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k-FACTORIZATION*)

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ABSTRACT

I review the k-factorization method to combine the high-energy behaviour in QCD with the renormalization group. Resummation formulas for coefficient functions and anomalous dimensions are derived, and their applications to small-x scaling violations in structure functions are briefly discussed.

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1 Introduction

Small-x hard processes are characterized by a large scale $Q^2 \gg \Lambda^2$, and by a much larger energy $s = Q^2/x \gg Q^2$. Therefore, renormalization group (RG) factorization and high-energy, $\boldsymbol{k}$-dependent, factorization should both apply to deep inelastic scattering (DIS) in the small Bjorken x region explored, e.g., at HERA[1)]. This talk is about their consistency and its consequences for QCD perturbative predictions.

The approach of $\boldsymbol{k}$-factorization[2)−6)], developed by Camici, Catani, Hautmann and myself, and triggered by a work of Nason, Dawson and Ellis[7)], leads to resummation formulas of QCD perturbative results for coefficient functions[2),3)] and for anomalous dimensions[4)]. It also leads to a generalization of the effective W-approximation in electroweak fusion processes[5)]. Here I will briefly review its basic results, by adding a few comments[6)] on their use to explain the HERA data[8)].

The high-energy approach and the RG one have a long history[9)−11)] and are, to start with, largely different.

The former is valid for $s \gg Q^2$ and hadronic or partonic masses, and is based on a diagrammatic analysis[9),10)] of gauge boson exchange processes, leading to quasi-constant high-energy cross-sections, of type $(\alpha_s \log 1/x)^n \quad f_n \quad (Q^2, Q_0^2)$, where $Q^2 (Q_0^2)$ denotes the probe (external parton) virtuality.

The main outcome of this approach is the resummation of all leading $\log x$ (LL) terms, provided by the BFKL equation[9)] which predicts increasing cross-sections[10)], due to a singularity in the t-channel angular momentum $J > 1$ (the "perturbative Pomeron"), located at

$$J - 1 = \omega_P = \left(\frac{12}{\pi} \log 2 \right) \alpha_s \ . \qquad (1)$$

It is natural to think[12)] that such increase is related to the small-x rise of structure functions at HERA.

On the other hand, the RG approach is valid for $s, Q^2 \gg \Lambda^2$ and $x = Q^2/s$ fixed, and is based on the structure of collinear singularities for $Q^2 \gg Q_0^2$. It resums all terms $\alpha_s^n (\log Q^2/Q_0^2)^m \;\; g_{nm}(x)$ for $n \geq m > 0$, and it leads to the generalized GLAP equations[11)], which yield the $\log Q^2$-evolution in terms of the QCD anomalous dimensions $\gamma_N(\alpha_s)$, where N is the Mellin transform index in the x variable.

For single hard scale processes and high energies ($s \gg Q^2 \gg Q_0^2$), the two methods above have to merge, with identification of J and N at leading s level. On one hand, the BFKL equation develops collinear singularities, related to the $\log(Q^2/Q_0^2)$'s which have to be factorized out and, on the other hand, the anomalous dimensions (and coefficient functions) develop $\alpha_s/(N-1)$ singularities, related to the $\log x$'s, which have to be resummed. Since we are dealing with the same perturbative series, the consistency requirement provides powerful constraints and new results.

For instance, it is known that DIS structure functions, related to total cross-sections, have no $\alpha_s(\log x)^2$ terms in their perturbative expansion. This is almost trivial from the high-energy point of view: there is at most one emitted gluon per power of α_s, and thus at most one power of $\log(1/x)$ from rapidity integration. Instead, from the RG angle, this fact requires refined cancellations[13)], and eventually implies that, unlike the timelike case[14)], the spacelike anomalous dimension γ_N is a function of the effective variable $\alpha_s/(N-1)$.

It is interesting to notice that such double $\log x$ terms are instead present in angular distributions associated to DIS due to a new kind of form factor[15),16)] and of small-x equation[15)]. They are of phenomenological interst for DIS event generators based on small-x branching schemes[17)] and call for quantitative studies[18)] in the HERA energy range.

Limiting myself to total cross-sections, I will now summarize the resummation formulas due to $\boldsymbol{k}$-factorization, for both leading $(\alpha_s/N-1)^n$ terms, and for some next-to-leading (NL) ones.

2 Resummation Formulas

I will discuss only the case of single-$\boldsymbol{k}$ processes (of DIS type) in which the hard probe scale is denoted by Q (or $2M$ if heavy-quark production is considered) and the corresponding total cross-section for photon ($\boldsymbol{Q}$)-hadron (A) scattering is denoted by $\sigma_N^{HA}(Q^2)$, where N is the moment index.

Then, at high energies, the exchange of a (Regge) gluon, of transverse momentum $\boldsymbol{k}$ in the electron-hadron rest frame and of virtuality $t \simeq -\boldsymbol{k}^2$, yields the following factorized expression (Fig. 1a)

$$Q^2\sigma_N^{HA}(Q^2) = \int d^2k \;\; \hat{\sigma}_N^H \; (\boldsymbol{k}^2/Q^2)\mathcal{F}_N^A(\boldsymbol{k}) \tag{2}$$

where $\hat{\sigma}_N^H$ is an off-shell, gauge invariant $\gamma g(\boldsymbol{k}) \to q\bar{q}$ (or $Q\bar{Q}$) cross-section defined by the high-energy limit of the squared five-point vertex in Fig. 1b, and $\mathcal{F}_N^A(\boldsymbol{k})$ is the unintegrated gluon density in hadron A, satisfying the BFKL integral equation[9)], and related to the usual density by

$$g^A(x,Q^2) = \int_0^{Q^2} \; d^2k \; \mathcal{F}^A(x,\boldsymbol{k}) \; . \tag{3}$$

Consistency of Eq. (2) – which is $\boldsymbol{k}$-dependent – with the RG factorization – which is not, is demanded by requiring that the $\boldsymbol{k}$- dependence of $\mathcal{F}$ be determined by the BFKL anomalous dimension

$$\gamma_N(\alpha_s) \equiv \gamma\left(\frac{\bar{\alpha}_s}{N-1}\right) = \frac{\bar{\alpha}_s}{N-1} + 2\zeta(3)\left(\frac{\bar{\alpha}_s}{N-1}\right)^4 + \ldots \;, \quad (\bar{\alpha}_s = \frac{3\alpha_s}{\pi}) \;, \tag{4}$$

as follows

$$\mathcal{F}_N^A(\boldsymbol{k}) = \frac{1}{\pi \boldsymbol{k}^2}\gamma_N(\alpha_s) \quad \left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^{\gamma_N(\alpha_s)} \quad g_N^A(\mu^2) \ . \tag{5}$$

Inserting Eq. (5) into Eq. (2) allows performing the $\boldsymbol{k}$-integration in terms of the calculable $\boldsymbol{k}^2$-moments

$$h_N(\gamma) \equiv \gamma \int_0^\infty \frac{d\boldsymbol{k}^2}{\boldsymbol{k}^2} \left(\frac{\boldsymbol{k}^2}{Q^2}\right)^\gamma \hat{\sigma}_N^H \left(\frac{\boldsymbol{k}^2}{Q^2}\right) \tag{6}$$

and provides the final result[2)]

$$Q^2 \sigma_N^{HA}(Q^2) = h_N\left(\gamma\left(\frac{\bar{\alpha}_s(Q^2)}{N-1}\right)\right) \ g_N^A(Q^2) \ . \tag{7}$$

Equation (7) is indeed consistent with the RG, with a coefficient function

$$C_N(\alpha_s(Q^2)) = h_N\left(\gamma\left(\frac{\bar{\alpha}_s(Q^2)}{N-1}\right)\right) \ , \tag{8}$$

which automatically resums all powers of $\alpha_s/(N-1)$ in terms of the anomalous dimension (5). Thus, a lowest-order calculation of the off-shell cross-section $\hat{\sigma}^H$ in Eq. (6) provides an all-order resummation of the coefficient function C_N in Eq. (8)!

In deriving Eq. (7) from Eq. (5) we have kept, for simplicity, α_s frozen and we have used the expression (5) even for $\boldsymbol{k}^2 < Q_0^2$, where Q_0 is a scale defining the boundary of the perturbative approach ($\alpha_s(Q_0^2) \lesssim 1$). It can be proved[2),19)], however, that using a RG improved expression (5) and/or a full solution of the BFKL equation[2),20)] including higher twists, and possibly running coupling[21),22)], *does not change* [1] the final result (7), except for subleading terms of relative order $\alpha_s(Q^2)$, which are not considered here.

Several examples of resummed formulae of type (8) are by now available[2)−5)]. Here I will only quote two:

(i) Heavy flavour photoproduction[2)]:

$$h^{Q\bar{Q}}(\gamma) = \frac{\alpha_s}{3\pi} \ \frac{7-5\gamma}{3-2\gamma} \ \frac{\Gamma(1-\gamma)^3\Gamma(1+\gamma)}{\Gamma(2-2\gamma)} \, 4^{-\gamma} \ , \quad (N=0) \ . \tag{9}$$

(ii) DIS structure function $F_2 \equiv Q^2\sigma^{2p}$. For scaling violations, the following function is of particular interest[2),4)]:

$$\begin{aligned} \gamma h_2(\gamma) \equiv \tilde{h}_2(\gamma) \ &\equiv \ \gamma \int \frac{d\boldsymbol{k}^2}{\boldsymbol{k}^2} \left(\frac{\boldsymbol{k}^2}{Q^2}\right)^\gamma \frac{\partial}{\partial \log Q^2} \ \hat{\sigma}^2_{N=0} \left(\frac{\boldsymbol{k}^2}{Q^2}\right) \\ &= \ \frac{\alpha_s}{3\pi} \ \frac{1+\frac{3}{2}\gamma(1-\gamma)}{1-\frac{2}{3}\gamma} \ \frac{[\Gamma(1-\gamma)\Gamma(1+\gamma)]^3}{\Gamma(2-2\gamma)\Gamma(2+2\gamma)} \ , \quad (N=0) \ . \end{aligned} \tag{10}$$

[1] I thus partially disagree with the emphasis of Ref. (20), which seems to cast doubt on this point.

Resummation effects in Eqs. (9) and (10) can be estimated by the enhancement ratios [2]

$$\frac{h^{Q\bar{Q}}(\frac{1}{2})}{h^{Q\bar{Q}}(0)} = \frac{27}{28} \left(\frac{\pi}{2}\right)^2 , \qquad \frac{\tilde{h}_2(\frac{1}{2})}{\tilde{h}_2(0)} = \frac{33}{32} \left(\frac{\pi}{2}\right)^3 , \tag{11}$$

in which $\gamma = 1/2$ is the asymptotic value of the anomalous dimension γ_N at the BFKL Pomeron (1). The enhancement is thus rather large *asymptotically*, ranging between 2.5 and 4, and is still sizeable with respect to one-loop results [8),23)] in the HERA range.

Such important effects are due to the opening of the $\boldsymbol{k}$ phase space (away from the collinear region) which occurs in the high-energy regime, and is also responsible for the asymptotic Pomeron singularity (1).

Is this worrying for the convergence of the perturbative series? Here let me notice that a large fraction of the enhancement (11) is washed out if we compare the heavy-quark process before with the light-quark one: indeed, their ratio is only enhanced, asymptotically, by about 50%. This is because the enhancement comes from the "disordered" $\boldsymbol{k}$ region ($\boldsymbol{k}^2 \gg Q^2$) mentioned before, which is independent of quark masses. This remark suggests to test, experimentally, cross-section ratios, which are much smoother, and to investigate, theoretically, the universality properties of the disordered $\boldsymbol{k}$ region.

3 Towards next-to-leading anomalous dimensions

In order to apply the procedure above to DIS structure functions, recall that the singlet anomalous dimension matrix mixes quark and gluon entries, and that the gluon entries

$$\frac{C_A}{C_F} \;\gamma_{gq} \simeq \gamma_{gg} = \gamma_L(\bar{\alpha}_s(N-1)) \;\; (1 + 0(\alpha_s))$$

are leading, while the quark ones γ_{qg}, γ_{qq} start at NL level, because the quark (spin $\frac{1}{2}$) exchange is subleading at high energies.

Nevertheless, since the quark couples directly to the photon, while the gluon coefficient carries an additional factor of α_s, it turns out that quark entries are as important as gluon entries. For instance, in a partonic DIS-scheme, scaling violations are directly given by γ_{qg} as follows:

$$\frac{\partial F_2^N}{\partial \log Q^2} \equiv \sum_f \; e_f^2 \;\; \dot{q}_N(Q^2) = \sum_f \; e_f^2 \gamma_{qg}^N \; g_N(Q^2) \;\; (1 + 0(\alpha_s)) \; . \tag{12}$$

Therefore, by applying $\boldsymbol{k}$-factorization to the light-quark loop as in Eq. (10), Catani and Hautmann[4)] found a resummed expression for γ_{qg}^N, essentially given by

$$\gamma_{qg}^N = 2N_f \; \tilde{h}_N^2 \;\left(\gamma\left(\frac{\bar{\alpha}_s}{N-1}\right)\right) \tag{13}$$

[2]The normalization of Eq. (9) differs by a factor $4^{-\gamma}$ from Ref. 2, being referred to scale $Q^2 = 4M^2$. The corresponding enhancement in Eq. (11) is smaller by a factor of 2.

Similar considerations apply to the full GLAP equations, and yield also

$$\gamma_{qq}^N = \frac{C_F}{C_A} \quad \left(\gamma_{qg}^N - \frac{2N_f\alpha_s}{3\pi} \right) \ ,$$

thus completing the NL resummation at quark level.

On the other hand, NL terms in the gluon channel (which mix, in principle, with the ones considered) are much harder to obtain by $\boldsymbol{k}$-factorization, because they involve gluon loops and subleading gluon emission vertices, which are still under investigation[24)]. By neglecting them, several authors have applied the resummed formulas (11) and (13) to HERA data, with encouraging results[8),25)]. In the present energy range, they find that resummation effects in γ_{qg} [Eq. (13)] are more important than the ones in γ_{gg} [Eq. (4)].

There are, however, a few subtleties related to Eq. (13). It holds as it stands in the so-called Q_0-scheme[6)] in which the initial parton virtuality Q_0 is fixed [3], and is used to factorize the collinear singularities for $Q_0 \ll Q$. In fact, if $Q_0 \neq 0$, the $|\boldsymbol{k}| < Q_0$ integration occurring in Eq. (2) is automatically suppressed. Instead, in the $Q_0 = 0$ limit, dimensional regularization (and a MS-type scheme) is needed to regularize and factorize all collinear singularities. As a consequence, in the latter case Eq. (13) carries an additional renormalization factor[19)] $R_N = 1 + 0(\alpha_s/(N-1))^3$, as stated in Ref. 4). R_N departs slowly from unity, but is soon sizeable, and is singular at the asymptotic value $\gamma = \frac{1}{2}$.

The ambiguity related to the presence or absence of the R_N factor in γ_{qg} (or, even, of any resummation effect at all[26)]) can be regarded just as a factorization scheme dependence of γ_{qg}, related to a different definition of the gluon density. Keep in mind, however, that in any scheme, we should consider NL terms in the gluon channel also[24)] (which change with the scheme) and also different probes (like, e.g., $F_L, F_3, Q\bar{Q}$ production, and so on) in order to constrain the gluon density. Thus, the present ambiguity will not last for ever, and it is mandatory to discuss *all* NL terms before reaching firm conclusions on resummation effects.

I personally think that the R_N-type factors can safely be reabsorbed in the initial gluon density, because they are related to small-x evolution at fixed scale in the leading BFKL equation, and are thus presumably factorizable at NL level also.

More precisely, if we consider the Green's function matrix of the BFKL equation $G_{ab}^N(Q^2, Q_0^2)$, $a, b = q, g$, for parton evolution from scale Q_0 to scale Q, it differs from the GLAP's one by a normalization matrix K^N, as follows[6)]

$$G^N(Q^2, Q_0^2) = \exp\left(\int_{\log \frac{Q_0^2}{\Lambda^2}}^{\log \frac{Q^2}{\Lambda^2}} \gamma^N(\alpha_s(t)) \right) \quad K^N(\alpha_s(Q_0^2)) + \text{higher twists} \ , \tag{14}$$

where K^N carries the information due to small-x evolution around the scale Q_0.

Thus we see that the high-energy properties of the BFKL equation determine, on the one hand, the anomalous dimensions γ_{ab}^N providing the QCD evolution, but also modify, on the

[3]This can be done in the BFKL framework, because the relevant vertices are defined in a gauge invariant way. Extension to gluons at NL level requires some work.

other hand, the initial parton densities for the GLAP equation, through the occurrence of the matrix K^N. The latter carries a hard Pomeron singularity, and possibly unitarity corrections to it[21),27)]. Equation (14) summarizes the leading twist consistency properties of the BFKL equation with the renormalization group.

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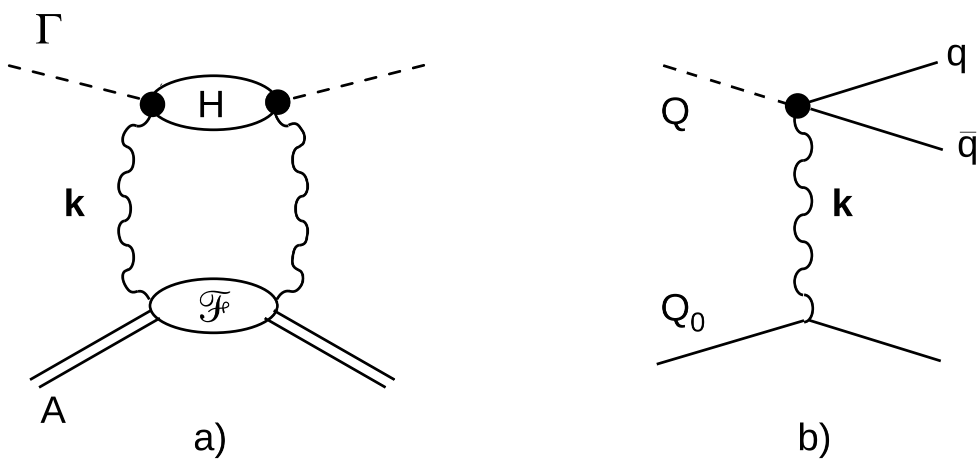

Figure 1: (a) Single-$\boldsymbol{k}$ factorization diagram for σ^{HA} and (b) high-energy vertex for the hard cross-section $\hat{\sigma}^{H}$. Dashed (wavy) lines denote photon (gluon) exchange.