Updated resonance photo-decay amplitudes to 2 GeV

Richard A. Arndt, Igor I. Strakovsky† and Ron L. Workman

Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061

(September 5, 1995)

Abstract

We present the results of an energy-dependent and set of single-energy partial-wave analyses of single-pion photoproduction data. These analyses extend from threshold to 2 GeV in the laboratory photon energy, and update our previous analyses to 1.8 GeV. Photo-decay amplitudes are extracted for the baryon resonances within this energy range. We consider two photoproduction sum rules and the contributions of two additional resonance candidates found in our most recent analysis of πN elastic scattering data. Comparisons are made with previous analyses.

PACS numbers: 11.80.Et, 14.20.Gk, 25.20.Lj

Typeset using REVTEX

I. INTRODUCTION

The study of baryon resonances and their electromagnetic decays is expected to see a resurgence as CEBAF becomes fully operational. The resulting flood of very precise photo- and electro-production data will provide a challenge for data analysts and model-builders alike. For many of these reactions, single-pion photoproduction serves as a "bench mark". In addition to providing the photo-decay amplitudes for resonances with appreciable couplings to the πN channel, it fixes the $Q^2 = 0$ point for pion electroproduction analyses. The amplitudes from these analyses have also been utilized in evaluating a number of sum rules which test the predictions of Chiral Perturbation Theory (ChPT) and extended current algebra. We will briefly discuss the status of two such sum rules in Section IV.

The present analysis is a significant improvement on our previously published result [1] for three main reasons. The database was carefully reexamined in order to check the assignments of statistical and systematic errors. This resulted in a number of changes which are discussed in Section II. The upper limit of our energy range was increased from 1.8 GeV to 2 GeV (in order to better regulate the solution near 1.8 GeV). Finally, the effect of two new resonance candidates was considered. These were found in a search for possible "missing resonances", as described in our most recent analysis [2] of elastic πN scattering data. In Section III, we give the results of our multipole analyses as well as the photo-decay amplitudes for resonances within our energy region. Finally, in Section V, we summarize our results and consider what improvements can be expected in the future.

II. THE DATABASE

The data compilations of Ukai and Nakamura [3] and an earlier compilation by Menze, Pfeil and Wilcke [4] were the main sources used in constructing our database. In the present study we have attempted to verify all references contained in these earlier compilations (more than 200 papers). As a result, we have corrected some data, photon energies, and systematic uncertainties according to the publications and/or the authors' suggestions. For example, the threshold π^0p differential cross sections produced by MAMI at Mainz now consist of 11 angular points (instead of 21), and have an increased normalization factor of 7.5% [5]. Total cross sections produced by ALS at Saclay were corrected by a factor of 0.94 [6]. We have added some data missed in our previous analyses [1], [7]. Other data were removed when found to be duplicated in our database or according to the authors' suggestions. A small number of points were removed when no reliable source was found.

As in our previous analyses, not all of the available data were used. Data taken before 1960 were not analyzed, nor were those single-angle and single-energy points measured prior to 1970. Some individual data points were also removed from the analysis in order to resolve database conflicts. Our previous published pion photoproduction scattering analysis [1] (SP93) was based on 4015 π^0p, 6019 π^+n, 2312 π^-p, and 120 π^0n data. Since then we have added 698 π^0p and 351 π^+n data. Through the checks described above, the total number of π^-p data actually decreased by 96.

The new low-energy data have been produced mainly by TRIUMF, for radiative pion capture on protons (39 differential and 10 total cross sections [8] and 10 measurements of P [9]), and by LEGS at BNL for π^0p photoproduction (12 differential cross sections and 97 measurements of Σ) [10]. We also have 9 Σ measurements from LEGS for π^-p photoproduction [11], and a small number of SAL π^+n photoproduction data (16 differential and 3 total cross sections) [12], [13].

Medium-energy differential cross section data for π^+n (245 data) [14] and T measurements for π^+n (216 data) and π^0p (52 data) were produced by ELSA at Bonn [15]. We have added 18 missing polarization measurements for π^0p in the 1 GeV region from Yerevan [16] and π^0p (7 P [17] and 14 Σ [18], [19]), π^+n (18 Σ, P, and T) [20], and π^-p (16 T) [21] measurements between 230 and 700 MeV from Kharkov. The distribution of recent (post-1993) data is given in Fig. 1.

Other experimental efforts will soon provide data in the low to intermediate energy region. These include a precise measurement of π^0p differential cross sections made in a

LEGS experiment. This experiment spanned the Δ isobar region and was completed at BNL in 1992 [22]. The region between 145 and 200 MeV was covered by MAMI at Mainz in 1991 [23], and the first phase of a measurement from threshold to about 25 MeV has been completed at SAL [24], [25]. A double polarization (beam–target) experiment at PHOENICS below 1150 MeV is planned at Bonn [26]. We also expect that the 1 to 2 GeV region will be extensively studied at CEBAF [27], [28].

III. MULTIPOLE ANALYSES AND PHOTO-DECAY AMPLITUDES

As in our previous studies [1], [7], we have performed both energy-dependent and single-energy analyses. The single-energy analyses were done mainly in order to check for structure missing in the energy-dependent form. However, these results were also used in Breit-Wigner fits to extract photo-decay amplitudes, as described below. The methods used to generate these solutions have been discussed previously [1], [7]. In the present analysis, one further degree of freedom was allowed. Some multipoles were given an overall phase $e^{i\Phi}$ where the angle Φ was proportional to $(\mathrm{Im}T_{\pi N} - T^2_{\pi N})$. This form satisfies Watson's theorem for elastic πN amplitudes ($T_{\pi N}$) while exploiting the undetermined phase for inelastic amplitudes.

Our energy-dependent solution (SM95) has a χ^2 of 31810 for 13415 data to 2 GeV. The overall χ^2/datum (about 2.4) is considerably lower than that found in our previously published [1] analysis to 1.8 GeV. While the number of data has increased by about 1000 points, the χ^2/datum has decreased significantly from the value (3.6) reported for the SP93 energy-dependent solution. This result mainly reflects the database changes discussed in Section II. Our present and previous solutions are compared in Table I.

The very low energy region is complicated by different thresholds for $\pi^0 p$ and $\pi^+ n$ production. While we have obtained a reasonable fit to the available differential and total cross sections, the multipole amplitudes should not used in the $\pi^+ n$ threshold region. We have concentrated on the extraction of resonance parameters, whereas the threshold region requires a detailed study.

The results from our first analysis [1] (SP93) to 1.8 GeV are compared with the present (SM95) energy-dependent and single-energy multipoles in Fig. 2. Significant deviations from SP93 are visible in multipoles connected to the πN S_{11}, S_{31}, and P_{11} partial-waves. Table II compares the energy-dependent and single-energy fits from threshold to 1.8 GeV.

In our most recent analysis [2] of elastic πN scattering data, we found evidence for two small structures on the high-energy tails of the $S_{11}(1650)$ and $F_{15}(1680)$ resonances. These structures remain small in the photoproduction reaction as well. In fact, they are too small for a reliable estimate of their photo-decay amplitudes.

A set of $N\gamma$ decay couplings has been extracted from our multipole amplitudes. We have fit these couplings using a background plus Breit-Wigner form, as is described in Ref. [7]. We analyzed both the energy-dependent and single-energy solutions over a variety of energy ranges in order to estimate uncertainties. Our results are listed in Table III. Here the resonance mass (W_R) and width (Γ) values were obtained from fits to our multipole amplitudes. The values of W_R remained quite consistent with estimates from our elastic πN analysis. The results for Γ tended to show more variation. Values of Γ_π/Γ, where Γ_π is the decay width to πN final states, were taken from the elastic πN analysis and were not varied. This ratio is required in calculating the photo-decay amplitudes.

As expected, there was little change in the photo-decay amplitudes for resonances strongly coupled to πN final states. These include the $P_{33}(1232)$, $D_{13}(1520)$, $S_{11}(1650)$, $F_{15}(1680)$, and $F_{37}(1950)$. The $D_{15}(1675)$ and $D_{33}(1700)$, have also remained stable. The most significant changes were found in the $S_{11}(1535)$ and $P_{11}(1440)$ $A_{1/2}$ γn couplings. As these resonances reflect complicated structures in the complex plane, uncertainty in the γn coupling is not surprising. We should also note that the $S_{11}(1535)$ γp $A_{1/2}$ coupling remains considerably below the value extracted from a recent analyses [29] of eta photoproduction data. A detailed analysis of both pion and eta photoproduction data in this region would be useful. A listing of our resonance couplings is given in Table III.

IV. SUM RULES

The development of Chiral Perturbation theory (ChPT) and extended current algebra has led to a renewed interest in a number of sum rules derived in the 1960's. Examples include the Gerasimov-Drell-Hearn [34] (GDH) and Weinberg [35] sum rules, as well as sum rules for the nucleon electric, magnetic, and spin-dependent polarizabilities [36]. Here we will briefly consider the status of two sum rules which require input from photoproduction amplitudes. These are the GDH sum rule and a sum rule [37], due to Fubini, Furlan and Rossetti (FFR), which has not attracted as much attention.

While the GDH sum rule was first derived from a dispersion relation (unsubtracted) and the low-energy theorem (LET) for Compton scattering, it was later obtained from the commutation relations of vector current densities. In Ref. [38], the extended current algebra of Chang and Liang [39] was found to imply a modified GDH sum rule. (It was observed [40] that modified currents would lead to modified sum rules soon after the original GDH sum rule appeared.) An estimation of this modification was shown to account for the apparent discrepancy [41] in the original sum rule.

In their discussion of modified sum rules, the authors of Ref. [40] mentioned in passing that a similar procedure could be used to determine modifications to the FFR sum rule. This sum rule relates nucleon magnetic moments to an integral over the invariant amplitude (A_1) for single-pion photoproduction. The FFR sum rule has the form [42]

$$g_A \left(\frac{e\kappa^{V,S}}{2M} \right) = \frac{2f_\pi}{\pi} \int Im\ A_1^{(+,0)}(\nu)\ \frac{d\nu}{\nu} \tag{1}$$

where $\kappa^{V,S}$ is the isovector (isoscalar) anomalous magnetic moment of the nucleon, given by $(\kappa_p \mp \kappa_n)/2$. The invariant amplitude A_1 corresponds to the amplitude associated with $\gamma_5 \gamma \cdot \epsilon \gamma \cdot k$ in the paper of Chew, Goldberger, Low, and Nambu [43]. The required isospin combinations are given [43], in terms of charge-channel information, by

$$A_1^{(+,0)} = \Big(A_1 \left(\gamma p \rightarrow \pi^0 p \right) \ \pm \ A_1 \left(\gamma n \rightarrow \pi^0 n \right) \Big) \ / \ 2. \tag{2}$$

Here the amplitude for photoproduction of $\pi^0 n$ states is inferred from measurements in the three other charge channels.

Empirical evaluation of the integral in Eq.(1) is (in principle) much simpler than the integral in the GDH sum rule – which involves contributions from multi-pion final states. Unfortunately, there are two problems which make a precise check of the FFR sum rule more difficult. Unlike the GDH sum rule, the FFR sum rule is not exact. It requires use of the Goldberger-Treiman relation [44]. In addition, convergence of the associated integral is expected to be less rapid than was found in the GDH sum rule.

Regardless of the above qualifications, early attempts to evaluate the integral in Eq.(1) were encouraging. An analysis [37] using the $P_{33}(1232)$ and $D_{13}(1520)$ resonances found good agreement for both κ^V and κ^S. A subsequent study [45], using an early multipole analysis [46], found 85% of the prediction for κ^V but did not present results for the isoscalar combination. In Ref. [45] the threshold behavior of the multipoles was modified by a factor to account for a non-zero pion mass [47].

This brings us to the reason for re-examining the FFR sum rule. If the FFR sum rule is valid, as the early studies suggest, it puts a constraint on the contribution to the GDH sum rule coming from single-pion photoproduction alone. Other tests of the GDH integral (including the $\pi\pi N$ contributions) have been made recently by Sandorfi et al. [36]. In reference [36], the multipole input to the GDH and spin-dependent polarizability sum rules was compared to predictions from ChPT [48]. The integrals in these sum rules involve the difference of helicity 3/2 and 1/2 total cross sections weighted by different powers of the photon energy. The difference of proton and neutron spin-dependent polarizabilities was found to agree with ChPT while the difference of proton and neutron GDH sum rules is known to have a problem [41]. The qualitative behavior found in Refs. [36], [41] is preserved in the present analysis. The isovector-isovector component of the GDH sum rule receives a single-pion production contribution very near the old estimate of Karliner [41] while the isovector-isoscalar (VS) component retains its sign and magnitude discrepancy.

While such comparisons are interesting, our poor knowledge of the $\pi\pi N$ contribution is an impediment. Early estimates of the $\pi\pi N$ contributions were based upon the resonance spectrum found in analyses of πN elastic scattering data. This neglects contributions from possible "missing states" which couple very weakly to the πN channel. (Though the FFR sum rule is not exact, we at least understand the approximation (PCAC) we are making.)

The integral giving κ^V is heavily dominated by the $P_{33}(1232)$ contribution, while the integral corresponding to κ^S appears to have important contributions from a wider range of energies. The result for κ^V was found to vary between 1.8 and 2.0, remarkably close to the predicted value. The integral corresponding to κ^S, however, shows considerable sensitivity to uncertainties in the high energy region. Here we find only qualitative agreement (correct sign and order of magnitude). The energy dependence of the isovector FFR integrand is displayed in Fig. 3.

In summary, we find the FFR sum rule for κ^V to be well satisfied, as was the case for isovector GDH sum rule. We also see that the FFR integral does not converge as quickly as the analogous GDH integral. The isoscalar result is less certain. The existence of significant structure apart from the D_{13} resonance suggests that early success [37] with the isoscalar FFR component was fortuitous. However, we should note that the isoscalar component of the FFR sum rule appears to have less problems than the VS component of the GDH sum rule. This tends to weaken arguments that require a large discrepancy in the single-pion photoproduction multipoles in order to explain the GDH discrepancy. It would be helpful if high-quality photoproduction measurements could be extended a further 1 GeV in order to test the convergence of both the FFR and GDH sum rules.

If extended current algebra does indeed contribute to the FFR sum rule (as suggested in Ref. [40]), the results presented here should provide a useful test for the form proposed by Chang and Liang [39]. While the isoscalar FFR sum rule would likely provide the most sensitive check on any such contribution, the phenomenological evaluation of the associated integral is not yet sufficiently stable for more than an order-of-magnitude test.

V. SUMMARY AND CONCLUSIONS

We have extensively checked the pion photoproduction database for missing, duplicated, and inconsistent measurements. This has resulted in a significantly reduced χ^2. The extracted photo-decay couplings generally remain, for dominant resonances, in good agreement with the older analysis of Crawford and Morton [30]. The γn $A_{1/2}$ coupling for the $S_{11}(1535)$ proved difficult to fit. The present value is quite different from the results of both Crawford and Morton [30] and our previous analysis [1] to 1.8 GeV. The uncertainty in this coupling is likely much greater than we previously estimated [1]. As mentioned above, the γp coupling could also have a problem given the discrepancy between the present value and the result of eta photoproduction analyses.

The quark model results of Capstick [32] reproduce most features of the photo-decay couplings. The $P_{33}(1232)$ couplings are underestimated, but this is an old problem. The $P_{11}(1440)$ couplings have the wrong sign and magnitude. There have been suggestions [49] that this state, and also the $P_{33}(1600)$, could be hybrids in which case a comparison with the conventional quark model is inappropriate. It is unfortunate that the weak resonance candidates, found in our analysis of elastic pion-nucleon scattering data, were not clearly evident here. These states should be considered in future analyses of other-meson photoproduction databases.

We briefly examined two sum rules which require photoproduction input. Those components dominated by the $P_{33}(1232)$ resonance seem to be reasonably well satisfied. The isoscalar components of the GDH sum rule and the FFR sum rule for κ^S are less certain. We are currently exploring the use of fixed-t dispersion relations which may help to constrain our analyses.

The results of these analyses, and the associated databases, are available [50] via either Telnet or the Internet, or from the authors upon request.

ACKNOWLEDGMENTS

The authors express their gratitude to G. Anton, B. Bassalleck, R. Beck, M. Blecher, H. Dutz, K. G. Fissum, P. Galumian, G. Hakopian, M. Khandaker, M. A. Kovash, E. Mazzucato, D. F. Measday, W. Meyer, A. S. Omelaenko, A. M. Sandorfi, P. V. Sorokin, J. C. Staško, K. Ukai, and H. B. van den Brink for providing experimental data prior to publication or for clarification of information already published. R.W. acknowledges useful discussions regarding sum rules with Lay Nam Chang. I. S. acknowledges the hospitality extended by the Physics Department of Virginia Tech. This work was supported in part by the U. S. Department of Energy Grant DE–FG05–88ER40454 and a NATO Collaborative Research Grant 921155U.

REFERENCES

[†] On leave from St. Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188350 Russia.

[1] Z. Li, R. A. Arndt, L. D. Roper, and R. L. Workman, Phys. Rev. **C 47**, 2759 (1993).

[2] R. A. Arndt, I. I. Strakovsky, R. L. Workman, and M. M. Pavan, to be published in Phys. Rev. **C** (1995).

[3] K. Ukai and T. Nakamura, Institute for Nuclear Study of University of Tokyo, Japan, Report INS–TEC–22, 1985. We have obtained an updated tape of this compilation from INS, University of Tokyo.

[4] D. Menze, W. Pfeil, and R. Wilcke, Physikalisches Institut der Universität Bonn, Germany, Physics Daten 7–1, 1977.

[5] R. Beck, F. Kalleicher, B. Schoch, J. Vogt, G. Koch, H. Ströher, V. Metag, J. C. McGeorge, J. D. Kellie, and S. J. Hall, Phys. Rev. Lett. **65**, 1841 (1990); R. Beck (private communication, 1994).

[6] E. Mazzucato, P. Argan, G. Audit, A. Bloch, N. de Botton, N. d'Hose, J.-L. Faure, M. L. Ghedira, C. Guerra, J. Martin, C. Schuhl, G. Tamas, and E. Vincent, Phys. Rev. Lett. **57**, 3144 (1986); E. Mazzucato (private communication, 1994).

[7] R. A. Arndt, R. L. Workman, Z. Li, and L. D. Roper, Phys. Rev. **C 42**, 1853 (1990); **C 42**, 1864 (1990).

[8] M. Wang, Ph.D. Thesis, University of Kentucky, 1992; M. A. Kovash (private communication, 1993).

[9] J. C. Staško, B. Bassalleck, E. C. Booth, M. B. Certok, D. Healey, R. Jacot-Guillarmod, M. A. Kovash, D. J. Mack, D. F. Measday, J. P. Miller, M. A. Moinester, D. Ottewell, M. A. Pickar, and P. Weber, Phys. Rev. Lett. **72**, 973 (1994).

[10] G. Blanpied, M. Blecher, A. Catacappa, C. Djalali, M.-A. Duval, G. Giordano, S. Hoblit, M. Khandaker, O. C. Kistner, G. Matone, L. Miceli, W. K. Mize, B. M. Preedom, A. M. Sandorfi, C. Schaerf, R. M. Sealock, C. E. Thorn, S. T. Thornton, K. Vaziti, C. S. Whisant, X. Zhao, and M. A. Moinester, Phys. Rev. Lett. **69**, 1880 (1992); A. M. Sandorfi (private communication, 1994).

[11] A. M. Sandorfi (private communication, 1995).

[12] K. G. Fissum, H. S. Caplan, E. L. Hallin, D. M. Skopik, J. M. Vogt, M. Frodyma, D. P. Rosenzweig, D. W. Storm, G. V. O'Rielly, and K. R. Garrow, submitted to Phys. Rev. **C** (1995); K. G. Fissum (private communication, 1994).

[13] H. B. van den Brink, H. P. Blok, H. T. Chung, H. Choi, T. Gresko, E. L. Hallin, W. H. A. Hesselink, T. Kobayashi, J. Mitchell, B. Norum, J. Shaw, D. M. Skopik, P. Stoler, J. M. Vogt, and T. P. Welch, Nucl. Phys. **A587**, 657 (1995).

[14] K. Büchler, K. H. Althoff, G. Anton, J. Arends, W. Beulertz, M. Breuer, P. Detemple, H. Dutz, E. Kohlgarth, D. Krämer, W. Meyer, G. Nöldeke, W. Schneider, W. Thiel, and B. Zucht, Nucl. Phys. **A570**, 580 (1994).

[15] K.-H. Althoff, G. Anton, J. Arends, W. Beulertz, A. Bock, M. Breuer, P. Detemple, H. Dutz, R. Gehring, M. Gemander, S. Goertz, K. Heibing, J. Hey, D. Krämer, W. Meyer, G. Nöldeke, G. Reicherz, A. Thomas, and B. Zucht, in: *"Few-Body Problems in Physics, Williamsburg, Virginia, USA, May 1994"*, ed. F. Gross, AIP Conf. Proc. **334**, 505 (1994); submitted to Nucl. Phys. **A**; G. Anton (private communication, 1994).

[16] R. O. Avakyan, R. A. Asatryan, A. S. Bagdasaryan, Yu. A. Garibyan, K. R. Dallakyan, S. S. Danagulyan, V. S. Eganov, A. P. Karapetyan, I. A. Keropyan, O. S. Kizogyan, G. O. Marukyan, E. M. Matevosyan, A. A. Oganesyan, G. M. Petrosyan, Zh. V. Petrosyan, I. P. Prokhorenko, Zh. S. Saakyan, Yu. Z. Susiakyan, E. M. Skho-

toryan, S. P. Taroyan, and G. M. Elbakyan, Yad. Fiz. **53**, 717 (1991) [Phys. At. Nucl. (former Sov. J. Nucl. Phys.) **53**, 448 (1991)].

[17] A. S. Bratashevski, A. A. Zybalov, S. P. Karasiov, O. K. Konovalov, A. S. Omelaenko, P. V. Sorokin, Yu. O. Storozhenko, and A. E. Tenishev, Ukrain. Phys. J. **31**, 1306 (1986); A. S. Omelaenko (private communication, 1994).

[18] V. G. Gorbenko, Yu. V. Zhebrovsky, L. Ya. Kolesnikov, A. L. Rubashkin, and P. V. Sorokin, in: *Voprosu Atomnoi Nauki i Techniki (Proceedings of Atomic Science and Technique) High Energy and Atomic Nucleus Physics, Kharkov*, **2**, 36 (1977); A. S. Omelaenko (private communication, 1994).

[19] V. G. Gorbenko, Yu. V. Zhebrovsky, L. Ya. Kolesnikov, A. L. Rubashkin, P. V. Sorokin, and V. F. Chechetenko, in: *Voprosu Atomnoi Nauki i Techniki (Proceedings of Atomic Science and Technique) High Energy and Atomic Nucleus Physics, Kharkov*, **2**, 31 (1977); A. S. Omelaenko (private communication, 1994).

[20] V. A. Getman, V. G. Gorbenko, A. Ya. Derkach, Yu. V. Zhebrovsky, I. M. Karnaukhov, L. Ya. Kolesnikov, A. A. Lukhanin, A. S. Omelaenko, A. L. Rubashkin, P. V. Sorokin, E. A. Sporov, and Yu. N. Telegin, *Voprosu Atomnoi Nauki i Techniki (Proceedings of Atomic Science and Technique) High Energy and Atomic Nucleus Physics, Kharkov*, **8**, 3 (1989); A. S. Omelaenko (private communication, 1994).

[21] V. L. Agranovich, A. A. Belyaev, B. G. Getman, A. V. Glamazdin, V. G. Gorbenko, A. P. Dzyubak, I. M. Karnaukhov, A. A. Lukhanin, I. L. Semisalov, P. V. Sorokin, E. A. Sporov, and I. A. Tolmachev, *Voprosu Atomnoi Nauki i Techniki (Proceedings of Atomic Science and Technique) High Energy and Atomic Nucleus Physics, Kharkov*, **8**, 5 (1989); A. S. Omelaenko (private communication, 1994).

[22] A. M. Sandorfi (private communication, 1994).

[23] R. Beck (private communication, 1994).

[24] D. A. Hutcheon (private communication, 1995).

[25] J. Bergstrom (private communication, 1995).

[26] G. Anton (private communication, 1994).

[27] W. J. Briscoe, J. Ficenec, D. Jenkins *et al. The photoproduction of pions"*, CEBAF Proposal, E–94–103, 1994.

[28] R. J. Holt, R. Gilman *et al. Measurement of photoproton polarization in the* $H(\gamma,\vec{p})\pi^0$ *reaction*, CEBAF Proposal, E–94–012, 1994.

[29] M. Benmerrouche, N.C. Mukhopadhyay, and J.F. Zhang, Phys. Rev. **D 51**, 3237 (1995).

[30] R. L. Crawford and W. T. Morton, Nucl. Phys. **B211**, 1 (1983); Particle Data Group, Phys. Lett. **B239**, 1 (1990).

[31] I. Arai and H. Fujii, Nucl. Phys. **B194**, 251 (1982).

[32] S. Capstick, Phys. Rev. **D 46**, 2864 (1992).

[33] L. Montanet, R. M. Barnet, D. E. Groom, T. G. Trippe, C. G. Wohl, H. Murayama, J. Stone, J. J. Hernandez, F. C. Porter, R. J. Morrison, A. Manohar, M. Anduilar-Benitez, C. Caso, R. L. Crawford, M. Roos, N. A. Törnqvist, K. G. Hayes, G. Höhler, S. Kawabata, D. M. Manley, K. Olive, R. E. Shrock, S. Eidelman, R. H. Schindler, A. Gurtu, K. Hikasa, G. Conforto, R. L. Workman, and C. Grab, *Review of Particle Properties*, Phys. Rev. **D 50**, 1 (1994).

[34] S.B. Gerasimov, Yad. Fiz. **2**, 598 (1965) [Sov. J. Nucl. Phys. **2**, 430 (1966)]; S.D. Drell and A.C. Hearn, Phys. Rev. Lett. **16**, 908 (1966). For an earlier related work, see L. I. Lapidus and Chou Kuang-chao, JETP **14**, 357 (1962).

[35] S. Weinberg, Phys. Rev. Lett. **18**, 188 (1967); see also a recent study by J.F. Donoghue and B.R. Holstein, Phys. Rev. **D 46**, 4076 (1992).

[36] The status of these sum rules is discussed in A.M. Sandorfi, C.S. Whisnant and M. Khandaker, Phys. Rev. **D 50**, R6681 (1994).

[37] S. Fubini, G. Furlan and C. Rossetti, Nuov. Cim. **40**, 1171 (1965); S. Fubini, C. Rossetti and G. Furlan, Nuov. Cim. **A 43**, 161 (1966).

[38] L.N. Chang, Y. Liang and R.L. Workman, Phys. Lett. **B 329**, 514 (1994).

[39] L.N. Chang and Y. Liang, Phys. Lett. **B 268**, 64 (1991); Phys. Rev. **D 45**, 2121 (1992).

[40] K. Kawarabayashi and M. Suzuki, Phys. Rev. **152**, 1383 (1966).

[41] R.L. Workman and R.A. Arndt, Phys. Rev. **D 45**, 1798 (1992); I. Karliner, Phys. Rev. **D 7**, 2717 (1973); G.C. Fox and D.Z. Freedman, Phys. Rev. **182**, 1628 (1969).

[42] This relation has appeared with different signs for the integral, and additional factors of 2 or M depending upon the definition of A_1. The sum rule is evaluated at t=0, though this is not necessary [see the discussion by V. De Alfaro, S. Fubini, G. Furlan, and C. Rossetti, in their book: *Currents in Hadron Physics* (North Holland, New York, 1973). Expected convergence properties of the FFR and GDH sum rules are also discussed in this reference.]

[43] G.F. Chew, M.L. Goldberger, F.E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

[44] M. Goldberger and S. Treiman, Phys. Rev. **110**, 1478 (1958).

[45] S.L. Adler and F.J. Gilman, Phys. Rev. **152**, 1460 (1966).

[46] Ref. [45] cites a private communication from R.L. Walker. The published version of Walker's analysis appeared later [R.L. Walker, Phys. Rev. **182**, 1729 (1969)].

[47] As the extrapolation $m_\pi \to 0$ is not well defined, we have used the physical photoproduction amplitudes.

[48] See V. Bernard, N. Kaiser, J. Kambor, and Ulf-G. Meissner, Nucl. Phys. **B388**, 315

(1992).

[49] Z. Li, V. Burkert, and Z. Li, Phys. Rev. **D 46**, 70 (1992).

[50] Those with access to TELNET can run the SAID program with a link to VTINTE.PHYS.VT.EDU (128.173.176.61). The login (password) is: PHYSICS (QUANTUM). The user may view the current database and compare our solutions to those of other groups. A WWW server is also available (http://clsaid.phys.vt.edu).

Figure captions

Figure 1. Energy-angle distribution of recent (post-1993) data. (a) $\pi^0 p$, (b) $\pi^+ n$, and (c) $\pi^- p$. $\pi^0 p$ data are [observable (number of data)]: $d\sigma/d\Omega$ (12), Σ (111), T (52), P (6), O_x (7), and O_z (7). $\pi^+ n$ data are: $d\sigma/d\Omega$ (261), σ^{tot} (3), Σ (6), T (222), and P (6). $\pi^- p$ data are: $d\sigma/d\Omega$ (39), σ^{tot} (10), Σ (9), T (16), and P (10). Total cross sections are plotted at zero degrees.

Figure 2. Partial-wave amplitudes ($L_{2I,2J}$) from 0 to 2 GeV. Solid (dashed) curves give the real (imaginary) parts of amplitudes corresponding to the SM95 solution. The real (imaginary) parts of single-energy solutions are plotted as filled (open) circles. The previous SP93 solution [1] is plotted with long dash-dotted (real part) and short dash-dotted (imaginary part) lines. Plotted are the multipole amplitudes (a) ${}_{\rm p}E_{0+}^{1/2}$, (b) ${}_{\rm n}E_{0+}^{1/2}$, (c) ${}_{\rm p}E_{0+}^{3/2}$, (d) ${}_{\rm p}M_{1-}^{1/2}$, (e) ${}_{\rm n}M_{1-}^{1/2}$, (f) ${}_{\rm p}E_{1+}^{1/2}$, (g) ${}_{\rm p}M_{1+}^{1/2}$, (h) ${}_{\rm n}E_{1+}^{1/2}$, (i) ${}_{\rm n}M_{1+}^{1/2}$, (j) ${}_{\rm p}M_{1-}^{3/2}$, (k) ${}_{\rm p}E_{1+}^{3/2}$, (l) ${}_{\rm p}M_{1+}^{3/2}$, (m) ${}_{\rm p}E_{2-}^{1/2}$, (n) ${}_{\rm p}M_{2-}^{1/2}$, (o) ${}_{\rm n}E_{2-}^{1/2}$, (p) ${}_{\rm n}M_{2-}^{1/2}$, (q) ${}_{\rm p}E_{2-}^{3/2}$, (r) ${}_{\rm p}E_{2+}^{3/2}$, (s) ${}_{\rm p}E_{3-}^{1/2}$, (t) ${}_{\rm p}M_{3-}^{1/2}$, (u) ${}_{\rm n}E_{3-}^{1/2}$, (v) ${}_{\rm n}M_{3-}^{1/2}$, (w) ${}_{\rm p}E_{3-}^{3/2}$, and (x) ${}_{\rm p}M_{3+}^{3/2}$ in millifermi units. The subscript p (n) denotes a proton (neutron) target.

Figure 3. Integrand for the FFR sum rule giving κ^V.

TABLES

TABLE I. Comparison of present (SM95) and previous (SP93 and SP89) energy-dependent partial-wave analyses of charged and neutral pion photoproduction data. N_{prm} is the number parameters varied in the fit.

Solution	Limit (MeV)	χ^2/data $\pi^0 p$	χ^2/data $\pi^+ n$	χ^2/data $\pi^- p$	χ^2/data $\pi^0 n$	N_{prm}
SM95	2000	13087/4711	12284/6359	6156/2225	282/120	135
SP93 [1]	1800	14093/4015	22426/6019	8280/2312	275/120	134
SP89 [7]	1000	13073/3241	11092/3847	4947/1728	461/120	97

TABLE II. Comparison of single-energy (binned) and energy-dependent analyses of pion photoproduction data. N_{prm} is the number parameters varied in the single-energy fits. χ^2_E is due to the energy-dependent fit (SM95) taken over the same energy interval.

E_{lab} (MeV)	Range (MeV)	N_{prm}	χ^2/data	χ^2_E
154	150 – 156	6	119/50	276
165	154 – 176	12	217/73	416
185	175 – 195	14	87/91	128
205	194 – 213	14	125/98	158
225	220 – 235	15	202/152	371
245	234 – 256	15	544/258	670
265	254 – 275	15	540/311	639
285	275 – 296	16	778/361	1000
305	294 – 316	16	722/431	866
325	314 – 336	17	902/423	1075
345	333 – 356	17	721/478	902
365	354 – 376	17	556/395	727
385	374 – 396	17	443/361	578
405	393 – 416	18	633/381	729
425	414 – 436	18	440/311	606
445	433 – 456	18	409/280	494
465	454 – 476	18	271/227	344
485	474 – 496	18	255/189	391
505	494 – 516	19	449/257	593
525	514 – 536	19	202/177	257
545	533 – 556	19	221/222	321
565	554 – 576	19	342/190	643
585	573 – 596	19	372/250	480

605	594 – 616	19	313/257	374
625	614 – 636	19	345/271	399
645	634 – 656	20	480/315	577
665	654 – 676	20	385/272	453
685	673 – 696	20	407/249	460
705	694 – 716	21	983/468	1139
725	714 – 736	21	290/221	468
745	733 – 756	21	766/409	1005
765	753 – 776	22	420/245	678
785	774 – 796	22	223/213	421
805	793 – 816	20	543/344	797
825	814 – 836	23	252/176	337
845	834 – 856	23	523/325	735
865	854 – 876	23	212/144	357
885	873 – 896	23	282/155	453
905	893 – 916	24	719/329	931
925	913 – 936	25	174/145	320
945	934 – 956	25	459/252	629
965	954 – 975	25	230/126	374
985	974 – 996	25	140/124	334
1005	994 – 1016	25	763/283	1051
1025	1014 – 1036	25	251/128	406
1045	1034 – 1056	25	394/195	622
1065	1054 – 1076	25	131/123	299
1085	1074 – 1096	25	92/97	286
1105	1094 – 1115	25	524/217	801
1125	1115 – 1136	25	140/98	283

1145	1134 – 1155	25	233/159	314
1165	1154 – 1176	25	127/97	199
1185	1174 – 1194	25	90/82	148
1205	1194 – 1216	25	276/174	433
1225	1214 – 1236	25	69/80	167
1245	1234 – 1255	25	168/104	249
1265	1254 – 1276	16	68/62	102
1285	1275 – 1296	16	31/40	84
1305	1294 – 1315	16	326/128	454
1325	1314 – 1335	16	52/45	129
1345	1335 – 1355	26	137/90	210
1365	1355 – 1375	16	37/36	92
1385	1375 – 1395	16	78/42	167
1405	1395 – 1416	26	496/136	669
1425	1415 – 1436	16	66/53	105
1445	1435 – 1456	26	104/78	148
1465	1455 – 1475	16	37/15	64
1485	1474 – 1495	16	63/32	121
1505	1494 – 1515	26	226/107	432
1525	1515 – 1535	16	69/33	148
1545	1535 – 1555	26	85/55	132
1565	1555 – 1575	16	18/17	39
1585	1575 – 1595	16	35/30	51
1605	1595 – 1616	26	122/92	217
1625	1614 – 1635	16	48/23	75
1645	1635 – 1655	16	199/79	243
1665	1655 – 1675	16	29/35	48

1685	1675 – 1695	16	20/28	37
1705	1694 – 1715	26	206/92	275
1725	1715 – 1735	16	9/14	18
1745	1735 – 1755	16	172/46	213
1765	1754 – 1775	16	49/34	65
1785	1775 – 1796	16	20/19	34
1805	1795 – 1815	16	224/75	308

Table III. Resonance couplings from a Breit-Wigner fit to the SM95 solution [VPI], the analysis of Crawford and Morton [CM83] [30], Arai and Fujii [AF82] [31], recent quark model [32] predictions [CAP92], and an average from the Particle Data Group [PDG] [33]. A † indicates the quantity was not fitted.

		$\gamma p (GeV)^{-1/2} * 10^{-3}$		$\gamma n (GeV)^{-1/2} * 10^{-3}$	
Resonance State	**Reference**	$A_{1/2}$	$A_{3/2}$	$A_{1/2}$	$A_{3/2}$
$\mathbf{S_{11}(1535)}$	VPI	60 ± 15		-20 ± 35	
$W_R = 1525(10)\ MeV$	CM83	65 ± 16		-98 ± 26	
$\Gamma_\pi/\Gamma = 0.31$	AF82	80 ± 7		-75 ± 8	
$\Gamma = 103(5)\ MeV$	PDG	68 ± 10		-59 ± 22	
	CAP92	76		-63	
$\mathbf{S_{11}(1650)}$	VPI	69 ± 5		-15 ± 5	
$W_R = 1677(8)\ MeV$	CM83	33 ± 15		-68 ± 40	
$\Gamma_\pi/\Gamma \approx 1$	AF82	61 ± 5		8 ± 19	
$\Gamma = 160(12)\ MeV$	PDG	52 ± 17		-11 ± 28	
	CAP92	54		-35	
$\mathbf{P_{11}(1440)}$	VPI	-63 ± 5		45 ± 15	
$W_R = 1463(7)\ MeV$	CM83	-69 ± 18		56 ± 15	
$\Gamma_\pi/\Gamma = 0.68$	AF82	-66 ± 4		19 ± 12	
$\Gamma = 360(20)\ MeV$	PDG	-72 ± 9		52 ± 25	
	CAP92	4		-6	
$\mathbf{P_{11}(1710)}$	VPI	7 ± 15		-2 ± 15	
$W_R = 1720(10)\ MeV$	CM83	6 ± 18		-17 ± 20	
$\Gamma_\pi/\Gamma = 0.15$	AF82	-12 ± 5		11 ± 21	

$\Gamma = 105(10)\ MeV$	PDG	-6 ± 27		16 ± 29	
	CAP92	13		-11	
$\mathbf{P_{13}(1720)}$	VPI	-15 ± 15	7 ± 10	7 ± 15	-5 ± 25
$W_R = 1713(10)\ MeV$	CM83	44 ± 66	-24 ± 36	-3 ± 34	18 ± 28
$\Gamma_\pi/\Gamma = 0.16$	AF82	71 ± 10	-11 ± 11	1 ± 38	-134 ± 44
$\Gamma = 153(15)\ MeV$	PDG	27 ± 24	-26 ± 10	18 ± 29	-33 ± 59
	CAP92	-11	-31	4	11
$\mathbf{D_{13}(1520)}$	VPI	-20 ± 7	167 ± 5	-48 ± 8	-140 ± 10
$W_R = 1516(10)\ MeV$	CM83	-28 ± 14	156 ± 22	-56 ± 11	-144 ± 15
$\Gamma_\pi/\Gamma = 0.61$	AF82	-32 ± 5	162 ± 3	-71 ± 11	-148 ± 9
$\Gamma = 106(4)\ MeV$	PDG	-22 ± 18	163 ± 7	-62 ± 6	-137 ± 13
	CAP92	-15	134	-38	-114
$\mathbf{D_{15}(1675)}$	VPI	15 ± 10	10 ± 7	-49 ± 10	-51 ± 10
$W_R = 1673(5)\ MeV$	CM83	21 ± 11	15 ± 9	-59 ± 15	-59 ± 20
$\Gamma_\pi/\Gamma = 0.38$	AF82	6 ± 5	29 ± 4	-25 ± 27	-71 ± 26
$\Gamma = 154(7)\ MeV$	PDG	18 ± 10	18 ± 9	-50 ± 14	-70 ± 6
	CAP92	2	3	-35	-51
$\mathbf{F_{15}(1680)}$	VPI	-10 ± 4	145 ± 5	30 ± 5	-40 ± 15
$W_R = 1679(5)\ MeV$	CM83	-17 ± 18	132 ± 10	44 ± 12	-33 ± 15
$\Gamma_\pi/\Gamma = 0.68$	AF82	-28 ± 3	115 ± 12	26 ± 5	-24 ± 9
$\Gamma = 124(4)\ MeV$	PDG	-14 ± 8	135 ± 17	27 ± 10	-35 ± 11
	CAP92	-38	56	19	-23

S$_{31}$(1620)	VPI	35 ± 20	
$W_R = 1672(5)\ MeV$	CM83	35 ± 10	
$\Gamma_\pi/\Gamma = 0.29$	AF82	-26 ± 8	
$\Gamma = 147(8)\ MeV$	PDG	30 ± 14	
	CAP92	81	
P$_{31}$(1910)	VPI	-2 ± 8	
$W_R = 1910^\dagger\ MeV$	CM83	14 ± 30	
$\Gamma_\pi/\Gamma = 0.26$	AF82	-31 ± 4	
$\Gamma = 250^\dagger\ MeV$	PDG	13 ± 22	
	CAP92	-8	
P$_{33}$(1232)	VPI	-141 ± 5	-261 ± 5
$W_R = 1232.5(0.5)\ MeV$	CM83	-145 ± 15	-263 ± 26
$\Gamma_\pi/\Gamma = 0.99$	AF82	-147 ± 1	-264 ± 2
$\Gamma = 117(2)\ MeV$	PDG	-141 ± 5	-257 ± 8
	CAP92	-108	-186
P$_{33}$(1600)	VPI	-18 ± 15	-25 ± 15
$W_R = 1672(15)\ MeV$	CM83	-39 ± 30	-13 ± 14
$\Gamma_\pi/\Gamma = 0.17$	AF82	-	-
$\Gamma = 315(20)\ MeV$	PDG	-26 ± 20	-6 ± 17
	CAP92	30	51
D$_{33}$(1700)	VPI	90 ± 25	97 ± 20
$W_R = 1690(15)\ MeV$	CM83	111 ± 17	107 ± 15
$\Gamma_\pi/\Gamma = 0.16$	AF82	112 ± 6	47 ± 7

$\Gamma = 285(20)\ MeV$	PDG	114 ± 13	91 ± 29
	CAP92	82	68
$\mathbf{D_{35}(1930)}$	VPI	-7 ± 10	5 ± 10
$W_R = 1955(15)\ MeV$	CM83	-38 ± 47	-23 ± 80
$\Gamma_\pi/\Gamma = 0.11$	AF82	-	-
$\Gamma = 350(20)\ MeV$	PDG	-15 ± 17	-10 ± 22
	CAP92	$-$	$-$
$\mathbf{F_{35}(1905)}$	VPI	22 ± 5	-45 ± 5
$W_R = 1895(8)\ MeV$	CM83	21 ± 10	-56 ± 28
$\Gamma_\pi/\Gamma = 0.12$	AF82	31 ± 9	-45 ± 6
$\Gamma = 354(10)\ MeV$	PDG	37 ± 16	-31 ± 30
	CAP92	26	-1
$\mathbf{F_{37}(1950)}$	VPI	-79 ± 6	-103 ± 6
$W_R = 1947(9)\ MeV$	CM83	-67 ± 14	-82 ± 17
$\Gamma_\pi/\Gamma = 0.49$	AF82	-83 ± 5	-100 ± 5
$\Gamma = 302(9)\ MeV$	PDG	-85 ± 17	-101 ± 14
	CAP92	-33	-42