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Comment on “A New Symmetry for QED” and “Relativistically Covariant Symmetry in QED”

Victor O. Rivelles*

Instituto de Física, Universidade de São Paulo,

C.Postal 20516, CEP 01498-970, São Paulo, SP, Brazil

Abstract

We show that recently found symmetries in QED are just non-local versions of standard BRST symmetry.

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Recently it was found a non-local and non-covariant symmetry of QED in the Feynman gauge by Lavelle and McMullan [1] which was cast in a covariant form by Tang and Finkelstein [2]. It was claimed that they are new symmetries of QED and give rise to new Ward identities. We would like to point out that these symmetries are standard BRST symmetries and therefore they can not give rise to any new Ward identity.

In the Hamiltonian formulation of QED besides the gauge field and its momenta we have a pair of ghost fields $(c, \bar{c})$ and its momenta $(\overline{\mathcal{P}}, \mathcal{P})$ [3] (we leave out the fermion fields since they are not essential for our purposes and can be easily included). The ghost Lagrangian of QED (still in the Hamiltonian form) which implements the Lorentz condition is then found to be [3] $L_{gh}^{(ham)} = \dot{\mathcal{P}}\bar{c} + \dot{c}\overline{\mathcal{P}} - i\bar{c}\nabla^2 c + i\overline{\mathcal{P}}\mathcal{P}$. Usually the next step is to perform the integration over the ghost momenta to get the usual ghost Lagrangian $L_{gh} = i\bar{c}\Box c$. However we can perform the integration over the ghost fields instead of their momenta. Performing the integration over $\bar{c}$ we get a delta functional $\delta(i\nabla^2 c + \dot{\mathcal{P}}) = det\nabla^2 \ \delta(ic + \frac{1}{\nabla^2}\dot{\mathcal{P}})$. Now performing the integration over c we get the non-local ghost Lagrangian $L_{gh}^{(non-local)} = -i\overline{\mathcal{P}}\frac{1}{\nabla^2}\ddot{\mathcal{P}} + i\overline{\mathcal{P}}\mathcal{P}$ and the non-local BRST transformations $\delta A_i = i\frac{\partial_i}{\nabla^2}\dot{\mathcal{P}}$, $\delta A_0 = i\mathcal{P}$, $\delta\mathcal{P} = 0$, $\delta\overline{\mathcal{P}} = \nabla^2 A_0 - \partial_i \dot{A}_i$. We can now perform the following change of variables $\overline{\mathcal{P}} = \nabla^2\overline{\mathcal{R}}$ in order to get a local action and to get rid of the term $det\nabla^2$ in the path integral measure (which came from the integration over $\bar{c}$). After this change of variables we get the usual ghost action $L_{gh} = i\bar{c}\Box c$ and the non-covariant and non-local transformations of Lavelle and McMullan [1] after identifying $\mathcal{P}$ with $\bar{c}$ and $\overline{\mathcal{R}}$ with c. Since we have a standard BRST symmetry we get the usual constraints on the physical states and no further independent Ward identities can be found.

We now turn to the Tang and Finkelstein transformations. The ghost Lagrangian of QED $L_{gh} = i\bar{c}\Box c$ has a huge freedom when we perform field redefinitions in the ghost fields c and $\bar{c}$. If we consider, e.g., the following non-local redefinitions $c = \frac{1}{\nabla^2}\partial_0 d$, $\bar{c} = \frac{1}{\partial_0}\nabla^2\bar{d}$, the Lagrangian and the path integral measure remain invariant and the usual BRST transformations become $\delta A_\mu = \partial_\mu\frac{1}{\nabla^2}\partial_0 d$, $\delta d = 0$, $\delta\bar{d} = -\frac{i}{\xi}\frac{1}{\nabla^2}\partial_0\partial_\mu A^\mu$. These are the covariant non-local transformations presented in Ref. [2] (written in an arbitrary gauge, i.e., arbitrary

ξ) after identifying d and $\overline{d}$ with $\overline{c}$ and c respectively. Of course this procedure can be generalized to any (local or non-local) redefinition of the ghost fields which leave the action and the path integral measure invariant.

The gauge fixed QED action is also invariant under anti-BRST transformations which anticommute with the BRST transformations. We can then perform an arbitrary field redefinition (which leaves the action and the path integral measure invariant) and consider the BRST transformations of the redefined fields. Then perform a second arbitrary field redefinition and consider the anti-BRST transformations of the redefined fields. Since the original action is invariant under these field redefinitions the BRST and anti-BRST transformations of the redefined fields are still symmetries of the action. The sum of these two transformations are precisely the Tang and Finkelstein transformations Eqs.(5) and (9). Originally the BRST and anti-BRST transformations are anticommutating but now, since they are acting after field redefinitions they no longer need to anticommute. This explains why the transformations Eqs.(5) and (9) of Ref. [2] are no longer nilpotent. In fact the anticommutator gives rise to a new field redefinition which is also a symmetry of the action as can be easily verified.

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* Partially supported by CNPq. E-mail: vrivelles@if.usp.br.

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