The SU(2) Confining Vacuum as a Dual Superconductor

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We investigate the dual superconductivity hypothesis in pure SU(2) lattice gauge theory. We find evidence of the dual Meissner effect both in the maximally Abelian gauge and without gauge fixing. We also obtain a rather good extimation of the string tension using the value of the London penetration length.

1. INTRODUCTION

Understanding the mechanism of quark confinement is a central problem in the high energy physics. According to a model conjectured long time ago by G. 't Hooft [1] and S. Mandelstam [2] the confining vacuum behaves as a coherent state of color magnetic monopoles, or, equivalently, the vacuum resembles a magnetic (dual) superconductor.

Up to now there is some some numerical evidence in favour of this model [3–8]. There are also efforts [9] towards the detection of monopoles condensation in the vacuum.

We analyzed the distribution of the color fields due to static $q\bar{q}$ pair in SU(2) lattice gauge theory, moreover we studied the gauge dependence of the London penetration length by working both without gauge fixing and in the maximally Abelian gauge [10]. The full details of this work can be found in Ref. [8]

We performed Monte Carlo simulation using the overrelaxed Metropolis algorithm in the range $2.45 \leq \beta \leq 2.7$ on lattices of sizes 16^4 , 20^4 , and 24^4 . After a number of thermalization sweeps ≥ 3000 we collect 1 measurement every 100 upgrades in the case of SU(2) without gauge fixing, for a total amount of 100 measurements. In order to reduce the quantum fluctuations we cooled the lattice configurations. This way quantum fluctuations are reduced by a few order of magnitude, while the relevant physical observables survives and show a plateau vs. cooling steps. In the case of maximally Abelian projected SU(2) we took 1 measurement every 50 upgrades for a total amount of 700 measurements. Remarkably in this case no cooling is needed to get a good signal/noise ratio. We handled statistical errors with jackknife algorithm modified to take into account correlations.

2. COLOR FIELDS

We can measure the color fields by means of the correlation ρ_W of a plaquette U_p with a Wilson loop W [11]. The plaquette is connected to the Wilson loop by a Schwinger line L (see Fig.1):

$$\rho_W = \frac{\left\langle \operatorname{tr}\left(WLU_PL^{\dagger}\right)\right\rangle}{\left\langle \operatorname{tr}(W)\right\rangle} - \frac{1}{2} \frac{\left\langle \operatorname{tr}(U_P)\operatorname{tr}(W)\right\rangle}{\left\langle \operatorname{tr}(W)\right\rangle} .$$
(1)

The chromoelectromagnetic field strength tensor is defined as

$$F_{\mu\nu}(x) = \frac{\sqrt{\beta}}{2} \rho_W(x) .$$
 (2)

2.1. Maximally Abelian Projection

In the 't Hooft formulation the dual superconductor model is elaborated through the Abelian projection. To perform the Abelian projection on the lattice we fix the gauge by diagonalizing an operator X(x) which transforms according to the adjoint representation and then extract the diagonal part $U^A_{\mu}(x)$ out of the gauge transformed links $\tilde{U}_{\mu}(x) = V(x)U_{\mu}(x)V^{\dagger}(x + \hat{\mu})$:

$$U^{A}_{\mu}(x) = \operatorname{diag}\left[e^{i\theta^{A}_{\mu}(x)}, e^{-i\theta^{A}_{\mu}(x)}\right], \qquad (3)$$

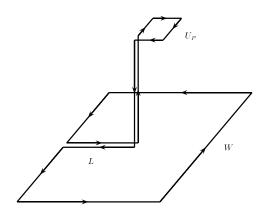


Figure 1. The connected correlator Eq.(1) between the plaquette U_p and the Wilson loop. The subtraction appearing in the definition of correlator is not explicitly drawn.

where $\theta^A_\mu(x) = \arg\left[\widetilde{U}_\mu(x)\right]_{11}$. The Abelian field strength tensor is

$$F^{A}_{\mu\nu}(x) = \frac{\sqrt{\beta}}{2}\rho^{A}_{W}(x) \tag{4}$$

with

$$\rho_W^A = \frac{\left\langle \operatorname{tr}\left(W^A U_P^A\right)\right\rangle}{\left\langle \operatorname{tr}\left(W^A\right)\right\rangle} - \frac{1}{2} \frac{\left\langle \operatorname{tr}\left(U_P^A\right) \operatorname{tr}\left(W^A\right)\right\rangle}{\left\langle \operatorname{tr}\left(W^A\right)\right\rangle} . (5)$$

We perform the Abelian projection in the maximally Abelian gauge (in the continuum: $D_{\mu}A^{\pm}_{\mu}(x) = 0$). On the lattice the gauge is fixed by maximizing (over all SU(2) gauge transformations g(x)) the lattice functional

$$R_l = \sum_{x,\hat{\mu}} \frac{1}{2} \operatorname{tr} \left[\sigma_3 U_{\mu}(x) \sigma_3 U_{\mu}^{\dagger}(x) \right] \,. \tag{6}$$

This is equivalent to diagonalize $(X(x) \in SU(2))$ algebra):

$$X(x) = \sum_{\mu} \left[U_{\mu}(x)\sigma_{3}U_{\mu}^{\dagger}(x) + U_{\mu}^{\dagger}(x-\hat{\mu})\sigma_{3}U_{\mu}(x-\hat{\mu}) \right].$$
(7)

Gauge fixing is performed by an iterative overrelaxed algorithm [12]. Once we found the SU(2) matrix g(x) which locally maximizes R_l , we keep $g_{\text{over}}(x) = g(x)^{\omega}$ where the ω parameter has to be properly tuned to increase the efficacy of the

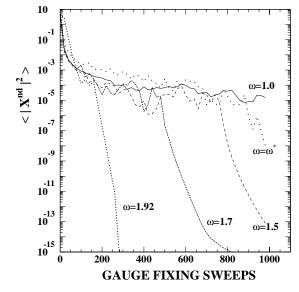


Figure 2. Efficacy of gauge fixing defined by Eq. (8) as a function of the overrelaxation parameter ω for the L = 16 lattice. The case $\omega = \omega^*$ corresponds to alternate $\omega = 1.0$ with $\omega = 2.0$ in the gauge fixing sweeps.

gauge fixing (Fig. 2). To establish a convergence criterion for the iterative gauge fixing we consider the average size of the non-diagonal matrix elements of X, Eq.(7), over the whole lattice:

$$\left\langle \left| X^{\text{nd}} \right|^2 \right\rangle = \frac{1}{L^4} \sum_x \left[\left| X_1 \right|^2 + \left| X_2 \right|^2 \right]$$
 (8)

where $X = X_1\sigma_1 + X_1\sigma_2 + X_1\sigma_3$. The optimal overrelaxation parameter agrees with the one relevant to the Landau gauge fixing [12]:

$$\omega_c = \frac{2}{1 + \frac{c}{L}}, \quad c \simeq 0.7.$$
(9)

2.2. Results

In both cases of SU(2) without gauge fixing and SU(2) in the maximally Abelian gauge we obtained that (see Fig. 3 for the case of SU(2) without gauge fixing) the longitudinal chromoelectric field $E_x \equiv E_l$ is sizeable (the other field strength components are a few order of magnitude smaller). Furthermore E_l is almost constant along the $q\bar{q}$ flux tube and decreases fast along the direction transverse to the flux tube.

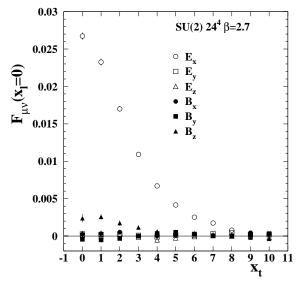


Figure 3. The field strength tensor $F_{\mu\nu}(x_l, x_t)$ evaluated at $x_l = 0$ on a 24⁴ lattice at $\beta = 2.7$, using Wilson loops of size 10 × 10 in Eq. (1).

3. THE LONDON PENETRATION LENGTH

If the dual superconductor scenario holds, the transverse shape of the longitudinal chromoelectric field E_l should resemble the dual version of the Abrikosov vortex field distribution. Hence we expect that $E_l(x_t)$ can be fitted according to

$$E_l(x_t) = \frac{\Phi}{2\pi} \mu^2 K_0(\mu x_t) , x_t > 0$$
 (10)

where K_0 is the modified Bessel function of order zero, Φ is the external flux, and $\lambda = 1/\mu$ is the London penetration length. We try the fit outside the coherence region $x_t > \xi$, ξ being the coherence length, i.e. ξ measures the coherence of the magnetic monopole condensate. We fit our data for $x_t \geq 2$ obtaining $\chi^2/f \leq 1$ and check the stability of the fit parameters μ and Φ by fitting the data with the cuts $x_t \geq x_t^{\min} = 2, 3, 4, 5$. Since in the case of SU(2) without gauge fixing we adopted a cooling procedure, we verified [8] the stability of the fit parameter μ versus the number of cooling steps.

We found that the London penetration length extracted from the gauge invariant correlator ρ_W

$$\frac{\mu}{\Lambda_{\overline{MS}}} = 8.96(31) , \chi^2/f = 2.11$$
 (11)

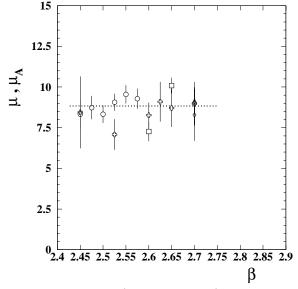


Figure 4. μ and μ_A (in units of $\Lambda_{\overline{MS}}$) versus β for square Wilson loops. Circles, squares, and triangle refer to L = 16, 20, 24 respectively. Crosses and diamond refer to the Abelian projected correlator ρ_W^A with L = 16, 20 respectively.

agrees with the one extracted from the Abelian projected correlator ρ_W^A (Fig. 4)

$$\frac{\mu_A}{\Lambda_{\overline{MS}}} = 8.26(67) \ , \chi^2/f = 0.41 \ . \tag{12}$$

4. STRING TENSION

We have shown that the longitudinal chromoelectric field E_l is almost constant along the flux tube (i.e. the long distance potential which feel the color charges is linear). We have also ascertained that the longitudinal chromoelectric field E_l is the only sizeable component of the field strength tensor, and its transverse distribution $E_l(x_t)$ can be fitted by Eq.(10). Since the string tension is given by the energy stored into the flux tube per unit length, using the above results and extrapolating the K_0 -distribution up to $x_t \to 0$ (with negligible error if $\lambda = 1/\mu \gtrsim \xi$), we get the simple relation between the string tension and the penetration length:

$$\sqrt{\sigma} \simeq \frac{\Phi}{\sqrt{8\pi}} \mu \ . \tag{13}$$

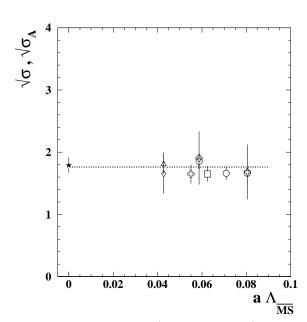


Figure 5. String tension (in units of $\Lambda_{\overline{MS}}$) evaluated through Eq. (14). Star refers to the continuum extrapolated value of the string tension obtained using Wilson loops on lattice larger than ours. Points and crosses refer to L = 16, squares and diamond to L = 20, triangles to L = 24. Crosses and diamond correspond to the maximally Abelian gauge. For figure readability not all the available data are displayed.

The main uncertainty comes out from the fit parameter Φ . However we observe that in the maximally Abelian projection $\Phi_A \simeq 1$, and $\Phi \simeq \Phi_A$ when $\beta \to \infty$. We feel that the external flux Φ is strongly affected by lattice artifacts which, however, are strongly reduced in the maximally Abelian projection. We can try to get rid of these effects by assuming that in the limit $\beta \to \infty$: $\Phi \simeq \Phi_A \simeq 1$. This way we get:

$$\sqrt{\sigma} \simeq \frac{\mu}{\sqrt{8\pi}} \,. \tag{14}$$

Fitting all together the data to a constant we get (purely statistic error)

$$\frac{\sqrt{\sigma}}{\Lambda_{\overline{MS}}} = 1.76(6), \qquad \chi^2/f = 1.44.$$
 (15)

Our extimation of the string tension is consistent with the linear asymptotic extrapolation of the string tension data extracted from Wilson loops on lattices larger than ours [13]

$$\frac{\sqrt{\sigma}}{\Lambda_{\overline{MS}}} = 1.79(12) . \tag{16}$$

Moreover note that, due to $\mu \simeq \mu_A$, we have

$$\sqrt{\sigma} \simeq \sqrt{\sigma_A} \,.$$
 (17)

5. CONCLUSIONS

We found evidence that the SU(2) vacuum behaves like a dual superconductor. In particular, we verified that the flux tube color field is composed by the chromoelectric component parallel to the line joining the static charges. This longitudinal chromoelectric field is almost constant far from the color sources and decreases rapidly in the direction transverse to the flux tube. The transverse distribution of the longitudinal chromoelectric field behaves according to the dual London equation Eq. (10). The London penetration length extracted from the Monte Carlo data using Eq. (10) is a physical quantity $\lambda_{\text{max. Ab. proj.}} = \lambda_{\text{SU}(2)}$. So that the long range properties of the SU(2) confining vacuum can be described by an effective Abelian theory. Moreover we established a simple relation between the string tension and the penetration length which gives an extimate of $\sqrt{\sigma}$ close to the extrapolated continuum limit available in the literature.

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