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**Lowest tensor-meson resonances contributions
to the chiral perturbation theory
low energy coupling constants**

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Abstract

The contributions of the lightest tensor-meson resonances to the low-energy coupling constants of second order chiral perturbation theory with two flavours are evaluated and compared with the available phenomenological information as well as with similar results for other resonances.

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1 Introduction

The physics of pions at low energy can be described by an effective Lagrangian given by the chiral perturbation theory (ChPT). This Lagrangian depends on some low-energy coupling constants which are fixed in principle by the underlying QCD, although their exact determination is still inaccessible.

In the framework of 2nd-order ChPT [1], the contributions to the coupling constants of all meson resonances of spin ≤ 1 have been established and their importance has been shown [2]. In fact, it is the vector-meson which dominates, but the others cannot be ignored. The tensor-meson resonance has a higher mass than those analyzed in [2] but, because of its multiplicity, it is hard to tell whether its contribution is trifling or not.

The $SU(2)$ ChPT to second order in the momenta and the external fields is presented briefly in Section 2. The kinematics of a spin-2 field is depicted in Section 3 using a construction made by Fierz and Pauli [3]. In Section 4, the coupling of the tensor-meson resonances with the pions and their contribution to the low-energy coupling constants is shown, following the general trend used in [2], and our way of describing the tensor-meson is compared with the one developed in [4]. Section 5 is devoted to a comparison of Pauli-Fierz scheme with other ones and finally in Section 6 a general overview of the contributions of the different meson-resonances and a comparison with the phenomenological values of the coupling constants are given.

2 Second-order ChPT with two flavours

To make the presentation simpler, we shall only give those results we consider necessary here, and the reader is invited to consult [1] or [2] for a more detailed discussion.

ChPT is an effective theory describing QCD at low energies. The two-flavours massless-quarks QCD Lagrangian is symmetric under $SU(2)_R \times SU(2)_L$, the chiral group. It is assumed that a spontaneous chiral symmetry breakdown occurs,

$$SU(2)_R \times SU(2)_L \rightarrow SU(2)_V,$$

whose Goldstone bosons are identified as the pions.

The QCD Lagrangian can be approximated at a given order in the momentum using an effective Lagrangian expressed in terms of a field $U \in SU(2)$ which transforms linearly under $SU(2)_R \times SU(2)_L$,

$$U \rightarrow g_R U g_L^+,$$

and contains the fields of the three pseudoscalar Goldstone bosons,

$$U = u^2 = e^{i\phi^a \tau_a / F}, \quad \phi^a \tau_a = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}, \quad (2.1)$$

where F is the pion decay constant in the chiral limit: $F_\pi = F(1 + O(m_{\text{quark}}))$.

Coupling U with external fields and expanding the effective Lagrangian in powers of the external momenta and of quark masses, gives

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \dots \quad (2.2)$$

The local nature of the chiral symmetry requires the introduction of a derivative $D_\mu U$ which is covariant with respect to the external vectorial and axial gauge fields.

It is found that \mathcal{L}_2 depends on two parameters and \mathcal{L}_4 on seven:

$$\mathcal{L}_4 = \sum_{i=1}^7 l_i P_i, \quad (2.3)$$

where the terms P_i contain external fields, pion fields and their derivatives of order p^4 . In particular, it will be shown that only the first two parameters receive contributions from the tensor-meson resonance; the corresponding P_i are:

$$P_1 = \frac{1}{4} \langle u^\mu u_\mu \rangle^2 \quad (2.4)$$

$$P_2 = \frac{1}{4} \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle, \quad (2.5)$$

using the notation of [4],

$$u_\mu = iu^\dagger D_\mu U u^\dagger = -iu D_\mu U^\dagger u = u_\mu^\dagger,$$

and $\langle \dots \rangle$ stands for the trace.

The generating functional of second-order ChPT consists of the trees and one-loop graphs generated by \mathcal{L}_2 and the trees of \mathcal{L}_4 . The divergences of the one-loop functional are absorbed by the l_i , hence they will depend on a renormalization scale μ , which drops out in all observable quantities. Denoting the renormalization parameters by $l_i^r(\mu)$, scale-independent parameters \bar{l}_i can be defined:

$$l_i^r(\mu) = \frac{\gamma_i}{32\pi^2} \left(\bar{l}_i + \ln \frac{m^2}{\mu^2} \right), \quad (2.6)$$

where γ_i is a given real number [1] and m is the pion mass.

3 Kinematics of a spin-2 field

The description of a spin-2 field requires a tensor-field $T_{\mu\nu}$ that is symmetric. However, as this object still contains 10 degrees of freedom, we add further that its trace should vanish and that $\partial^\mu T_{\mu\nu}$ should obey a simple condition implied by the classical equations of motion, to get the simplest interaction term arising when the coupling of the tensor-field with the Goldstone bosons of ChPT are considered.

The attempt to obtain the equations of motion from the most general quadratic Lagrangian by a variational principle using the holonomic constraints $T_{\mu\nu} = T_{\nu\mu}$ and $T^\mu{}_\mu = 0$ produces a condition on $\partial^\mu T_{\mu\nu}$ which is not very simple. For this reason we prefer to follow [3] and introduce an auxiliary scalar field C , independent of $T_{\mu\nu}$ which gives a simple additional condition on $\partial^\mu T_{\mu\nu}$ derived by variation from the Lagrange function. We take C and $T_{\mu\nu}$ to be real fields.

Denoting the mass of the tensor-field by M and using Fierz and Pauli's choice, the Lagrangian becomes

$$\begin{aligned} \mathcal{L}_T = & -\frac{1}{4}M^2 T_{\mu\nu} T^{\mu\nu} + \frac{1}{4}\partial_\rho T^{\mu\nu} \partial^\rho T_{\mu\nu} - \frac{1}{2}\partial^\rho T_{\rho\mu} \partial_\nu T^{\nu\mu} \\ & + \frac{3}{16}M^2 C^2 - \frac{3}{32}\partial_\rho C \partial^\rho C + \frac{1}{4}\partial^\mu T_{\mu\nu} \partial^\nu C \\ & + J_{\mu\nu} T^{\mu\nu} + CJ, \end{aligned} \quad (3.1)$$

where $T_{\mu\nu} = T_{\nu\mu}$ and $T^\mu{}_\mu = 0$; $J_{\mu\nu}$ and J are external fields.

Taking the second derivative of the classical equation of motion for $T_{\mu\nu}$, with respect to x_μ and x_ν , together with the classical equation of motion for C , gives a linear system of equations for $T_S \equiv \partial^{\mu\nu} T_{\mu\nu}$ and C :

$$\mathcal{M} \begin{pmatrix} T_S \\ C \end{pmatrix} = \begin{pmatrix} j_1 \\ j_2 \end{pmatrix}, \quad (3.2)$$

where

$$\mathcal{M} = \begin{pmatrix} M^2 - \frac{1}{2}\square & \frac{3}{8}\square\square \\ -4 & 3\square + 6M^2 \end{pmatrix} \quad (3.3)$$

and

$$\begin{cases} j_1 &= 2(\partial^{\mu\nu} J_{\mu\nu} - \frac{1}{4}\square J^\rho{}_\rho) \\ j_2 &= -16J. \end{cases} \quad (3.4)$$

Because $\det(\mathcal{M}) = 6M^4 \neq 0$, (3.2) can be inverted:

$$\begin{pmatrix} T_S \\ C \end{pmatrix} = \mathcal{M}^{-1} \begin{pmatrix} j_1 \\ j_2 \end{pmatrix}. \quad (3.5)$$

Introducing (3.5) into the classical equation of motion of $T_{\mu\nu}$, differentiated once with respect to x_μ , gives

$$\partial^\mu T_{\mu\nu} = \frac{1}{4M^2} \left(8\partial_\mu J_{\mu\nu} - 2\partial_\nu J_\rho^\rho + 2\partial_\nu j_1 - \frac{1}{4}\square\partial_\nu j_2 \right). \quad (3.6)$$

The equation of motion becomes

$$(\square + M^2) T_{\mu\nu} = \Theta_{\mu\nu} \quad (3.7)$$

$$\begin{aligned} \text{where } \Theta_{\mu\nu} = & 2(J_{\mu\nu} - \frac{1}{4}g_{\mu\nu}J_\rho^\rho) \\ & + \frac{1}{6M^4}(4\partial_\mu\partial_\nu - g_{\mu\nu}\square)(j_1 - \frac{1}{8}(\square + M^2)j_2) \\ & + \frac{1}{M^2}(2\partial_\mu\partial^\rho J_{\rho\nu} + 2\partial_\nu\partial^\rho J_{\rho\mu} - \partial_{\mu\nu}J_\rho^\rho - \frac{1}{2}g_{\mu\nu}j_1). \end{aligned} \quad (3.8)$$

Thus the effective action can be written as:

$$\begin{aligned} S_T = & \frac{1}{2} \int dx dy [J^{\mu\nu}(x) \Pi_{\mu\nu\rho\sigma}(x-y) \Theta^{\rho\sigma}(y) + j_1(x) K_1(x-y) j_1(y) \\ & + j_1(x) K_{12}(x-y) j_2(y) + j_2(x) K_2(x-y) j_2(y)] \end{aligned} \quad (3.9)$$

$$\begin{aligned} \text{with } \Pi_{\mu\nu\rho\sigma}(x) = & \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{k^2 - M^2 + i\epsilon} \left[\frac{1}{2}(\bar{g}_{\mu\rho}(k)\bar{g}_{\mu\sigma}(k) + \bar{g}_{\mu\sigma}(k)\bar{g}_{\mu\rho}(k)) \right. \\ & \left. - \frac{1}{3}\bar{g}_{\mu\nu}(k)\bar{g}_{\rho\sigma}(k) \right], \end{aligned}$$

$$\bar{g}_{\mu\nu}(k) = g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2},$$

$$K_1(x) = -\delta^4(x) \frac{1}{36M^8} (3\square + 6M^2),$$

$$K_{12}(x) = -\delta^4(x) \left(\frac{1}{24M^4} + \frac{1}{144M^8} (-3\square\square + 6M^4) \right),$$

$$\text{and } K_2(x) = \delta^4(x) \left(\frac{1}{96M^4} + \frac{1}{384M^8} \square\square \right) (M^2 - \frac{1}{2}\square).$$

The coupling with the auxiliary scalar field C does not generate poles.

Switching off the external fields, the Lagrangian (3.1) describes the kinematics of a spin-2 field using a tensor-field $T_{\mu\nu}$ which satisfies the conditions

$$\begin{cases} T_{\mu\nu} = T_{\nu\mu}, \\ g^{\mu\nu}T_{\mu\nu} = 0, \\ \partial^\mu T_{\mu\nu} = 0, \end{cases} \quad (3.10)$$

with the usual equations of motion,

$$(\square + M^2) T_{\mu\nu} = 0. \quad (3.11)$$

4 Chiral coupling of the tensor-meson resonance and its contribution to the ChPT Lagrangian

As with the other meson resonances treated in [2], the tensor-meson effect on second-order ChPT generates a contribution to some low-energy coupling constants.

All resonances carry non-linear realizations of the chiral group $SU(2)_R \times SU(2)_L$ which depend on their transformation properties under the subgroup $SU(2)_V$.

The lightest spin-2 resonances are around the same energy [5]: $f_2(1270)$, $a_2(1320)$, which is not coupled with two pions but with three because of G -parity, $f'_2(1525)$, that mainly decays into $K\bar{K}$ ($\Gamma(f'_2 \rightarrow \pi\pi) \cong 0.6$ MeV), and $\pi_2(1670)$, which has again wrong G -parity to interact with two pions.

Hence the lightest spin-2 field which has a significant coupling with the pions is the singlet $f_2(1270)$ ($J^{PC} = 2^{++}$) [5], the realization of $SU(2)_R \times SU(2)_L$ is trivial

$$T_{\mu\nu} \xrightarrow{SU(2)_R \times SU(2)_L} T_{\mu\nu}.$$

To obtain the contributions of the tensor-meson resonance to the low-energy coupling constants, the lowest-order couplings in the chiral expansion are needed. These are linear in the resonance fields and at most of order p^2 .

Because of the symmetries and of the null trace property of $T_{\mu\nu}$, the only interaction that can occur is with the external current

$$J_{\mu\nu} = K_T \langle u_\mu u_\nu \rangle. \quad (4.1)$$

The coupling constant K_T determines the width,

$$\Gamma(f_2 \rightarrow \pi\pi) = \frac{K_T^2 M^3}{40\pi F^4} \left(1 - \frac{4m^2}{M^2}\right)^{\frac{5}{2}}. \quad (4.2)$$

Using the experimental results ¹, equation (4.2) gives

$$|K_T| = 28.5 \text{ MeV}. \quad (4.3)$$

The external scalar current J contains all possible coupling respecting the symmetries of the effective chiral Lagrangian. Since the pions cannot interact at rest, J must be of order p^2 .

¹ $F = 93.2$ MeV is the pion decay constant;
 $m = 140$ MeV is the pion mass;
 $M = 1275$ MeV is the f_2 mass;
and $\Gamma(f_2 \rightarrow \pi\pi) = 157$ MeV is the measured width [5]

At order p^4 the Lagrangian (3.1) may receive polynomial corrections that can be expressed in terms of the P_i of (2.3) in order to be consistent with QCD [6]. Only the P_i with the right symmetry have to be taken into account. Moreover the terms generated by the auxiliary field C in (3.9) at this order are precisely of this type. Hence both can be treated together.

On the one hand, using (3.9) and (4.1), the $\pi^0\pi^0$ scattering amplitude in the Mandelstam variables reads:

$$\begin{aligned}
T^{\pi^0\pi^0}(s, t, u) = & \frac{8K_T^2}{F^4} \left[\frac{1}{M^2 - s} \left\{ -5m^4 + 3m^2s - \frac{1}{3} \frac{m^4s}{M^2} - \frac{1}{4}s^2 + \frac{1}{2}(t^2 + u^2) - \frac{1}{2} \frac{m^2s^2}{M^2} \right. \right. \\
& \left. \left. - \frac{1}{6} \frac{s^3}{M^2} + \frac{1}{6} \frac{s^3}{M^2} + \frac{1}{6} \frac{m^2s^3}{M^4} + \frac{1}{12} \frac{s^4}{M^4} \right\} + \frac{1}{M^2 - t} \left\{ (s, t, u) \rightarrow (t, u, s) \right\} \right. \\
& \left. + \frac{1}{M^2 - u} \left\{ (s, t, u) \rightarrow (u, s, t) \right\} \right] + \text{polynomial}. \quad (4.4)
\end{aligned}$$

Due to the Froissart bound, a once-subtracted fixed t dispersion relation can be written for this amplitude:

$$T^{\pi^0\pi^0}(s, t, u) = \mu(t) + \frac{1}{\pi} \int_{4m^2}^{\infty} dx \frac{1}{x^2} \left(\frac{s^2}{x-s} + \frac{u^2}{x-u} \right) \text{Im}T^{\pi^0\pi^0}(s, t, u), \quad (4.5)$$

implying that for high s the contribution of the tensor-meson to the forward $\pi^0\pi^0$ scattering amplitude must be a constant. Then (4.4) is made compatible with QCD by subtracting $K_T^2 P_1/3$ to the Lagrangian (3.1).

On the other hand, the tensor-meson cannot contribute to the scalar form-factor of the pion, which is the case for the corrected Lagrangian. This imply that there is no other correction involving the remaining P_i .

The corrected Lagrangian, $\bar{\mathcal{L}}_T$, is now compatible with QCD. It will be used to compute the contributions of the tensor-meson to the low-energy coupling constants of ChPT.

With the external current (4.1), the equation of motion reads

$$(\square + M^2) T_{\mu\nu} = 2K_T \left(\langle u_\mu u_\nu \rangle - \frac{1}{4} g_{\mu\nu} \langle u_\mu u^\mu \rangle \right) + S_{\mu\nu}, \quad (4.6)$$

where $S_{\mu\nu} = O(p^4)$ and contains only pion fields, scalar and pseudoscalar external fields and their derivatives (typically, $\langle D_\mu D^\tau D_\tau (U^2)^\dagger D^\mu (U^2) \rangle$). Since we only want the Lagrangian at order p^4 , $T_{\mu\nu}$ only needs to be known at order p^2 . Equation (4.6) implies that

$$T_{\mu\nu} = \frac{2K_T}{M^2} \left(\langle u_\mu u_\nu \rangle - \frac{1}{4} g_{\mu\nu} \langle u_\tau u^\tau \rangle \right) + O(p^4) \quad (4.7)$$

and the corrected action at order p^4 is

$$\begin{aligned}\bar{S}_T &= \int d^4x \bar{\mathcal{L}}_T^{(4)}(x) + O(p^6), \\ \bar{\mathcal{L}}_T^{(4)}(x) &= -\frac{4K_T^2}{3M^2} \left(\frac{1}{4} \langle u_\rho u^\rho \rangle^2 \right) + \frac{4K_T^2}{M^2} \left(\frac{1}{4} \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle \right).\end{aligned}\quad (4.8)$$

Comparing (4.8) with the P_i from (2.4-5), the contributions of the tensor-meson resonance to \bar{l}_i are found to be

$$\begin{aligned}\bar{l}_1^T &= -\frac{32\pi^2}{\gamma_1} \frac{4K_T^2}{3M^2} = -0.63 \\ \bar{l}_2^T &= \frac{32\pi^2}{\gamma_2} \frac{4K_T^2}{M^2} = 0.95,\end{aligned}\quad (4.9)$$

with $\gamma_1 = \frac{1}{3}$ and $\gamma_2 = \frac{2}{3}$. These results are compatible with those obtained from another effective Lagrangian describing the tensor-meson interaction with the pions developed in [4].

5 Comparison of Pauli-Fierz scheme with other ones

This polynomial correction procedure to make a theory compatible with QCD can be applied to the Pauli-Fierz type of Lagrangians, as may be seen by using a description of a spin-1 field in their way and the conditions developed in [6]. The Lagrangian

$$\begin{aligned}\mathcal{L}_V^{\text{kin}} &= \left\langle \frac{1}{4} (V_{\mu\nu} V^{\mu\nu} - 2M_V^2 V_\mu V^\mu) - \alpha \bar{C}^2 + \beta^2 \nabla_\mu \bar{C} \nabla^\mu \bar{C} + \beta M_V \bar{C} V \right\rangle \\ \text{with } V_{\mu\nu} &= \frac{1}{\sqrt{2}} (\nabla_\mu V_\nu - \nabla_\nu V_\mu), \\ V_S &= \nabla_\mu V^\mu, \\ \bar{C} &\text{ an auxiliary scalar field,} \\ \alpha \neq 0 \text{ and } \beta \in \mathbb{R},\end{aligned}$$

describes a spin-1 field of mass M_V because

$$\begin{pmatrix} V_S \\ \bar{C} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{and} \quad (\square + M_V^2) V_\mu = 0,$$

can be derived from Euler-Lagrange equations in the same way as in Section 3. The single condition $V_S = 0$ does not fix all the parameters of $\mathcal{L}_V^{\text{kin}}$, whereas for \mathcal{L}_T the four conditions on $\partial_\mu T^{\mu\nu}$ fix them all.

On the introduction of the couplings with the pseudoscalar mesons,

$$\mathcal{L}_V^{\text{int}} = \langle \bar{J}^{\mu\nu} V_{\mu\nu} + \bar{J} \bar{C} \rangle,$$

the effective action becomes

$$S_V = \frac{1}{2} \int dx dy \langle \bar{J}^{\mu\nu}(x) \Delta_{\mu\nu\rho\sigma}(x-y) \bar{J}^{\rho\sigma}(y) + \bar{J}(x) \bar{K}(x-y) \bar{J}(y) \rangle$$

$$\text{with } \Delta_{\mu\nu\rho\sigma}(x) = \int \frac{d^4 k}{(2\pi)^4} \frac{e^{-ikx}}{k^2 - M_V^2 + i\epsilon} [g_{\mu\rho} k_\nu k_\sigma - g_{\mu\sigma} k_\nu k_\rho - (\mu \leftrightarrow \nu)]$$

$$\text{and } \bar{K}(x) = \delta^4(x) \left(\frac{1}{2\alpha} - \frac{3\beta^2}{2\alpha^2} \square \right).$$

As in the tensor meson case, the coupling of \bar{C} with the pions generates polynomial terms that can be expressed in terms of the P_i of (2.3).

This effective model *à la* Pauli-Fierz is different from the two presented in [6]. But imposing the same conditions to make it compatible with QCD (the behaviour of the two-point functions, of the pion form factors and of the forward amplitude for the elastic scattering of pseudoscalar mesons) implies its equivalence with the two other descriptions (the antisymmetric tensor and the vector field formulation) to $O(p^4)$.

6 Overview of the various contributions of the meson-resonances and conclusion

The roles played by different resonances can now be quantified. Following from (2.6) and (4.9), we have

$$\bar{l}_i = -\ln \frac{m^2}{\mu^2} + \sum_R \bar{l}_i^R, \quad (6.1)$$

where

- $m = 140$ MeV, is the pion mass,
- $\mu = M_\rho = 770$ MeV is the renormalization scale,
- \bar{l}_i^R is the contribution of the resonance R .

From [2], the contributions to \bar{l}_1 and \bar{l}_2 of all the meson-resonances of spin ≤ 1 and of mass $\lesssim 1$ GeV can be extracted, with (4.9) it gives:

$$\begin{aligned} \bullet \text{ vector: } \quad \bar{l}_1^V &= -4.5 \\ \bar{l}_2^V &= 2.2 \end{aligned} \quad (6.2)$$

- scalar (singlet and octet): $\bar{l}_1^S = 1.2$
 $\bar{l}_2^S = 0$ (6.3)

- tensor: $\bar{l}_1^T = -0.63$
 $\bar{l}_2^T = 0.95.$ (6.4)

Thus, in absolute value, the vector-meson $\rho(770)$ contributions are about twice as big as the sum of the others.

It must be noted that although $\rho'(1450)$ is a vector-meson with a mass comparable to that of $f_2(1270)$, its contributions to the low energy coupling-constants is only about $\frac{\Gamma_{\pi\pi}^{\rho'}}{\Gamma_{\pi\pi}^{\rho}} \frac{M_{\rho}}{M_{\rho'}} \cong \frac{1}{20}$ of those of $\rho(770)$, mainly because $\Gamma_{\pi\pi}^{\rho'} \cong \frac{1}{10} \Gamma_{\pi\pi}^{\rho}$ [5].

We therefore assume that a good estimate of the contributions of the resonances is obtained by considering their spectrum up to the tensor-meson mass. This estimate can be compared with the experimental information:

$$\begin{aligned} \bar{l}_1^{\text{reson}} &\cong -0.5 & \text{as } \bar{l}_1^{\text{exp}} &= -0.7 \pm 2.0 \\ \bar{l}_2^{\text{reson}} &\cong 6.6 & \text{as } \bar{l}_2^{\text{exp}} &= 5.3 \pm 1.3. \end{aligned}$$

The intervals for the phenomenological values given above cover all the published values together with their uncertainties [4, 7, 8, 9, 10], at least to the best of our knowledge.

Besides this, the vector-meson cannot contribute to $\bar{l} = 2\bar{l}_1 + 4\bar{l}_2$ (low-energy combination involved in $\pi^0\pi^0$ scattering, for instance), so that

$$\bar{l}^{\text{reson}} \cong 25.4 \quad \text{as} \quad \bar{l}^{\text{exp}} = 19.8 \pm 9.2.$$

Thus it has been shown that although the tensor-meson resonance is 50% heavier than the dominating vector-meson one, it still contributes to the renormalized coupling constants of chiral perturbation theory. This analysis supports the statement that their phenomenological values may be understood on the basis of the spectrum of low lying excited states.

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