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## PERTURBATIVE EVOLUTION OF POLARIZED STRUCTURE FUNCTIONS

Richard D. Ball\*

*Theory Division, CERN  
CH-1211 Genève 23, Switzerland*

and

*Department of Physics and Astronomy  
University of Edinburgh, EH9 3JZ, Scotland*

### Abstract

We review the perturbative evolution of the polarized structure functions  $g_1$  and their associated parton distribution functions, with particular emphasis on the anomalous coupling of the first moment of the polarized gluon distribution. We also describe the small  $x$  behaviour of polarized parton distributions, contrasting it with that of the unpolarized distributions. We then explain how this theoretical analysis affects the extraction of the singlet axial charge from experimental data on  $g_1$ , and show that it may be possible to use such data to infer the existence of polarized gluons in the nucleon.

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\* CERN Fellow and Royal Society University Research Fellow

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R.D. BALL

*Theory Division, CERN, CH-1211 Genève 23, Switzerland*

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*Department of Physics and Astronomy,  
University of Edinburgh, EH9 3JZ, Scotland*

We review the perturbative evolution of the polarized structure functions  $g_1$  and their associated parton distribution functions, with particular emphasis on the anomalous coupling of the first moment of the polarized gluon distribution. We also describe the small  $x$  behaviour of polarized parton distributions, contrasting it with that of the unpolarized distributions. We then explain how this theoretical analysis affects the extraction of the singlet axial charge from experimental data on  $g_1$ , and show that it may be possible to use such data to infer the existence of polarized gluons in the nucleon.

The publication of the EMC results<sup>1</sup> for the polarized proton structure function  $g_1^p$  has been directly responsible for a renewed interest in polarized deep inelastic scattering among the theoretical community. In particular the implication that in the naive parton model the total helicity carried by quarks and antiquarks in the proton was consistent with zero led to a reexamination of the role played by the axial anomaly and polarized gluons in the perturbative evolution of the first moment of polarized structure functions.<sup>2-7</sup> Since then, more precise measurements<sup>8-11</sup> of both  $g_1^p$  and  $g_1^d$  over a wider range of both  $x$  and  $Q^2$  have been made, and from them seemingly very precise values of the first moments deduced.<sup>12,13</sup> These first moments are generally obtained by extrapolating the experimental data to a common scale (which is done in practice by assuming that the asymmetries measured in the experiments are  $Q^2$  independent), and further by extrapolating from the measured region to small  $x$  (using Regge behaviour, and assuming that the small  $x$  contribution is then  $Q^2$  independent). However if the polarized gluon distribution were large, both of these approximations could turn out to be very poor,<sup>14</sup> because of the anomalously large coupling of polarized gluons to the first moment of  $g_1$ . It thus becomes necessary to examine in detail the theoretical errors implicit in our present ignorance of the size of the polarized gluon distribution, and conversely whether the  $x$  and  $Q^2$  dependence of existing or future structure function data may be used to infer the existence of a large gluonic contribution to the nucleon spin.

## 1 Polarized Partons

We begin by reviewing the relation between polarized structure functions and parton densities in the parton model, and its relation to the operator product expansion and renormalization group approach. In the parton model<sup>15</sup> the polarized structure function  $g_1$  is decomposed in terms of polarized quark and gluon distributions  $\Delta q_i$  and  $\Delta g$  according to

$$g_1(x, Q^2) = \frac{\langle e^2 \rangle}{2} \int_x^1 \frac{dy}{y} \left\{ C_{\text{NS}}\left(\frac{x}{y}, \alpha_s(t)\right) \Delta q_{\text{NS}}(y, t) + C_{\text{S}}\left(\frac{x}{y}, \alpha_s(t)\right) \Delta q_{\text{S}}(y, t) + C_g\left(\frac{x}{y}, \alpha_s(t)\right) \Delta g(y, t) \right\} + O(1/Q^2), \quad (1.1)$$

where  $t \equiv \ln \frac{Q^2}{\Lambda^2}$ , the various coefficient functions  $C(x, \alpha_s)$  are directly related to hard cross-sections calculable in perturbative QCD, and  $\Delta q_{\text{NS}}$  and  $\Delta q_{\text{S}}$  are respectively the nonsinglet and singlet quark distributions:

$$\begin{aligned} \Delta q_{\text{NS}}(x, t) &\equiv \sum_{i=1}^{n_f} \frac{e_i^2 - \langle e^2 \rangle}{\langle e^2 \rangle} (\Delta q_i(x, t) + \Delta \bar{q}_i(x, t)), \\ \Delta q_{\text{S}}(x, t) &\equiv \sum_{i=1}^{n_f} (\Delta q_i(x, t) + \Delta \bar{q}_i(x, t)), \end{aligned} \quad (1.2)$$

where  $n_f$  is the number of active flavours, each with electric charge  $e_i$ , and  $\langle e^2 \rangle \equiv \sum e_i^2/n_f$ . Although they are themselves intrinsically nonperturbative, the perturbative part of the  $x$  and  $t$  dependence of the polarized quark and gluon distributions is given by Altarelli-Parisi equations:<sup>16</sup> the nonsinglet quark evolves independently as

$$\frac{d}{dt} \Delta q_{\text{NS}}(x, t) = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dy}{y} P_{qq}^{\text{NS}}\left(\frac{x}{y}, \alpha_s(t)\right) \Delta q_{\text{NS}}(y, t), \quad (1.3)$$

while the singlet quark and the gluon mix according to

$$\begin{aligned} \frac{d}{dt} \Delta q_{\text{S}}(x, t) &= \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dy}{y} \left[ P_{qq}^{\text{S}}\left(\frac{x}{y}, \alpha_s(t)\right) \Delta q_{\text{S}}(y, t) + P_{qg}\left(\frac{x}{y}, \alpha_s(t)\right) \Delta g(y, t) \right], \\ \frac{d}{dt} \Delta g(x, t) &= \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dy}{y} \left[ P_{gq}\left(\frac{x}{y}, \alpha_s(t)\right) \Delta q_{\text{S}}(y, t) + P_{gg}\left(\frac{x}{y}, \alpha_s(t)\right) \Delta g(y, t) \right]. \end{aligned} \quad (1.4)$$

The splitting functions  $P(x, \alpha_s)$  are again computable perturbatively in terms of hard cross-sections. In the naive parton model  $C_{\text{NS}} = C_{\text{S}} = \delta(1-x)$ ,  $C_g = 0$ , so the gluons decouple. In LO perturbation theory the gluons couple through the singlet evolution equations (1.4), while at NLO they also couple directly.

Heavy quark contributions are generated radiatively as thresholds are crossed. All this works in just the same way as in the unpolarized case.

Taking the Mellin transform of eq.(1.1) gives the leading twist component of the operator product expansion of the moments of  $g_1$ :

$$\begin{aligned}\Gamma_N(Q^2) &\equiv \int_0^1 dx x^{N-1} g_1(x, Q^2) \\ &= \frac{\langle e^2 \rangle}{2} [C_N^{\text{NS}} \Delta q_N^{\text{NS}} + C_N^{\text{S}} \Delta q_N^{\text{S}} + C_N^g \Delta g_N] + O(1/Q^2).\end{aligned}\quad (1.5)$$

Here  $C_N(\alpha_s) \equiv \int_0^1 dx x^{N-1} C(x, \alpha_s)$  are the Wilson coefficients and  $\Delta q_N(t) \equiv \int_0^1 dx x^{N-1} \Delta q(x, t)$  may, for some values of  $N$ , be related to forward matrix elements of local operators. In the unpolarized case moments of the distributions  $q_i + \bar{q}_i$  and  $g$  correspond to matrix elements of local twist two operators for  $N = 2, 4, \dots$ , while for  $q_i - \bar{q}_i$  suitable local operators exist for  $N = 1, 3, \dots$ : all the other moments are well defined, but can only be obtained by analytic continuation in  $N$  (a good example<sup>17</sup> being the Gottfried sum  $q_1^{\text{NS}}$ ). Here however the situation is reversed:<sup>18</sup>

$$\begin{aligned}s_\mu p_{\nu_1} \dots p_{\nu_{N-1}} \Delta q_N^{\text{NS}}(t) &= \langle p, s | \mathcal{O}_{\mu, \nu_1, \dots, \nu_{N-1}}^{\text{NS}, N} | p, s \rangle_t, & N = 1, 3, 5, \dots \\ s_\mu p_{\nu_1} \dots p_{\nu_{N-1}} \Delta q_N^{\text{S}}(t) &= \langle p, s | \mathcal{O}_{\mu, \nu_1, \dots, \nu_{N-1}}^{\text{S}, N} | p, s \rangle_t, & N = 3, 5, 7, \dots \\ s_\mu p_{\nu_1} \dots p_{\nu_{N-1}} \Delta g_N(t) &= \langle p, s | \mathcal{O}_{\mu, \nu_1, \dots, \nu_{N-1}}^{\text{g}, N} | p, s \rangle_t, & N = 3, 5, 7, \dots,\end{aligned}\quad (1.6)$$

where  $|p, s\rangle$  is some hadronic state carrying momentum  $p$ , with polarization vector  $s_\mu$ , and the twist two local operators  $\mathcal{O}_N^{\text{g}}$  are purely gluonic. Although there exist local operators for even  $N = 2, 4, \dots$ , these have opposite charge conjugation and thus correspond to moments of the valence distributions  $q_i - \bar{q}_i$ . Again all other moments are well defined, but can only be obtained by analytic continuation. They are necessarily gauge invariant since the matrix elements (1.6) are gauge invariant, but will in general be scheme dependent. The case  $N = 1$  in the singlet channel is peculiar, in that there is just one local operator, the axial singlet current: the identification of matrix elements of this operator with the first moments  $\Delta q_1^{\text{S}}$  and  $\Delta g_1$  is then rather subtle and will be discussed in the next section.

Taking moments of the Altarelli-Parisi equations (1.3),(1.4) yields the renormalization group equations for the matrix elements (1.6):

$$\begin{aligned}\frac{d}{dt} \Delta q_N^{\text{NS}}(t) &= \gamma_N^{qq, \text{NS}}(\alpha_s(t)) \Delta q_N^{\text{NS}}(t) \\ \frac{d}{dt} \begin{pmatrix} \Delta q_N^{\text{S}}(t) \\ \Delta g_N(t) \end{pmatrix} &= \begin{pmatrix} \gamma_N^{qq, \text{S}}(\alpha_s(t)) & \gamma_N^{qg}(\alpha_s(t)) \\ \gamma_N^{gq}(\alpha_s(t)) & \gamma_N^{gg}(\alpha_s(t)) \end{pmatrix} \begin{pmatrix} \Delta q_N^{\text{S}}(t) \\ \Delta g_N(t) \end{pmatrix},\end{aligned}\quad (1.7)$$

where  $\gamma_N(\alpha_s) \equiv \frac{\alpha_s}{2\pi} \int_0^1 dx x^{N-1} P(x, \alpha_s)$  are the anomalous dimensions of the various local operators.

## 2 First Moments

Due to interest in the Bjorken<sup>19</sup> and Ellis-Jaffe<sup>20</sup> sum rules, much of the literature on  $g_1(x, Q^2)$  has focussed on its first moment  $\Gamma_1(Q^2)$ . The proper interpretation<sup>2-5</sup> of the singlet first moments is complicated by the presence in this channel of the axial anomaly.<sup>21</sup>

Consider firstly the renormalization group equations (1.7) with  $N = 1$ . While some elements of the matrix of anomalous dimensions are nonvanishing at LO (one loop), this dependence turns out to be trivial, since the eigenvectors of the evolution

$$\Delta q_1^{\text{NS}}, \quad \Delta q_1^{\text{S}} \quad \text{and} \quad a_0 \equiv \Delta q_1^{\text{S}} - n_f \frac{\alpha_s}{2\pi} \Delta g_1, \quad (2.1)$$

only evolve at NLO (two loops).<sup>2</sup> It follows that there must exist factorization schemes in which both  $\Delta q_1^{\text{NS}}$  and  $\Delta q_1^{\text{S}}$  do not evolve at all, since multiplicative renormalizations which only begin at NLO can always be removed by a change of scheme. Thus in such schemes

$$\frac{d}{dt} \Delta q_1^{\text{NS}} = 0, \quad \frac{d}{dt} \Delta q_1^{\text{S}} = 0 \quad \text{and} \quad \frac{d}{dt} a_0 = \gamma_s a_0, \quad (2.2)$$

where  $\gamma_s = -n_f(\alpha_s^2/2\pi^2) + \dots$  has been calculated at two<sup>18</sup> and three<sup>22</sup> loops.

In the nonsinglet sector such schemes are essential in order to make the usual identification (1.6) of  $\Delta q_1^{\text{NS}}$  with forward matrix elements of (partially) conserved nonsinglet axial currents, i.e. with the nonsinglet axial charges. These may then be determined (assuming exact flavor symmetry) from weak decays of hyperons: below the charm threshold

$$\Delta q_1^{\text{NS}} = \frac{3}{4}g_A + \frac{1}{4}a_8, \quad (2.3)$$

where  $s_\mu g_A = \langle p, s | (J_\mu^5)_3 | p, s \rangle$ , etc. Above heavy quark thresholds nonsinglet charges  $a_{15}$ ,  $a_{24}$ , etc. must also be added. The nonsinglet combinations  $g_A = \Delta u - \Delta d$  and  $a_8 = \Delta u + \Delta d - 2\Delta s$ , etc. of the quark contributions to the spin of the hadron are then both well defined and scale independent (above threshold). The Bjorken sum rule<sup>19</sup> follows immediately from eqn.(1.5).

Similarly in the singlet sector the conservation of  $\Delta q_1^{\text{S}}$  makes it the natural candidate<sup>2-15</sup> for the singlet quark contribution  $\Delta u + \Delta d + \Delta s + \dots$  to the hadron spin. Individual quark contributions may then be disentangled, and in particular the Zweig rule  $0 \simeq \Delta s \simeq \Delta c \simeq \dots$  acquires a scale independent meaning.<sup>23</sup> In fact although there is no local gauge invariant conserved current in the axial singlet channel,  $\Delta q_1^{\text{S}}$  may still be formally identified with singlet quark helicity.<sup>24</sup> The other evolution eigenvector  $a_0$  may then be identified with forward matrix elements  $\langle p, s | j_\mu^5 | p, s \rangle_t = s_\mu a_0(t)$  of the singlet axial current

$j_\mu^5$ , whose conservation is violated at one loop by the (purely gluonic) axial anomaly,

$$\partial_\mu j_\mu^5 = n_f \frac{\alpha_s}{2\pi} \epsilon_{\mu\nu\rho\sigma} \text{tr} G_{\mu\nu} G_{\rho\sigma}, \quad (2.4)$$

which is thus directly responsible for the NLO evolution (2.2) of  $a_0$ . Since the anomaly (2.4) is unaffected by higher order perturbative corrections,<sup>25</sup> it is possible to find schemes in which both the decomposition (2.1) of  $a_0$  and its evolution (2.2) hold to all orders in perturbation theory:<sup>2</sup> in such schemes

$$-n_f \frac{\alpha_s}{2\pi} \gamma_1^{gg}(\alpha_s) = \gamma_s(\alpha_s), \quad \gamma_1^{gg}(\alpha_s) = \gamma_s(\alpha_s) + \beta(\alpha_s), \quad (2.5)$$

where  $\beta(\alpha_s) \equiv d \ln \alpha_s / dt$ .

Now consider the Wilson coefficients in the operator product expansion (1.5). When these are calculated in a factorization scheme in which first moments evolve according to (2.2), their first moments are

$$C_1^{\text{NS}} = 1 - \frac{\alpha_s}{\pi} + \dots, \quad C_1^{\text{S}} = 1 - \frac{\alpha_s}{\pi} + \dots, \quad C_1^{\text{g}} = -n_f \frac{\alpha_s}{2\pi} + \dots, \quad (2.6)$$

at NLO (in fact  $C_1^{\text{NS}}$  is known to<sup>26</sup>  $O(\alpha_s^3)$ ,  $C_1^{\text{S}}$  to<sup>27</sup>  $O(\alpha_s^2)$ ). When combined with (2.1), this implies that when  $N = 1$  there are indeed only two terms on the right hand side of (1.5):

$$\Gamma_1(Q^2) = \frac{\langle e^2 \rangle}{2} [C_1^{\text{NS}}(\alpha_s) \Delta q_1^{\text{NS}}(t) + C_1^{\text{S}}(\alpha_s) a_0(t)], \quad (2.7)$$

in accordance with the fact that there is only one local gauge invariant singlet operator with twist two and spin one, the axial singlet current  $j_\mu^5$ , which must then be multiplicatively renormalized. Although (1.5),(2.6) and (2.1) only imply (2.7) at NLO, in schemes in which the Adler-Bardeen theorem is satisfied it must be true to all orders in perturbation theory: in such schemes

$$C_1^{\text{g}}(\alpha_s) = -n_f \frac{\alpha_s}{2\pi} C_1^{\text{S}}(\alpha_s). \quad (2.8)$$

In practice all of these results may be obtained by regularizing infrared collinear divergences by putting external particles off-shell.<sup>18</sup> Alternatively the infrared divergences may be regulated by giving the quarks a mass;<sup>2</sup> this has the disadvantage that chiral symmetry is broken, so a finite renormalization must be performed to ensure that the quark helicities  $\Delta q_1^{\text{NS}}$  and  $\Delta q_1^{\text{S}}$  remain scale independent. More seriously, if dimensional regularization alone is used, with massless quarks and all external particles on-shell,  $C_1^{\text{g}}$  vanishes, so  $\Delta q_1^{\text{S}}$  is identified with the axial singlet charge and thus can no longer be related to quark helicity. Such factorization schemes are inappropriate because soft contributions are included in the coefficient function, rather than being factorized into the parton densities.<sup>3,6,7</sup>

Finally we consider the implications of this analysis. Combining (2.1) and (2.2), we see that  $\Delta g_1(t) \sim t$  as  $t \rightarrow \infty$ , so asymptotically the gluonic contribution to  $a_0(t)$  does not decouple, but may be as large as the quark contribution.<sup>2</sup> Thus even if  $\Delta q_1^S \sim a_8$  as suggested by the Zweig rule,  $a_0(t)$  may still be very different from  $a_8$ , and the Ellis-Jaffe sum rule<sup>20</sup> may fail.

To actually explain why experimentally<sup>1</sup>  $a_0(t)$  is small in the perturbative region we would nevertheless require some nonperturbative mechanism. Various attempts at natural explanations exist, in which either  $\Delta q_1^S$  is suppressed by instantons,<sup>28</sup> and thus the Zweig rule is strongly violated, or else the Zweig rule holds but  $a_0(t)$  is suppressed, either by strong evolution at low scales,<sup>29</sup> or due to the smallness of the first derivative of the topological susceptibility.<sup>30</sup> The latter explanations are both target independent, so might be tested by measuring  $g_1$  for other targets such as the photon. Meanwhile, it would be useful to have some way of determining  $\Delta q^S$  or  $\Delta g$  independently.

Since the gluonic contribution to the first moment of  $g_1$  is effectively LO, it might be reasonable to expect that if  $\Delta g$  were indeed large the scale dependence of  $g_1(x, Q^2)$  might also be anomalously large in certain regions of  $x$ .<sup>14</sup> If this were so it might be possible conversely to determine  $\Delta g$  by studying scaling violations of  $g_1$ . We will return to this idea in section 4, but first will discuss another way of arriving at the same conclusion.

### 3 Small $x$ Evolution

The behaviour of structure functions at small  $x$  is governed both by the qualitative form of the non-perturbative input at some scale  $t_0 \equiv \ln Q_0^2/\Lambda^2$ , given in principle by Regge theory, and the structure of the perturbative evolution to  $Q^2 > Q_0^2$ , which can then often overwhelm it. Indeed the form and evolution of structure functions at small  $x$  depends on the outcome of a competition between Regge behaviour, LO evolution and higher order perturbative evolution. We will begin by describing how this works for unpolarized distributions, and then consider the polarized distributions.

According to Regge theory the behaviour as  $x \rightarrow 0$  of the unpolarized singlet distributions is controlled by the pomeron intercept, so  $xg_S(x, t_0) \sim xg(x, t_0) \sim x^{-\lambda}$ , with  $\lambda \simeq 0.08$ . However this essentially flat (or ‘soft’) behaviour is substantially modified by perturbative evolution, determined by the leading singularity which for singlet anomalous dimensions is at  $N = 1$ . At one loop,  $\gamma_N^{gg} \sim \gamma_N^{qq} \sim \alpha_s/(N-1)$  as  $N \rightarrow 1$ , while  $\gamma_N^{gg}$  and  $\gamma_N^{qq}$  are both regular. It follows that in the double limit  $x \rightarrow 0$  and  $t \rightarrow \infty$  the gluon distribution grows in a precisely determined way, faster than any power of  $\ln 1/x$  but slower than any power of  $x$ .<sup>31</sup>

$$xg(x, t) \sim \mathcal{N} \sigma^{-1/2} \exp(2\gamma\sigma - \delta\zeta), \quad (3.1)$$

where if  $\xi \equiv \ln \frac{x_0}{x}$ ,  $\zeta \equiv \ln \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \sim \ln \frac{t}{t_0}$ ,  $\sigma \equiv \sqrt{\xi\zeta}$ , and  $\gamma$  and  $\delta$  are (known) numerical constants. This growth in  $xg$  drives a similar ‘double scaling’ behaviour in  $xq_S$ , which has recently been confirmed by measurements of  $F_2^p$  at small  $x$  and large  $t$  at HERA.<sup>32</sup>

Two loop corrections to double scaling are small<sup>33</sup> essentially because the singularity in the two loop singlet anomalous dimensions is of the form  $\alpha_s^2/(N-1)$ , and thus no stronger than that at one loop. Naively one expects that at  $l$  loops  $\gamma_N^{gg} \sim \alpha_s^l/(N-1)^{2l-1}$  because at each order there is both an extra mass singularity and an extra collinear singularity,<sup>34</sup> which would wreck double scaling by inducing a strong power-like growth. However many of the singularities cancel,<sup>35,36</sup> so that the true behaviour at  $l$  loops is  $\gamma_N^{gg} \sim (\alpha_s/(N-1))^l$ . The (scheme independent) coefficients of these remaining singularities can be calculated<sup>37</sup> and in fact turn out to be very small; indeed the coefficients at 2, 3 and 5 loops actually vanish. This means that although the higher singularities may eventually produce a power-like growth in the Regge limit like  $x^{-\lambda_S}$ , where  $\lambda_S = 12 \ln 2 \alpha_s / \pi$  is the radius of convergence of the sum of singularities, no indication of such behaviour has yet been seen at HERA.<sup>32,38</sup>

The behaviour of nonsinglet unpolarized parton distributions at small  $x$  is rather different, however. To begin with, the leading singularity in  $\gamma_N^{qq,NS}$  is not at  $N=1$  but at  $N=0$ : all singularities at  $N=1$  cancel. According to Regge behaviour as  $x \rightarrow 0$   $q_{NS}(x, t_0) \sim x^{-\lambda}$ , with  $\lambda \simeq 0.5$ , which now counts as a ‘hard’ boundary condition. So instead of the double scaling behaviour (3.1), we now have

$$q_{NS}(x, t) \sim \tilde{\mathcal{N}} \exp(\lambda\xi + (\tilde{\gamma}^2/\lambda - \tilde{\delta})\zeta), \quad (3.2)$$

for  $\rho \equiv \sqrt{\xi/\zeta} \gtrsim \tilde{\gamma}/\lambda$ ,  $\tilde{\gamma}$  and  $\tilde{\delta}$  constants. It follows immediately that at small  $x$   $q_{NS}$  is considerably smaller  $q_S$ , and thus  $F_2^p - F_2^n$  will be much more difficult to measure than  $F_2^p$  alone. The two loop correction is now rather larger since to  $O(\alpha_s^2)$   $\gamma_N^{qq,NS}$  behave as  $\alpha_s^2/N^3$  as  $N \rightarrow 0$ , and indeed at  $l$  loops there is now no cancellation of double logarithmic singularities,<sup>39</sup> so  $\gamma_N^{qq,NS} \sim \alpha^l/N^{2l-1}$ . Again the coefficients of the singularities can be computed,<sup>40</sup> and turn out to be large.<sup>a</sup> It follows that even if  $q_{NS}(x, t_0)$  had been soft, asymptotically  $q_{NS}(x, t)$  will eventually grow as  $x^{-\lambda_{NS}}$ , where<sup>39</sup>  $\lambda_{NS} = \sqrt{8\alpha_s/3\pi} \sim \frac{1}{2}$ . As it is, this behaviour should set in very quickly, and indeed it seems to be in agreement with existing NMC and CCFR data.

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<sup>a</sup> Although there is now no factorization theorem to guarantee that the resulting leading singularity anomalous dimension is scheme independent, the results at two loops (both unpolarized and polarized<sup>41</sup>) are correct,<sup>42</sup> and the behaviour of the  $\overline{\text{MS}}$  coefficient functions at  $O(\alpha_s)$  and<sup>43</sup>  $O(\alpha_s^2)$  is not so singular as to introduce scheme dependence into the anomalous dimension (at least at NNLO). It thus seems that the implications of ref.39 deserve to be taken seriously.



Polarized distributions, both nonsinglet and singlet, have a soft Regge behaviour:<sup>44,45</sup>  $\Delta q_{NS}(x, t_0) \sim \Delta q_S(x, t_0) \sim x^{-\lambda}$ , with  $-0.5 \lesssim \lambda \lesssim 0$  while again all perturbative singularities at  $N = 1$  cancel. Indeed at LO all anomalous dimensions behave as  $\alpha_s/N$ , and thus one might expect a double scaling growth for both nonsinglet and singlet distributions:<sup>14,46–49</sup>

$$\begin{aligned}\Delta q_{NS}(x, t) &\sim \mathcal{N}_{NS} \sigma^{-1/2} \exp(2\gamma_{NS}\sigma - \delta_{NS}\zeta), \\ v_{\pm}(x, t) &\sim \mathcal{N}_{\pm} \sigma^{-1/2} \exp(2\gamma_{\pm}\sigma - \delta_{\pm}\zeta),\end{aligned}\tag{3.3}$$

where  $\gamma_{NS}$ ,  $\gamma_{\pm}$ ,  $\delta_{NS}$  and  $\delta_{\pm}$  are all (known) constants, and  $v_{\pm}(x, t)$  are eigenvectors of the evolution:  $v_{\pm} = (\Delta q_S^{\pm}, \Delta g^{\pm})$ ,  $\Delta q_S^{\pm} = -c_{\pm} \Delta g^{\pm}$ , with  $c_{\pm}$  both positive constants. Thus rather than the gluon driving the singlet quark, as happens in the unpolarized case, here both gluon and singlet quark grow together, but with opposite signs. It follows that in general while  $\Delta q_{NS}$  and  $\Delta g$  become large and positive as  $x \rightarrow 0$  and  $t \rightarrow \infty$ ,  $\Delta q_S$  becomes large and negative, and  $g_1(x, Q^2)$  can then exhibit strong fluctuations due to interference between the various contributions.<sup>14</sup> Furthermore  $g_1^p$  and  $g_1^n$  can behave rather differently, since  $\Delta q_{NS}$  grows on the same footing as  $\Delta q_S$ .

However, these results must be interpreted with great care since at small enough  $x$  higher order singularities can quickly become important. All the two loop anomalous dimensions behave as  $\alpha_s^2/N^3$ , and although the two loop corrections are still numerically small in the small  $x$  region accessible currently,<sup>49</sup> and in particular leave the singlet eigenvectors unchanged at NLO, at yet smaller  $x$  they soon become comparable to the LO term. Just as in the nonsinglet unpolarized case, there is no evidence that the double logarithmic singularities cancel: indeed the leading singularities in the nonsinglet polarized channel have already been computed,<sup>39</sup> and the resulting power behaviour of  $\Delta q_{NS}(x, t)$  turns out to be even stronger<sup>50</sup> than that of  $q_{NS}(x, t)$ . The same seems to be true in the polarized singlet channel,<sup>51</sup> though the matrix of anomalous dimensions is not yet known.

To summarise, as  $1/x$  and  $Q^2$  increase  $|g_1(x, Q^2)|$  grows rapidly due to perturbative evolution, and may go negative if  $\Delta g$  is large enough. The precise details of the behaviour at very small  $x$  have yet to be calculated, but will probably involve rather complicated fluctuations in the  $(x, Q^2)$  plane. However what is already clear is that the small  $x$  contribution to the first moment of  $g_1$  may have a strong  $Q^2$  dependence. Since the scale dependence of the complete first moment (2.7) is perturbatively rather weak, this would necessarily imply a compensating  $Q^2$  dependence at larger  $x$  driven by  $\Delta g$ , which may be visible in existing data.

#### 4 Polarized Gluons?

Fixed target experiments with a fixed beam energy measure the polarization asymmetries, and thus indirectly  $g_1(x, Q^2)$ , along a curve  $Q_{\text{exp}}^2(x)$ . For

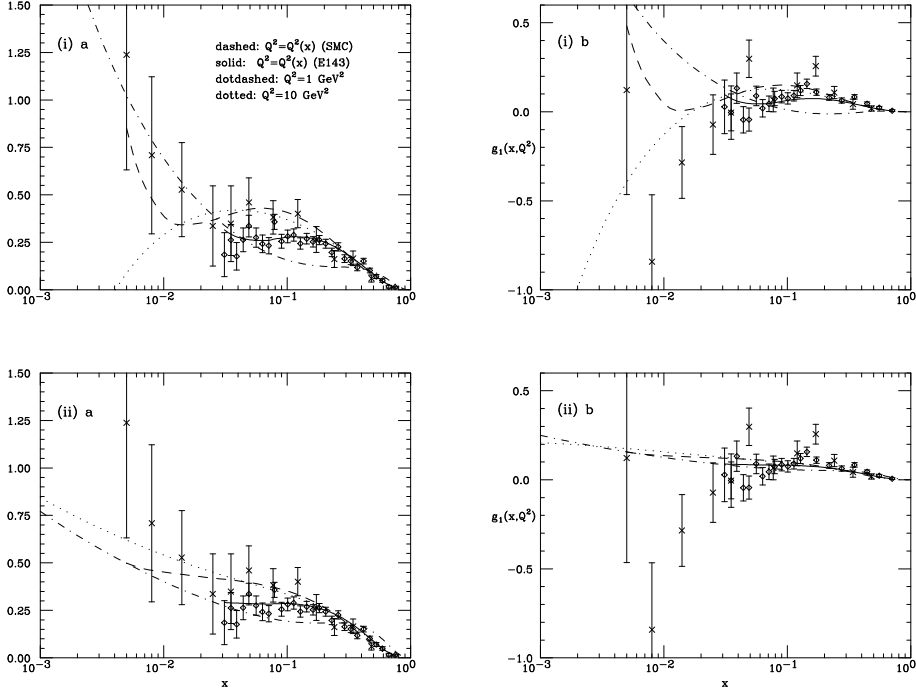


Figure 1: The SMC<sup>8,9</sup> (crosses) and E143<sup>10,11</sup> (diamonds) data for (a)  $g_1^p$  and (b)  $g_1^d$  plotted against  $x$ . The curves correspond to (i) maximal gluon and (ii) minimal gluon fits.<sup>14</sup>

the SMC experiments<sup>8,9</sup> this curve goes from  $(0.7, 50\text{GeV}^2)$  to  $(0.003, 1\text{GeV}^2)$ , while for the E143 experiments the beam energy is lower, so the  $Q^2$  is lower for all  $x$ , the curve reaching from  $(0.7, 9\text{GeV}^2)$  to  $(0.03, 1\text{GeV}^2)$ . Combining both experiments thus gives us two values of  $Q^2$  for each value of  $x$ , making it possible to search for purely evolutionary effects.

Several fits to the available data have been made, by evolving a standard parameterization of  $\Delta q_{\text{NS}}(x, t_0)$ ,  $\Delta q_{\text{S}}(x, t_0)$  and  $\Delta g(x, t_0)$  from the starting scale (usually taken to be  $Q_0^2 = 1\text{GeV}^2$ ) up to the data. Until recently the evolution was performed at LO<sup>52,53</sup> or in some hybrid ‘nLO’ approximation<sup>54,14</sup> in which the one loop anomalous dimensions are used together with one loop coefficient functions in some sensible scheme (one in which the first moments are given by (2.6)), in an effort to assess the effect of the direct coupling of  $\Delta g$  to  $g_1$ . The calculation<sup>41</sup> of the two loop anomalous dimensions has now made complete NLO calculations<sup>49</sup> possible.

At LO the effects of evolution are generally small, and in particular the asymmetries are reasonably scale independent.<sup>52</sup> However at nLO the direct gluon coupling can have a dramatic effect, especially if  $\Delta g$  is large. In fig. 1a

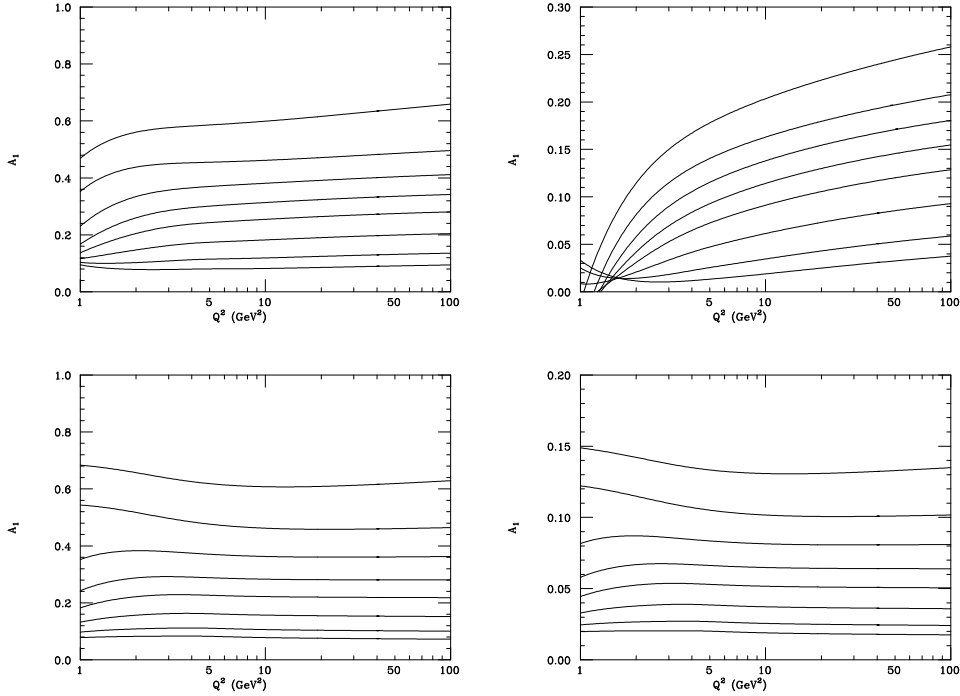


Figure 2: The asymmetries corresponding to the four fits in fig. 1, plotted against  $Q^2$ . From top to bottom the curves correspond to  $x = 0.5, 0.35, 0.25, 0.175, 0.125, 0.08, 0.05, 0.035$ .

we show the result of two nLO fits<sup>14</sup> to the proton data<sup>8,10</sup> alone, one (the ‘minimal gluon’) in which  $\Delta g_1(t_0) = 0$ , while  $\Delta q_1^S(t_0)$  is fitted, the other (the ‘maximal gluon’) in which  $\Delta q_1^S(t_0) = a_8$  (as expected from the Zweig rule) while  $\Delta g_1(t_0) = 0$  is fitted.<sup>b</sup> The minimal gluon fit shows that there is considerable evolution from the E143 data up to the SMC data, but below the crossover (at  $x \sim 0.3$ ) this is fairly uniform in  $x$ . The maximal gluon fit is much more dramatic: there is a second crossover at  $x \sim 0.03$  while at yet smaller  $x$   $g_1^p(x, Q^2)$  becomes increasingly negative as  $Q^2$  increases. The evolution between the two crossovers is then correspondingly larger. This is particularly evident in the corresponding asymmetries (fig.2a), which are fairly flat for the minimal gluon but rise quite steeply with  $Q^2$  for the maximal gluon.

Although on the basis of these fits to the proton data alone it was not possible to distinguish between minimal and maximal gluon, the corresponding predictions fig 1b for the deuteron structure function are more distinct, essentially because the deuteron is predominantly singlet, and a large direct

<sup>b</sup> In both cases  $\Delta q_1^S$  is fixed by eqn.(2.3). Other details of the fits may be found in ref.14.

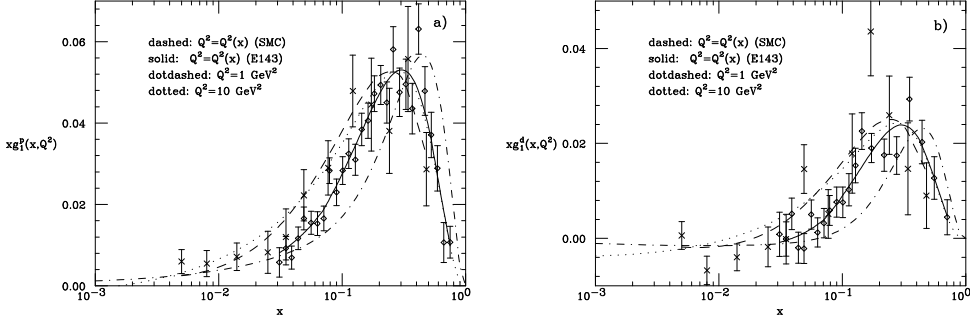


Figure 3: The structure functions  $xg_1^p(x, Q^2)$  and  $xg_1^d(x, Q^2)$  plotted against  $x$ . The notation of the data points<sup>8–11</sup> and fitted curves<sup>49</sup> is the same as in fig. 1.

gluonic contribution to  $g_1$  thus tends to make  $g_1^d$  negative at small  $x$ . The data<sup>9,11</sup> seem to prefer a maximal gluon. The strong growth in the deuteron asymmetry which this generates is apparent in fig. 2b.

This conclusion has recently been confirmed by a complete NLO calculation, with both proton and deuteron data<sup>8–11</sup> included in the fit.<sup>49</sup> The parton distributions turn out to be surprisingly well determined, with  $\Delta q_1^S(t_0) = 0.5 \pm 0.1$ ,  $\Delta g_1(t_0) = 1.5 \pm 0.8$ . The Zweig rule expectation  $\Delta s \simeq 0$  is thus confirmed experimentally, while the discrepancy in the Ellis-Jaffe sum rule<sup>20</sup> is accounted for almost entirely by a direct polarized gluon contribution, just as was conjectured in ref.2. The behaviour of the parton distributions at large  $x$  is roughly consistent with quark counting rules. At small  $x$  the non-singlet distribution is singular (behaving as  $x^{-\lambda}$ ,  $\lambda = 0.7 \pm 0.2$ ) while the singlet quark and gluon distributions are generally either flat or valencelike, as expected from the theoretical considerations reviewed in the previous section.

Using this fit (displayed in fig. 3) to determine the first moments  $\Gamma_1^p$ ,  $\Gamma_1^d$  and  $a_0$  (as defined in eqn.(2.1)) we find<sup>49</sup>

$$\begin{aligned}
 \Gamma_1^p &= 0.122 \pm 0.013 \text{ (exp.) } \begin{matrix} +0.011 \\ -0.005 \end{matrix} \text{ (th.)}, \\
 \Gamma_1^d &= 0.025 \pm 0.013 \text{ (exp.) } \begin{matrix} +0.012 \\ -0.004 \end{matrix} \text{ (th.)}, \\
 a_0 &= 0.14 \pm 0.10 \text{ (exp.) } \begin{matrix} +0.12 \\ -0.05 \end{matrix} \text{ (th.)},
 \end{aligned}
 \tag{4.1}$$

at  $Q^2 = 10\text{GeV}^2$ . The central values are lower than those given by the experimental collaborations because of the scale dependence both of the asymmetries in the measured region and of the small  $x$  extrapolations: the experimental uncertainty is larger because of the uncertainty in the size of the polarized gluon distribution which drives this evolution. There is also a theoretical error which is predominantly due to an estimate of NNLO corrections: these can be large because the two loop coupling of  $\Delta g_1$  to  $\Gamma_1$  is effectively NLO (compare (2.7) with (1.5) when  $N = 1$ ).

## 5 Desiderata

In conclusion, the effects of perturbative evolution have to be taken into account when extracting axial charges from polarization asymmetries, since they may be large due to the anomalous coupling to polarized gluons. Conversely, the structure of the evolution seen in the combined proton and deuteron data sets indicates that the polarized gluon distribution may indeed be large, making a substantial contribution to the nucleon spin. In order to make this conclusion more definite, more data over a range of  $Q^2$  at moderate values of  $x$  from one experiment are needed. Polarization of the protons at HERA,<sup>55</sup> enabling a measurement of  $A_1^p$  (and perhaps eventually  $A_1^d$ ) would further pin down the small  $x$  contribution and the size of the gluon. Data for moderately small  $x$  (say  $0.001 \lesssim x \lesssim 0.01$ ) but over a wide range of  $Q^2$  would be most useful. Indeed the behaviour of polarized structure functions at small  $x$  promises to be a very rich subject, for both theorists and experimentalists alike.

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