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DILATON GRAVITY
WITH A NON-MINIMALLY COUPLED
SCALAR FIELD

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ABSTRACT

We discuss the two-dimensional dilaton gravity with a scalar field as the source matter. The coupling between the gravity and the scalar, massless, field is presented in an unusual form. We work out two examples of these couplings, and solutions with black-hole behaviour are discussed and compared with those found in the literature.

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1 INTRODUCTION

It is widely recognized that two-dimensional models of gravity can give us a better understanding of the gravitational quantum effects. These models, derived from a string motivated effective action [1] or from some low-dimensional version of the Einstein equations [2], have a rich structure in spite of their relative simplicity. Gravitational collapse, black holes and quantum effects are examples of subjects whose description is rather complicated in four-dimensional gravity while their lower-dimensional version turns out to be more treatable, sometimes completely solved.

This is the case of the seminal work of Callan, Giddings, Harvey and Strominger (CGHS)[1], where the Hawking effect is analysed semiclassically. Black-hole solutions are found and used to obtain an expression for the Hawking radiation. Some improvements have been done [3] to circumvent difficulties that arise from its quantum version, but leaving the initial purpose unchanged.

In the CGHS model, one starts with the four-dimensional Einstein-Hilbert action in the spherically symmetric metric. Then, the assumption of dependence in two variables of all the fields is done. With a suitable form of the metric, it is possible to integrate the angular dependence. The resulting equations can be derived from a two-dimensional action, the dilaton being the relic of the integrated coordinates. Then this action is modified, rendering a simpler one, which is called the two-dimensional dilaton gravity.

With this modified action, black-hole solutions are found to be formed from non-singular initial conditions, namely a source of scalar matter, coupled minimally with the two-dimensional gravity sector (the terminology will be clarified later). It is shown that there is also a linear vacuum region, which imposes conditions to the expression for the Hawking radiation.

In this model, the scalar matter is coupled only with the two-dimensional gravity sector, without interaction with the dilaton field. Since this field is part of the formerly four-dimensional world and, as pointed out in [1], some results obtained are model- dependent, it could be interesting to analyse some consequences in considering the coupling between the scalar matter and the dilaton field, as originally derived from the dimensional reduction. This is the main purpose of the present work. It is organized as follows: we begin with the construction of the 2d dilaton gravity, but in an unusual way, and then modifications are introduced with the respective solutions. Discussion and final remarks are in the conclusion.

2 THE 2d DILATON GRAVITY AND THE DIMENSIONAL REDUCTION

Intending to compare results, we present in this section the CGHS model. However, this will be done in a slightly different way, closer to the dimensional reduction, giving more motivations for the modifications we will work out.

The starting point is the Einstein action on $\bar{D}$ dimensions coupled with a dilaton field $\bar{\phi}$ [4]

$$S = \int d^{\bar{D}}x\sqrt{-\bar{g}}\, e^{-\bar{\phi}}\Big\{\bar{g}^{\mu\nu}\partial_\mu\bar{\phi}\partial_\nu\bar{\phi} + \bar{R}\Big\}, \tag{1}$$

where $\bar{R}$ is the curvature scalar in $\bar{D}$ dimensions and $\bar{D} = D + d$. When $\bar{D} = 10$, this is the bosonic part of the heterotic string effective action in the critical dimension [4]. The antisymmetric part was omitted, since our interest is in the 2d case.

The theory is considered in a spacetime $M \times K$, where M and K are D and d dimensional spacetime, respectively. If the fields are independent of the d coordinates, the $\bar{D}$ metric can be decomposed as [4]

$$\bar{g}_{\bar{\mu}\bar{\nu}} = \begin{pmatrix} g_{\mu\nu} + A^\gamma_\mu A_{\nu\gamma} & A_{\mu\beta} \\ A_{\nu\alpha} & G_{\alpha\beta} \end{pmatrix}. \tag{2}$$

The resulting dimensionally reduced action is

$$S = \int d^D x\sqrt{-g}\, e^{-\phi}\left\{R + g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{4}g^{\mu\nu}\partial_\mu G_{\alpha\beta}\partial_\nu G^{\alpha\beta} - \frac{1}{4}g^{\mu\rho}g^{\nu\lambda}G_{\alpha\beta}F^\alpha_{\mu\nu}F^\beta_{\rho\lambda}\right\} \tag{3}$$

with $\phi = \bar{\phi} - \frac{1}{2}\,log\,(det\, G_{\alpha\beta})$ being the shifted dilaton and $F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu$.

Now we make the assumptions to reach the action we are interested in, namely taking the simple case where the original dilaton field ϕ is zero and writing the metric in the most general spherically symmetric form:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + \frac{1}{\lambda^2}e^{-2\phi}d^2\Omega. \tag{4}$$

So, with $D = d = 2$ and for $A_{\mu\gamma} = 0$ we have the 2d dimensionally reduced action

$$S = \frac{1}{2\pi}\int d^2x\sqrt{-g}\, e^{-2\phi}\Big\{R + 2(\nabla\phi)^2 + 2\lambda^2 e^{2\phi}\Big\}. \tag{5}$$

It is worth to mentioning that this is the simplest example of the dimensional reduction giving an action with $O(d,d)_{d=2}$ symmetry. If one starts with a scalar field, action (1) changes to

$$S = \int d^{\bar{D}}x\sqrt{-\bar{g}}\, e^{-\phi}\Big\{\bar{g}^{\mu\nu}\partial_\mu\bar{\phi}\partial_\nu\bar{\phi} + \bar{R} - \frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\bar{f}\partial_\nu\bar{f}\Big\} \tag{6}$$

and we have, with the same assumptions:

$$S = \int d^2x\sqrt{-g}\, e^{-2\phi}\Big\{R + 2(\nabla\phi)^2 + 2\lambda^2 e^{2\phi} - \frac{1}{2}(\nabla f)^2\Big\} \tag{7}$$

for the reduced version. Note that in (7), the scalar matter couples with both the 2d-gravity sector and the dilatonic field. We will consider this coupling as the first modification to the CGHS action. The second one concerns the semiclassical quantization and some explanation is needed.

The semiclassical quantization via functional method requires integrating over fields (the scalar one in this particular case). On the other hand, we are interested in theories that have the full quantized version free of anomalies. This paradigm can be worked out via the BRST analysis of the theory by using the Fujikawa's technique [6]. In this framework, the quantized theory is anomaly-free if the functional measure is BRST-invariant. It follows that this invariance requires some redefinition of the field variables, the so-called gravitational dressing, and we must consider it as the field variable of the model that we are studying. It is instructive to show how the gravitational dressing works in a very simple example, the 2d conformal scalar field, minimally coupled with gravity.

The first-order quantum-corrected version for the 2d scalar field gives a traceless vacuum expected value (VEV) for the energy momentum tensor

$$\langle T^{\mu}_{\mu} \rangle = g^{\mu\nu} \langle T_{\mu\nu} \rangle = 0 \tag{8}$$

but non covariantly conserved

$$\nabla^{\nu} \langle T_{\mu\nu} \rangle = \langle \partial_{\mu} f \nabla^{\nu} \partial_{\nu} f \rangle \tag{9}$$

since (9) is an ill-defined quantity. However, most of the quantities of interest arise from non-classical effects (e.g. the Hawking radiation) it could be useful to consider the theory within this scope. At the same time, we hope that the model (in the semiclassical version) agrees with some basic principles, one of them being the (semiclassical) EMT covariantly conserved [5], unlike the one given by (9).

This undesired result can be avoided by a redefinition of the field variable [7]:

$$f \to \tilde{f} = g^{(\frac{1}{4})} f. \tag{10}$$

It is straightforward to see that, considering $\tilde{f}$ as the variable,

$$\langle \tilde{T}^{\mu}_{\mu} \rangle = \langle f \nabla^{\mu} \partial_{\mu} f \rangle = \frac{1}{24\pi} R \tag{11}$$

and

$$\nabla^{\nu} \langle \tilde{T}_{\mu\nu} \rangle = 0, \tag{12}$$

to renormalized vacuum expected values. The classical action for the new field variable is obtained from the old one, using the definition (10):

$$S[\tilde{f}] = \int d^2x (\nabla \tilde{f})^2. \tag{13}$$

In the four-dimensional case, the redefinition of the field variable is not sufficient to give us the conservation of the VEV of the energy-momentum tensor. Instead of this, we must impose (12) as a condition to be satisfied. Actually, it is used to eliminate ambiguities from the definition of some numerical factors in the expression of the renormalized VEV of the four-dimensional energy-momentum tensor [5].

From the definition (10) and the coupling given by (7), these follows a modified action to the dilaton gravity

$$S = \frac{1}{2\pi} \int d^2x \, e^{-2\phi} \Big\{ \sqrt{-g}[R + 4(\nabla\phi)^2 + 4\lambda^2] - \frac{1}{2}(\nabla\tilde{f})^2 \Big\}, \tag{14}$$

where we have done the same modifications, regarding the numerical factors and the coupling between the dilaton and the constant λ, as those in the CGHS model. Since there are couplings other than the original dilaton gravity action, we call the theory given by (14) the non-minimally coupled one.

3 BLACK-HOLE SOLUTIONS FROM SCALAR MATTER

In this section, we show that there are black-hole-type solutions arising from a pulse of scalar matter, as in the CGHS model. However, due to the new couplings with both the dilaton field and the two-dimensional gravity sector, these solutions have new features with respect to those from the usual dilatonic gravity model.

The equations of motion, derived from (14), are

$$e^{-2\phi}[\nabla_\mu\nabla_\nu\phi + g_{\mu\nu}((\nabla\phi)^2 - \nabla^2\phi - \lambda^2) - \frac{1}{2}\nabla_\mu f \nabla_\nu f] = 0, \tag{15}$$

$$e^{-2\phi}[R + 4\lambda^2 + 4\nabla^2\phi - 4(\nabla\phi)^2] = 0 \tag{16}$$

and

$$\nabla^2 f = 2\nabla_\mu f \nabla^\mu \phi \, , \tag{17}$$

for the dilaton, the metric, and the scalar field, respectively. The first two equations imply

$$e^{-2\phi}[R + 2\nabla^2\phi] = 0. \tag{18}$$

It is useful to work with light-cone coordinates

$$x^{\pm} = x^0 \pm x^1 \tag{19}$$

and choose the conformal gauge $g_{\mu\nu} = e^{2\rho}\eta_{\mu\nu}$ that, in the light-cone coordinates, gives

$$g_{+-} = -\frac{1}{2}e^{2\rho}, \quad g_{++} = g_{--} = 0. \tag{20}$$

Now, using (17), the $(+-)$ component of the equation for the dilaton (15) becomes

$$\partial_+\partial_-\phi - 2\partial_+\phi\partial_-\phi = \frac{1}{2}\lambda^2 e^{2\rho} - \frac{1}{2}\partial_+ f\partial_- f \tag{21}$$

or

$$\partial_+\partial_-(e^{-2\phi}) = -\lambda^2 + \partial_+ f\partial_- f e^{-2\phi} \tag{22}$$

and eq.(18)

$$\partial_+\partial_-\phi = \partial_+\partial_-\rho. \tag{23}$$

We can use the residual gauge freedom [1] to make

$$\rho = \phi. \tag{24}$$

This relation, besides rendering calculations easier, has some implications on the original four-dimensional metric (4). We will comment on this point later.

The equations of motion for the gauge-fixed components g_{++} and g_{--} must be imposed as constraints:

$$\nabla_+\nabla_+\phi = \frac{1}{2}\nabla_+ f\nabla_+ f \quad and \quad \nabla_-\nabla_-\phi = \frac{1}{2}\nabla_- f\nabla_- f. \tag{25}$$

Using (24) and $\Gamma^+_{++} = 2\partial_+\rho$ and $\Gamma^-_{--} = 2\partial_-\rho$ these relations can be written as

$$\partial_+^2\phi - 2(\partial_+\phi)^2 = \frac{1}{2}(\partial_+ f)^2 \tag{26}$$

and

$$\partial_-^2\phi - 2(\partial_-\phi)^2 = \frac{1}{2}(\partial_- f)^2. \tag{27}$$

Let us consider, as a source of matter, a shock-wave travelling in the x^- direction. As in [1], this will be described by the stress tensor

$$\frac{1}{2}\partial_+ f\partial_+ f = a\delta(x^+ - x_0^+), \tag{28}$$

where a is its magnitude.

Defining $h(x^+, x^-) = e^{-2\phi}$ eqs. (26) and (27) can be written as

$$\partial_+^2 h = -a\delta(x^+ - x_0^+)h \tag{29}$$

and

$$\partial_-^2 h = 0. \tag{30}$$

We note that there is a difference between eq.(28) and the corresponding one from the dilaton gravity. This difference is due to the coupling we have introduced in the action (14). The solution of this new equation has the following general form:

$$h(x^+, x^-) = -ah(x_0^+, x^-)x^+ + A(x^-)x^+ + K, \tag{31}$$

where $A(x^-)$ is a function of only x^- and K, a constant. The quantity $h(x_0^+, x^-)$ can be evaluated taking (31) at $x^+ = x_0^+$, giving

$$h(x_0^+, x^-) = \frac{K}{1 + a x_0^+} + \frac{x_0^+}{1 + a x_0^+} A(x^-). \tag{32}$$

Using the eq.(22), the final expression for the solution is

$$h(x^+, x^-) = -\lambda^2 x^+ x^{-\prime} + \frac{M}{\lambda}, \tag{33}$$

with $x^{-\prime} = x^- + a\frac{c_1}{\lambda^2}$ and $c_1 = \frac{M}{\lambda(1+a x_0^+)}$.

Comparing this expression with the one from the dilaton gravity, we see that (33) corresponds to a black-hole solution with mass given by $M = K\lambda$ and the coordinate x^- shifted to

$$x^{-\prime} = x^- + a\frac{M}{\lambda^3(1 + a x_0^+)}. \tag{34}$$

When $a = 0$, which means no source, we have a situation called the linear vacuum solution, also present in the CGHS model, in the same situation.

Another situation can be easily analysed, namely when there are two matter sources, one of them being like (28) and the other given by

$$\partial_- f \partial_- f = b\delta(x^- - x_0^-) \tag{35}$$

The only modification with respect to the previous case is in the constraint equation (27), for the g_{--} component, written as

$$\partial_+^2 h(x^+, x^-) = -b\delta(x^- - x_0^-). \tag{36}$$

It is straightforward to show that the solution is

$$h(x^+, x^-) = -\lambda^2 x^+ x^- + K, \tag{37}$$

that is the eternal black-hole solution, with $K = \frac{M}{\lambda}$, found in the CGHS model, when there are no sources. This result is not completely unexpected: as seen at the first case, the non-minimal coupling introduces solutions with the same behaviour in both $x^+ < x_0^+$

and $x^+ > x_0^+$ regions. When the source (35) is present, the regions are separated by this front wave that turns to be unchanged, and no horizon is presented. We can see this directly in eq.(37).

4 CONCLUSIONS AND FINAL REMARKS

In the last section, we have analysed the solutions generated from the non-minimal coupling between the scalar matter and both the dilaton field and the 2-d gravity sector. This coupling was derived from the dimensional reduction of the string motivated models, discussed without details in section 2, and gives us, in a more natural way, the dilaton-scalar matter coupling. The second modification was motivated by the off-shell relations, which can be used in a semiclassical version of the model. The resulting theory is solved in a way very similar to the CGHS model, showing in the two studied cases black-hole type solutions. We mention that the relation (24), present in both the CGHS and the modified version presented here, it is important to make the solutions in this simple way. In the new version, however, this happens only when we consider the two modifications together.

When there are two matter sources, as given by (28) and (35) the situation is the one in which the solution is called eternal black-hole, already discussed at the end of the section 3. However, it is the first case, the one with the same source as CGHS model, that seems to have more interesting features. As mentioned before, there is also an horizon, defined by the shift in the x^- coordinate, but all the spacetime has a black-hole behaviour, in contrast to the CGHS model, where only in the $x^+ > x_0^+$ region has this solution. The linear vacuum region $x^+ < x_0^+$ is important in the CGHS model because this condition is used to determine the Hawking radiation. It seems interesting to calculate this effect in the picture given by this new solution. This work is in progress.

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