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Monte Carlo program KORALW 1.02 for W-pair production at LEP2/NLC energies with Yennie-Frautschi-Suura exponentiation[†]

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Abstract

The Monte Carlo program KORALW version 1.02 for W-pair production is presented. Its main features are: multiple initial state photonic radiation with finite transverse photon momenta generated according to the Yennie-Frautschi-Suura Monte Carlo technique and massive kinematics for all final particles. Matrix element features second-order leading-logarithmic initial-state QED corrections.

The dedicated semi-analytical program KORWAN, with structure-function based leading-logarithmic initial-state photonic radiation up to third order, is also included in the distribution package.

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PROGRAM SUMMARY

Title of the program: KORALW, version 1.02 .
Computer: HP-9000/735,
Operating system: UNIX,
Programming language used: FORTRAN 77
High speed storage required: $<$ 1MB
No. of bits in a word: 32
Peripherals used: Line printer
No. of cards in combined program and test deck: about 8988 plus 2272+5958 of physic generators libraries.
Keywords: Radiative corrections, heavy boson W, Monte Carlo simulation, quantum electrodynamics, spin polarization, electro-weak theory, LEP2.
Nature of the physical problem: The W pair production and decay will be used as an important data point for precise tests of the standard electroweak theory at LEP2 and higher energies. The effects due to QED bremsstrahlung and apparatus efficiency have to be subtracted from the data. The program includes also effects of the secondary decays, i.e. hadronization of quarks and τ lepton decays.
Method of solution: The Monte Carlo simulation of the combined W production and decay process including subsequent decays of τ lepton and/or hadronization of quarks is used to calculate the effects of spin and the effects of radiative corrections, including hard bremsstrahlung, simultaneously. Any experimental cut and apparatus efficiency may be introduced easily by rejecting some of the generated events.
Restrictions on the complexity of the problem: See introduction
Typical running time: Efficiency is about 30 CPU sec. of HP-9000/735/99 per 1000 unweighted events, for the parameter setting as in the demonstration output.

Contents

1 Introduction

In the next years the LEP energy will be increased to pass the production threshold of the pair of W bosons and we shall enter a new exciting era in the experimental tests of the electroweak Standard Model (SM). As is well known, the very successful Standard Model of electrowek interactions is deeply incomplete and we expect to find that it fails at certain moment, in confrontation with the experimental data. This would give us a hint to find its extension or replacement. The new experimental investigation of the W-pair production process at LEP is our next chance for this objective. In the new experiments at energies up to 200 GeV in the centre of mass, we may find new particles or deviations in fine measurements of the total or differential cross section. In the second task of the maximally precise measurements of the total/differential cross section of the W-pair production and decay process we expect to reach the 1% experimental precision level. The corresponding theoretical predictions of the SM have to be given with at least 0.5% precision – possibly at the 0.3% level. Furthermore, the W^-W^+ production and decay process involves a complicated production mechanism with delicate gauge cancellations and multichannel W decays into light leptons, heavy tau lepton and/or quark jets. The experimental measurements of the W^-W^+ process will be complicated because of at least four particle/jet final states in the presence of experimental cuts. Also, due to limited statistics, it will be of critical importance to exploit maximally the detector efficiency/resolution. This inevitably means that high quality Monte Carlo event generator(s) for the W-pair production and decay process will be extremely helpful in the data analysis. For example, in the measurement of the W mass, which will be one of the primary measurements at LEP, this W mass will be measured either close to the threshold from the total cross section or further from the threshold, at the peak cross section, using final jet/lepton mass and/or angular distributions. For this and other measurements, elimination of all the effects due to the Initial State Radiation of photons (ISR) and proper evaluation of the apparatus inefficiencies will require extensive use of Monte Carlo (MC) event generators.

In this paper we present the first version of the Monte Carlo event generator KORALW, which is aimed to help the experimental data analysis for the W-pair production and decay process. The present version features:

- Matrix element for W-pair production and W-pair decay into four fermions (quarks and/or leptons) with proper W spin treatment and finite W width,
- Initial-state multi-photon emission with full photon phase space (i.e. with finite transverse photon momenta),
- Simmulation of the decay of polarized heavy τ lepton (decay product of W) in all possible channels, taking into account spin polarization and QED bremsstrahlung,

- Emission by leptons, the decay products of W, of photons (up to double bremsstrahlung),
- Arrangement of quarks (decay products of W) into strings and fragmentation into hadrons according to the LUND model using JETSET,
- Massive kinematics with exact four momentum conservation for the entire W^-W^+ production and decay process.

It is obvious from the above list that the present version of the program is already well advanced and may be useful for a realistic Monte Carlo simulation of the W-pair production and decay process. The most important limitations of the present program are:

- Lack of so-called "background process" Feynman diagrams; in particular Z-pair production,
- Lack of optional additional "new physics" matrix elements,
- Simplified matrix element for QED photon emission,
- Lack of Coulomb correction close to threshold,
- Lack of electroweak non-QED corrections[1],
- Simplified "colour arrangement" for four quark jets.

The above and other shortcomings of the program will be addressed in the next versions of the program.

The outline of the paper is the following: In section 2 we describe the matrix element for W-pair production and decay process and we lay down the MC algorithm of our MC program KORALW, which leads to efficient generation in spite of strong peaks in the differential distributions. In section 3 we briefly describe the simple semi-analytical calculation which is included in the package and was instrumental in debugging normalization of the total cross section of our KORALW at the technical (relative) precision level of 2×10^{-4}, see ref. [1]. In section 4 we describe the structure of the program, important subroutines, etc. In section 5 we instruct the reader on how to use the program, in particular we present and explain the simple "demo" program. The corresponding source codes and outputs are shown. After the paper, there are three appendices: (A) a description of the Born level matrix element used in the generation, (B) a listing of the demonstration program, (C) printouts from this demonstration program.

[1] Most probably these corrections are small in comparison with the experimental precision and it is not necessary to include them in the Monte Carlo program – it is enough if they are in the auxiliary semi-analytical program.

2 The Monte Carlo Algorithm

In this section we present the Monte Carlo algorithm implemented in the KORALW program. The philosophy adopted by us in the construction of the algorithm can be summarized in the following steps:

- Write down the general, exact *master formula* for the cross section in terms of the standard Lorentz-invariant phase-space integration element times the appropriate matrix element. It can be arbitrarily complicated and must be, from the physics point of view, as complete as possible.
- Find a series of approximations of the *master formula*, until a fairly simple form is reached. The main constraint in this simplification is that we keep all the dominant singularities/peaks intact.
- Integrate analytically the crude distribution step by step until the so-called "crude" total cross section is reached.
- Going "backward" from the crude total cross section, find generation methods for all the necessary variables and reconstruct the Monte Carlo event in terms of four-momenta.
- Calculate the total cross section as the product of the average of the appropriate weights and the crude total cross section.

In the following subsections we discuss in detail the procedure outlined above for the KORALW Monte Carlo generator. We start with the complete, exact *master formula* for the total cross section. We explain it in detail and then introduce step by step all the *simplifications* that lead to the *crude distribution*, used by the generator. Finally we discuss the actual *event generation* algorithm. All the corresponding *weights* are also listed.

2.1 Notation

We adopt the following four-momenta notation convention: p_1, p_2 are the incoming e^-, e^+ momenta with masses m_e; q_1, q_2 are momenta of the decay products of the W^- resonance, with q_1 being the fermion momentum and q_2 the antifermion momentum of the decay pair. Their masses are denoted by m_1 and m_2; q_3, q_4, m_3, m_4 are the analogous momenta and masses of the W^+ decay; k_i are the momenta of the radiative photons. The components of a four-vector are denoted by $p = (p_0, \mathbf{p})$. The invariants s and s' are defined in the usual way: $s = (p_1 + p_2)^2$ and $s' = (\sum_{i=1}^{4} q_i)^2$.

2.2 Master Formula

Apart from the Born matrix element the photonic part of the master formula of KORALW is identical to that of YFS2 (see ref. [2]; YFS2 was also used in KORALZ [3]). Both are based on the principle of the Yennie-Frautschi-Suura (YFS) [4] resummation of the soft photons to all orders in perturbative QED. Let us note that in the presented version of KORALW only the Initial State Radiation (ISR) is considered whereas in the latest version of YFS3, ref. [5], also the Final State Radiation was included. We want to stress very clearly and strongly that the "truncation" of the YFS scheme to the leading-logarithmic version in the present work is not a genuine limitation of the YFS scheme but rather a temporary solution forced upon the authors by the fact that complete $\mathcal{O}(\alpha^1)$ QED matrix element for the W-pair production (with off-shell W's) is not yet available[2]. The master formula is the following

$$
\begin{aligned}
\sigma = & \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{4} \frac{d^3 q_i}{q_i^0} \left(\prod_{i=1}^{n} \frac{d^3 k_i}{k_i^0} \tilde{S}(p_1,p_2,k_i) \right) \delta^{(4)} \left(p_1 + p_2 - \sum_{i=1}^{4} q_i - \sum_{i=1}^{n} k_i \right) \Theta_\epsilon^{cm} \\
& \exp\left(2\alpha \Re B + \int \frac{d^3 k}{k^0} \tilde{S}(p_1,p_2,k)(1-\theta_\epsilon^{cm}) \right) \Bigg[\bar{\beta}_0^{(2)}(\mathcal{R}p_1,\mathcal{R}p_2,\mathcal{R}q_1,\mathcal{R}q_2,\mathcal{R}q_3,\mathcal{R}q_4) \\
& + \sum_{i=1}^{n} \frac{\bar{\beta}_1^{(2)}(\mathcal{R}p_1,\mathcal{R}p_2,\mathcal{R}q_1,\mathcal{R}q_2,\mathcal{R}q_3,\mathcal{R}q_4,\mathcal{R}k_i)}{\tilde{S}(p_1,p_2,k_i)} \\
& + \sum_{\substack{l,j=1 \\ l\neq j}}^{n} \frac{\bar{\beta}_2^{(2)}(\mathcal{R}p_1,\mathcal{R}p_2,\mathcal{R}q_1,\mathcal{R}q_2,\mathcal{R}q_3,\mathcal{R}q_4,\mathcal{R}k_l,\mathcal{R}k_j)}{\tilde{S}(p_1,p_2,k_l)\tilde{S}(p_1,p_2,k_j)} \Bigg],
\end{aligned}
\qquad (1)
$$

where $\tilde{S}(p_1,p_2,k) = -(\alpha/4\pi^2)\big((p_1/kp_1) - (p_2/kp_2)\big)^2$ is the real photon infrared (IR) factor and $\Theta_\epsilon^{cm} = \prod_{i=1}^{n} \theta\left(2k_i^0/\sqrt{s} - \epsilon\right)$ cuts out the singular IR region, already included to all orders in the YFS form factor F_{YFS}. The YFS form factor is equal to

$$
\begin{aligned}
F_{YFS}(p_1,p_2,\epsilon) & = \exp\left[2\alpha\Re B + \int \frac{d^3k}{k^0} \tilde{S}(p_1,p_2,k) \left(1 - \theta\left(k^0 - \frac{2\epsilon}{\sqrt{s}} \right) \right) \right] \\
& = \exp\left[2\frac{\alpha}{\pi} \left(\left(\ln\frac{s}{m_e^2} - 1 \right) \ln\epsilon + \frac{1}{2} \ln\frac{s}{m_e^2} - 1 + \frac{\pi^2}{3} \right) \right]. \qquad (2)
\end{aligned}
$$

The $\bar{\beta}_0, \bar{\beta}_1$ and $\bar{\beta}_2$ functions will be discussed later on. For all the other details concerning eq. (1) we refer the reader to ref. [2].

[2] Examples of incomplete preliminary calculations can be found in refs. [6, 7].

2.3 Crude Photonic Distribution

In the following few subsections we discuss simplifications of the master formula of eq. (1) leading to the crude distributions used as the basis of the actual MC generation. The series of simplifications of eq. (1) starts with the $\bar{\beta}$-terms. We drop completely the $\bar{\beta}_1$ and $\bar{\beta}_2$ contributions.

$$\bar{\beta}_1^{(2)}, \bar{\beta}_2^{(2)} \rightarrow 0. \tag{3}$$

They are pure higher-order corrections, relatively small, and it is easy to restore them by reweighting events at the end of the MC generation.

The case of $\bar{\beta}_0$ is more complicated. To the lowest order, $\bar{\beta}_0$ is equal to the differential Born cross-section, and therefore has to be kept, at least to some approximation. ¿From now on we replace $\bar{\beta}_0$ by its lowest-order term:

$$\begin{aligned} \bar{\beta}_0^{(2)} \rightarrow \bar{\beta}_0^{(0)} &= \frac{1}{2s'} \mathcal{N}_\pi |\mathcal{M}_B|^2, \\ \mathcal{N}_\pi &= \frac{(2\pi)^4}{\Big(2(2\pi)^3\Big)^r}, \end{aligned} \tag{4}$$

where r denotes the number of Born level particles in the final state ($r = 4$ in our case); see Sect. 2.4 for details on $\mathcal{M}_B$. With the help of the above approximations, see eqs. (3,4), we get from eq. (1) the first crude distribution:

$$\sigma_{crude,1} = F_{YFS}(p_1, p_2, \epsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \int \left(\prod_{i=1}^{n} \frac{d^3 k_i}{k_i^0} \tilde{S}(k_i) \right) \Theta_\epsilon^{cm} \; \sigma_{Born}(s') \tag{5}$$

with $\sigma_{Born}(s')$ denoting

$$\begin{aligned} \sigma_{Born}(s') =& \frac{1}{2s'} \int d\Phi_4 |\mathcal{M}_B|^2 \\ d\Phi_4 =& (2\pi)^4 \delta^{(4)}\Big(p_1 + p_2 - \sum_{i=1}^{n} k_i - \sum_{i=1}^{4} q_i\Big) \prod_{i=1}^{4} \frac{d^3 q_i}{2 q_i^0 (2\pi)^3} \; , \end{aligned} \tag{6}$$

All these simplifications in $\bar{\beta}$ functions are reintroduced by the appropriate w_β weight, defined as

$$\begin{aligned} w_\beta = \frac{2s'}{\mathcal{N}_\pi |\mathcal{M}_B|^2} \Bigg[& \bar{\beta}_0^{(2)}(\mathcal{R}p_1, \mathcal{R}p_2, \mathcal{R}q_1, \mathcal{R}q_2, \mathcal{R}q_3, \mathcal{R}q_4) \\ & + \sum_{i=1}^{n} \frac{\bar{\beta}_1^{(2)}(\mathcal{R}p_1, \mathcal{R}p_2, \mathcal{R}q_1, \mathcal{R}q_2, \mathcal{R}q_3, \mathcal{R}q_4, \mathcal{R}k_i)}{\tilde{S}(p_1, p_2, k_i)} \\ & + \sum_{\substack{l,j=1 \\ l \neq j}}^{n} \frac{\bar{\beta}_2^{(2)}(\mathcal{R}p_1, \mathcal{R}p_2, \mathcal{R}q_1, \mathcal{R}q_2, \mathcal{R}q_3, \mathcal{R}q_4, \mathcal{R}k_l, \mathcal{R}k_j)}{\tilde{S}(p_1, p_2, k_l) \tilde{S}(p_1, p_2, k_j)} \Bigg]. \end{aligned} \tag{7}$$

Now we reduce the photonic part of eq. (5) to a single integral over v variable. This procedure and algorithm for the generation of bremsstrahlung photon kinematics for a given v is described in detail in ref. [2]. For that reason we skip here all the details of the evaluation and refer the interested reader directly to this work for more information. Here we confine ourselves to the final result of this simplification.

After this simplification our $\sigma_{crude,1}$ distribution is reduced to:

$$\sigma_{crude,2} = e^{\delta_{YFS}} \int\limits_0^{v_{max}} dv \gamma v^{\gamma-1} \rho(v) \sigma_{Born}((1-v)s). \tag{8}$$

The v variable is defined as $v = 1 - s'/s$. The function $\rho(v)$ is defined as

$$\begin{aligned}
\rho(v) &= \frac{1}{2}\left(1+\frac{1}{\sqrt{1-v}}\right)\left(\theta(\epsilon-v)+\theta(v-\epsilon)\frac{\gamma'}{\gamma}\left(\frac{v}{\epsilon}\right)^{\gamma'-\gamma}\right), & (9)\\
\gamma &= 2\frac{\alpha}{\pi}\left(\ln\frac{s}{m_e^2}-1\right) \;, & \\
\gamma' &= 2\frac{\alpha}{\pi}\ln\frac{s}{m_e^2} \;, & \\
\delta_{YFS} &= \frac{\alpha}{\pi}\left(\frac{1}{2}\ln\frac{s}{m_e^2}-1+\frac{\pi^2}{3}\right). & (10)
\end{aligned}$$

Details corresponding to the $\sigma_{crude,2} \to \sigma_{crude,1}$ restoration, which include the generation of bremsstrahlung photon complete kinematics, are described in detail in ref. [2]. Let us note that the corresponding weight will be denoted as

$$w_{YFS}. \tag{11}$$

2.4 Born Cross Section

The matrix element for W-pair production at LEP2 of eq. (6) is a rather complicated object. For practical reasons it is usually written not in terms of the close analytical formula, but rather in terms of spin amplitudes calculated numerically, see e.g. refs. [8, 9]. On the other hand the Born cross-section exhibits a strongly varying energy dependence (WW threshold) as well as a non-trivial angular behaviour (s and t channels together); this, for the sake of constructing an efficient MC algorithm, has to be understood, i.e. some analytical (at least approximate) expressions are of interest. Let us discuss the distributions in eq. (6) in detail and find some approximate formula reflecting basic properties of the integrand in eq. (6).

We start by rewriting the expression for the Born cross section of eq. (6), showing explicitly the sum over all the open decay channels (a, b). We assume implicit summation over final-state spin states and average over initial-state spin:

$$\sigma_{Born}(s') = \frac{1}{2s'} \sum_{\substack{a,b\\ decay\\ chan.}} \int d\Phi_4 |\mathcal{M}_{ab}|^2. \tag{12}$$

Standard manipulations on the four-body phase-space lead to the expression

$$\begin{aligned}
d\Phi_4 &= \frac{1}{32} ds_1 ds_2\, d\Omega\, d\Omega_1\, d\Omega_2\, \mathcal{J}\, \Theta_S \mathcal{N}_\pi, && (13)\\
\mathcal{J} &= \sqrt{\lambda\Big(1, \frac{s_1}{s'}, \frac{s_2}{s'}\Big)} \sqrt{\lambda\Big(1, \frac{m_1^2}{s_1}, \frac{m_2^2}{s_1}\Big)} \sqrt{\lambda\Big(1, \frac{m_3^2}{s_2}, \frac{m_4^2}{s_2}\Big)}\,, && (14)\\
\lambda(x,y,z) &= x^2 + y^2 + z^2 - 2xy - 2xz - 2yz, && (15)\\
\Theta_S &= \theta(s' - s_1)\theta(\sqrt{s'} - \sqrt{s_1} - \sqrt{s_2}) && \\
&\quad \theta(\sqrt{s_1} - m_1 - m_2)\theta(\sqrt{s_2} - m_3 - m_4), && (16)
\end{aligned}$$

where Ω is a solid angle of the W^- in the CMS frame[3] with the $+z$-axis pointing in the direction of the incoming e^- beam, while $\Omega_{1(2)}$ is the solid angle of the $W^{-(+)}$ decay products in the $W^{-(+)}$ rest frame. The $s_{1(2)}$ denote invariant mass squared of the virtual $W^{-(+)}$. The detailed description of the matrix element $|\mathcal{M}_{ab}|^2$ is given in Appendix A.

2.5 Crude Born

In the next step we have to simplify $d\Phi_4$ and the matrix element $|\mathcal{M}_{ab}|^2$ into a form suitable for the Monte Carlo generation: $|\mathcal{M}_{ab}|^2 \to |\mathcal{M}_{ab}|^2_{crude}$. As a result of some numerical trials we have chosen the following representation for the "crude matrix element"[4]

$$\begin{aligned}
|\mathcal{M}_{ab}|^2_{crude} &= N_{ab}\, \frac{1}{\mathcal{N}_\pi}\, \sigma_0(s')\, W(s_1)\, W(s_2)\, T(\cos\theta)\, C(\cos\theta_1)\, C(\cos\theta_2), && (17)\\
N_{ab} &= \frac{BR_a}{BR_{(e\nu)}} \frac{BR_b}{BR_{(e\nu)}}, \quad \sum_i BR_i = 1, && (18)\\
W(s_i) &= \frac{1 + (s_i/M_W^2 - 1)\theta(s_i - M_W^2)}{BW^0(s_i)}, && (19)\\
BW^0(s_i) &= (s_i - M_W^2)^2 + (\Gamma_W M_W)^2 && (20)\\
T(\cos\theta) &= \left[\frac{1}{2}\Big((s' - s_1 - s_2) - \sqrt{\lambda(s', s_1, s_2)}\cos\theta\Big)\right]^{-1} && (21)\\
&\quad \frac{1}{2}\sqrt{\lambda(s', s_1, s_2)} \left[\ln \frac{(s' - s_1 - s_2) + \sqrt{\lambda(s', s_1, s_2)}}{(s' - s_1 - s_2) - \sqrt{\lambda(s', s_1, s_2)}}\right]^{-1}, && \\
C(\cos\theta_i) &= \frac{1 + \Delta(s') + \cos\theta_i}{1 + \Delta(s')}. && (22)
\end{aligned}$$

[3] In the actual FORTRAN code of KORALW we assume Ω to be in the CMS$'$ frame of the effective beams. This choice reflects part of the freedom we have in reducing the multiphotonic phase space into the Born phase space; see Sect. 2.8 for details.

[4] The purpose of this "crude matrix element" is purely technical. Strictly speaking it is to generate flat $d\Phi_4$ phase space weighted events with the weight including the $1/|M_{ab}|^2_{crude}$ factor necessary for compensating matrix element peaks.

The factor N_{ab} is the relative normalization of the different decay channels with respect to the $(e\bar{\nu}, \bar{e}\nu)$ channel. The BR_a is the branching ratio for the channel a. Except for the N_{ab}, the remaining part of eq. (17) is the same for all the decay channels. The angles θ, ϕ and θ_i, ϕ_i correspond to the solid angles Ω and Ω_i of eq. (13). The $\sigma_0(s')$ and $\Delta(s')$ are some complicated functions of s' optimized to approximate the true s' behaviour of the matrix element[5]. To the above simplifications of the matrix element corresponds the weight w_B defined as

$$w_B \cdot w_{BW} = \frac{|\mathcal{M}_{ab}|^2}{|\mathcal{M}_{ab}|^2_{crude}}\ , \tag{23}$$

where the w_{BW} weight explicitly accounts for the fact that the W propagator Breit-Wigner functions $BW(s_i)$ defined in Appendix A are replaced in the crude Born distribution by the "non-running" $BW^0(s_i)$

$$BW(s_i) \to BW^0(s_i). \tag{24}$$

The appropriate w_{BW} weight factor is defined as

$$w_{BW} = \frac{BW^0(s_1)}{BW(s_1)} \frac{BW^0(s_2)}{BW(s_2)}. \tag{25}$$

The next approximations are the phase-space limits on the s_1, s_2 variables, denoted collectively as Θ_S in eq. (13). We could in principle generate s_1, s_2 in the exact Θ_S domain. For the sake of simplicity of the algorithm, however, we replace Θ_S by a broader and easier to handle integration domain defined by the function Θ_S^{crude}

$$\Theta_S^{crude} = \theta(s - s_1)\theta(s_{max} - s_2),\ \ s_{max} = s\theta\Big(\frac{s'}{4} - s_1\Big) + \frac{s'}{4}\left(1 - \theta\Big(\frac{s'}{4} - s_1\Big)\right). \tag{26}$$

The corresponding weight, denoted by w_S, is defined as

$$w_S = \Theta_S. \tag{27}$$

Finally, to make the crude σ_{Born} distribution completely factorizable into independent distributions over s_1, s_2 and angles, the $\mathcal{J}$ function of eq. (14) is approximated by 1 and the corresponding weight we denote by $w_{\mathcal{J}}$, defined as

$$w_{\mathcal{J}} = \mathcal{J}(s', s_1, s_2). \tag{28}$$

[5] Let us remember that the actual form of the functions $\Delta(s')$ and $\sigma_0(s')$ is irrelevant for the final distributions and total cross sections, as it cancels out with the introduction of the weight $w_B \cdot w_{BW}$. It may reflect only upon the efficiency of the MC generation.

Putting together all the above simplifications we end up with the following crude cross section

$$\begin{aligned}\sigma^{crude}_{Born}(s') &= \left(\sum_{a,b} N_{ab}\right) \frac{1}{32}\frac{1}{2s'}\sigma_0(s') \int_0^s ds_1 \int_0^{s_{max}} ds_2\; W(s_1)\; W(s_2) \\ & \int_0^{2\pi} d\phi \int_{-1}^{1} d\cos\theta\; T(\cos\theta) \\ & \int_0^{2\pi} d\phi_1 \int_{-1}^{1} d\cos\theta_1\; C(\cos\theta_1) \int_0^{2\pi} d\phi_2 \int_{-1}^{1} d\cos\theta_2\; C(\cos\theta_2), \qquad (29)\end{aligned}$$

which, as we remember, is the simplified form of σ_{Born} of eq. (12).

2.6 Normalization of the Crude Born

The crude Born matrix element of eq. (29) was chosen in such a way that it can easily be integrated analytically. This is crucial for the generation of the phase-space points. The overall normalization is determined by the crude cross section which is known analytically. Specifically, one can immediately integrate out all the angular degrees of freedom in eq. (29) with the overall result $(4\pi)^3/2$:

$$\sigma^{crude}_{Born}(s') = \frac{(4\pi)^3}{2} \left(\sum_{a,b} N_{ab}\right) \frac{1}{32}\frac{1}{2s'}\sigma_0(s') \int_0^s ds_1 \int_0^{s_{max}} ds_2\; W(s_1)\; W(s_2)\;. \qquad (30)$$

Also the integration over s_1 and s_2 can be carried out analytically, yielding the result $S(s')$, while resummation over the decay channels (a, b) gives $\sum_{a,b} N_{ab} = 1/(BR_{(e\nu)})^2$

$$\begin{aligned}\sigma^{crude}_{Born}(s') &= \frac{\pi^3}{(BR_{(e\nu)})^2}\frac{1}{2s'}\sigma_0(s')\; S(s'), \qquad (31)\\ S(s') &= A_1(s')^2 + 2A_1(s')A_2(s'),\\ A_1(s') &= w\left(\frac{s'}{4}\right) - w(0) + \theta\left(\frac{s'}{4} - M_W^2\right)\left[u\left(\frac{s'}{4}\right) - u(M_W^2)\right],\\ A_2(s') &= w(s) - w\left(\frac{s'}{4}\right) + \theta(s - M_W^2)\Bigg[u(s) \\ & -\theta\left(\frac{s'}{4} - M_W^2\right)u\left(\frac{s'}{4}\right) - \left(1 - \theta\left(\frac{s'}{4} - M_W^2\right)\right)u(M_W^2)\Bigg],\\ w(s) &= \frac{1}{\Gamma_W M_W}\arctan\left(\frac{s - M_W^2}{\Gamma_W M_W}\right),\\ u(s) &= \frac{1}{2M_W^2}\log\left((s - M_W^2)^2 + (\Gamma_W M_W)^2\right).\end{aligned}$$

This completes the evaluation of the Born contribution to the master formula as well as different level "crude" distributions.

2.7 Algorithm

At this point we are ready to insert the integrated crude Born of eq. (31) into the crude photonic formula of eq. (8) and write down the final one-dimensional crude distribution:

$$\sigma_{crude} = e^{\delta_{YFS}} \frac{\pi^3}{2} \frac{1}{(BR_{(e\nu)})^2} \int\limits_0^{v_{max}} dv \gamma v^{\gamma-1} \rho(v) \frac{\sigma_0((1-v)s)}{(1-v)s} S((1-v)s). \tag{32}$$

Starting from the above formula, the actual generation is done in the following steps:

1. The variable v, the central variable of the Monte Carlo algorithm, is generated according to the one-dimensional v-distribution of eq. (32) with the help of the general-purpose Monte Carlo sampler (subroutine **vesk1w**). The auxiliary weight set by **vesk1w** in the course of constructing distribution (32) is denoted by
$$w_{VES}. \tag{33}$$
2. All variables necessary for the construction of a complete kinematic configuration of real bremsstrahlung photons are generated identically as in the algorithm of YFS2 program, see ref. [2]. The appropriate additional factor w_{YFS} is calculated. We refer the interested reader to ref. [2] for more details.
3. In the next step we choose the decay channels (a, b). This is done according to the branching ratios hidden in the function N_{ab}, defined in eq. (17); see also eq. (30).
4. Now, having fixed the decay channels and the variable v, we generate variables which will lead, after introduction of an appropriate weight, to a parametrization of the four-body phase space (see eq.(13)). We start with the distribution (30). The s_1, s_2 variables are generated according to the two-dimensional function $W(s_1)\ W(s_2)\ \Theta_S^{crude}$.
5. Next we generate the angular distributions of eq. (29). First the θ and ϕ variables are generated according to the $T(\cos\theta)$ function defined in eq. (21).
6. Finally, the angles θ_i and ϕ_i are generated according to the $C(\cos\theta_i)$ distributions of eq. (22). This completes the generation of the kinematic variables given in eq. (29), and in fact of all the variables necessary to reconstruct the whole event.

The total Monte Carlo weight w is a product of all the intermediate weights listed earlier. We have separated them in two groups: technical, w_{Crude}, related to the construction of the crude Monte Carlo algorithm, eqs. (33,11,27,25), and physical, w_{Model}, related to the matrix element used, eqs. (23,28,7)

$$w = w_{Crude} w_{Model}, \quad w_{Crude} = w_{VES} w_{YFS} w_S w_{BW}, \quad w_{Model} = w_{Born} w_{\beta}. \tag{34}$$

Furthermore, for practical reasons we merged the w_B and $w_{\mathcal{J}}$ weights into one weight denoted w_{Born}

$$w_{Born} = w_B w_{\mathcal{J}}. \tag{35}$$

Finally, let us note that a shift of the overall normalization constants, such as factors π or coupling constants, was executed, in the actual FORTRAN code, between $|\mathcal{M}_{ab}|^2$, $\sigma_0(s)$ used in the differential distribution and $\sigma_0(s)$ used in formula (32).

2.8 $\mathcal{R}$-Projection and Final-State Masses

We owe the reader some explanation about the meaning of the symbol $\mathcal{R}$ used frequently in connection with the $\bar{\beta}$-functions, see e.g. eq. (1).

The $\mathcal{R}$-operation is a projection operation from the multiphotonic phase space onto the four-body phase space with no photons. Some kind of operation of this type is a part inherent in the YFS scheme in which the $\bar{\beta}_i$, $i = 1 \dots n$ functions are defined for the phase space with n photons. It simply reflects the fact that the $\bar{\beta}_i$, $i = 1 \dots n$, functions are the residua after subtraction of the singular $\tilde{S}$ factors, and strictly speaking they are defined at the singularity point $k_j = 0, j > n$. In general such a procedure has to *extrapolate* a matrix element, as defined from Feynman rules, into phase space with additional real (even hard) photons. The $\mathcal{R}$-operation is a particular realization of such an *extrapolation*, which consists of some manipulations on four-momenta in order to remove additional photons (keeping four-momentum conservation). It is also sometimes called the “technique of effective beam”. It was used in the YFS2/YFS3 programs and it has certain advantages. However, it is not the only possible solution (and not the best one); for example, in the BHLUMI program of ref. [10] the *extrapolation* was done without referring to the technique of the effective beams, but simply by extending the validity of the analytical expressions for the matrix element to higher-dimensional phase space.

Also note that in our present implementation $\mathcal{R}$-projections for $\bar{\beta}_1$ and $\bar{\beta}_2$ are reduced/related completely to the $\mathcal{R}$-projection for $\bar{\beta}_0$. In general it is not true and here it was possible only because the present approach is restricted to the leading-logarithmic approximation. It must also be remembered that the $\tilde{S}$ factors are never affected by $\mathcal{R}$-projections or *extrapolation*.

How is the $\mathcal{R}$-operation implemented in KORALW? The final-state $\mathcal{R}q_i$ momenta are simply the q_i momenta transformed from the CMS frame to the new frame CMS$'$. This new frame is defined as the rest frame of the W^-W^+ pair. The orientation of the CMS$'$ frame is defined with the help of “effective beams” p_i^{eff}. The $+z$ direction of CMS$'$ is oriented along the $\mathbf{p_1^{eff}}$ three-vector. The “effective beams” p_i^{eff} are defined in the CMS frame as

$$p_1^{eff} = p_1 - \sum_{\substack{i \\ k_3 \geq 0}} k_i, \;\; p_2^{eff} = p_2 - \sum_{\substack{i \\ k_3 < 0}} k_i. \tag{36}$$

To construct the CMS$'$ frame we transform the p_i^{eff} to their rest frame. It is done by means of a boost along the vector $v = p_1^{eff} + p_2^{eff}$. The actual Lorentz transformation is the standard one: $x'_0 = \gamma(x_0 - \boldsymbol{\beta}\mathbf{x})$, $\mathbf{x}' = \mathbf{x} + ((\gamma - 1)/\boldsymbol{\beta}^2)(\boldsymbol{\beta}\mathbf{x})\boldsymbol{\beta} - \gamma\boldsymbol{\beta}x_0$, $\boldsymbol{\beta} = \mathbf{v}/(\gamma m_v)$, $\gamma = 1/\sqrt{1 - \boldsymbol{\beta}^2}$, see ref. [11]. To complete the construction we rotate the p_i^{eff} vectors around the y- and x-axes so that the $\mathbf{p_1^{eff}}$ would point into the $+z$ direction. We call this frame the CMS$'$ frame and all other four-vectors are transformed from CMS to CMS$'$ in the same way. Finally, we redefine the effective beams in the CMS$'$ frame to be $p_1^{eff} = (\sqrt{s'}/2, 0, 0, \sqrt{s'}/2)$ and $p_2^{eff} = (\sqrt{s'}/2, 0, 0, -\sqrt{s'}/2)$. This set of CMS$'$ momenta is used to compute the matrix element.

The actual KORALW algorithm reverses in some sense the procedure described above. Namely, in the course of constructing the Monte Carlo event KORALW assumes that all the angles θ and ϕ are defined in the CMS$'$ frame, i.e. they refer directly to the $\mathcal{R}$-projected final states. Instead, the transformation back to CMS involves the $\mathcal{R}$-procedure.

The other issue we want to address here is the way we adopt the massive final-state kinematics for the massless matrix element. We have to keep in mind that in principle one can use the complete, massive matrix element. In fact the KORALW program is prepared for that option. We regard it, however, as an unnecessary complication of the program, affecting the results well below the required accuracy of LEP2. Therefore, we stick to the relatively simple massless matrix element. To estimate the accuracy of this approximation we provide two different mass-reduction prescriptions on how to transform massive four-vectors into massless ones. The first, “brute-force” method makes final-state fermions massless by simple rescaling of their three-momenta. Rescaling by the factor $q_i^0/|\mathbf{q_i}|$ is done in the CMS$'$ frame, i.e. in the CMS frame of the effective beams. Of course this procedure violates the momentum conservation. The other, “sophisticated” method conserves four-momentum and preserves all the angles. In practice it is implemented by repeating the construction done for the true, massive final states, starting from the same set of angles $\theta, \phi, \theta_1, \phi_1, \theta_2, \phi_2$ of eqs. (13,29) but with substituted zero masses. Since these two mass-reduction procedures are so different, switching between them provides a good estimate of the size of the mass effect.

Two more comments are in order here. First, we want to stress that both these mass-reduction procedures are the internal parts of the matrix element calculation and, in other parts of the program, such as in the calculation of phase-space Jacobians, true massive kinematics is used. Therefore, not only kinematic configurations generated by our program will differ from the massless case, but so will the differential and total cross sections, even if the analytical form of the matrix elements is identical to the massless case.

2.9 $\bar{\beta}$-Functions

The $\mathcal{O}(\alpha^r)$ functions $\bar{\beta}_i^{(r)}$ are defined in the Yennie-Frautschi-Suura scheme [4] as residuals after the removal of infrared virtual and real singularities. They are therefore infrared-finite. In the YFS scheme they are obtained from the $\mathcal{O}(\alpha^r)$ "raw" differential distributions which originate from Feynman diagrams. This is the case in the $\mathcal{O}(\alpha^1)$ YFS-exponentiated matrix element of BHLUMI [10, 12, 13] or YFS2 [2] and YFS3 [5]. In the $\mathcal{O}(\alpha^2)$ the "raw" distributions are often constructed as a leading-logarithmic (LL) ansatz [2, 13] instead of being taken directly from the Feynman rules[6]. Here, in our calculation we employ an LL ansatz not only in $\mathcal{O}(\alpha^2)$ but already in $\mathcal{O}(\alpha^1)$. Let us stress immediately that we treat our present LL ansatz as a temporary solution on the way to a better $\mathcal{O}(\alpha^1)$ matrix element. In this way we set already now a well-defined framework for future development of the program. The other advantage of our scheme is that events simulated in our present version of the MC program feature realistic multiple photons with finite transverse momenta. Two things have to be borne in mind: (a) in our matrix element the $\mathcal{O}(\alpha^1)$ non-logarithmic contributions are already present in the YFS form factor[7], see eq. (10); (b) the γ parameter in eq. (9), contrary to other LL approaches, is not inserted by hand but results from the phase-space integration.

The actual expressions for $\bar{\beta}_i^{(r)}, r = 1,2$ functions are defined:

$$
\begin{aligned}
&\bar{\beta}_0^{(1)} = \bar{\beta}_0^{(0)}(1+\gamma/2), \quad \bar{\beta}_0^{(2)} = \bar{\beta}_0^{(0)}(1+\gamma/2+\gamma^2/8),\\
&\bar{\beta}_1^{(1)}(k_i) = \bar{\beta}_0^{(0)}\nu(x_i)\ \tilde{S}(p_1,p_2,k_i),\\
&\bar{\beta}_1^{(2)}(k_i) = \bar{\beta}_0^{(0)}(k_i)\left[\nu(x_i) + (\gamma/4)\ \chi(x_i)\ \ln(1-x_i)\right]\ \tilde{S}(p_1,p_2,k_i),\\
&\bar{\beta}_2^{(2)}(k_i,k_j) = \bar{\beta}_0^{(0)}(k_i,k_j)\Big\{\theta(-k_i^3k_j^3)\left[\chi(x_i)\chi(x_j) - \nu(x_i) - \nu(x_j) - 1\right] \qquad (37)\\
&\qquad\qquad + \theta(k_i^3k_j^3)\Big[\chi(x_i)\chi(x_j^*)/2 + \chi(x_i^*)\chi(x_j)/2 - \nu(x_i) - \nu(x_j) - 1\Big]\Big\}\\
&\qquad\qquad \times\ \tilde{S}(p_1,p_2,k_i)\tilde{S}(p_1,p_2,k_j),\\
&\nu(x) \equiv x(-1+x/2), \qquad \chi(x) \equiv (1+(1-x)^2)/2,
\end{aligned}
$$

where $\bar{\beta}_0^{(0)}$ is defined in eq. (4), $x_i = k_i^0/E_{beam}$, and in the $\bar{\beta}_2$ definition we have to employ $x_i^* = x_i/(1-x_j)$, $i \neq j$. The function $\theta(-k_i^3k_j^3)$ is non-zero for photons in the opposite hemispheres defined with respect to the initial beams, and similarly the function $\theta(k_i^3k_j^3)$ is non-zero for photons in the same hemisphere.

It is straightforward to check, by expanding the YFS form factor of eq. (10) and truncating all our differential distributions to $\mathcal{O}(\alpha^r)$ $r = 1,2$, that the dependence

[6] This solution is simple and economical in terms of the size (computation time) and of the resulting physical precision.

[7] For the sake of comparison with other programs, we provide a switch in the input data which sets to zero the $\mathcal{O}(\alpha^1)$ subleading terms in the YFS form factor.

of the distributions on the longitudinal photon momenta follows exactly the LL QED behaviour in the corresponding order; see for example the LL QED structure functions of ref. [14].

Note that in the present LL version of the QED matrix element the reduction procedure $\mathcal{R}$ described in the previous subsection is hidden entirely in $\bar{\beta}_0^{(0)}$, that is in the Born differential cross section. The soft factors $\tilde{S}$ in eq. (37) are not concerned by the reduction procedure.

3 Semi-analytical Calculation

The semi-analytical calculation implemented in the KORWAN routine is based on the formulas for the Born cross section taken from ref. [15]. The following modifications had to be done in order to match the normalization of KORALW: introduction of the Z width to the Z propagator, change of the Breit-Wigner functions of the W resonances to the form $BW(s_i)$ of eq. (46), and appropriate changes in the overall normalization scheme. The ISR corrections are implemented via the LL structure functions (SF) approach. The complete formula is given by the convolution of the two SF's with the Born cross section

$$\sigma^{SAN}(v_{min}, v_{max}) = N_S \int_{1-v_{max}}^{1-v_{min}} dx F(x,s) \int_0^{xs} ds_1 \int_0^{(\sqrt{xs}-\sqrt{s_1})^2} ds_2\, \rho(s_1)\, \rho(s_2)\, \sigma_0^S(xs, s_1, s_2) \tag{38}$$

with

$$\rho(s_i) = \frac{1}{\pi}\frac{1}{12}\frac{\alpha_W}{\sin^2\theta_W}\frac{s_i}{BW(s_i)}, \tag{39}$$

$$N_S = \left(\frac{1}{BR_{(e\nu)}}\right)^2, \tag{40}$$

$$F(x,s) = \int_0^1 dx_1 dx_2 \delta(x - x_1 x_2) D(x_1, s) D(x_2, s). \tag{41}$$

The Breit-Wigner $\rho(s_i)$ function of the W resonances is modified to agree with the KORALW choice – $BW(s_i)$. The overall normalization factor N_S is adjusted so that the $\sigma_0^S(xs, s_1, s_2)$ function, for the cross section summed over all channels, is identical to the function $\sigma(xs, s_1, s_2)$ of eq. (4) ref. [15]. The only modification added to σ_0^S with respect to σ is the introduction of the Z width to the Z propagators in σ_0^S. The α_W denotes the QED coupling constant at WW threshold and $\sin\theta_W$ is the sine of the Weinberg angle.

The function $F(x,s)$ is the convolution of the two electron non-singlet structure functions $D(x_i, s)$, see eq. (41). In the integrating routine KORWAN we have implemented a broad series of the possible $D(x_i, s)$'s, ranging from the third-order one exponentiated according to the Jadach-Ward prescription [16] to the second-order one exponentiated according to Kuraev-Fadin [17] prescription. Also a family of structure functions designed especially to mimic the YFS formulas of KORALW is provided in this routine. The actual formulas and discussion of the various above-mentioned structure functions can be found in ref. [14].

The actual threefold integration in eq. (38) is done numerically with the help of the Gaussian-type integrating routines, originating mostly from the CERN program library. Changes of variable as well as factorizing out of most peaked part of the distribution are also performed. This part of the distribution is integrated analytically down to two-dimensional numerical integration.

4 Structure of the Program

In this section we provide the reader with a brief guide over the KORALW program. We will describe its main routines, libraries and library interfaces.

The KORALW version 1.02 program is written in FORTRAN77 and, in principle, its source code could be stored in a single big file[8]. However, we prefer to keep it divided into parts grouped in 8 UNIX subdirectories. Each of them contains a functionally related and well separated set of FORTRAN subroutines:

- `korww` - The core of the program, a low-level Monte Carlo generating the phase-space points according to the prescription of Sect. 2.7.
- `model` - A part of the code calculating the Born matrix element and higher-order corrections in the $\bar{\beta}_i$-functions.
- `semian` - The semi-analytical routine KORWAN with its subprograms.
- `glib` - An auxiliary library for KORALW, including `glibk.f`, the HBOOK-like histogramming package of ref. [18].
- `interfaces` - Interfaces to TAUOLA, PHOTOS and JETSET. The standard common block /HEPEVT/ is filled in there.
- `tauola` - the TAUOLA library.
- `kwlund` - the PHOTOS and JETSET[9] codes.
- `demo` - The demonstration decks, input data files and sample outputs of the program.

[8] Such a file can be automatically produced with the help of the script `klej` which is located in the directory `demo`. This script creates also the new directory `koralw-source`.

[9] As sufficiently up-to-date version of this library is not available in the Computer Physics Communication library. We provide our CPC distribution file with the set of dummy routines named `jestet74.f`. Users of the KORALW program should replace this file by an appropriate link.

In the UNIX distribution each directory is equipped with the makefile. Of particular interest to the user are the commands make demo and make demo2 issued from the demo directory. These commands compile the program, run it with sample data files and compare outputs against the pre-prepared samples (for the moment only the *Hewlett-Packard* test outputs are provided).

4.1 korww

The main administering routine of the korww part (directory) is called koralw(mode, xpar,npar). Its first argument MODE frames the main loop of the generation. For MODE $= -1$ the initialization of the generator takes place. The input data parameters are transferred via the vectors xpar and npar. The actual initialization of the internal variables is done in the routine filexp. Here the user can find all the other parameters not defined in an input file. Any input parameter overwritten by filexp is then returned by koralw in xpar or npar. In that mode all necessary library initializations take place. MODE $= 0$ is the main generation mode. Each call generates one Monte Carlo event. The calculation of $\bar{\beta}$-functions (subroutine betax) and, in unweighted mode, the rejection according to the best weight wtmod (see Table 6 for definition of wtmod) take place in here. Also some internal monitoring is done here, including an important quantity – the value of the total cross section coming from the events with a total weight greater than the maximal weight set for rejection. Finally, MODE $= 1$ provides all sorts of important information, with the total cross section and cross section over the maximal weight among them. Various final library printouts are generated here. In optional MODE $= 2$ some further printouts are provided.

The heart of the generator is hidden in the routine karlud and its satellites. After initialization, the series of calls in karlud to various routines builds up a Monte Carlo event following all steps sketched in Sect. 2.7. First, the v variable is generated by the routine vesk1w, which is a general-purpose one-dimensional event generator where the generated function is supplemented by the FORTRAN function rhosko. The crude σ_0 function is encoded in bornsc and the additional overall Born normalization is set in case of formula (32) in bnorl2. The YFSgen does all generation of the photonic degrees of freedom. A detailed description of this generation can be found in ref. [2], and will be skipped here. Next, having the s' fixed, the variables s_1 and s_2 are generated by the routine resms2. Generation is done in the broader phase-space area defined by the function Θ_S^{crude} of eq. (26). The decay channels are chosen in the subroutine decay. The angular variables are generated by the routines cospro and cosdec. Finally, the routine kineww reconstructs the four-momenta of the final fermions, based on the earlier generated variables. kineww returns these four-momenta in the CMS frame. As a 'byproduct' kineww provides the effective beams and, stored in the common block bormom, the four-momenta of the final fermions transformed to the CMS$'$ frame, i.e. the CMS frame of the effective beams. The common block bormom and the effective beams are provided *exclusively* for the calculation of the matrix element in subroutine **wwborn**. Call to **wwborn**, supplying the exact Born matrix element completes the single-event generation in **karlud**.

4.2 model

The central routine for the calculation of the Born matrix element is the function **wwborn**. It does the mass reduction, either directly or by calling **kineww** with the masses set to 0. Next, it calls the **wwprod** and **wdecay** routines, calculating respectively the production and decay spin amplitudes of the W bosons. Finally, the spin amplitudes are summed and squared by **wwborn**. An important subroutine used in the course of constructing production and decay amplitudes is called **polvec**. It calculates the polarization vectors of the W boson in the rectangular basis.

The $\bar{\beta}$-functions are calculated in the routine **betax**. It returns separately all the LL components of the $\bar{\beta}_0$-, $\bar{\beta}_1$- and $\bar{\beta}_2$-functions up to the second order.

4.3 semian

Semi-analytical calculations are interfaced by the subroutine **korwan**. This routine is not designed as a stand-alone routine, i.e. apart from the arguments the user has to fill a few commons required by **korwan**. Normally, these commons are all set by the **filexp** routine, called during the initialization mode of the **koralw**. In case **korwan** is used separately from **koralw**, a possible way to fill these commons could be by calling **filexp** explicitly with the **xpar** and **npar** arguments prepared like for **koralw** routine. Amongst **korwan** arguments the **keymod** key defines which version of structure functions is to be used in the calculation and **keypre** sets the precision level (levels 1, 2, 3 and 4 are implemented). **korwan** returns the total cross section and its absolute error. The structure functions are coded in function **yfspho**. The **yfspho** takes care also of the photonic singularities in the structure functions. The Born total cross section is calculated by the function **xsmuta**. The corresponding one-dimensional differential cross section is provided by the function **d1muta**. The actual integrand function is coded in **d2muta**. Appropriate changes of variables are done in **xsmuta** and **d1muta**.

4.4 glib

This auxiliary package of programs (also included in the BHLUMI program) does histogramming, book-keeping of the MC weight, plots histograms in the LaTeX format, stores and retrieves histograms into/from the disk. Quite often its entries have names and input/output parameters similar to the **hbook** and **hplot** packages from the CERN library.

4.5 interfaces

The following three physics generation libraries are now interfaced to KORALW: (i) the τ decay library TAUOLA[10] [19], (ii) the generator of radiative corrections used in W and τ decays, PHOTOS [20], (iii) JETSET [21], used for hadronization of quark pairs produced in hadronic decays of W's. The form of the interfaces is constrained by the properties of the libraries. For the final-state particles TAUOLA and PHOTOS use a coding format of the common block advocated by the Particle Data Group PDG [22]. On the other hand JETSET has its own common block for the output, which is equally widespread. Fortunately, it is not a problem as JETSET includes a special routine that translates the common block of the PDG format into its own convention.

The natural solution is thus to write particles produced in the main part of KORALW into the common block of the PDG first. Later, if necessary, generate and update in the PDG common block the τ-decay products. The $\tau^{\pm}$ leptons are assumed to be fully polarized along the flight direction. All this is done in the routine **tohep** with the help of routine **filhep** from the TAUOLA library in a rather simple way. The **filhep** routine is well tested. It is used in TAUOLA, KORALZ [3] and KORALB [23]. Subroutine **filhep** sets also logical flags to be used later by PHOTOS, which is finally called for the generation of bremsstrahlung in W and τ decays.

In the routine **tohad**, hadronization of the quarks originating from W decay can be performed. To this end, the contents of the (PDG) common block is translated into the JETSET proper common block with the help of its routine **luhepc**. Note that this is the first call to the JETSET library. Then, the position of quarks from W^-/W^+ is found and **call lujoin(2,ijoin)** is executed twice. Finally parton shower and hadronization are performed with the help of **call lushow** and **call luexec**. See the description of JETSET [21] for more details.

The two interfaces are stored in files **hepface.f** and **lundface.f** in the directory **interfaces**. In this directory there is also the **tauola-photos-ini.f** file, which contains initialization routines for the TAUOLA and PHOTOS libraries; see refs. [19, 20] for more details. Note that, as usual, TAUOLA initialization parameters, such as τ decay branching ratios have to be set to realistic values by the TAUOLA/KORALW user. The predefined boost routine **tralo4** from the τ rest frame to the laboratory frame used by TAUOLA is also stored in this file.

4.6 demo

This directory contains two demonstration programs: **demo.f** and **demo2.f**. The first, shorter one is described in detail in Sect. 5.2. In both cases, the **main** program performs initialization. The input data are read from the data file and then **koralw** arguments – vectors **npar** and **xpar** are filled. After initialization, the events are gen-

[10] Version higher than 2.5 or that of the KORALW distribution should be used.

erated. For demo2.f, histogramming is done with the help of the glibk package. The Monte Carlo events are decoded from the /hepevt/ common block [22]. At the end of the calculation, a final printout of the Monte Carlo results is provided. In demo.f two calls to the semi-analytical routine korwan are performed; Born and second-order ISR cross-sections are calculated. The data files pro.data.10k.keywgt.eq.0 and pro.data.50k.keywgt.eq.1, together with the corresponding output files pro.output.10k.keywgt.eq.0.hp and pro.output.50k.keywgt.eq.1.hp are located in the subdirectory 190gev. These output files were obtained on a *Hewlett-Packard* installation.

5 How to Use the Program?

In this section we discuss in some detail the input and output parameters of the KORALW, we show how to write a simple program calling KORALW and discuss the contents of the standard KORALW printout. We also do a similar description of the semi-analytical routine KORWAN.

5.1 Input/Output Parameters

The input parameters are sent to koralw via the vector arguments npar and xpar in MODE $= -1$. They are collected in Tables 1 and 2. The additional internal-input parameters are set in MODE $= -1$ with the help of the routine filexp. They are collected in Table 3. In the generation mode (MODE $= 0$) all the four-momenta of the event are provided in the standard common block /hepevt/. For those users who do not want to use the /hepevt/ common block, the alternative, internal commons /momset/ and /momdec/ can provide all four-momenta, as described in Tables 4 and 5. In the case of weighted events, the weights available for the user in the common /wgtall/ are listed in Table 6. To reconstruct the cross section out of the wtset weights one needs to multiply the crude cross section (xpar(30) in MODE $= 1$) by the average of the wtcrud times chosen wtset. In Table 7 the post-generation (MODE $= 1$) output parameters sent as arguments of koralw are collected. Finally, Table 8 summarizes the meanings of the arguments of the semi-analytical routine korwan.

5.2 Simple User Program

In this section we provide the user with a very simple demo program. It shows how to do initialization of the koralw routine, how to generate events and read its output. Also a call to the korwan routine is explicitly shown. The listing of the demo program is given in Appendix B and the actual file demo.f is provided in the directory demo.

Parameter	Meaning
npar(1)=KeyRad	=1000*KeyCul+100*KeyNLL+10*KeyFSR+KeyISR – *General radiation switch* KeyISR=0 Initial State Radiation is OFF, KeyISR=1 Initial State Radiation is ON. KeyFSR Final State Radiation switch, INACTIVE. KeyNLL=0 sets to zero the Next-to Leading α/π terms in YFS form-factor, useful for comparisons, KeyNLL=1 the α/π terms are kept in YFS form-factor. KeyCul Coulomb correction, INACTIVE.
npar(2)=KeyPhy	=10000*KeyRed+1000*KeySpn+100*KeyZet+10*KeyMas+KeyBra – *General physics switch* KeyBra=0 sets branching ratios to Born values, no mixing assumed $ud, cs = 1/3,\ \ e\nu, \mu\nu, \tau\nu = 1/9,$ others $= 0$ KeyBra=1 Branching ratios with CKM mixing and QCD, preset in filexp. KeyMas=0 Massless kinematics for W decay products; τ decay, radiative correction in decay and hadronization must be switched off. KeyMas=1 Massive kinematics for W decay products, masses are preset in filexp. KeyZet=0 Z width in Z propagator: $(s/M_Z)\Gamma_Z$, KeyZet=1 Z width in Z propagator: $M_Z\Gamma_Z$, KeyZet=2 no Z width in Z propagator. KeySpn=0 spin effects are switched OFF in W decays, for tests only, KeySpn=1 spin effects are switched ON in W decays, normal setting. KeyRed=0 "sophisticated" reduction of massive to massless Matrix Element, KeyRed=1 "brute force" reduction of massive to massless Matrix Element, four-momentum NON-conserving.
npar(3)=KeyTek	=10*KeyRnd +KeyWgt – *General technical switch* KeyWgt=0 unweighted events, wtmod=1, useful for apparatus Monte Carlo, KeyWgt=1 weighted events, wtmod varying. KeyRnd NOT IMPLEMENTED.

npar(4)=KeyMis	=KeyMix,
	– *Miscellaneous options switch*
	KeyMix=0 $\sin^2\theta_W$ calculated from α_W, G_μ, M_W (LEP2 Workshop prescription)
	KeyMix=1 $\sin^2\theta_W$ calculated from α_W, G_μ, M_Z (as in 1.00 version)

Table 1: *List of input parameters of the KORALW generator.*

Parameter	Meaning
`npar(5)=KeyDwm`	Sets decay channel of W^- resonance
`npar(6)=KeyDwp`	Sets decay channel of W^+ resonance $0 =$ all channels according to branching ratios $1 = ud, 2 = cd, 3 = us, 4 = cs, 5 = ub, 6 = cb, 7 = e, 8 = \mu, 9 = \tau$
`npar(7)=nout`	Output unit number for the generator.
`npar(21)=jak1`	For TAUOLA, sets τ of W^+ decay channel
`npar(22)=jak2`	For TAUOLA, sets τ of W^- decay channel `jak1,jak2=-1` TAUOLA is OFF, `jak1,jak2=0` all channels, `jak1,jak2>0` single specific channel, see TAUOLA manual.
`npar(23)=itdkrc`	For TAUOLA, radiative corrections in leponic τ decays switch, `itdkrc=1` corrections are ON, see TAUOLA manual, `itdkrc=0` corrections are OFF, see TAUOLA manual.
`npar(24)=ifphot`	PHOTOS, activation switch, `ifphot=1` PHOTOS is ON, `ifphot=0` PHOTOS is OFF.
`npar(25)=ifhadm`	W^- hadronization activation switch (JETSET)
`npar(26)=ifhadp`	W^+ hadronization activation switch (JETSET) `ifhadm,ifhadp=1` hadronization is ON, `ifhadm,ifhadp=0` hadronization is OFF. For the moment `ifhadm` and `ifhadp` have to be equal!
`xpar(1)=cmsene`	$\sqrt{s}$, centre-of-mass (CMS) energy in GeV units.
`xpar(2)=gmu`	G_F, Fermi constant.
`xpar(3)=alfwin`	$1/\alpha_W$ inverse QED coupling constant at M_W scale, see Appendix A.
`xpar(4)=amaz`	M_Z, mass of Z boson in GeV.
`xpar(5)=gammz`	Γ_Z, width of Z boson in GeV.
`xpar(6)=amaw`	M_W, mass of W boson in GeV.
`xpar(7)=gammw`	Γ_W, width of W boson in GeV, for (`gammw < 0`) Γ_W is recalculated from G_μ and M_W.
`xpar(8)=vvmin`	Minimum v-variable (dimensionless), Infra-Red cut-off.
`xpar(9)=vvmax`	Maximum v-variable.
`xpar(10)=wtmax`	Maximum weight for rejection, for `wtmax < 0` default settings of `koralw` are used.

Table 2: *List of input parameters of the KORALW generator.*

Parameter	Meaning
`alfinv`	$1/\alpha(0)$ Inverse QED coupling constant at scale Q^2 =0.
`gpicob`	Conversion from GeV^{-2} to pb.
`amel`	Electron mass used by the bremsstrahlung generator.
`br(20)`	W branching ratios, numbering of entries is: $1 = ud, 2 = cd, 3 = us, 4 = cs, 5 = ub, 6 = cb, 7 = e, 8 = \mu, 9 = \tau$; br(7) is always used for normalization and should not be set to 0.
`amafin(20)`	Masses of the W decay products; used entries are $1 = d, 2 = u, 3 = s, 4 = c, 5 = b$ $11 = e, 12 = \nu_e, 13 = \mu, 14 = \nu_\mu, 15 = \tau, 16 = \nu_\tau$; masses of τ and ν_τ have to be independently set to the same numerical values in intialization of TAUOLA.

Table 3: *List of input parameters of the KORALW generator set in the routine* **`filexp`**.

Parameter	Meaning
`qeff1(4)`	Effective parameter for matrix element (e^- effective beam, see formula (36)), CMS.
`qeff2(4)`	Effective parameter for matrix element (e^+ effective beam, see formula (36)), CMS.
`sphum(4)`	Sum of four-momenta of IS photons, CMS.
`sphot(100,4)`	Four-momenta of IS photons, CMS.
`nphot`	Multiplicity of IS photons

Table 4: *List of four-momenta in the internal common block* `/momset/` *of the KORALW generator.*

Parameter	Meaning
`q1(4)`	Four-momentum of the W^- resonance, CMS.
`q2(4)`	Four-momentum of the W^+ resonance, CMS.
`p1(4)`	Four-momentum of the fermion from W^- decay, CMS.
`p2(4)`	Four-momentum of the antifermion from W^- decay, CMS.
`p3(4)`	Four-momentum of the fermion from W^+ decay, CMS.
`p4(4)`	Four-momentum of the antifermion from W^+ decay, CMS.

Table 5: *List of four-momenta in the internal common block* `/momdec/` *of the KORALW generator.*

Parameter	Meaning
wtcrud	Crude weight, necessary to build the total weight out of wtset.
wtmod	Best weight
	For KeyISR=0 wtmod=wtcrud*wtset(1),
	For KeyISR=1 wtmod=wtcrud*wtset(3).
wtset(100)	*– Born matrix el. with various $\bar{\beta}$ contributions;*
	to get total weight must be multiplied by wtcrud.
wtset(1)	Zero-order complete ($\bar{\beta}_0$)
wtset(2)	First-order complete ($\bar{\beta}_0 + \bar{\beta}_1$)
wtset(3)	Second-order complete ($\bar{\beta}_0 + \bar{\beta}_1 + \bar{\beta}_2$)
wtset(20)	First-order $\bar{\beta}_0$ contribution alone
wtset(21)	First-order $\bar{\beta}_1$ contribution alone
wtset(30)	Second-order $\bar{\beta}_0$ contribution alone
wtset(31)	Second-order $\bar{\beta}_1$ contribution alone
wtset(32)	Second-order $\bar{\beta}_2$ contribution alone

Table 6: *List of output weights in the common block* **/wgtall/** *of the KORALW generator.*

Parameter	Meaning
npar(10)=nevtru	For KeyWgt=0 number of accepted *unweighted* events,
	For KeyWgt=1 number of generated *weighted* events.
npar(11)=nevtot	For KeyWgt=0 number of generated events before rejection,
	For KeyWgt=1 npar(11)=npar(10)
xpar(20)=xsecpb	*Best* cross section in pb.
	For KeyISR=0 Born cross section in pb,
	For KeyISR=1 second-order exponentiated cross section in pb.
xpar(21)=errpb	Error of total cross section xsecpb, in pb.
xpar(30)=xcrude	Cross section in pb:
	For KeyWgt=0 best cross section xcrude*nevtru/nevtot*wtmax,
	For KeyWgt=1 crude cross section in pb.
xpar(31)=wtmax	Maximal weight used for rejection.

Table 7: *List of output parameters of the KORALW generator in Mode* $= 1$*.*

Parameter	Meaning
	INPUT
svar	s, CMS energy squared [GeV].
vvmin	v_{min}, minimal v variable, in most cases should be set to 0.
vvmax	v_{max}, maximal v variable.

keymod	Defines type of Structure Functions used for IS Radiation,
	keymod= 0 No ISR, Born,
	keymod=300 Zero Order, YFS style,
	keymod=301 First Order, YFS style,
	keymod=302 Second Order, YFS style,
	keymod=303 Third Order, YFS style,
	keymod=502 Second Order, Gribov-Kuraev-Fadin style,
	keymod=310 First Order YFS Beta0 only,
	keymod=311 First Order YFS Beta1 only,
	keymod=320 Second Order YFS Beta0 only,
	keymod=321 Second Order YFS Beta1 only,
	keymod=322 Second Order YFS Beta2 only,
	keymod<0 as (-keymod) but multiplied by v, $(d\sigma/d\log v)$, for tests only.

keypre	Defines precision level of the computation.
	For keymod=0 (No ISR, Born), in $(e\nu_e)$ channel:
	keypre=1 absolute error 1×10^{-5} pb,
	keypre=2 absolute error 1×10^{-6} pb,
	keypre=3 absolute error 1×10^{-7} pb,
	For keymod>0 (ISR), in $(e\nu_e)$ channel:
	keypre=1 absolute error 3×10^{-5} pb,
	keypre=2 absolute error 1×10^{-5} pb,
	keypre=3 absolute error 1×10^{-6} pb,
	keypre=4 absolute error 1×10^{-7} pb.

	OUTPUT
xsect	Cross section in pb.
errabs	Absolute error in pb.

Table 8: *List of arguments of the KORWAN routine.*

In this short demo we have explicitly called the LUND listing routine `lulist`. All the LUND part is marked by _* (((_ and _*)))_, and can be commented out if necessary.

Demo starts by initializing the histogramming package GLIBK used by KORALW. Also appropriate common `cglib` is set. Next, the vectors `npar` and `xpar` are loaded with parameters required to initialize the `koralw` routine. The `koralw` routine is then called in the initialization mode ($MODE = -1$). After that, `koralw` is ready to generate events ($MODE = 0$). This takes place in the main loop of the program. Call to the `koralw` in the post generation mode ($MODE = +1$) provides output parameters in the vectors `npar` and `xpar`. Two of them, the total cross section and its error, are explicitly extracted and later on printed out.

Finally, the semi-analytical routine `korwan` is called for the second-order ISR total cross section and the result is printed out. An accuracy is set to a low value in order to make the execution time short. There is no separate initialization mode for `korwan`. One has to remember however, that all the necessary common blocks must be filled before calling the routine `korwan`. If `korwan` is used together with the `koralw` generator, the `filexp` routine called by `koralw` for $MODE = -1$ takes care of this initialization. Otherwise the solution might be to call `filexp` explicitly, with the arguments `xpar` and `npar` prepared as for `koralw`. Important excerpts from the output of this program are listed in Appendix C.

Printout starts with the detailed specification of the actually used input parameters. Also logos of all the activated libraries (TAUOLA, PHOTOS, JETSET) are printed here. Next, a printout of the one full event is shown. The other events, 10 printed by default, are not shown. The chosen event is the second generated one. It is chosen because one of the W bosons decays into $\tau\bar{\nu}$, and one can see TAUOLA doing the τ decay, whereas the other W decays hadronically with the JETSET acting on the cascade.

The final reports of KORALW are collected in four windows: V, A, B and C. Window V provides technical information on the performance of the Monte Carlo sampler `vesk1w`, generating the vital v-distribution. The approximate integral of the v-function, as approximated by `vesk1w` is printed in line $V4$, the reweighted (exact) value in line $V5$ and the exact value by Gaussian integration in line $V6$. Corresponding ratios follow in lines $V7$ and $V8$.

Window A provides a technical internal report of the `karlud` routine, i.e. information on the crude distribution and the Born matrix element. Line $A0$ gives the total number of the generated *weighted* events. Line $A1$ shows the number of events with the negative weight `wtcrud`. This entry should be equal to zero. Line $A2$ provides the value of the master crude distribution of eq. (32). The difference with line $V6$ is in the YFS form-factor of eq. (10) which is not included in $V6$. Line $A3$ shows the average of the weight `wtcrud`, and the line $A4$ – the corresponding "cross section" (no matrix element is included here – only the crude distribution formula). Lines $A5$-$A8$ repeat the information of lines $A0$-$A1$, $A3$-$A4$, but for the weight *with* the Born matrix element, i.e. `wtcrud*wtset(1)`. Lines $A9$-$A11$ provide information on how many of the events have the weight wtcrud*wtset(1) over the wtmax, the maximal weight for rejection. Their contribution to the cross section in absolute units (picobarns) and relative to the Born cross section is listed in $A10$ and $A11$, respectively. Note that there are no $\bar{\beta}$-functions included in any of the printouts in Window A.

Window B is devoted to the technical information on the first order $\bar{\beta}_0$- and $\bar{\beta}_1$-functions. Entry $B0$ is the number of negative $\bar{\beta}_0$ weights wtbe01. Entry $B1$ is the average of the wtbe01 weight and line $B2$ the average of the wtset(20)*wtcrud weight. Entry $B3$ is the contribution of $\bar{\beta}_0$ to the cross section (in picobarns). Entries $B4$-$B6$ are similar to the $B0$-$B1$, $B3$ ones, but for the $\bar{\beta}_1$-function.

Window C is the most important from the users point of view. It provides the total cross sections of the first and second orders ($C2$ and $C5$), in picobarns, with absolute errors in picobarns. Also the average of the $\bar{\beta}_0 + \bar{\beta}_1$ weight wtbe01+wtbe10 (up to the first order) is provided in line $C1$, whereas the number of negative weights wtbe01+wtbe10 is given in line $C0$. Similar entries for $\bar{\beta}_0 + \bar{\beta}_1 + \bar{\beta}_2$ the weight wtbe02+wtbe11+wtbe20 (up to the second order) are given in $C4$ and $C3$. A very important entry, the number of events with a total weight wtmod (wtmod is the Born weight for KeyISR=0 and the second order ISR weight for KeyISR=1) over wtmax, is shown in line $C6$. The corresponding total cross section (in picobarns) from above the wtmax is printed in line $C7$. Since wtmod is used in actual rejection for the unweighted events, the smallness of the $C7$ entry is a good indication that the value of wtmax is not chosen too low.

This completes the description of the output of KORALW. The remaining two entries shown in the demo output are produced by the demo program.

6 Acknowledgements

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A Matrix Element

The full amplitude (accordingly to the normalization convention of ref. [24]) for the W-pair production and decay into channel (kl) can be written as

$$\begin{aligned}\mathcal{M}_{kl} =&\mathcal{M}_{kl}(\Delta\sigma;\sigma_1,\sigma_2,\sigma_3,\sigma_4)\\ =&\frac{1}{G_W(s_1)}\frac{1}{G_W(s_2)}\sum_{\lambda_1}\sum_{\lambda_2}\mathcal{M}_1(\Delta\sigma;\lambda_1,\lambda_2)\mathcal{M}_2^k(\lambda_1,\sigma_1,\sigma_2)\mathcal{M}_3^l(\lambda_2,\sigma_3,\sigma_4),\end{aligned} \tag{42}$$

where $\Delta\sigma$ is the difference between e^- and e^+ helicities[11], λ_1, λ_2 are polarizations of the intermediate W^- and W^+, respectively, and σ_i are helicities of the W-decay products.

The amplitude $\mathcal{M}_1$ is the polarization amplitude for the W-pair production in e^+e^- annihilation. For this amplitude we use the expression given in ref. [9][12]:

$$\begin{aligned}\mathcal{M}_1(\Delta\sigma=-;\lambda_1,\lambda_2) = -e^2\sqrt{s}\Bigg\{2\left[\frac{1}{s}-\frac{\eta_L}{s-M_Z^2+is\Gamma_Z/M_Z}\right]&\\ \times\Big[(Q_2\epsilon_1)F_1(\epsilon_2)-(Q_1\epsilon_2)F_1(\epsilon_1)-(\epsilon_1\epsilon_2)F_1(Q_2)\Big]&\\ +\frac{\zeta_L}{(p_1-Q_1)^2}F_3(\epsilon_2,p_1-Q_1,\epsilon_1)\Bigg\},&\end{aligned} \tag{43}$$

where

$$\begin{aligned}\eta_\sigma &= \begin{cases}1-\frac{1}{2\sin^2\theta_W} & \text{for } \sigma=L,\\ 1 & \text{for } \sigma=R,\end{cases}\\ \zeta_\sigma &= \begin{cases}\frac{1}{2\sin^2\theta_W} & \text{for } \sigma=L,\\ 0 & \text{for } \sigma=R,\end{cases}\end{aligned} \tag{44}$$

and

$$G_W(s_i) = s_i - M_W^2 + is_i\Gamma_W/M_W, \tag{45}$$

$$BW(s_i) = |G_W(s_i)|^2, \tag{46}$$

M_W and Γ_W denote the mass and the width of the W bosons, Q_1,ϵ_1 and Q_2,ϵ_2 are four-momenta and polarization vectors of W^- and W^+, respectively, while $s_1 = Q_1^2$ and $s_2 = Q_2^2$.

[11] Since the masses of the initial-state leptons are neglected in the matrix-element calculation ($m_e^2 \ll s$), $\Delta\sigma = -$ or $+$ only.

[12] Note that our WW-production amplitude differs from that in ref. [9] by a normalization factor and by the Z boson propagator, where we have included the Z width Γ_Z, and M_Z denotes the Z mass.

The constant e in eq. (43) denotes as usual the positron electric charge, for which in practical calculations we substitute

$$e \longrightarrow e_W = \sqrt{4\pi\alpha_W}, \tag{47}$$

whith α_W being the effective coupling constant at the $2M_W$ scale related to the coupling constant α in the Thomson limit through the relation

$$\alpha_W = \frac{\sqrt{2}G_F M_W^2 \sin^2\theta_W}{\pi} = \frac{\alpha}{1-\Delta r}, \tag{48}$$

where G_F is the Fermi coupling constant, $\sin^2\theta_W$ is the weak mixing parameter, and Δr is a radiative correction to the relation between W and Z bosons masses. Using the effective coupling constant α_W instead of α in the Born distribution[13] takes into account a major portion of radiative non-bremsstrahlung corrections [25].

The coefficients F_i in eq. (43) are some functions of four-vectors contracted with Dirac matrices defined in ref. [9]. In order to write them down explicitly, let us introduce some auxiliary notation. For any a being a four-vector in the Minkowski space, $a = (a^0, a^1, a^2, a^3)$, we define

$$a_\pm = a^1 \pm i a^2, \quad \bar{a}_\pm = a^0 \pm a^3. \tag{49}$$

Then the functions F_i read

$$F_1(a) = a_-, \tag{50}$$

$$F_3(a,b,c) = \bar{a}_+\,(\,\bar{b}_- c_- - b_-\bar{c}_-\,) - a_-\,(\,\bar{b}_+\bar{c}_- - b_+ c_-\,). \tag{51}$$

The amplitude for opposite values of e^+ and e^- helicities, i.e. $\Delta\sigma = +$, can be obtained from eq. (43) by means of the interchange: $F_i \rightarrow -F_i^*$ and $L \rightarrow R$. The above expression for the production amplitude $\mathcal{M}_1$ has been derived within the spinor formalism, using the Weyl representation for Dirac matrices and a rectangular basis for polarization vectors (for more details see ref. [9]). It is given in the CMS' of the initial lepton beams with the $+z$-axis pointing in the e^- beam direction. Three vector boson couplings according to the Standard Model are only included, however extensions to the general type couplings at the $WW\gamma$ and WWZ vertices, see e.g. ref. [8], are possible within the above formalism.

The amplitudes $\mathcal{M}_2^k$ and $\mathcal{M}_3^l$ describe respectively W^- and W^+ decays into corresponding channels. Following ref. [8], the polarization amplitudes for W^- decay into massless fermions can be written in the form

$$\mathcal{M}^{W^-\to f_1\bar{f}_2}(\lambda_1,\sigma_1,\sigma_2) = e g_-^{Wf_1f_2} C\,\delta_{\sigma_1,-}\delta_{\sigma_2,+} 2\sqrt{q_1^0 q_2^0}\,S(q_1,\epsilon(Q_1,\lambda_1),q_2)^-_{--}. \tag{52}$$

[13] We include α_W among the KORALW input parameters.

The coupling factors for the W decay are

$$g_-^{W\nu e} = g_-^{We\nu} = \frac{1}{\sqrt{2}\sin\theta_W},$$
$$g_-^{Wu_id_j} = (g_-^{Wd_ju_i})^* = \frac{U_{ij}}{\sqrt{2}\sin\theta_W}, \qquad (53)$$

where $(u_1,u_2,u_3) = (u,c,t)$, $(d_1,d_2,d_3) = (d,s,b)$, and U_{ij} denotes the Cabibbo-Kobayashi-Maskawa matrix elements. C is the effective colour factor: $C = \sqrt{3}$ for quarks and $C = 1$ for leptons. In practical calculations of the cross section the combination CU_{ij} is absorbed into the W-decay branching ratios. For the electric charge e we use again the substitution of eq. (47).

$S(q_1, \epsilon(Q_1,\lambda_1), q_2)^-_{--}$ is a spinorial string function defined in Appendix C of ref. [8]. The polarization four-vector $\epsilon(Q_1,\lambda_1)$ of the W^- boson is given in the rectangular (real) basis, see Appendix C of ref. [8]. Using the notation of eq. (49), the spinorial string function corresponding to various final-state momenta configurations can be expressed as

1. $q_1^3 \neq -q_1^0$ and $q_2^3 \neq -q_2^0$:

$$S(q_1,\epsilon,q_2)^-_{--} = \frac{1}{2\sqrt{q_1^0 q_2^0 \bar{q}_{1+}\bar{q}_{2+}}}\,[\bar{q}_{1+}(\bar{q}_{2+}\bar{\epsilon}_- - q_{2-}\epsilon_+) + q_{1+}(q_{2-}\bar{\epsilon}_+ - \bar{q}_{2+}\epsilon_-)], \qquad (54)$$

2. $q_1^3 = -q_1^0$ and $q_2^3 \neq -q_2^0$:

$$S(q_1,\epsilon,q_2)^-_{--} = \frac{1}{\sqrt{2q_2^0\bar{q}_{2+}}}\,(q_{2-}\bar{\epsilon}_+ - \bar{q}_{2+}\epsilon_-), \qquad (55)$$

3. $q_1^3 \neq -q_1^0$ and $q_2^3 = -q_2^0$:

$$S(q_1,\epsilon,q_2)^-_{--} = \frac{1}{\sqrt{2q_1^0\bar{q}_{1+}}}\,(\bar{q}_{1+}\epsilon_+ - q_{1+}\bar{\epsilon}_+), \qquad (56)$$

4. $q_1^3 = -q_1^0$ and $q_2^3 = -q_2^0$:

$$S(q_1,\epsilon,q_2)^-_{--} = \bar{\epsilon}_+. \qquad (57)$$

For the W^+ decay one merely needs to substitute $\epsilon(Q_1,\lambda_1) \longrightarrow \epsilon(Q_2,\lambda_2)$. The formulae for the decay amplitudes are given in the massless fermion limit, which is justified by the fact that all W-decay products are relatively light when compared to the W-boson mass. In such a case expressions for the polarization amplitudes are particularly simple and fast in numerical evaluations. However, including the fermion masses, or any other modifications, in the above amplitudes does not pose a major problem and can be done in the future, if necessary. Care must be taken of the normalization and relative amplitude phases, especially if they originate from calculations based on different conventions.

The polarization-averaged Born cross section for the W-pair production and decay into the channel (kl) can then be obtained from the formula:

$$d\sigma_{kl} = \frac{1}{2s}\frac{1}{4}\sum|\mathcal{M}_{kl}|^2 d\Phi_4, \tag{58}$$

with $d\Phi_4$ being the invariant four-body phase space

$$d\Phi_4 = (2\pi)^4\,\delta^{(4)}\left(p_1+p_2-\sum_{i=1}^{4}q_i\right)\prod_{i=1}^{4}\frac{d^3q_i}{(2\pi)^3 2q_i^0}, \tag{59}$$

and the sum $\sum|\mathcal{M}_{kl}|^2$ running over all initial- and final-states polarizations. The factor $\frac{1}{4}$ in formula (58) stands for the initial-state spin average.

B Listing of the Simple Program

```
      PROGRAM  MAIN
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION XPAR(100), NPAR(100)

* ((((       LUND stuff
      COMMON / LUDAT1 / MSTU,PARU,MSTJ,PARJ
      REAL PARU(200),PARJ(200)
      INTEGER MSTU(200),MSTJ(200)
* ))))  end LUND stuff

* For internal histogramming package GLIBK
      COMMON / CGLIB / BLIBK(20000)
      SAVE   / CGLIB /
      SAVE
      NOUT=16
      OPEN(NOUT,file='pro.output')
      CALL GLIMIT(20000)
      CALL GOUTPU(NOUT)
* End of initialization of internal histogramming package GLIBK

* ((((       LUND stuff
      mstu(11)=nout
* ))))  end LUND stuff

* ============================================
* Prepare the input parameters for KORALW
      NPAR(1)=1    ! KeyRad =1000*KeyCul+100*KeyNLL+10*KeyFSR+KeyISR
      NPAR(2)=1011 ! KeyPhy =10000*KeyRed+1000*KeySpn+100*KeyZet
                   !                                  +10*KeyMas+KeyBra
      NPAR(3)=10   ! KeyTek =10*KeyRnd+KeyWgt

      NPAR(4)=0    ! KeyMis =  KeyMix
      NPAR(5)=0    ! KEYDWM    W- decay: 7=(ev), 0=all ch.
      NPAR(6)=0    ! KEYDWP    W+ decay: 7=(ev), 0=all ch.

      NPAR(7)=NOUT ! NOUT      Output unit number, for Nout.LE.0, Nout=16

      NPAR(21)=0   ! JAK1      Decay mode tau+
      NPAR(22)=0   ! JAK2      Decay mode tau-
      NPAR(23)=1   ! ITDKRC    Bremsstrahlung switch in Tauola
      NPAR(24)=1   ! IFPHOT    PHOTOS switch
      NPAR(25)=1   ! IFHADM    Hadronisation W-
      NPAR(26)=1   ! IFHADP    Hadronisation W+IFHADP

      XPAR(1)=180D0      ! CMSENE CMS total energy
      XPAR(2)=1.16639D-5 ! GFERMI Fermi Constant
      XPAR(3)=128.07D0   ! ALFWIN alpha QED at WW tresh. scale (inverse)
      XPAR(4)=91.1888d0  ! AMAZ   Z mass
      XPAR(5)=2.4974d0   ! GAMMZ  Z width
      XPAR(6)=80.230d0   ! AMAW   W mass
      XPAR(7)=-2.03d0    ! GAMMW  W with,
                         !        For GAMMW<0 RECALCULATED inside program
      XPAR(8)=1D-6       ! VVMIN  Photon spectrum parameter
      XPAR(9)=0.99D0     ! VVMAX  Photon spectrum parameter
      XPAR(10)=-1D0      ! WTMAX  max weight for reject.
                         !        WTMAX<0 = default setting
      cmsene=xpar(1)     ! input for korwan
* End of setting parameters

*      KORALW initialisation            ================
      CALL KORALW(-1,XPAR,NPAR)
* end KORALW initialisation             ================
* KORALW main loop
      DO 100 I=1,1000
        CALL KORALW( 0,XPAR,NPAR)

* ((((        LUND stuff
        IF(I.LE.10) CALL lulist(2)
* ))))   end LUND stuff

  100 CONTINUE
* End KORALW main loop

*     final printouts of KORALW        =================
      CALL KORALW( 1,XPAR,NPAR)
* some output parameters of KORALW
      XSECT = xpar(20)
      ERSECT= xpar(21)
* end final printouts of KORALW        =================

* SemiAnalytical O(alf2)exp. , KORWAN
      keypho=302
      keyacc=1
      call korwan(cmsene**2,0d0,1d0,keypho,keyacc,Xtot2e,errabs)
* End SemiAnalytical O(alf2)exp. , KORWAN

* Final printouts of demo

      WRITE(NOUT,'(A60)') '============ DEMO ============'
      WRITE(NOUT,*)
      WRITE(NOUT,'(1X,F17.8, 4H  +-, F11.8, 1X, A30)')
     $     xsect,ersect, 'MC Best, XPAR, KORALW'
      WRITE(NOUT,*)
      WRITE(NOUT,'(1X,F17.8, 4H  +-, F11.8, 1X, A30)')
     $     Xtot2e,errabs,'SemiAnal O(alf2)exp.LL, KORWAN'
      WRITE(NOUT,*)
      WRITE(NOUT,'(A60)') '========== End DEMO =========='

      END
```

C Output of the Simple Program

```
         ***************************************************************
         *  ###   ###                                   ###       ###  *
         *  ###  ###  ####    ######      ##     ##     ###       ###  *
         *  ### ###  ##  ##   ##   ##    ####    ##     ###       ###  *
         *  ######  ##    ##  ##   ##   ##  ##   ##     ###       ###  *
         *  ######  ##    ##  #####    ##    ##  ##     ###   #   ###  *
         *  ### ###  ##  ##   ##  ##   ########  ##      ### ### ###   *
         *  ###  ###  ####    ##   ##  ##    ##  #######  #### ####    *
         *  ###   ###            version 1.02              ##   ##     *
         ***************************************************************
         ************************** July 95 ****************************
         ***************************************************************
         *  Written by:                                                *
         *    M. Skrzypek    (skrzypek@hpjmiady.ifj.edu)               *
         *    S. Jadach      (jadach@cernvm.cern.ch),                  *
         *    W. Placzek     (placzek@hephp02.phys.utk.edu),           *
         *    Z. Was         (wasm@cernvm.cern.ch)                     *
         *  Papers:                                                    *
         *    preprint CERN-TH/95-205, July 1995                       *
         *  WWW:                                                       *
         *    http://hpjmiady.ifj.edu.pl/                              *
         *  Acknowledgements:                                          *
         *    We acknowledge warmly very useful help of M. Martinez   *
         *    in testing the program.                                  *
         ***************************************************************
***************************************************************************
*                KORALW input parameters used                             *
*     180.00000000                 CMS energy total         CMSENE    I.0 *
* ***********************************************                         *
*                1                 QED super-switch         KeyRad    IQ1 *
*                1                 Init. state Rad.         KeyISR    IQ2 *
*                0                 Final state Rad.         KeyFSR    IQ3 *
*                0                 Next. To Leading         KeyNLL    IQ4 *
*                0                 Coulomb, INACTIVE        KeyCul    IQ5 *
* ***********************************************                         *
*             1011                 Physics super-switc      KeyPhy    IP1 *
*                0                 FS mass reduction        KeyRed    IP2 *
*                1                 Spin in W decays         KeySpn    IP3 *
*                0                 Z propag.                KeyZet    IP4 *
*                1                 Mass kinematics.         KeyMas    IP5 *
*                1                 Branching Rat.           KeyBra    IP6 *
* ***********************************************                         *
*               10                 Technical super-swi      KeyTek    IT1 *
*                1                 rand Numb type           KeyRnd    IT2 *
*                0                 weighting  switch        KeyWgt    IT3 *
* ***********************************************                         *
*                0                 Miscelaneous             KeyMis    IM1 *
*                0                 sinW2 input type         KeyMix    IM2 *
* ***********************************************                         *
*       1.16639000                 G_mu * 1d5               GMU       I.1 *
*     128.07000000                 inv alpha_w              ALFWIN    I.2 *
*      91.18880000                 Z mass   [GeV]           AMAZ      I.3 *
*       2.49740000                 Z width  [GeV]           GAMMZ     I.4 *
*      80.23000000                 W mass   [GeV]           AMAW      I.5 *
*       2.03367033                 W width  [GeV]           GAMMW     I.6 *
*        .00000100                 dummy infrared cut       VVMIN     I.7 *
*        .99000000                 v_max ( =1 )             VVMAX     I.8 *
*                0                 W- decay mode            KEYDWM    I.9 *
*                0                 W+ decay mode            KEYDWP    I10 *
*       4.00000000                 max wt for rejectn.      WTMAX     I11 *
* ***********************************************                         *
*        .23103091                 sin(theta_W)**2          SINW2     I12 *
* ***********************************************                          *
*      -.01409737              LEP200 workshop             GVE     *** *
*      -.18579403              LEP200 workshop             GAE     *** *
*       .23040993              LEP200 workshop             GWF     *** *
*       .31324289              LEP200 workshop             GWWG    *** *
*       .57147945              LEP200 workshop             GWWZ    *** *
* ***********************************************                          *
*         Z width in Z propagator: s/M_Z *GAMM_Z                           *
* ***********************************************                          *
*                                                                          *
*                                          DECAYS:                         *
*                               branching ratios:                          *
*       .32110000                                    ud       BR(1)    IB1 *
*       .01630000                                    cd       BR(2)    IB2 *
*       .01635000                                    us       BR(3)    IB3 *
*       .32043000                                    cs       BR(4)    IB4 *
*       .00002000                                    ub       BR(5)    IB5 *
*       .00070000                                    cb       BR(6)    IB6 *
*       .10840000                                     e       BR(7)    IB7 *
*       .10840000                                    mu       BR(8)    IB8 *
*       .10830000                                   tau       BR(9)    IB9 *
*                                          masses:                         *
*       .01000000                                     d   AMAFIN(1)    IM1 *
*       .00500000                                     u   AMAFIN(2)    IM2 *
*       .20000000                                     s   AMAFIN(3)    IM3 *
*      1.30000000                                     c   AMAFIN(4)    IM4 *
*      4.30000000                                     b   AMAFIN(5)    IM5 *
*       .00051100                                     e  AMAFIN(11)    IM6 *
*       .00000000                                    ve  AMAFIN(12)    IM7 *
*       .10565839                                    mu  AMAFIN(13)    IM8 *
*       .00000000                                   vmu  AMAFIN(14)    IM9 *
*      1.77710000                                   tau  AMAFIN(15)   IM10 *
*       .00000000                                  vtau  AMAFIN(16)   IM11 *
*                                                                          *
*                                  DECAY LIBRARIES                         *
*              0                         TAUOLA for W+         JAK1    IL1 *
*              0                         TAUOLA for W-         JAK2    IL2 *
*              1                     TAUOLA Ord(alpha)       ITDKRC    IL3 *
*              1                                PHOTOS       IFPHOT    IL4 *
*              1                         JETSET for W-       IFHADM    IL5 *
*              1                         JETSET for W+       IFHADP    IL6 *
****************************************************************************


****************************************************************************
*                         *****TAUOLA LIBRARY: VERSION 2.6 ******          *
*                         ***********August   1995***************          *
*                         **AUTHORS: S.JADACH, Z.WAS*************          *
*                         **R. DECKER, M. JEZABEK, J.H.KUEHN*****          *
*                         **AVAILABLE FROM: WASM AT CERNVM ******          *
*                         ***** PUBLISHED IN COMP. PHYS. COMM.***          *
*                         *******CERN-TH-5856 SEPTEMBER 1990*****          *
*                         *******CERN-TH-6195 SEPTEMBER 1991*****          *
*                         *******CERN-TH-6793 NOVEMBER  1992*****          *
*                         **5 or more pi dec.: precision limited           *
*                         ******DEXAY ROUTINE: INITIALIZATION****          *
*                   0     JAK1   = DECAY MODE FERMION1 (TAU+)              *
*                   0     JAK2   = DECAY MODE FERMION2 (TAU-)              *
****************************************************************************
 RANMAR INITIALIZED:   54217137          0         0
```

```
*******************************************************************************
*                          ==========================                         *
*                            PHOTOS, Version:  2. 0                           *
*                            Released at:  16/11/93                           *
*                          ==========================                         *
*                                                                             *
*                  PHOTOS QED Corrections in Particle Decays                  *
*                                                                             *
*         Monte Carlo Program - by E. Barberio, B. van Eijk and Z. Was        *
*         From version 2.0 on - by E.B. and Z.W.                              *
*                                                                             *
*******************************************************************************
*                                                                             *
*                  Internal input parameters:                                 *
*                                                                             *
*                  INTERF= T  ISEC= T  IFTOP= T                               *
*                  ALPHA_QED=  .00730   XPHCUT=  .01000                       *
*                                                                             *
*                  option with interference is active                         *
*                  option with double photons is active                       *
*                  emision in t tbar production is active                     *
*******************************************************************************

******************************************************************************
**                                                                          **
**  PPP  Y   Y TTTTT H   H III   A        JJJJ EEEE TTTTT  SSS  EEEE TTTTT  **
**  P  P  Y Y    T   H   H  I   A A         J E      T   S     E      T    **
**  PPP    Y     T   HHHHH  I  AAAAA        J EEE    T    SSS  EEE    T    **
**  P      Y     T   H   H  I  A   A     J  J E      T       S E      T    **
**  P      Y     T   H   H III A   A      JJ  EEEE   T    SSS  EEEE   T    **
**                                                                          **
**              *......*                  Welcome to the Lund Monte Carlo!  **
**         *:::!!:::::::::::*                                               **
**      *::::::!!::::::::::::::*           Warning: PYTHIA is not loaded!   **
**    *::::::::!!::::::::::::::::*        Did you remember to link PYDATA?  **
**   *:::::::::!!:::::::::::::::::*                                         **
**   *:::::::::!!:::::::::::::::::*         This is JETSET version 7.401    **
**    *::::::::!!::::::::::::::::*!       Last date of change: 11 Feb 1994  **
**      *::::::!!::::::::::::::* !!                                         **
**      !! *:::!!:::::::::::*    !!                                         **
**      !!     !* -><- *         !!                 Main author:            **
**      !!     !!                !!              Torbjorn Sjostrand         **
**      !!     !!                !!            Theory Division, CERN,       **
**      !!                       !!              CH-1211 Geneva 23,         **
**      !!        ep             !!                 Switzerland             **
**      !!                       !!          phone +41 - 22 - 767 28 20     **
**      !!                 pp    !!         E-mail TORSJO@CERNVM.CERN.CH    **
**      !!   e+e-                !!                                         **
**      !!                       !!         Copyright Torbjorn Sjostrand    **
**      !!                                     and CERN, Geneva 1993        **
**                                                                          **
** When you cite these programs, priority should always be given to the     **
** latest published description.                                            **
** Currently this is, for JETSET                                            **
** T. Sjostrand and M. Bengtsson, Computer Physics Commun. 43 (1987) 367,   **
** and for PYTHIA                                                           **
** H.-U. Bengtsson and T. Sjostrand, Computer Physics Commun. 46 (1987) 43. **
** The most recent long description (unpublished) is:                       **
** T. Sjostrand, CERN-TH.7112/93 (1993).                                    **
** Also remember that the programs, to a large extent, represent original   **
** physics research. Other publications of special relevance to your        **
** studies may therefore deserve separate mention.                          **
******************************************************************************
```

```
   ...  ...  ...
    first event skipped
   ...  ...  ...

====================dumpw====================
               p(1)          p(2)          p(3)          p(4)  pdg-code
PHO       .00012717    -.00127282    -.00238594     .00270720     22
PHO       .00000061     .00000032    -.00017966     .00017966     22
 W-    -27.73493798  -10.83832552   26.10287894   90.69685708    -24
 W+     27.73481021   10.83959801  -26.10031334   89.30025606     24
 p1    -48.01722190  -18.46743658   -6.14744728   51.84254050     15
 p2     20.28228392    7.62911106   32.25032621   38.85431659    -16
 p3    -13.00605304   -9.29593565   19.27808861   25.07800900      4
 p4     40.74086324   20.13553366  -45.37840195   64.22224706     -3
sum       .00000000     .00000000     .00000000  180.00000000      0
=============================================
```

```
                            Event listing (standard)

   I  particle/jet  K(I,1)   K(I,2) K(I,3)      K(I,4)      K(I,5)       P(I,1)       P(I,2)       P(I,3)       P(I,4)       P(I,5)
   1  !e-!              21       11      0           3           6       .00000       .00000     90.00000     90.00000       .00051
   2  !e+!              21      -11      0           3           6       .00000       .00000    -90.00000     90.00000       .00051
   3  (W-)              11      -24      1           7           8    -27.73494    -10.83833     26.10288     90.69685     81.59573
   4  (W+)              11       24      1           9          10     27.73481     10.83960    -26.10031     89.30025     80.04119
   5  gamma              1       22      1           0           0       .00013      -.00127      -.00239       .00271       .00000
   6  gamma              1       22      1           0           0       .00000       .00000      -.00018       .00018       .00000
   7  (tau-)            11       15      3          11          12    -48.01722    -18.46744     -6.14745     51.84254      1.77710
   8  nu_tau~            1      -16      3           0           0     20.28228      7.62911     32.25033     38.85432       .00000
   9  (c)               14        4      4   3  10  17   0   0  17    -13.00605     -9.29594     19.27809     25.07801      1.30000
  10  (s~)              14       -3      4   0   0  18   3   9  18     40.74086     20.13553    -45.37840     64.22224       .20000
  11  nu_tau             1       16      7           0           0    -12.76560     -4.97430     -1.42791     13.77473       .00063
  12  (W-)              11      -24      7          13          15    -35.25161    -13.49314     -4.71954     38.06780      1.46381
  13  (eta)             11      221     12          33          34    -21.84054     -8.43183     -3.26327     23.64434       .54880
  14  pi-                1     -211     12           0           0     -9.94241     -3.51432      -.93589     10.58760       .13957
  15  (pi0)             11      111     12          35          36     -3.46867     -1.54698      -.52038      3.83586       .13496
  16  (CHshower)        11       94      9          17          18     27.73481     10.83960    -26.10031     89.30025     80.04119
  17  (c)               14        4     16   3   9  20   0   9  19    -12.77663     -9.17593     19.01150     25.33461      5.74152
  18  (s~)              14       -3     16   0  10  21   3  10  22     40.51144     20.01553    -45.11182     63.96565      3.83799
  19  (c)               14        4     17   3  20  24   0  17  23    -12.88282     -9.11275     19.03638     25.20857      4.90701
  20  (g)               13       21     17   2  17   0   2  19   0       .10619      -.06318      -.02488       .12604       .00000
  21  (s~)              13       -3     18   0  18   0   2  22   0     20.74255     10.22071    -20.38377     30.82619       .19900
  22  (g)               13       21     18   2  21   0   2  18   0     19.76889      9.79482    -24.72804     33.13946       .00000
  23  (c)               14        4     19   3  24  26   0  19  25    -13.05451     -9.12877     18.90506     24.99181      3.66534
  24  (g)               13       21     19   2  19   0   2  23   0       .17170       .01602       .13133       .21676       .00000
  25  (c)               13        4     23   2  26   0   0  23   0    -12.67742     -9.43382     18.29591     24.21315      1.35000
  26  (g)               13       21     23   2  23   0   2  25   0      -.37710       .30505       .60915       .77866       .00000
  27  (s~)        A     12       -3     21          37          37     20.74255     10.22071    -20.38377     30.82619       .19900
  28  (g)         I     12       21     22          37          37     19.76889      9.79482    -24.72804     33.13946       .00000
  29  (g)         I     12       21     20          37          37       .10619      -.06318      -.02488       .12604       .00000
  30  (g)         I     12       21     24          37          37       .17170       .01602       .13133       .21676       .00000
  31  (g)         I     12       21     26          37          37      -.37710       .30505       .60915       .77866       .00000
  32  (c)         V     11        4     25          37          37    -12.67742     -9.43382     18.29591     24.21315      1.35000
  33  gamma              1       22     13           0           0    -18.61424     -7.01143     -2.66166     20.06825       .00000
  34  gamma              1       22     13           0           0     -3.22630     -1.42040      -.60160      3.57609       .00000
  35  gamma              1       22     15           0           0     -1.95753      -.85912      -.22651      2.14972       .00000
  36  gamma              1       22     15           0           0     -1.51114      -.68787      -.29387      1.68614       .00000
  37  (string)          11       92     27          38          49     27.73481     10.83960    -26.10031     89.30025     80.04120
  38  (K0)              11      311     37          50          50      7.68898      3.59828     -7.98609     11.66591       .49770
  39  n~0                1    -2112     37           0           0     10.02494      5.13412    -11.08608     15.83170       .93960
  40  (Lambda0)         11     3122     37          51          52      9.08600      4.49971     -9.40533     13.87471      1.11560
  41  (K*+)             11      323     37          53          54      9.80283      4.73202    -12.05487     16.26360       .83495
  42  (pi0)             11      111     37          55          56      1.85638       .68310     -1.20573      2.32051       .13500
  43  (omega)           11      223     37          57          59       .28113      1.01223      -.69439      1.48162       .78063
  44  (rho0)            11      113     37          60          61      1.02011      -.12029     -1.46830      1.95576       .78357
  45  (omega)           11      223     37          62          64       .06061       .12238      -.07067       .78798       .77283
  46  pi-                1     -211     37           0           0       .34901       .27016       .23146       .51755       .13960
  47  (rho+)            11      213     37          65          66      -.27300       .22344      -.26637       .85588       .73289
  48  K-                 1     -321     37           0           0     -5.16155     -4.56596      7.96976     10.54752       .49360
  49  (D_s+)            11      431     37          67          69     -7.00065     -4.74960      9.93631     13.19751      1.96880
  50  (K_S0)            11      310     38          70          71      7.68898      3.59828     -7.98609     11.66591       .49770
  51  p+                 1     2212     40           0           0      7.59320      3.85463     -7.97283     11.70305       .93830
  52  pi-                1     -211     40           0           0      1.49280       .64508     -1.43249      2.17166       .13960
  53  K+                 1      321     41           0           0      7.75578      3.50193     -9.50857     12.76998       .49360
  54  (pi0)             11      111     41          72          73      2.04705      1.23009     -2.54630      3.49362       .13500
  55  gamma              1       22     42           0           0       .81035       .36544      -.56757      1.05468       .00000
  56  gamma              1       22     42           0           0      1.04602       .31766      -.63816      1.26583       .00000
  57  pi+                1      211     43           0           0       .30309       .55123      -.32158       .72015       .13960
  58  pi-                1     -211     43           0           0       .11908       .14578       .00185       .23436       .13960
  59  (pi0)             11      111     43          74          75      -.14104       .31522      -.37465       .52711       .13500
```

```
 60  pi-             1    -211    44          0          0      .15507     -.17319     -.82227      .86583      .13960
 61  pi+             1     211    44          0          0      .86505      .05290     -.64603     1.08993      .13960
 62  pi-             1    -211    45          0          0      .04608      .05101     -.20498      .25735      .13960
 63  pi+             1     211    45          0          0      .16278     -.06672      .17382      .28399      .13960
 64  (pi0)          11     111    45         76         77     -.14824      .13809     -.03951      .24664      .13500
 65  pi+             1     211    47          0          0     -.09925      .45889     -.18436      .52336      .13960
 66  (pi0)          11     111    47         78         79     -.17375     -.23544     -.08202      .33252      .13500
 67  (pi0)          11     111    49         80         81    -3.98092    -2.11017     5.79007     7.33783      .13500
 68  (omega)        11     223    49         82         83    -2.44734    -2.08769     3.10957     4.54583      .80448
 69  pi+             1     211    49          0          0     -.57239     -.55174     1.03667     1.31386      .13960
 70  pi-             1    -211    50          0          0     5.87131     2.83530    -6.31194     9.07586      .13960
 71  pi+             1     211    50          0          0     1.81767      .76298    -1.67416     2.59005      .13960
 72  gamma           1      22    54          0          0     1.13028      .66950    -1.49755     1.99209      .00000
 73  gamma           1      22    54          0          0      .91677      .56059    -1.04875     1.50154      .00000
 74  gamma           1      22    59          0          0      .00686      .02864     -.09762      .10196      .00000
 75  gamma           1      22    59          0          0     -.14790      .28657     -.27703      .42514      .00000
 76  gamma           1      22    64          0          0     -.01012      .06728      .03265      .07547      .00000
 77  gamma           1      22    64          0          0     -.13812      .07081     -.07217      .17117      .00000
 78  gamma           1      22    66          0          0     -.14093     -.11124     -.09549      .20335      .00000
 79  gamma           1      22    66          0          0     -.03282     -.12420      .01347      .12917      .00000
 80  gamma           1      22    67          0          0    -1.91607     -.95766     2.68075     3.43145      .00000
 81  gamma           1      22    67          0          0    -2.06484    -1.15251     3.10932     3.90637      .00000
 82  gamma           1      22    68          0          0     -.91460    -1.29394     1.48769     2.17348      .00000
 83  (pi0)          11     111    68         84         85    -1.53274     -.79375     1.62187     2.37235      .13500
 84  gamma           1      22    83          0          0    -1.05165     -.47797     1.05048     1.56138      .00000
 85  gamma           1      22    83          0          0     -.48109     -.31578      .57140      .81096      .00000
             sum charge:   .00  sum momentum and inv. mass:      .00000      .00000      .00001   180.00000   180.00000



  ...   ...   ...
    other events skipped
  ...   ...   ...
```

```
***************************************************************************
*                             Window V                                    *
*                        VESKO FINAL REPORT                               *
*                              mode  1                                    *
*      18.41936258                approx xs_crude VESK      XCVESK     V4 *
*      18.22119473  +-  .01813815 exact  xs_crude VESK      XSVE       V5 *
*      18.23931434  +-  .00000000 exact  xs_crude Gaus      XSGS       V6 *
*        .00987144                XCVESK_appr/XSGS-1                   V7 *
*       -.00099344  +-  .00099644  XSVE_exact/XSGS-1                   V8 *
***************************************************************************

***************************************************************************
*                          KARLUD  FINAL  REPORT                          *
*                                window A                                 *
*                                                                         *
*                 Xsect with NO Matrix Element                            *
*             4913                 total no of events       NEVTOT     A0 *
*                0                WTcrud < 0 evts           NEVNEG     A1 *
*      18.95197787                 sigma_crude              XCRUDE     A2 *
*        .85612580  +-  .00507362    <WTcrud> rel err       WTKACR     A3 *
*      16.22527727  +-  .08232088 sigma, no Matrix El.        XSKR     A4 *
*                                                                         *
*                                                                         *
*       Xsect with Born Matrix El. only, NO Betas                         *
*             4913                 total no of events       NEVTOT     A5 *
*                0                WTcrud*WTborn <0 evt      NEVNEG     A6 *
*        .78111712  +-  .01158884 <WTcrud*WTborn>, rel      WTKAB0     A7 *
*      14.80371432  +-  .17155790   sigma (Born M.El.)       XSKA0     A8 *
*                                                                         *
*                         Xsect OVER Wtmax                                *
*                  Born Matrix El. only, no Betas                         *
*                0                 no of ev OVER wtmax      NEVACC     A9 *
*        .00000000  +-  .00000000     sigma OVER wtmax      XSKAB0    A10 *
*        .00000000  +-  .00000000     relat sigma OVER      XSKAB0    A11 *
***************************************************************************
```

```
***************************************************************************
*                          KORALW  FINAL  REPORT                          *
*                                window B                                 *
*                                                                         *
*       Beta0  1st Order contrib. to Total Xsect.                         *
*                0                   WTbe01<0  events       NEVNEG     B0 *
*        .96772620  +-  .00433352 <WTbe01>virt rel err      WTKABO     B1 *
*        .82564997  +-  .01158884 <WTSET(20)*WTCRUD>vi      WTKABO     B2 *
*      15.64769995  +-  .18133872  sigma (born*bevi01)      XSKABO     B3 *
*                                                                         *
*       Beta1  1st Order contrib. to Total Xsect.                         *
*             3408                   WTbe10<0  events       NEVNEG     B4 *
*       -.01836212  +-  .03235363 <WTbe10>real rel err      WTKABO     B5 *
*       -.24631365  +- -.00939180  sigma (born*bere10)      XSKABO     B6 *
***************************************************************************

***************************************************************************
*                          KORALW  FINAL  REPORT                          *
*                                window C                                 *
*                                                                         *
*                         1st Order  Total Xsect.                         *
*                0                  WTbe0+1<0  events       NEVNEG     C0 *
*       1.03704350  +-  .00059499 <WTbe0+1> 1ord rel e      WTKABO     C1 *
*      15.40138629  +-  .17945791       sigma O(alpha)      XSKABO     C2 *
*                                                                         *
*                         2nd Order  Total Xsect.                         *
*                0                WTbe0+1+2<0  events       NEVNEG     C3 *
*       1.03816929  +-  .00060765 <WTbe0+1+2> 2ord rel      WTKABO     C4 *
*      15.41909801  +-  .17969116    sigma O(alpha**2)      XSKABO     C5 *
*                                                                         *
*      Best Order (WTMOD) Total Xsect. OVER wtmax                         *
*                0                 no of ev OVER wtmax      NEVACC     C6 *
*        .00000000  +-  .00000000  sigma MODEL OVER         XSKABO     C7 *
***************************************************************************

  ...  ...  ...
    TAUOLA final report skipped
  ...  ...  ...
                             ============ DEMO ============

      15.41909801  +-  .17969116          MC Best, XPAR, KORALW
      15.39192353  +-  .00255307 SemiAnal O(alf2)exp.LL, KORWAN

                             ========== End DEMO ==========
```

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