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Chiral behaviour of the lattice B_K-parameter with the Wilson and Clover Actions at $\beta = 6.0$

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Abstract

We present results for the kaon B-parameter B_K from a sample of 200 configurations using the Wilson action and 460 configurations using the SW-Clover action, on a $18^3 \times 64$ lattice at $\beta = 6.0$. We compare results obtained by renormalizing the relevant operator with different "boosted" values of the strong coupling constant α_s. In the case of the SW-Clover action, we also use the operator renormalized non-perturbatively. In the Wilson case, we observe a strong dependence of B_K on the prescription adopted for α_s, contrary to the results of the Clover case which are almost unaffected by the choice of the coupling. We also find that the matrix element of the operator renormalized non-perturbatively has a better chiral behaviour. This gives us our best estimate of the renormalization group invariant B-parameter, $\hat{B}_K = 0.86 \pm 0.15$.

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1 Introduction

The kaon B-parameter B_K is a quantity of great phenomenological interest, being related to the ϵ-parameter which measures CP-violation in the K^0–$\bar{K}^0$ system. With a large top quark mass [1], an accurate prediction of B_K enables us to limit the range of values of the CP-violation phase δ; see for example ref. [2]. B_K is obtained from the calculation of the matrix element of the relevant four-fermion operator $\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle$, where $\hat{O}^{\Delta S=2} = \bar{s}\gamma^L_\mu d \bar{s}\gamma^L_\mu d$; s and d stand for strange and down quarks and $\gamma^L_\mu = \frac{1}{2}\gamma_\mu(1-\gamma_5)$. A non-perturbative estimate of the matrix element may be obtained in the framework of lattice QCD [3, 4]. Several lattice calculations of B_K have been performed in recent years, both with staggered [5, 6] and Wilson [7]–[10] fermions.

With the Wilson or SW-Clover quark lattice actions, due to the presence of the chiral symmetry breaking term, $\hat{O}^{\Delta S=2}$ mixes with operators of different chirality [11, 12]. For this reason, it is possible to define a renormalized operator with definite chiral properties only in the continuum limit, i.e. when $a \to 0$. At finite a, one can improve the chiral behaviour of the matrix element of $\hat{O}^{\Delta S=2}$ by subtracting a suitable set of dimension-6 operators. The mixing coefficients have been computed so far only in one-loop perturbation theory [12]–[15]. In this way, the systematic error in the value of the matrix element determined on the lattice is of $O(\alpha_s^2)$ and, due to the finitess of the lattice spacing, of $O(a)$. Following the Symanzik proposal, one can reduce the discretization errors from $O(a)$ to $O(\alpha_s a)$ by using the tree-level "improved" SW-Clover quark action [16, 17]. Using this action, the improvement has been shown to be effective for two-fermion operators, at values of β currently used in numerical simulations [18]–[20]. It remains true, however, that ignorance of higher-order perturbative corrections to the mixing coefficients, which are expected to be large [21], can distort the chiral behaviour of the operator and hence induce a large systematic error in the determination of B_K. For this reason the kaon matrix element of $\hat{O}^{\Delta S=2}$ does not vanish in the chiral limit [22]. This renders the evaluation of B_K more problematic with Wilson or SW-Clover fermions than with staggered fermions.

In this paper, we present an extended study of $\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle$, using different sets of quenched gauge field configurations, at $\beta = 6.0$. We address the two systematic effects which afflict its chiral behaviour: corrections of $O(\alpha_s^2)$ or higher in the mixing coefficients and contributions of $O(a)$ due to the finiteness of the lattice spacing a. In order to estimate both sources of error, we have performed a high statistics analysis of these effects. Discretization errors have been studied by comparing the results for B_K obtained with the Wilson and SW-Clover actions. The systematic error due to the use of perturbation theory in the calculation of the mixing coefficients has also been analysed by studying the effects of a different choice of the expansion parameter, as proposed in ref. [21]. We also used, in the Clover case, the coefficients computed in [23] with the Non-Perturbative Method (NPM) of ref. [24]. The quality of our results is somewhat limited by the use of the "thinning" procedure, imposed by memory limitations of the APE 6-Gflops machine [25]. Our tentative conclusion is that the most appreciable improvement of the chiral behaviour of the matrix element comes from the implementation of the non-perturbative calculation of the mixing coefficients, whereas Clover improvement and Boosted Perturbation Theory (BPT) seem to be less influential. Preliminary results of this analysis have been presented in [26].

Section 2 introduces the notation necessary for the renormalization of the lattice operator; in section 3 we spell out all the details of the numerical simulation; the main results of our work are contained in section 4; in particular the various fitting procedures are explained and the effects of Clover improvement, boosting and non-perturbative subtraction are analysed; in section 5 we extract the renormalization group invariant value of B_K; section 6 contains our conclusions.

2 The renormalized operator

The weak effective Hamiltonian relevant to K^0–$\bar{K}^0$ mixing is

$$\mathcal{H}_{eff}^{\Delta S=2} = C\left(M_W/\mu\right) \hat{O}^{\Delta S=2}(\mu) \tag{1}$$

where $\hat{O}^{\Delta S=2}(\mu)$ is the renormalized operator; $C(M_W/\mu)$ is the corresponding Wilson coefficient in the Operator Product Expansion (OPE) and μ is the renormalization scale. Our aim is to measure the matrix element $\langle \bar{K}^0|\hat{O}^{\Delta S=2}(\mu)|K^0\rangle$, parametrized by the B_K parameter as follows:

$$\langle \bar{K}^0|\hat{O}^{\Delta S=2}(\mu)|K^0\rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)\,, \tag{2}$$

where

$$\langle \bar{K}^0|\hat{O}^{\Delta S=2}(\mu)|K^0\rangle \to \frac{8}{3}\langle \bar{K}^0|\bar{s}\gamma_\mu d|0\rangle\langle 0|\bar{s}\gamma_\mu d|K^0\rangle = \frac{8}{3} f_K^2 m_K^2 \tag{3}$$

is the matrix element in the vacuum saturation approximation.

Since the lattice regularization with Wilson fermions explicitly breaks chiral symmetry, the lattice $\hat{O}^{\Delta S=2}$ mixes under renormalization with other "effervescent" operators with the same flavour numbers but with different chirality [11]–[15]:

$$\hat{O}^{\Delta S=2}(\mu) = Z_+(\mu a, \alpha_s)\left[\hat{O}^{\Delta S=2}\left(a\right) + Z_{SP}\hat{O}_{SP}\left(a\right) + Z_{VA}\hat{O}_{VA}\left(a\right) + Z_{SPT}\hat{O}_{SPT}\left(a\right)\right]\,, \tag{4}$$

where $Z_{SP,VA,SPT} = Z_{SP,VA,SPT}(\alpha_s)$; $\hat{O}(a)$ denotes the bare lattice operators

$$\begin{aligned}
\hat{O}_{SP}\left(a\right) &= -\frac{1}{16N}[\bar{s}d\bar{s}d - \bar{s}\gamma_5 d\bar{s}\gamma_5 d]\,; &(5)\\
\hat{O}_{VA}\left(a\right) &= -\frac{N^2+N-1}{32N}[\bar{s}\gamma_\mu d\bar{s}\gamma_\mu d - \bar{s}\gamma_\mu\gamma_5 d\bar{s}\gamma_\mu\gamma_5 d]\,; &(6)\\
\hat{O}_{SPT}\left(a\right) &= \frac{N-1}{16N}[\bar{s}d\bar{s}d + \bar{s}\gamma_5 d\bar{s}\gamma_5 d + \bar{s}\sigma_{\mu\nu} d\bar{s}\sigma_{\mu\nu} d]\,. &(7)
\end{aligned}$$

and N is the number of colours. In the chiral limit, the basis of operators appearing on the r.h.s. of eq. (4) remains unchanged beyond one-loop, due to CPS symmetry

[27]. Note that, since there are no $\Delta S = 2$ operators of lower dimension, the mixing takes place between dimension-6 operators only. This implies that the mixing coefficients are at most logarithmically divergent and can be computed on the lattice using Perturbation Theory (PT) [11]. This calculation has been performed to one-loop for both the Wilson and Clover actions [12, 13, 15]; the results are expressed in terms of two constants F_+ and F^* defined as

$$Z_+ = 1 + \frac{\alpha_s}{4\pi} F_+, \qquad Z_{SP} = Z_{VA} = Z_{SPT} = \frac{\alpha_s}{4\pi} F^*. \tag{8}$$

Notice that the equality between the off-diagonal mixing coefficients holds only at one-loop. Their values are given in table 1. In the Wilson case, F_+ is the constant necessary to relate the bare lattice operator to the continuum operator renormalized at $\mu = 1/a$ in the $\overline{\text{MS}}$ Dimensional Reduction scheme (DRED) [12]. The perturbative result for F_+ in the Clover case requires some explanation. In ref. [15], $F_+ = -10.9$ is the constant necessary to relate the bare lattice operator to the continuum operator renormalized at $\mu = 1/a$ in DRED. The value of F_+ reported in the table, instead, refers to the overall lattice renormalization constant necessary to normalize the operator in the Regularization Independent (RI) scheme, defined in ref. [2] and used in ref. [23] in the Landau gauge ($F_+ \to F_+ - 8\ln 2 + 5/3 + L(\mu)$). This is the same scheme used in the non-perturbative case, to which we want to compare the perturbative results. In the RI scheme, there is a dependence of F_+ on the momentum $p^2 = \mu^2$ of the external states, which is reflected in the term $L(\mu) = 2/(16\pi^2)\ln(\mu^2 a^2)$, appearing in the table. This term is numerically small in the range of $\mu^2 a^2$ used in our study: it gives a contribution of order 0.01 to Z_+, if one uses the bare coupling, and of order 0.02 if one uses some boosted coupling, see below; for this reason, it will be ignored in the following.

Although the renormalized operator $\hat{O}^{\Delta S=2}(\mu)$ is constructed so as to have the correct chiral properties in the continuum limit, the perturbative estimate of the Z's is bound to introduce a systematic distorsion arising from the one-loop truncation. BPT should improve the behaviour of the perturbation series, thus providing better estimates of the Z's [21]. On the other hand, the NPM for the calculation of the Z's, as proposed in ref. [24], should in principle account for all perturbative and non-perturbative contributions. In order to compare the perturbative mixing coefficients to the non-perturbative ones, in table 1 we also give the effective F's obtained non-perturbatively in the Clover case [23]. Since in the non-perturbative case there is some dependence on the renormalization scale $\mu^2 a^2$, see also below, we give the results at two values of $\mu^2 a^2$. The F's are defined from $Z_{SP,VA,SPT} = \alpha_s^{(1)}/(4\pi) F_{SP,VA,SPT}$, with the Z's taken from the non-perturbative calculation. We denote by $\alpha_s^{(1)} = 1.68\,\alpha_s$ the coupling constant in the Boosted Scheme (BS) 1, to be introduced in section 4, and by $\alpha_s = 3/(2\pi\beta)$ the bare lattice coupling constant.

Were the coefficients of the mixing with the dimension 6 operators to be known with infinite accuracy, a further source of systematic error would still be present, due to the finiteness of the lattice spacing. The SW-Clover action [16] removes $O(a)$ effects from

Method	F_+	F_{SP}	F_{VA}	F_{SPT}
W-PT	-47.6	9.6	9.6	9.6
SW-PT	$-14.8 + L(\mu)$	19.4	19.4	19.4
NPM-$\mu^2 a^2 = 0.96$	$-15(3)$	13(7)	28(2)	23(4)
NPM-$\mu^2 a^2 = 2.47$	$-14(2)$	21(6)	31(2)	22(3)

Table 1: *Renormalization constants for the Wilson (W) and SW-Clover (SW) actions. The first two rows are the results from one-loop perturbation theory, denoted by F_+ and F^* in the text. The last two lines are the results for the effective constants $F_{+,SP,VA,SPT}$, obtained from the corresponding $Z_{+,SP,VA,SPT}$, calculated with the NPM in the Clover case [23], at two different values of the renormalization scale.*

all matrix elements, leaving us with $O(\alpha_s a)$ and $O(a^2)$ systematic corrections [17]. It must be noted that in the Clover case all the results refer to the "improved-improved" operators introduced in refs. [15, 28].

3 Computational details

The calculations have been done in the quenched approximation, generating 460 gauge configurations on a $18^3 \times 64$ lattice at $\beta = 6.0$ with a standard Metropolis algorithm, using the 6-Gflops version of APE. The light fermion propagators have been computed on the full set of configurations using the SW-Clover action, and on a subset of 200 configurations using the Wilson action. The reason for this difference is the following. Results obtained with the two actions on the same ensemble of 200 configurations demonstrated that the Wilson action is characterised by smaller statistical fluctuations. In order to have comparable errors, we then increased the statistics to 460 configurations in the Clover case. The values of the hopping parameter used were $K = 0.1550, 0.1540, 0.1530$ for the Wilson case and $K = 0.1440, 0.1432, 0.1425$ for the Clover case. All two- and three-point correlation functions have been calculated at degenerate quark masses. The statistical error has been estimated by the jacknife method, by decimating 20 (46) configurations at a time, for the ensembles of 200 (460) configurations.

The $\hat{O}^{\Delta S=2}$ operator was placed at the origin. In what follows, $\hat{O}^{\Delta S=2}$ denotes the renormalized operator defined in eq. (4). One kaon source was inserted at fixed time $t_y = 12$ or 16, whereas the second kaon source was allowed to vary over the full time-range $t_x = 0, 63$; each kaon source carries a spatial momentum denoted by $\vec{p}$ or $\vec{q}$. In lattice units of $2\pi/L$ (L is the number of sites in each spatial direction) the values considered were $\vec{p}_0 = (0,0,0), \vec{p}_\pm = (\pm 1, 0, 0), \vec{p}_{11} = (1,1,0)$ and the corresponding ones equivalent under the cubic group. The same values have also been attributed to $\vec{q}$. The combinations of momenta examined were $(\vec{p}_0, \vec{q}_0)$; $(\vec{p}_0, \vec{q}_\pm)$; $(\vec{p}_0, \vec{q}_{11})$; $(\vec{p}_\pm, \vec{q}_\pm)$. The last case includes three possibilities of relative momentum orientation, namely parallel, antiparallel and perpendicular. Cases which are equivalent under the cubic group have been averaged.

The meson two-point functions were fitted in the time interval $t_x = 10$–22, which guarantees a good isolation of the lightest pseudoscalar state. By extrapolating (linearly in the quark mass) to the vanishing pseudoscalar mass, we computed the critical hopping parameter $K_{cr} = 0.15697(3)$ for the Wilson action and $K_{cr} = 0.14546(2)$ for the SW-Clover action. The lattice spacing determined from the ρ mass is $a^{-1} = 2.19(4)$ GeV (W) and $a^{-1} = 2.06(4)$ GeV (SW). Using the kaon mass and interpolating the lattice pseudoscalar masses we find the hopping parameter value at the strange quark mass, $K_s = 0.15468(9)$ (W) and $K_s = 0.14366(7)$ (SW).

In order to extract B_K from the matrix element $\langle \bar{K}^0(\vec{p})|\hat{O}^{\Delta S=2}(\mu)|K^0(\vec{q})\rangle$, using the method exposed in section 4, we calculated the following two- and three-point correlation functions

$$
\begin{aligned}
G_5(t_x;\vec{p}) &\equiv \sum_{\vec{x}} \langle \bar{s}(x)\gamma_5 d(x) \bar{d}(0)\gamma_5 s(0)\rangle e^{-i\vec{p}\cdot\vec{x}}\,, \\
G_A(t_x) &\equiv \sum_{\vec{x}} \langle \bar{s}(x)\gamma_0\gamma_5 d(x) \bar{d}(0)\gamma_5 s(0)\rangle\,, \\
G_{\hat{O}}(t_x,t_y;\vec{p},\vec{q}) &\equiv \sum_{\vec{x},\vec{y}} \langle \bar{d}(y)\gamma_5 s(y)\hat{O}^{\Delta S=2}(0)\bar{d}(x)\gamma_5 s(x)\rangle e^{-i\vec{p}\vec{y}}e^{i\vec{q}\vec{x}}\,,
\end{aligned}
\tag{9}
$$

and the ratios

$$
\begin{aligned}
R_3 &\equiv \frac{G_{\hat{O}}(t_x,t_y;\vec{p},\vec{q})}{G_5(t_x;\vec{p})G_5(t_y;\vec{q})} \to \frac{\langle \bar{K}^0(\vec{p})|\hat{O}^{\Delta S=2}|K^0(\vec{q})\rangle}{Z_5}\,, \\
X &\equiv \frac{8}{3}\frac{G_A^2(t_x)}{G_5^2(t_x;\vec{0})} \to \frac{8}{3}\frac{f_K^2}{Z_A^2 Z_5} m_K^2\,,
\end{aligned}
\tag{10}
$$

where $Z_5 \equiv \langle 0|\bar{s}(0)\gamma_5 d(0)|K^0\rangle$; f_K is the kaon decay constant and Z_A is the axial current renormalization constant.

The asymptotic behaviour indicated by the arrows in the above equations is valid in the limit $t_y \ll 0 \ll t_x$. In the absence of final state interaction [29] and finite volume effects, the ratio R_3 with time ordering $0 \ll (t_y, t_x)$ would isolate the matrix element $\langle \bar{K}^0(\vec{p})\bar{K}^0(-\vec{q})|\hat{O}^{\Delta S=2}|0\rangle$. On our lattice with periodic boundary conditions both signals appear as two distinct plateaux. In fig. 1, we show two typical cases among the best (fig. 1a) and the worst (fig. 1b) plateaux for R_3. Two plateaux are clearly visible; the rightmost one corresponds to the case of interest $\langle \bar{K}^0|\hat{O}^{\Delta S=2}|K^0\rangle$, whereas the leftmost one corresponds to $\langle \bar{K}^0\bar{K}^0|\hat{O}^{\Delta S=2}|0\rangle$.

Since one can average those cases characterised by spatial momenta equivalent under the cubic symmetry, we are faced with two options: either average the correlation functions before taking their ratios or average the ratios. Having found no significant difference between the results obtained from the two methods, we have opted for the first one. For example, in fig. 1b, we show the ratio of the average of three correlations.

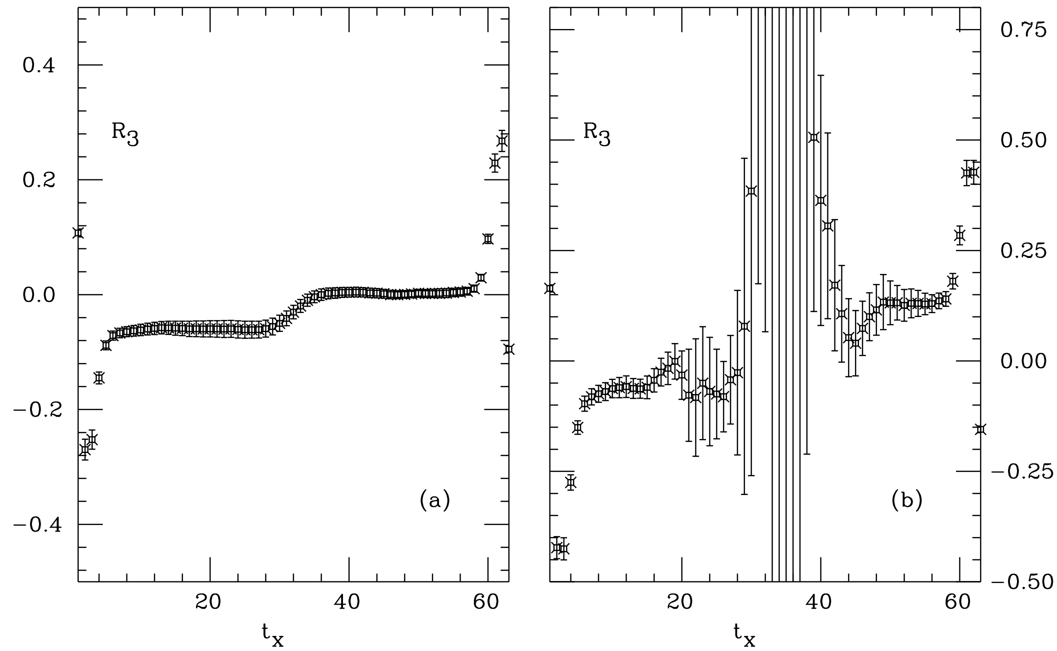

Figure 1: R_3 *for the SW-Clover action at* $K = 0.1440$ *(a)* $\vec{p} = \vec{q} = \vec{p}_0$*; (b) average of three cases:* $\vec{p} = \vec{q} = \vec{p}_+ = (1,0,0)$*;* $\vec{p} = \vec{q} = (0,1,0)$ *;* $\vec{p} = \vec{q} = (0,0,1)$*. The renormalized operators have been obtained in perturbation theory, by using* $\alpha_s^{(1)}$ *as the boosted coupling, cf. eq. (17).*

We note that the results obtained from the data with $t_y = 16$ are fully compatible to those obtained with $t_y = 12$; we only report results from the latter case. In order to extract the matrix elements and errors, we have performed a weighted average of R_3 and X over t_x. The intervals in t_x were chosen on the basis of a study of the time-dependence of the effective energy (mass) obtained from G_5 for a meson at rest. In order to have a good isolation of the lightest pseudoscalar state, and avoid exceedingly large statistical errors, the optimal time interval was found to be $10 \leq t_x \leq 22$. This was the case for both the Wilson and SW-Clover actions and for all the values of the hopping parameter. We estimate that the contamination from the excited states induces a systematic error of some per cent on the relevant quantities for all values of the quark masses considered in this study. Since as meson sources we used local operators, whose couplings to the physical states are independent of the momentum, the same interval in t_x has been used for all meson momenta. Due to periodic boundary conditions, we also used the interval $42 \leq t_x \leq 54$. For R_3, this corresponds to a different matrix element. In the following, X and R_3 will always denote the weighted averages on t_x. As mentioned above, computer memory limitations have obliged us to use the so-called "thinning" approximation. Thinning has been implemented here exactly as in [25] (which may be consulted for details). Although we have explicitly tested that thinning does not influence G_5, we have no way of testing its effect on more complicated correlations, such as $G_{\hat{O}}$. We strongly suspect that this approximation results in an appreciable worsening of the statistical quality of our data. The rather large errors are also due to the relatively small size of our lattice.

4 Chiral behaviour of the B_K parameter

We now examine the chiral behaviour of the matrix element, which we parametrize as

$$\begin{aligned}\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle = \alpha + \beta m_K^2 \quad &+ \quad \gamma (p \cdot q) + \\ + \delta m_K^4 + \epsilon m_K^2 (p \cdot q) \quad &+ \quad \zeta (p \cdot q)^2 + \dots\end{aligned} \tag{11}$$

where all the quantities are expressed in lattice units and the ellipses stand for higher-order terms in $(p \cdot q)$ and m_K^2. The matrix element on the l.h.s. of eq. (11) is obtained by multiplying R_3 of eq. (10) by Z_5, which is extracted in the standard way from the asymptotic behaviour of the pseudoscalar two-point correlation G_5 of eq. (9). The parameters α and β are lattice artefacts that should vanish in the continuum limit. As we have already stressed in section 2, their origin may be attributed to two effects: (i) discretization errors of the lattice matrix elements arising from the subtraction of operators with the wrong "naive" chirality; (ii) higher-order perturbative corrections to the renormalization constants $Z_+, Z_{SP}, Z_{VA}, Z_{SPT}$ of eq. (4). Similar artefacts also contaminate the other parameters $\gamma, \delta, \epsilon, \zeta$. We investigate the differences in the values of α and β, obtained by fitting the dependence of the matrix element on the kaon masses and momenta. Since in the continuum α and β vanish [30], we consider a reduction of their values as a measure of the correctness of the chiral behaviour and of the accuracy in the determination of the matrix element.

There are several possibilities for extracting the parameters $\alpha, \beta, \dots, \zeta$. We have verified that they all give compatible results. In agreement with the observation of ref. [7], we found that, in order to minimize higher-order terms in m_K^2, the most convenient procedure is to substitute the fitting variables as follows

$$m_K^2 \quad \Rightarrow \quad X = \frac{8}{3} \frac{f_K^2}{Z_5 Z_A^2} m_K^2 \tag{12}$$

$$(p \cdot q) \quad \Rightarrow \quad Y = \frac{X (p \cdot q)}{m_K^2} = \frac{8}{3} \frac{f_K^2}{Z_5 Z_A^2} (p \cdot q) \tag{13}$$

and fit directly the ratio R_3 with respect to X and Y. Up to quadratic terms, with $\alpha, \beta, \dots, \zeta$ appropriately redefined, eq. (11) becomes

$$R_3 = \frac{1}{Z_5} \langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle = \alpha + \beta X + \gamma Y + \delta X^2 + \epsilon XY + \zeta Y^2 \,. \tag{14}$$

Another advantage is that R_3 (and X) are obtained directly from ratios of Green functions which have correlated fluctuations. This has to be contrasted with $\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle$ which is obtained by multiplying R_3 by Z_5, computed from a fit of $G_5(t_x; \vec{0})$. The reduction of the size of the higher-order terms in m_K^2, obtained by using X and Y, is reflected in the stability of the results for α, β and γ. The values obtained using eq. (14) differ by only about 20% from those obtained from the linear fit

$$R_3 = \frac{1}{Z_5} \langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle = \alpha + \beta X + \gamma Y \,. \tag{15}$$

Using eq. (11) instead, the results, although compatible within the errors, would differ by about 50%. Even though eq. (14) more than halves the discrepancy between linear and quadratic fits, the remaining systematic effect due to higher-order corrections requires further investigation. Below, we only present results from eqs. (15). The B-parameter is computed from the relation

$$B_K(\mu) = \gamma/Z_A^2 \,. \tag{16}$$

In the remainder of this section, we discuss possible systematic effects on the chiral behaviour of the matrix element of $\hat{O}^{\Delta S=2}$.

4.1 Chiral behaviour in perturbation theory

One possible source of systematic error is that the one-loop perturbative determination of the renormalization constants, at current values of β, is not accurate enough to correct the chiral behaviour. Thus, it is essential to improve the accuracy of the mixing renormalization constants which are calculated in PT. For this reason, the by now standard procedure of BPT [21] has been adopted for both actions; we have tried two different boosted schemes, BS1 and BS2, by using the following definitions of α_s [21]:

$$\text{BS1}: \qquad \alpha_s^{(1)} = \frac{1}{\langle \mathrm{Tr}\Box \rangle}\alpha_s \simeq 1.68\,\alpha_s \text{ (W)};\; 1.68\,\alpha_s \text{ (SW)} \tag{17}$$

$$\text{BS2}: \qquad \alpha_s^{(2)} = (8K_c)^4\alpha_s \simeq 2.49\,\alpha_s \text{ (W)};\; 1.84\,\alpha_s \text{ (SW)}. \tag{18}$$

As can be seen, the effective coupling depends quite significantly on the prescription in the Wilson case, while the variation in the Clover case is small. In the following, we will also give results with $\alpha_s^{(3)} = 3.1\,\alpha_s$ (BS3) [31], in the Wilson case, in order to compare with [8] and will call SPT the case in which we use the bare lattice α_s as expansion parameter. In fig. 2, we show all our results for R_3, plotted against $Y = X(p\cdot q))/m_K^2$, where $X = (8f_K^2 m_K^2)/(3Z_5 Z_A^2)$. $Y > 0$ corresponds to the matrix element $\langle \bar{K}^0|\hat{O}^{\Delta S=2}|K^0\rangle$, whereas $Y < 0$ corresponds to $\langle \bar{K}^0\bar{K}^0|\hat{O}^{\Delta S=2}|0\rangle$ (with the usual caveat of final state interaction [29] and finite size effects) [1]. Notice that the operator is renormalized in the $\overline{\mathrm{MS}}$ DRED scheme in the Wilson case, and in the RI scheme in the Clover case.

In the following, we will only analyse data related to $\langle \bar{K}^0|\hat{O}^{\Delta S=2}|K^0\rangle$. In fig. 2, the effective coupling of the BS1 (which is the same for the two quark actions) was used to define the renormalized operator. In table 2, we give the values of α, β and γ obtained from a fit of our data to eq. (15). Figure 2 and the values of α, β and γ in table 2 allow several observations:

[1]Lattice artefacts are clearly visible in the points corresponding to the highest momenta on the leftmost side of the figure.

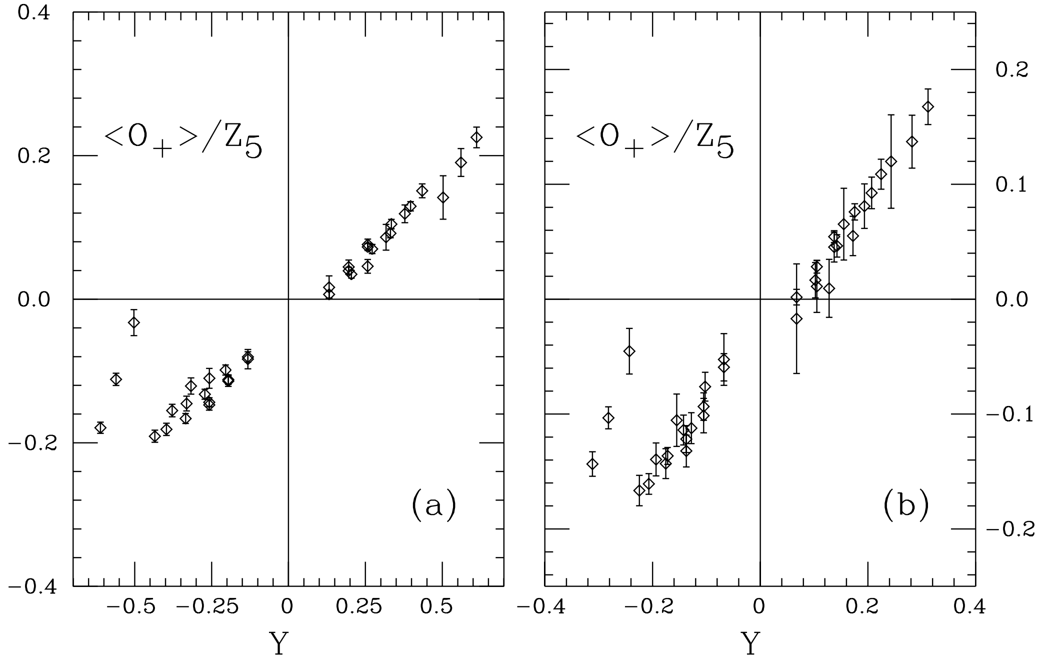

Figure 2: *Dependence of* $\langle 0|\hat{O}^{\Delta S=2}|K^0 K^0\rangle$ *(*$Y < 0$*) and* $\langle \bar{K}^0|\hat{O}^{\Delta S=2}|K^0\rangle$ *(*$Y > 0$*) on mass and momentum obtained with (a) the Wilson action (200 configurations) and (b) the SW-Clover action (460 configurations). The renormalized operators have been obtained by using* $\alpha_s^{(1)}$ *as the boosted coupling.*

Renormalization	α	β	γ
SPT-W	−0.100(7)	0.21(8)	0.56(7)
BS1-W	−0.068(5)	0.15(5)	0.40(5)
BS2-W	−0.032(2)	0.08(3)	0.21(3)
BS3-W	−0.0082(6)	0.020(7)	0.057(7)
SPT-SW	−0.067(12)	0.17(15)	0.62(11)
BS1-SW	−0.054(12)	0.17(15)	0.62(11)
BS2-SW	−0.052(12)	0.16(15)	0.62(11)
NPM-SW $\mu^2 a^2 = 0.96$	0.017(13)	0.21(17)	0.70(12)
NPM-SW $\mu^2 a^2 = 2.47$	0.022(14)	0.23(17)	0.72(12)

Table 2: *SW-Clover (SW) and Wilson (W) results with different choices of mixing renormalization constants. For comparison, in the Clover case the non-perturbative results at two values of the renormalization scale are also given.*

1. The use of the SW-Clover action by itself does not have a significant effect on the chiral behaviour of the matrix element.

2. For all the choices of boosted coupling, the fitted values of α in table 2 are different from zero.

3. A comparison of boosted and unboosted results shows that, in the Clover case, α, β and γ are almost unaffected by the boosting procedure. This is to be contrasted by the results in the Wilson case, which are rather unstable and strongly dependent on the precise choice of the effective coupling constant[2]. This is particularly relevant for γ, from which we estimate the B_K parameter. It implies that the estimate of B_K in the Wilson case is subject to a large systematic uncertainty, due to its sensitivity to the choice of the expansion parameter, i.e. to higher-order effects.

4. In the Wilson case, the instability is due to the large value of F_+ (see table 1): α, β and γ diminish rapidly with increasing α_s, roughly proportionally to Z_+.

   Inspired by mean field theory [21], we have used eq. (4), with Z_+ as an overall factor, also in the perturbative case. Alternatively, we can use the formula

$$\hat{O}^{\Delta S=2}(\mu) = Z_+(\mu a, \alpha_s)\hat{O}^{\Delta S=2}\,(a)+Z_{SP}\hat{O}_{SP}\,(a)+Z_{VA}\hat{O}_{VA}\,(a)+Z_{SPT}\hat{O}_{SPT}\,(a)\ , \tag{19}$$

   which is equivalent at one loop, but may lead to significant differences for the matrix elements at large values of α_s. Indeed, in the BS3 the two possibilities correspond to values of γ which differ by a factor of 2.5.

4.2 Chiral behaviour with the non-perturbative renormalization

We now examine the case in which the renormalization constants are determined non-perturbatively. This will only be done in the Clover case, where the mixing coefficients have already been computed. The data for the operator renormalized at $\mu^2 a^2 = 0.96$ are shown in fig. 3. In ref. [23], the non-perturbative method of ref. [24] was implemented for the calculation of Z_+, Z_{SP}, Z_{VA} and Z_{SPT}. For completeness, we reproduce the results of ref. [23] in table 3. With the NPM, the renormalization conditions for $\hat{O}^{\Delta S=2}$ can be imposed at any arbitrary scale $\mu^2 a^2$, subject to the conditions that lattice artefacts are small and that $\mu \gg \Lambda_{\rm QCD}$, so that continuum perturbation theory can be applied. In [23], a window of values around $\mu^2 a^2 \simeq 1$ was established, for which the non-perturbative estimates of the Z's are fairly stable, see table 3; the results in table 4 show that, with the non-perturbative renormalization, α and β are always compatible with zero and γ is very stable (for $\mu^2 a^2 \geq 0.66$) in all the range of scales considered in our study.

[2]Notice that in the literature boosting factors varying between 1.6 and 3.1 have been used for this case, at $\beta = 6.0$.

$\mu^2 a^2$	Z_+	Z_{SP}	Z_{VA}	Z_{SPT}
0.46	0.91 ± 0.05	0.08 ± 0.14	0.34 ± 0.03	0.34 ± 0.07
0.66	0.84 ± 0.05	0.07 ± 0.09	0.33 ± 0.03	0.34 ± 0.06
0.81	0.83 ± 0.04	0.11 ± 0.07	0.31 ± 0.03	0.28 ± 0.05
0.96	0.84 ± 0.03	0.14 ± 0.07	0.30 ± 0.02	0.24 ± 0.04
1.27	0.80 ± 0.04	0.17 ± 0.05	0.29 ± 0.02	0.21 ± 0.03
1.54	0.82 ± 0.02	0.19 ± 0.04	0.27 ± 0.02	0.16 ± 0.03
1.89	0.83 ± 0.03	0.22 ± 0.05	0.30 ± 0.02	0.18 ± 0.03
2.47	0.85 ± 0.02	0.22 ± 0.06	0.33 ± 0.02	0.23 ± 0.03
SPT	0.91	0.12	0.12	0.12
BPT	0.84	0.21	0.21	0.21

Table 3: *Values of the Z's for several renormalization scales $\mu^2 a^2$. We also give the results obtained at $\mu^2 a^2 = 1$, by using "standard" perturbation theory (SPT) and "boosted" perturbation theory (BPT) with the BS1 effective coupling $\alpha_s^{(1)}$.*

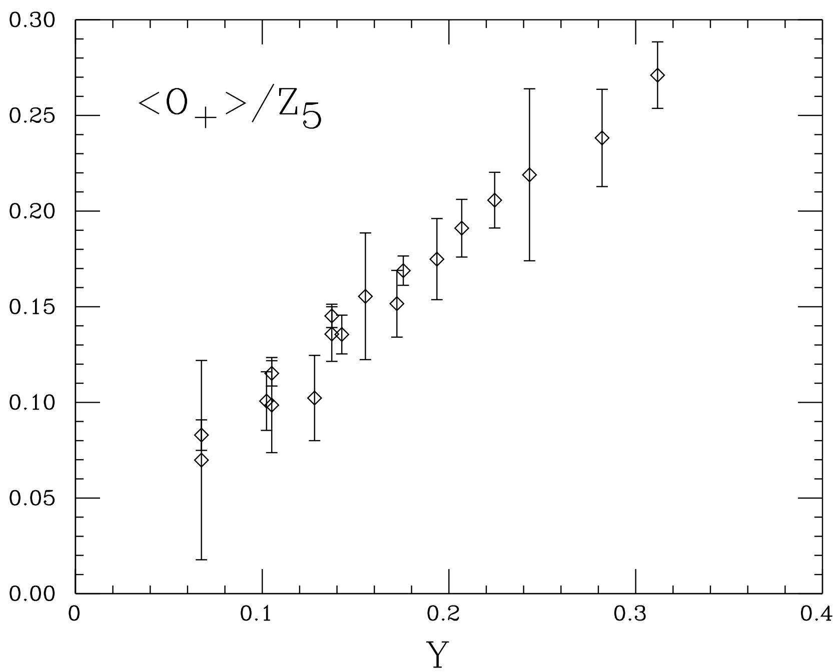

Figure 3: *Dependence of $\langle \bar{K^0} | \hat{O}^{\Delta S=2} | K^0 \rangle$ on mass and momentum, with the non-perturbative renormalization constants computed at $\mu^2 a^2 = 0.96$. The data can be compared with the corresponding perturbative ones in fig. 2.*

The use of the values of the Z's in table 3 is subject at present to some limitations that we briefly want to discuss[3].

1. The calculation of ref. [23] was performed on 36 configurations at $\beta = 6.0$ on a $16^3 \times 32$ lattice, using the SW-Clover action and "improved-improved" operators. The renormalization constants were determined for a single quark mass only, corresponding to $K = 0.1425$. We used these constants at all values of the quark masses. Even though for light quarks we expect very little dependence of the Z's on the quark mass [18, 28], one has to bear in mind that this approximation may introduce a small systematic effect in our results.

2. The optimal strategy to obtain an accurate determination of the matrix element of the renormalized operator would be to compute the renormalization constants and the operator matrix elements on the same set of configurations and for the same values of the quark masses. This is not possible at the present stage, since the Z's and the matrix elements were computed on different sets of configurations, quark masses and lattice volumes. On the other hand, estimating the uncertainty of the final result by varying the Z's and the matrix elements independently within their errors is questionable because the values of the Z's, obtained by solving a linear system of equations on the same configurations, are strongly correlated. For this reason, we used the central values of the Z's from [23] and ignored their statistical error in the present analysis. To monitor the stability of the results, we varied over a wide range the renormalization scale at which the Z's were computed. Correspondingly, the Z's change by an amount comparable to their error, or even larger. A large set of results, obtained from a fit of the data to eq. (15), corresponding to $0.19 \leq \mu^2 a^2 \leq 2.47$ is given in table 4. Notice that for scales below 0.81, the values of the Z's become unstable and the use of perturbation theory (for matching) questionable; for $\mu^2 a^2 \geq 2.47$ lattice artefacts become clearly visible [23, 24].

3. The stability of the results for the matrix elements of the renormalized operator appears surprising, since the renormalization constants vary appreciably on the same range of $\mu^2 a^2$ [23]. The explanation for this is the following. The most unstable of the renormalization constants, according to ref. [23], is Z_{SP}. The $\hat{O}_{SP}$ matrix element carries, however, the least weight in the final result. On the other hand, the most stable of the Z's is Z_{VA}, which gives the largest contribution. The case of the operator $\hat{O}_{SPT}$ is intermediate between the two cases discussed before. We show this explicitly in table 5, where we present separately, for different quark masses, the contribution of the different operators to the final result. For comparison, we also give the different contributions in the perturbative case in the BS1. In the table, the contribution to R_3 of the bare operator $\hat{O}^{\Delta S=2}$, corresponding to $\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle / Z_5$, will be denoted by $R^{\Delta S=2}$; the contribution of the operator $\hat{O}_{SP}$, corresponding to $\langle \bar{K}^0 | \hat{O}_{SP} | K^0 \rangle / Z_5$, etc., will be denoted by R_{SP}, etc. We

[3]These limitations are not intrinsic to the method and will be easily eliminated in future calculations.

$\mu^2 a^2$	α	β	γ
0.19	0.03(2)	0.2(2)	0.99(17)
0.46	0.022(16)	0.23(19)	0.78(13)
0.66	0.013(15)	0.20(17)	0.70(12)
0.81	0.012(13)	0.21(17)	0.69(12)
0.96	0.017(13)	0.21(17)	0.70(12)
1.27	0.015(13)	0.21(16)	0.66(11)
1.54	0.018(13)	0.22(16)	0.67(12)
1.89	0.023(13)	0.22(16)	0.69(12)
2.47	0.022(14)	0.23(17)	0.72(12)

Table 4: *Values of the parameters of the fit of* $\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle$ *to eq. (15).*

Method	K	$R^{\Delta S=2}$	R_{SP}	R_{VA}	R_{SPT}
Wilson	0.1550	−0.004(5)	−0.0154(4)	0.0575(13)	−0.0313(7)
BS1	0.1540	0.027(4)	−0.0161(3)	0.0620(10)	−0.0331(5)
	0.1530	0.058(4)	−0.0165(2)	0.0654(8)	−0.0345(4)
Clover	0.1440	−0.024(5)	−0.0491(8)	0.178(3)	−0.1026(15)
BS1	0.1432	−0.002(5)	−0.0515(7)	0.190(3)	−0.1083(13)
	0.1425	0.020(4)	−0.0529(6)	0.199(3)	−0.1123(12)
NPM-Clover	0.1440	−0.024(5)	−0.0332(6)	0.261(5)	−0.1202(17)
$\mu^2 a^2 = 0.96$	0.1432	−0.002(5)	−0.0348(5)	0.279(4)	−0.1315(14)
	0.1425	0.020(4)	−0.0358(4)	0.292(4)	−0.1315(14)
NPM-Clover	0.1440	−0.025(5)	−0.0540(9)	0.284(5)	−0.1161(17)
$\mu^2 a^2 = 2.47$	0.1432	−0.002(5)	−0.0567(8)	0.305(5)	−0.1225(15)
	0.1425	0.021(4)	−0.0582(7)	0.319(4)	−0.1270(14)

Table 5: *Separate contributions to* R_3 *from the bare operator* $R^{\Delta S=2}$ *and the off-diagonal matrix elements,* R_{SP}*,* R_{VA} *and* R_{SPT}*, for different quark masses. We give the non-perturbative results at two different renormalization scales, and the perturbative result in the BS1. All the matrix elements correspond to the case where the two mesons are at rest.*

see that the largest subtraction comes from the operator $\hat{O}_{VA}$, which is multiplied by the constant Z_{VA}, which is quite well determined. This explains the stability of the fitting parameters α, β and γ with varying renormalization scale.

4. The improvement in the chiral behaviour of the matrix element arises from the difference in $Z_{SP} : Z_{VA} : Z_{SPT}$ between the non-perturbative and perturbative case. Since Z_{VA} is much larger than Z_{SP} and Z_{SPT} in the non-perturbative case, most of the beneficial effect on the chiral behaviour is due to the enhanced contribution of $\hat{O}_{VA}$. Boosted perturbation theory, on the other hand, can only change all the coefficients by the same amount, leaving $Z_{SP} : Z_{VA} : Z_{SPT}$ unaltered, so that the correction is much less effective. A very large boost of the coupling would be necessary in order to obtain the same effect as in the non-perturbative case.

5 Physics results

In order to compare the results for the renormalized B-parameter obtained from γ, cf. eq. (16), with other theoretical predictions of the same quantity, it is useful to refer to a "standard" definition, which we will take to be the renormalization group invariant B-parameter $\hat{B}_K$. In the standard perturbative approach, by a suitable choice of Z_+, the B-parameter is defined in the $\overline{\mathrm{MS}}$ scheme at a scale $\mu \sim 1/a$. Let us call this B-parameter $B^{\overline{MS}}(\mu)$. At the next-to-leading order (NLO), $\hat{B}_K$ is written in terms of $B^{\overline{MS}}(\mu)$ as

$$\hat{B}_K = \alpha_s(\mu)^{-\gamma^{(0)}/2\beta_0} \left[1 - \frac{\alpha_s(\mu)}{4\pi} \left(\frac{\gamma^{(1)}\beta_0 - \gamma^{(0)}\beta_1}{2\beta_0^2} \right) \right] B^{\overline{MS}}(\mu)\,, \tag{20}$$

where $\beta_{0,1}$ and $\gamma^{(0,1)}$ are the leading and next-to-leading coefficients of the β-function and anomalous dimension. $\beta_{0,1}$ and $\gamma^{(0)}$ are universal while $\gamma^{(1)}$ depends on the renormalization scheme. The explicit expressions of all the quantities appearing in eq. (20), $\gamma^{(0),(1)}$ and $\beta_{0,1}$, can be found for example in refs. [32, 33]. Notice that the renormalization group invariant B-parameter defined in this way is also regularization-scheme independent, up to next-to-next-to-leading order terms.

We compute $\hat{B}_K$ only for the Clover case using the NPM, since this is our best result. By using the non-perturbative estimate of $Z_A = 1.06$[19], we obtain the B-parameter corresponding to the operator renormalized in the Regularization Independent (RI) scheme [2, 24]. The matching to the $\overline{\mathrm{MS}}$ scheme can be done in continuum perturbation theory (this is one of the main advantages of the non-perturbative approach), by computing the operator matrix element in the same gauge and on the same external quark states as those used for the non-perturbative calculation of the lattice renormalization constants [23, 24]. At the next-to-leading order, the NDR–RI matching coefficient for $\hat{O}^{\Delta S=2}$ is given by [2]

$$B^{\overline{MS}}(\mu) = \left(1 + \frac{\alpha_s(\mu)}{4\pi}\Delta r_+^{\overline{MS}}\right) B_K(\mu) \tag{21}$$

where

$$\Delta r_+^{\overline{MS}} = -14/3 + 8\log(2)\,. \tag{22}$$

In numerical estimates, we have used $\alpha_s(\mu) = 0.3$, at the renormalization scale $\mu^2 a^2 = 0.96$, corresponding to $\mu \sim 2$ GeV. Given the still large error on B_K, we have not included the uncertainty due to the choice of $\alpha_s(\mu)$ in the result given below. At fixed $\alpha_s(\mu)$, there is still a variation of ± 0.03 when varying n_f, the number of active flavours, between 0 and 4 in eq. (20). With the above parameters, we get

$$\hat{B}_K = 0.86 \pm 0.15\,. \tag{23}$$

Within large errors, our results agree with other lattice calculations and with other results obtained with a different theoretical approach, namely the $1/N_c$ expansion [34, 35]. It may be noted that, in spite of our increased statistics, the final errors for our results are comparable (or even larger) to those of previous lattice studies. We attribute this to our thinning approximation, which we plan to remove in future calculations.

6 Conclusions

We have presented a parallel lattice calculation of the kaon B parameter B_K with the Wilson and the SW-Clover actions, at $\beta = 6.0$. In the Wilson case, using perturbation theory to determine the renormalized four-fermion operator, we found that the value of B_K strongly depends on the precise choice of the expansion parameter α_s and on higher-order corrections. Consequently the results are subject to a rather large uncertainty. In the Clover case, the results were more stable with respect to a change of α_s. The most appreciable improvement in the chiral behaviour of the operator is, however, found with the operator renormalized non-perturbatively, by using the mixing coefficients computed in ref. [23].

The accuracy of our results was somehow limited by two reasons: the precision in the calculation of the mixing coefficients and the memory of the APE 6-Gflops machine, which forced us to work on a relatively small lattice volume and to use the "thinning" approximation. These limitations are not intrinsic to the non-perturbative method and will be easily eliminated in future calculations. In particular, we expect significant improvements by computing renormalization constants and operator matrix elements on the same set of configurations and for the same values of the quark masses (and, of course, by eliminating the "thinning" and by working on a larger volume). We also plan to compute non-perturbatively the mixing coefficients in the Wilson case, although, in this case, discretization errors could still play an important role. In the non-perturbative case, with more accurate results, it will be possible to check whether α and β are so small as to give a negligible contribution to the matrix element for quark masses corresponding to the kaon. Then, similarly to the case of staggered fermions, it will be possible to compute the kaon matrix element directly, without the fitting procedures used so far. This will allow us then to reduce the error in the determination of the quenched $\hat{B}_K$ to about 5%, as in the staggered case.

The results of our study are encouraging, and motivate us to extend the calculation of matrix elements of operators renormalized non-perturbatively to the operators relevant to $\Delta I = 1/2$ transitions and to the penguin and electro-penguin operators which control CP-violation in kaon systems. Our strategy is to achieve an accurate determination of the physical weak amplitudes, by combining the improvement of the action *à la* Symanzik, which reduces $O(a)$ effects, with the non-perturbative method of ref. [24].

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