The Rice Meeting*, B. Bonner, H. Miettinen, eds, World Scientific (1990); J.A. Casas, Z. Lalak, C. Muñoz, G.G. Ross, Nucl. Phys. B347 (1990) 243.
- [11] D. Lüst, T. Taylor, Phys. Lett. 253B (1991) 335.
- [12] B. de Carlos, J.A. Casas, C. Muñoz, Nucl. Phys. B399 (1993) 623.

- [13] R. Brustein, P. Steinhardt, Phys. Lett. B302 (1993) 196.
- [14] T. Banks, D. Kaplan, A. Nelson, Phys. Rev. D49 (1994) 779; B. de Carlos, J.A. Casas, F. Quevedo, E. Roulet, Phys. Lett. B318 (1993) 447.
- [15] A. de la Macorra, G.G. Ross, Nucl. Phys. B404 (1993) 321; R. Peschanski, C. Savoy, hep-ph/9504243.
- [16] C. Kounnas, M. Porrati, Phys. Lett. 191B (1987) 91.
- [17] V. Kaplunovsky and J.Louis, Nucl. Phys. B422 (1994) 57.
- [18] C.P. Burgess, J.-P. Derendinger, F. Quevedo, M. Quirós, hep-th/9505171.
- [19] E. Cremmer, S. Ferrara, L. Girardello, A. van Proeyen, Nucl. Phys. B212 (1983) 413.
- [20] J.-P. Derendinger, S. Ferrara, C. Kounnas F. Zwirner, Nucl. Phys. B372 (1992) 145; Phys. Lett. 271B (1991) 307.
- [21] C.P. Burgess, J.-P. Derendinger, F. Quevedo and M. Quirós, Phys. Lett. 348B (1995) 428.
- [22] J.-P. Derendinger, F. Quevedo, M. Quirós, Nucl. Phys. B428 (1994) 282; I. Gaida, D. Lüst, Int. J. Mod. Phys. A10 (1995) 2769; hep-th/9510022; P. Binetruy, M.K. Gaillard, T. Taylor, hep-th/9504143.
- [23] C.P. Burgess, L.E. Ibáñez, F. Quevedo, unpublished.
- [24] A. Font, L.E. Ibáñez, D. Lüst, F. Quevedo, Phys. Lett. 249B (1990) 35.
- [25] M. Cvetič, A. Font, L.E. Ibáñez, D. Lüst, F. Quevedo, Nucl. Phys. B361 (1991) 194.
- [26] Z. Lalak, A. Niemeyer, H.P. Nilles, Phys. Lett. 349B (1995) 99; hep-th/9503170.
- [27] J. Horne, G. Moore, Nucl. Phys. B432 (1994) 109.
- [28] P. Binetruy, M.K. Gaillard, hep-th/9506207.
- [29] L.E. Ibáñez, D. Lüst, F. Quevedo, unpublished.
- [30] T. Banks, M. Dine, Phys. Rev. D50 (1994) 7454.
- [31] M. Dine, A. Nelson, Y.Nir, Y. Shirman, hep-ph/9507378.

- [32] T. Banks, M. Berkooz, G. Moore, S. Shenker, P. Steinhardt, Phys. Rev. D52 (1995) 3548; T. Banks, M. Berkooz, P. Steinhardt, Phys. Rev. D52 (1995) 705; A. de la Macorra, hep-ph/9501250.