LUNFD6/(NFFL-7082)-Rev. 1994

hep-ph/9509213

Bose-Einstein Correlations for Three-Dimensionally Expanding Cylindrically Symmetric Finite Systems

T. Csörgő[1,2] and B. Lörstad[3] *

[1] *Department of Physics, Columbia University, New York, NY 10027, USA,*

[2] *MTA KFKI RMKI, H–1525 Budapest 114, P.O. Box 49. Hungary,*

[3] *Physics Institute, University of Lund, Sölvegatan 14, S - 223 62 Lund, Sweden*

Abstract

There are *two type of scales* present simultaneously in the space-like as well as in the time-like directions in a model-class describing a cylindrically symmetric, finite, three-dimensionally expanding boson source. One type of the scales is related to the finite lifetime or geometrical size of the system, the other type is governed by the rate of change of the local momentum distribution in the considered temporal or spatial direction. The parameters of the Bose-Einstein correlation function may obey an *M_t-scaling*, as observed in $S+Pb$ and $Pb+Pb$ reactions at CERN SPS. This M_t-scaling implies that the Bose-Einstein correlation functions view only a small part of the big and expanding system. The full sizes of the expanding system at the last interaction are shown to be measurable with the help the invariant momentum distribution of the emitted particles. A vanishing duration parameter can also be generated, with a specific M_t dependence, in the considered model-class.

*csorgo@sunserv.kfki.hu, bengt@quark.lu.se

Introduction. The method of intensity interferometry has recently become a widely used tool for determining the space-time picture of high energy heavy ion collisions. Originally, the method was invented [1] to measure angular diameters of distant stars. The objects under study were approximately static and the length-scales astronomical. In principle the same method is applied to measure space-time characteristics of high energy heavy ion collisions, where the objects are expanding systems, with life-times of a few fm/c (10^{-23} sec) and length-scales of a few fm (10^{-15} m).

In the case of high energy heavy-ion collisions intensity interferometry is pursued to infer the equation of state and identify the possible formation of a transient Quark-Gluon Plasma state from a precise determination of the freeze-out hyper-surface, as scanned by the Bose-Einstein correlation function (BECF), see e. g. the contributions of the NA35, NA44 and WA80 collaborations in ref. [2]. For an introduction and review on Bose-Einstein correlations see ref. [3].

The recent $^{32}S +\ ^{197}Pb$ reactions at 200 AGeV laboratory bombarding energy resulted in a non-expected, symmetrical BECF-s if measured in the LCMS, the longitudinally co-moving system of the boson pairs [2]. The longitudinal component was shown to measure a thermal length-scale, $R_L = \tau_0\sqrt{T_0/m_t}$, introduced first in ref. [4] for an infinite, longitudinally expanding Bjorken tube. The *side* radius parameter was thought to measure the geometrical radius and the *out* component to be sensitive to the duration of the particle freeze-out times [5,6]. The radius parameters turned out to be equal within the experimental errors. Although this might be just a coincidence, in this Letter we show that such a behavior, valid in a wide m_t interval, may be a natural consequence of a cylindrically symmetric three-dimensional hydrodynamic expansion. In this case the local temperature, the gradients of the temperature distribution and the flow-gradients generate 'thermal' length-scales in all these space-like directions. Changes in the local temperature during the particle emission induce a temporal scale, the thermal duration. Recently it became clear that the parameters of the BECF-s measure the lengths of homogeneity [4,7–9] which in turn were shown to be expressible in terms of the geometrical and the thermal lengths, [10,6,9].

We shall derive here model-independent relationships among the functional forms of the BECF-s as given in the laboratory (LAB) frame and the LCMS frame. We introduce the longitudinal saddle-point system (LSPS) in which the functional form of the BECF-s turns out to be the simplest one.

A new class of analytically solvable models is introduced thereafter, describing a three-dimensionally expanding, cylindrically symmetric system for which the geometrical sizes and the duration of the particle emission are finite. In this class of the models there are two length-scales present in all directions, including the temporal one. The BECF is found to be dominated by the shorter, while the momentum distribution by the longer of these scales. The interplay between the finite "geometrical scales" of the boson-emitting source and the finite "thermal scales" shall be considered in detail.

Formalism. Both the momentum spectra and the BECF-s are prescribed in the applied Wigner-function formalism [11,3]. In this formalism the BECF is calculated from the two-body Wigner-function assuming chaotic particle emission. In the final expression the time-derivative of the Wigner function is approximated [11,3] by a classical emission function $S(x,p)$, which is the probability that a boson is produced at a given $x = (t, \mathbf{r}\,) = (t, r_x, r_y, r_z)$ point in space-time with the four-momentum $p = (E, \mathbf{p}\,) = (E, p_x, p_y, p_z)$. The off-shell two-particle Wigner functions shall be approximated by the off-shell continuation of the on-shell Wigner-functions [11,10,6,9]. The particle is on the mass shell, $m^2 = E^2 - \mathbf{p}^2$. Please note the difference between x indicating a four-vector in space-time and the script-size $_x$ which indexes a direction in coordinate space.

A useful auxiliary function is the Fourier-transformed emission function

$$\tilde{S}(\Delta k, K) = \int d^4x\, S(x,K)\, \exp(i\Delta k \cdot x), \tag{1}$$

where

$$\Delta k = p_1 - p_2, \qquad\qquad K = \frac{p_1 + p_2}{2} \tag{2}$$

and $\Delta k \cdot x$ stands for the inner-product of the four-vectors. Then the one-particle inclusive invariant momentum distribution (IMD) of the emitted particles, $N_1(\mathbf{p})$ is given by

$$N_1(\mathbf{p}) = \tilde{S}(\Delta k = 0, K = p) \;=\; \frac{E_1}{\sigma_{tot}} \frac{d\sigma}{d\mathbf{p}}, \tag{3}$$

where σ_{tot} is the total inelastic cross-section. This IMD is normalized to the mean multiplicity $\langle n \rangle$ as

$$\int \frac{d\mathbf{p}}{E} N_1(\mathbf{p}) = \langle n \rangle. \tag{4}$$

In the present Letter effects arising from the final state Coulomb and Yukawa interactions shall be neglected. The two-particle BECF can be calculated from the emission function with the help of the well-established approximation

$$C(\Delta k; K) = \frac{\langle n(n-1)\rangle}{\langle n \rangle^2} \frac{N_2(\mathbf{p}_1, \mathbf{p}_2)}{N_1(\mathbf{p}_1)\, N_1(\mathbf{p}_2)} \simeq 1 + \frac{\mid \tilde{S}(\Delta k, K) \mid^2}{\mid \tilde{S}(0, K) \mid^2}, \tag{5}$$

see e.g. ref. [11] for further details. The corrections to this expression are known to be small [9]. Note that among the eight components of Δk and K only six are independent due to the two constraints $p_1^2 = p_2^2 = m^2$. These constraints can be formulated alternatively as $\Delta k \cdot K = 0$ and $K^2 = m^2 - \Delta k^2/4$. Thus the two-particle BECF depends on the off-shell emission function, which we approximate by the off-shell continuation of the on-shell emission functions.

A similar but not identical approximation is to replace $\tilde{S}(\Delta k, K)$ with $\tilde{S}(\Delta k, K')$ exchanging the off-shell K to the on-shell K'. The latter mean momentum is defined to be on-shell as $K'^0 = m^2 - \mathbf{K}'^2$ where $\mathbf{K}' = \mathbf{K} = (\mathbf{p_1} + \mathbf{p_2})/2$. The differences between these two approximation schemes are of $\mathcal{O}(\Delta k/m)$. The above two approximation schemes coincide in the $\Delta k \to 0$ limit when the Bose-Einstein correlations are maximal. Since we shall make use of the $\Delta k \cdot K = 0$ constraint which is exact only if the K four-vector is off-shell, we shall approximate the off-shell emission function in eq. (5) with the off-shell continuation of the on-shell emission function.

$\underline{\text{General considerations}}$. We model the emission function in terms of the longitudinally boost-invariant variables: the (longitudinal) proper-time, $\tau = \sqrt{t^2 - r_z^2}$, the space-time rapidity $\eta = 0.5 \ln\left((t+z) \,/\, (t-z) \right)$, the transverse mass $m_t = \sqrt{E^2 - p_z^2}$ and the momentum-space rapidity $\mathrm{y} = 0.5 \ln\left((E+p_z) \,/\, (E-p_z) \right)$. In the transverse direction, the transverse radius, $r_t = \sqrt{r_x^2 + r_y^2}$ is introduced. We have

$$t = \tau \cosh(\eta), \qquad\qquad z = \tau \sinh(\eta). \tag{6}$$

For systems undergoing a boost-invariant longitudinal expansion, the emission function may be a function of boost-invariant variables only. These are τ, r_x, r_y, p_x, p_y and $\eta - y$. However, for finite systems the exact longitudinal boost-invariance cannot be achieved and the emission function becomes a function of $\eta - y_0$ too, where y_0 stands for the mid-rapidity. Thus approximate boost-invariance is recovered only in the mid-rapidity region, where terms proportional to $\eta - y_0$ can be neglected. Thus for finite systems undergoing a boost-invariant longitudinal expansion the emission function can be given in terms of these variables as

$$S(x;K)\, d^4x = S_*(\tau, \eta, r_x, r_y)\, d\tau\, \tau_0 d\eta\, dr_x\, dr_y. \tag{7}$$

Here we introduced the constant τ_0 in front of $d\eta$ due to dimensional reasons and included the Jacobian from the d^4x to the $d\tau\, d\eta\, dr_x\, dr_y$ variables into the emission function $S_*(\tau, \eta, r_x, r_y; K)$. The subscript $_*$ indicates that the functional form of the emission function is changed with the change of the variables. Further, dependences on the mean momentum K as well as on the mid-rapidity y_0 are also indicated with the subscript $_*$. The effective, momentum-dependent parameters of the emission function $S_*(\tau, \eta, r_x, r_y)$ shall also be indexed with $_*$ in the forthcoming. Suppose that the Fourier-transform $\tilde{S}(\Delta k; K)$ can be evaluated in terms of the τ and η variables in the small Δk region relevant for the analysis of the BECF-s, since the region around $x_s(K)$, where the Fourier integrals pick up the dominant contribution from, is sufficiently small so that within this region the τ and η dependence of t and z can be linearized as

$$t \simeq \tau \cosh[\eta_s] + (\eta - \eta_s)\tau_s \sinh[\eta_s] \tag{8}$$

$$z \simeq \tau \sinh[\eta_s] + (\eta - \eta_s)\tau_s \cosh[\eta_s] \qquad (9)$$

with negligible second-order corrections. The subscript $_s$ stands for the point where the emission function is maximal (we assume that $S(x;K)$ has only one maximum for any values of K). We do not assume at this point whether the function $-\ln S(x;K)$ is expandable into a (multi-variate) Taylor series [9] around its unique minimum at the saddle point x_s or not, merely we assume that the Fourier-transformed $\tilde{S}(\Delta k;K)$ exists. See the Appendix for a clarifying example.

The principal directions for the decomposition of the relative momentum at a given value of the mean four-momentum K are given as [12,5]: the *out* direction is parallel to the component of $\mathbf{K}$, which is perpendicular to the beam, indexed with $_{out}$, the *longitudinal* or *long* direction is parallel to the beam-axis r_z, this component of the relative momentum is indexed with $_L$, and the remaining direction orthogonal to both *longitudinal* and *out* is called the *side* direction, indexed with $_{side}$. Thus the mean and the relative momenta are decomposed as $K = (K_0, K_{out}, 0, K_L)$ and $\Delta k = (Q_0, Q_{out}, Q_{side}, Q_L)$.

Since the particles are on mass-shell, we have

$$0 = K \cdot \Delta k = K_0 Q_0 - K_L Q_L - K_{out} Q_{out}. \qquad (10)$$

Thus the energy difference Q_0 can be expressed as

$$Q_0 = \beta_L Q_L + \beta_{out} Q_{out} \qquad (11)$$

where we have introduced the longitudinal and the outward component of the velocity of the pair, $\beta_L = K_L/K_0$ and $\beta_{out} = K_{out}/K_0$, respectively. These relations become further simplified in the LCMS, the longitudinally co-moving system, introduced first in ref. [5]. The LCMS is the frame where $K_L = 0$ thus $\beta_L = 0$. We also have $\beta_{out} = \beta_t$ where $_t$ stands for transverse, e.g. $r_t = \sqrt{r_x^2 + r_y^2}$ and $m_t = \sqrt{m^2 + r_x^2 + r_y^2}$. Let us express the Fourier integrals in terms of the τ and η variables in the laboratory reference frame (LAB), utilizing eqs. (8,9). The results in LCMS can be obtained from the more complicated results in the

LAB frame by the substitution $\beta_L = 0$ and $\beta_{out} = \beta_t$. To simplify the notation, let us rewrite

$$\Delta k \cdot x = Q_0 t - Q_{out} r_x - Q_{side} r_y - Q_L r_z \simeq Q_\tau \tau - Q_{out} r_x - Q_{side} r_y - Q_\eta \tau_s (\eta - \eta_s), \qquad (12)$$

utilizing the linearized eqs. (8,9). We have introduced the coefficients of the τ and the $\tau_s(\eta - \eta_s)$ as new variables given by

$$Q_\tau = Q_0 \cosh[\eta_s] - Q_L \sinh[\eta_s] = (\beta_t Q_{out} + \beta_L Q_L) \cosh[\eta_s] - Q_L \sinh[\eta_s], \qquad (13)$$

$$Q_\eta = Q_L \cosh[\eta_s] - Q_0 \sinh[\eta_s] = Q_L \cosh[\eta_s] - (\beta_t Q_{out} + \beta_L Q_L) \sinh[\eta_s] \qquad (14)$$

From these relations it follows that

$$C(\Delta k; K) \simeq 1 + \frac{\mid \tilde{S}(\Delta k, K) \mid^2}{\mid \tilde{S}(0, K) \mid^2} \simeq 1 + \frac{\mid \tilde{S}_*(Q_\tau, Q_\eta, Q_{out}, Q_{side}) \mid^2}{\mid \tilde{S}_*(0,0,0,0) \mid^2}. \qquad (15)$$

Note that this expression contains a four-dimensional Fourier-transformed function, and among the four variables Q_τ, Q_η, Q_{out} and Q_{side} only three are independent due to eq. (11). Note also that at this point the BECF may have a non-Gaussian structure, and its dependence on its variables does not factorize.

The core/halo model. If the system under consideration consists of a *core* characterized by a hydrodynamical expansion and small regions of homogeneity, and a surrounding *halo* of long-lived resonances, then the above general expression can be further evaluated if the halo is characterized by sufficiently large regions of homogeneity. Indeed, the long lived resonances may decay in a large volume proportional to their lifetime, and the decay products are emitted with a given momentum distribution from the whole volume of the decay. Thus the halo of long-lived resonances is characterized by large regions of homogeneity. (In case of the pionic halo the dominant long-lived resonances are ω, η, η' and K^0, all with life-times greater than 20 fm/c). If the emission function is a sum of the emission function of the core and the halo,

$$S_*(\tau, \eta, r_x, r_y) = S_{*,c}(\tau, \eta, r_x, r_y) + S_{*,h}(\tau, \eta, r_x, r_y) \qquad (16)$$

and the Fourier-transformed emission function of the halo is sufficiently narrow to vanish at the finite resolution of the relative momentum Δk in a given experiment, then one can show [23] that

$$N_1(\mathbf{p}) = N_{1,c}(\mathbf{p}) + N_{1,h}(\mathbf{p}), \tag{17}$$

$$C(\Delta k; K) = 1 + \lambda_* \frac{\mid \tilde{S}_{*,c}(Q_\tau, Q_\eta, Q_{out}, Q_{side}) \mid^2}{\mid \tilde{S}_{*,c}(0,0,0,0) \mid^2}, \tag{18}$$

where $N_{1,i}(\mathbf{p})$ indicates the number of particles emitted from the halo or from the core for $i = h, c$ and the effective intercept parameter,

$$\lambda_* = \lambda_*(K) = \left[\frac{N_{1,c}(\mathbf{p})}{N_1(\mathbf{p})} \right]^2 \tag{19}$$

is the square of the ratio of the number of particles emitted from the core to the number of all the emitted particles with a given momentum $\mathbf{p}$. This effective intercept parameter arises due to the finite relative momentum resolution and the comparably large region of homogeneity characterizing the halo part of the system.

If the emission function of the core can be factorized,

$$S_{*,c}(\tau, \eta, r_x, r_y) = H_*(\tau)\, G_*(\eta)\, I_*(r_x, r_y) \tag{20}$$

where $H_*(\tau)$ stands for the effective emission function in proper-time, $G_*(\eta)$ stands for the effective emission function in space-time rapidity, and $I_*(r_x, r_y)$ stands for the effective emission function in the transverse directions, then the expression for the BECF can be further simplified as

$$C(\Delta k; K) = 1 + \lambda_* \frac{\mid \tilde{H}_*(Q_\tau) \mid^2 \mid \tilde{G}_*(Q_\eta) \mid^2 \mid \tilde{I}_*(Q_{out}, Q_{side}) \mid^2}{\mid \tilde{H}_*(0) \mid^2 \mid \tilde{G}_*(0) \mid^2 \mid \tilde{I}_*(0,0) \mid^2}. \tag{21}$$

If the $I_*(r_x, r_y)$ function is symmetric for rotations in the (r_x, r_y) plane around its maximum point $r_{x,s}$ then one may introduce $Q_t = \sqrt{Q_{side}^2 + Q_{out}^2}$ to find

$$C(\Delta k; K) = 1 + \lambda_* \frac{\mid \tilde{H}_*(Q_\tau) \mid^2 \mid \tilde{G}_*(Q_\eta) \mid^2 \mid \tilde{I}_*(Q_t) \mid^2}{\mid \tilde{H}_*(0) \mid^2 \mid \tilde{G}_*(0) \mid^2 \mid \tilde{I}_*(0) \mid^2}. \tag{22}$$

Such factorization around the saddle-point happens e.g. for the new class of analytically solvable models discussed in the subsequent part. From the above expression it is clear that for this type of models the dependence of the BECF on the components of the relative momentum can be diagonalized with appropriate choice of the three independent components of the relative momentum. Note that the assumed existence of the Fourier-transformed distribution functions is a weaker condition than the assumption of the analyticity of the Fourier-transformed function, see Appendix for an example. Another example was given e.g. in ref. [13] for a $H(\tau)$ distribution for which $\tilde{H}(Q_\tau)$ is not analytic function at $Q_\tau = 0$ and $\mid H(Q_\tau) \mid^2$ starts with a linear term. In mathematical statistics it is well known that the Fourier-transformed stable distributions are not analytic at $Q = 0$ [14]. On the other hand, there are many physically interesting Gaussian models which correspond to the multivariate second order Taylor expansion of the above general results. The out-longitudinal cross term [9] has been recently discovered also in this context. To study its properties let us apply a Gaussian approximation to the effective distribution functions as

$$H_*(\tau) \propto \exp(-(\tau - \tau_s)^2/(2\Delta\tau_*^2)), \tag{23}$$

$$G_*(\eta) \propto \exp(-(\eta - \eta_s)^2/(2\Delta\eta_*^2), \tag{24}$$

$$I_*(\eta) \propto \exp(-((r_x - r_{x,s})^2 + (r_y - r_{y,s})^2)/(2R_*^2)) \tag{25}$$

Apart from the momentum-dependent parameters $\Delta\tau_*$, $\Delta\eta_*$ and R_* the mean emission point may also be momentum-dependent in the above expression, $\tau_s = \tau_s(K)$, $\eta_s = \eta_s(K)$, $r_{x,s} = r_{x,s}(K)$ and $r_{y,s} = r_{y,s}(K)$. For the sake of simplicity we do not specify the normalization constants in eq. (25) since they cancel from the BECF which is given by

$$C(\Delta k; K) = 1 + \lambda_* \exp(-Q_\tau^2 \Delta\tau_*^2 - Q_\eta^2 \tau_s^2 \Delta\eta_*^2 - Q_t^2 R_*^2). \tag{26}$$

This is a diagonal form of BECF-s for which the factorization property, eq. (20) and the Gaussian approximation for the core, eq. (25) are simultaneously satisfied. In the present form of the BECF, there are no cross-terms among the chosen variables. Now, let us rewrite this form using the standard HBT coordinate system [12] to find

$$C(\Delta k; K) = 1 + \lambda_* \exp(-R_{side}^2 Q_{side}^2 - R_{out}^2 Q_{out}^2 - R_L^2 Q_L^2 - 2R_{out,L}^2 Q_{out} Q_L) \tag{27}$$

$$R_{side}^2 = R_*^2, \tag{28}$$

$$R_{out}^2 = R_*^2 + \delta R_{out}^2, \tag{29}$$

$$\delta R_{out}^2 = \beta_t^2 (\cosh^2[\eta_s] \Delta\tau_*^2 + \sinh^2[\eta_s] \tau_s^2 \Delta\eta_*^2) \tag{30}$$

$$R_L^2 = (\beta_L \sinh[\eta_s] - \cosh[\eta_s])^2 \tau_s^2 \Delta\eta_*^2 + (\beta_L \cosh[\eta_s] - \sinh[\eta_s])^2 \Delta\tau_*^2 \tag{31}$$

$$R_{out,L}^2 = (\beta_t \cosh[\eta_s](\beta_L \cosh[\eta_s] - \sinh[\eta_s]))\Delta\tau_*^2 + (\beta_t \sinh[\eta_s](\beta_L \sinh[\eta_s] - \cosh[\eta_s]))\tau_s^2 \Delta\eta_*^2 \tag{32}$$

This result is valid in any frame. We see that the life-time information $\Delta\tau_*^2$ and the invariant measure of the longitudinal size along the $\tau_s = const$ hyperbola, $\tau_s^2 \Delta\eta_*^2$ appear in a mixed form in the R_{out}^2, R_L^2 and the $R_{out,L}^2$ source parameters. These results simplify a lot in the LCMS system, where $\beta_L = 0$:

$$\delta R_{out}^2 = \beta_t^2 (\cosh^2[\eta_s] \Delta\tau_*^2 + \sinh^2[\eta_s] \tau_s^2 \Delta\eta_*^2) \tag{33}$$

$$R_L^2 = \cosh[\eta_s]^2 \tau_s^2 \Delta\eta_*^2 + \sinh[\eta_s]^2 \Delta\tau_*^2 \tag{34}$$

$$R_{out,L}^2 = -\beta_t \sinh[\eta_s] \cosh[\eta_s] (\Delta\tau_*^2 + \tau_s^2 \Delta\eta_*^2) \tag{35}$$

If we study an expansion in terms of $\epsilon = \mid Y - y_0 \mid / \Delta\eta$, where Y is the rapidity belonging to K the mean momentum of the pair, and $\Delta\eta >> \Delta\eta_*$ is the geometrical size of the expanding system in the space-time rapidity variable, it is obvious that in the LAB frame $\eta_s(LAB) = Y + \mathcal{O}(\epsilon)$, since in the $\epsilon \to 0$ limit we recover boos-invariance and the particle emission must be centered around the only scale: the rapidity of the pair. Similarly we see that $\eta_s(LCMS) = \mathcal{O}(\epsilon)$. Follows that the cross-term and the crossing of temporal and longitudinal information in the LAB frame is a leading order effect,

$$\delta R_{out}^2 = \beta_t^2 (\cosh^2[Y] \Delta\tau_*^2 + \sinh^2[Y] \tau_s^2 \Delta\eta_*^2) \tag{36}$$

$$R_L^2 = \frac{\tau_s^2 \Delta\eta_*^2}{\cosh^2[Y]}, \tag{37}$$

$$R_{out,L}^2 = -\beta_t \frac{\sinh[Y]}{\cosh^2[Y]} \tau_s^2 \Delta\eta_*^2. \tag{38}$$

However, in the LCMS, the mixing of the temporal and longitudinal information is only next-to leading order according to eq. 35, i.e. $R_{out,L}^2(LCMS) = 0 + \mathcal{O}(\epsilon)$. However, if the $\mid Y - y_0 \mid << \Delta\eta$ condition is not satisfied, the out-long cross-term might be large even in LCMS, as has been demonstrated numerically in ref. [15].

Let us define the LSPS, the longitudinal saddle point system, to be the frame where $\eta_s = 0$. Since in a fixed frame $\eta_s = \eta_s(K)$, the LSPS frame may depend on K (e.g. on transverse mass of the pair). In the LSPS frame the out-long cross-term and the mixing of the temporal and time-like informations can be diagonalized. We have in LSPS

$$\delta R_{out}^2 = \beta_t^2 \Delta\tau_*^2, \tag{39}$$

$$R_L^2 = \tau_s^2 \Delta\eta_*^2 + \beta_L^2 \Delta\tau_*^2, \tag{40}$$

$$R_{out,L}^2 = \beta_t \beta_L \Delta\tau_*^2, \tag{41}$$

as follows from eqs. (33-e:lrol). Introducing the new variables $Q_0 = \beta_t Q_{out} + \beta_L Q_L$ and $Q_t = \sqrt{Q_{out}^2 + Q_{side}^2}$ we obtain for the correlation function

$$C(\Delta k; K) = 1 + \lambda_* \exp(-\Delta\tau_*^2 Q_0^2 - \tau_s^2 \Delta\eta_*^2 Q_L^2 - R_*^2 Q_t^2). \tag{42}$$

From this relationship we also see that $Q_0(LSPS) = Q_\tau, Q_L(LSPS) = Q_\eta$, c.f. eq. (26). This relationship clarifies the physical significance of the η_s, the space-time rapidity of the maximum of the emission function in any frame: η_s is the cross-term generating hyperbolic mixing angle for cylindrically symmetric, finite systems undergoing longitudinal expansion and satisfying the factorization property eq. (20). This cross-term generating mixing angle, η_s vanishes exactly in the LSPS frame, becomes a small parameter in the LCMS if $\mid Y - y_0 \mid / \Delta\eta << 1$ and becomes leading order in any frame significantly different from LSPS or

LCMS. Thus we confirm the recent finding of S. Chapman et al, that the out-longitudinal cross-term can be diagonalized away if one finds the (transverse mass dependent) longitudinal rest frame of the source [16].

Up to this point, we have reviewed the properties of BECF-s without reference to any particular model. Let us study the analytic properties of an analytically solvable model-class in the subsequent parts.

A new class of analytically solvable models. For central heavy ion collisions at high energies the beam or z axis becomes a symmetry axis. Since the initial state of the reaction is axially symmetric and the equations of motion do not break this pattern, the final state must be axially symmetric too. However, in order to generate the thermal length-scales in the transverse directions, the flow-field must be either three-dimensional, or the temperature distribution must have significant gradients in the transverse directions. Furthermore, the local temperature may change during the the duration of the particle emission either because of the re-heating of the system caused by the hadronization [17] and/or intensive re-scattering processes or the local temperature may decrease because of the expansion and the emission of the most energetic particles from the interaction region.

We study the following model emission function for high energy heavy ion reactions:

$$S(x,K)\,d^4x = \frac{g}{(2\pi)^3} m_t \cosh(\eta - y)\, \exp\left(- \frac{K \cdot u(x)}{T(x)} + \frac{\mu(x)}{T(x)} \right) H(\tau) d\tau\, \tau_0 d\eta\, dr_x\, dr_y, \quad (43)$$

Here g is the degeneracy factor, $u(x)$ stands for the four-velocity given by

$$u(x) \simeq \left(\cosh(\eta) \left(1 + b^2 \frac{r_x^2 + r_y^2}{2\tau_0^2} \right),\ b\frac{r_x}{\tau_0},\ b\frac{r_y}{\tau_0},\ \sinh(\eta) \left(1 + b^2 \frac{r_x^2 + r_y^2}{2\tau_0^2} \right) \right), \quad (44)$$

the local temperature distribution $T(x)$ at the last interaction point is given by

$$\frac{1}{T(x)} = \frac{1}{T_0} \left(1 + a^2 \frac{r_x^2 + r_y^2}{2\tau_0^2} \right) \left(1 + d^2 \frac{(\tau - \tau_0)^2}{2\tau_0^2} \right), \quad (45)$$

and the local rest density distribution is controlled by the chemical potential $\mu(x)$ for which we have the ansatz

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - \frac{r_x^2 + r_y^2}{2R_G^2} - \frac{(\eta - y_0)^2}{2\Delta\eta^2}. \tag{46}$$

The proper-time distribution of the last interaction points is assumed to have the following simple form:

$$H(\tau) = \frac{1}{(2\pi\Delta\tau^2)^{(1/2)}} \exp(-(\tau - \tau_0)^2/(2\Delta\tau^2))). \tag{47}$$

This emission function corresponds to a Boltzmann approximation to the local momentum distribution of a longitudinally expanding finite system which expands into the transverse directions with a transverse flow which is non-relativistic at the saddle-point. The transverse gradients of the local temperature at the last interaction points are controlled by the parameter a. The strength of the flow is controlled by the parameter b. The parameter $c = 1$ is reserved to denote the speed of light, and the parameter d controls the strength of the change of the local temperature during the course of particle emission.

For the case of $a = b = d = 0$ we recover the case of longitudinally expanding finite systems as presented in ref. [6]. The finite geometrical and temporal length-scales are represented by the transverse geometrical size R_G, the geometrical width of the space-time rapidity distribution $\Delta\eta$ and by the mean duration of the particle emission $\Delta\tau$. Effects arising from the finite longitudinal size were calculated analytically in ref. [19] in certain limited regions of the phase-space. We assume here that the finite geometrical and temporal scales as well as the transverse radius and proper-time dependence of the inverse of the local temperature can be represented by the mean and the variance of the respective variables i.e. we apply a Gaussian approximation, corresponding to the forms listed above, in order to get analytically trackable results. We have first proposed the $a = 0, b = 1$ and $d = 0$ version of the present model, and elaborated also the $a = b = d = 0$ model [6] corresponding to longitudinally expanding finite systems with a constant freeze-out temperature and no transverse flow. Soon the parameter b has been introduced [9] and it has been realized that the transverse flow has to be non-relativistic at the saddle-point corresponding to the maximum of the emission function. Yu. Sinyukov and collaborators classified the various classes of the ultra-relativistic transverse flows [8], [18], and introduced a parameter which controls the transverse temperature profile, corresponding to the $a \neq b = 0$ case. We have studied [20] the model-class $a \neq 0$, $b \neq 0$, $d = 0$ which we extend here to the $d \neq 0$ case too.

The integrals of the emission function are evaluated using the saddle-point method [4,7,9]. The saddle-point coincides with the maximum of the emission function, parameterized by $(\tau_s, \eta_s, r_{x,s}, r_{y,s})$. These coordinate values solve simultaneously the equations

$$\frac{\partial S}{\partial \tau} = \frac{\partial S}{\partial \eta} = \frac{\partial S}{\partial r_x} = \frac{\partial S}{\partial r_y} = 0 \tag{48}$$

These saddle-point equations are solved in the LCMS, the longitudinally comoving system, for $\eta_s(LCMS) << 1$ and $r_{x,s} << \tau_0$. The approximations are self-consistent if $\mid Y - y_0 \mid << 1 + \Delta\eta^2 m_t/T_0$ and $\beta_t = p_t/m_t << (a^2 + b^2)/b$. The transverse flow is non-relativistic at the saddle-point if $\beta_t << (a^2 + b^2)/b^2$. We assume that $\Delta\tau < \tau_0$ so that the Fourier-integrals involving $H(\tau)$ in the $0 \leq \tau < \infty$ domain can be extended to the $-\infty < \tau < \infty$ domain. The radius parameters are evaluated here up the leading order in $r_{x,s}/\tau_0$. Thus terms of $\mathcal{O}(r_{x,s}/\tau_0)$ are neglected, however we keep all the higher-order correction terms arising from the non-vanishing value of η_s in the LCMS.

We find that the saddle point approximation for the integrals leads to an effective emission function which can be factorized similarly to eq. (20), and the radius parameters are just expressible in terms of the homogeneity lengths $\Delta\eta_*, R_*$ and $\Delta\tau_*$, and the position of the saddle point η_s which is in turn the cross-term generating hyperbolic mixing angle. The saddle-point in LCMS is given by $\tau_s = \tau_0, \eta_s = (y_0 - Y)/(1 + \Delta\eta^2(1/\Delta\eta_T^2 - 1))$, $r_{x,s} = \beta_t b R_*^2/(\tau_0 \Delta\eta_T^2)$ and $r_{y,s} = 0$. The radius parameters or lengths of homogeneity are given in the LCMS by eqs. (27-29,33-35), and we obtain

$$\frac{1}{R_*^2} = \frac{1}{R_G^2} + \frac{1}{R_T^2} \cosh[\eta_s] \tag{49}$$

$$\frac{1}{\Delta\eta_*^2} = \frac{1}{\Delta\eta^2} + \frac{1}{\Delta\eta_T^2}\cosh[\eta_s] - \frac{1}{\cosh^2[\eta_s]}, \tag{50}$$

$$\frac{1}{\Delta\tau_*^2} = \frac{1}{\Delta\tau^2} + \frac{1}{\Delta\tau_T^2}\cosh^2[\eta_s]. \tag{51}$$

where the thermal length-scales are given by

$$R_T^2 = \frac{\tau_0^2}{a^2+b^2}\frac{T_0}{M_t}, \tag{52}$$

$$\Delta\eta_T^2 = \frac{T_0}{M_t}, \tag{53}$$

$$\Delta\tau_T^2 = \frac{\tau_0^2}{d^2}\frac{T_0}{M_t}. \tag{54}$$

Here $M_t = \sqrt{K_0^2 - K_L^2}$ is the transverse mass belonging to the mean momentum K. In the region of the Bose-Einstein enhancement, where the relative momentum of the pair is small, M_t satisfies $M_t = \frac{1}{2}(m_{t,1}+m_{t,2})(1+\mathcal{O}(y_1-y_2)+\mathcal{O}((m_{t,1}-m_{t,2})/(m_{t,1}+m_{t,2})))$.

The parameters of the BECF-s are dominated by the smaller of the geometrical and the thermal scales not only in the spatial directions but in the temporal direction too according to eqs. (49-54). These analytic expressions indicate that the BECF views only a part of the space-time volume of the expanding systems, which implies that *even a complete measurement of the parameters of the BECF as a function of the mean momentum K may not be sufficient to determine uniquely the underlying phase-space distribution.* We also can see that the LCMS frame approximately coincides with the LSPS frame for pairs with $\mid y_0 - Y \mid << 1 + \Delta\eta^2 M_t/T_0$ and the terms arising from the non-vanishing values of η_s can be neglected. In this approximation, the cross-term generating hyperbolic mixing angle $\eta_s \approx 0$ thus we find the leading order LCMS result:

$$C(\Delta k; K) = 1 + \lambda_* \exp(-R_L^2 Q_L^2 - R_{side}^2 Q_{side}^2 - R_{out}^2 Q_{out}^2), \tag{55}$$

with a vanishing out-long cross-term, $R_{out,L} = 0$. To leading order, the parameters of the correlation function are given by

$$R_{side}^2 = R_*^2, \tag{56}$$

$$R_{out}^2 = R_*^2 + \beta_T^2 \Delta\tau_*^2, \tag{57}$$

$$R_L^2 = \tau_0^2 \Delta\eta_*^2 \tag{58}$$

Observe that the difference of the side and the out radius parameters is dominated by the lifetime-*parameter* $\Delta\tau_*$. Thus a vanishing difference between the R_{out}^2 and R_{side}^2 can be generated dynamically in the case when the duration of the particle emission is large, but the thermal duration $\Delta\tau_T$ becomes sufficiently small. This in turn can be associated with intensive changes in the local temperature distribution during the course of the particle emission.

Please note, that the BECF in an arbitrary frame can be obtained from combining eqs. (49-54) with the general expressions given by eqs. (27 - 32). In these equations, the value of $\eta_s = Y + \eta_s^{LCMS} = Y + (y_0 - Y)/(1 + \Delta\eta^2(1/\Delta\eta_T^2 - 1))$ has to be used. Note that in our results higher order terms arising from the non-vanishing value of η_s in the LCMS are summed up, while in refs. [9] the first sub-leading corrections were found.

Invariant momentum distributions The IMD plays a *complementary role* to the measured Bose-Einstein correlation function [10,6,20]. Namely, the width of the rapidity distribution at a given m_t, $\Delta y(m_t)$ as well as T_*, the effective temperature at a mid-rapidity y_0 shall be dominated by the *longer* of the thermal and geometrical length-scales. Thus a *simultaneous analysis* of the Bose-Einstein correlation functions and the IMD may reveal information both on the temperature and flow profiles and on the geometrical sizes.

E.g. the following relations hold:

$$\Delta y(m_t)^2 = \Delta\eta^2 + \Delta\eta_T^2(m_t), \qquad \text{and} \qquad \frac{1}{T_*} = \frac{f}{T_0 + T_G(m_t = m)} + \frac{1-f}{T_0}, \tag{59}$$

where the geometrical contribution to the effective temperature is given by $T_G = T_0\, R_G^2/R_T^2$ and the fraction f is defined as $f = b^2/(a^2 + b^2)$, satisfying $0 \leq f \leq 1$.

For the considered model, the invariant momentum distribution can be calculated as

$$N_{1,c}(\mathbf{p}) = \frac{g}{(2\pi)^3} \exp\left(\frac{\mu_0}{T_0}\right) m_t \, (2\pi\Delta\eta_*^2\tau_0^2)^{1/2} \, (2\pi R_*^2) \, \frac{\Delta\tau_*}{\Delta\tau} \cosh(\eta_s) \, \exp(+\Delta\eta_*^2/2) \times$$
$$\times \exp\left(-\frac{(y-y_0)^2}{2(\Delta\eta^2 + \Delta\eta_T^2)}\right) \exp\left(-\frac{m_t}{T_0}\left(1 - f\frac{\beta_t^2}{2}\right)\right) \exp\left(-f \frac{m_t\beta_t^2}{2(T_0+T_G)}\right). \quad (60)$$

This IMD has a rich structure: it features both a rapidity-independent and a rapidity-dependent low-pt enhancement as well as a high-pt enhancement or decrease. The measured IMD can be obtained from the IMD of the core as given above and from the measured $\lambda_*(\mathbf{p})$ parameters as

$$N_1(\mathbf{p}) = \frac{1}{\sqrt{\lambda_*(\mathbf{p})}} N_{1,c}(\mathbf{p}), \quad (61)$$

as has been presented e.g. in ref. ([23]).

The invariant momentum distribution described by eq. 60 features two types of low transverse momentum enhancement. The *rapidity-independent low-pt enhancement* is a consequence of the transverse mass dependence of the effective volume, which particles with a given momentum are emitted from. We may introduce the *volume factor* or $V_*(y, m_t)$ which yields the momentum-dependent size of the region, from where the particles with a given momentum are emitted:

$$V_*(y, m_t) = (2\pi\Delta\eta_*^2\tau_0^2)^{1/2}(2\pi R_*^2)\frac{\Delta\tau_*}{\Delta\tau}. \quad (62)$$

This effective volume may depend on m_t for certain limiting cases in the following ways

$$V_*(y, m_t) \propto \left(\frac{T_0}{m_t}\right)^{k/2} \quad (63)$$

where $k = 0$ for a static fireball ($a = b = d = 0$ and $\Delta\eta_T^2 >> \Delta\eta^2$), the case $k = 1$ is satisfied for $a = b = d = 0$ and $\Delta\eta >> \Delta\eta_T^2$ see e.g. [6], the case $k = 2$ corresponds to $a = b = 0 \neq d$

and $\Delta\eta >> \Delta\eta_T^2$, describing longitudinally expanding systems with cooling, the case $k = 3$ corresponds to $a \neq 0$, $b \neq 0 = d$ i.e. three-dimensionally expanding, cylindrically symmetric, finite systems with transverse temperature profile, [20] and the $k = 4$ case corresponds to the same appended with a $d \neq 0$ parameter describing the temporal changes in the local temperature during the particle emission process appended with the condition $\Delta\tau_T << \Delta\tau$. Thus the inclusion of this effective m_t dependent volume factor into the data analysis not only would undoubtedly increase the precision of the measurements of the slope parameters, but in turn it also could shed light on the dynamics of the particle emission from such complex systems.

The *rapidity-dependent low-pt enhancement*, which is a generic property of the longitudinally expanding finite systems [24], reveals itself in the rapidity-dependence of the effective temperature, defined as the slope of the exponential factors in the IMD in the low-pt limit at a given value of the rapidity. The leading order [24] result is

$$T_{eff}(y) = \frac{T_*}{1 + a(y - y_0)^2} \qquad \text{with} \qquad a = \frac{T_0 T_*}{2m^2}\left(\Delta\eta^2 + \frac{T_0}{m}\right)^{-2}. \tag{64}$$

Please note that the derivation of this effect relies on the assumption $m_t >> T_0$ in the low transverse momentum region too. Thus it may be valid for kaons or heavier particles as well as locally very cold pionic systems. However, in case of pions, the self-consistency of the applied formulas and their region of validity, $m_t >> T_0$ has to be very carefully checked. Note also that the low transverse momentum region is populated by a number of resonance decays. For the long-lived resonances, thus a non-trivial $1/\sqrt{\lambda_*(\mathbf{p})}$ factor may appear and contribute to both the rapidity-dependent and the rapidity-independent low-p_t enhancement. Although this factor is measurable from the shape-analysis of the BECF, care is required to extract the contribution of the decay products of short lived resonances to the momentum distribution.

The *high-pt enhancement or decrease* refers to the change of the effective temperature at mid-rapidity with increasing m_t . The large transverse mass limit T_∞ shall be in general different from the effective temperature at low pt given by T_* since

$$T_\infty = \frac{2T_0}{2-f} \qquad \text{and} \qquad \frac{T_\infty}{T_*} = \frac{2}{2-f}\left(1 - f\frac{T_G(m)}{T_0 + T_G(m)}\right). \tag{65}$$

Utilizing $T_G/T_0 = R_G^2/R_T^2$, the high-pt enhancement or decrease turns out to be controlled by the ratio of the thermal radius $R_T(m_t = m)$ to the geometrical radius R_G. One obtains $T_\infty > T_*$ if $R_T^2(m) > R_G^2$ and similarly $T_\infty < T_*$ if $R_T^2(m) < R_G^2$. Since for large colliding nuclei R_G is expected to increase, a possible high-pt decrease in these reactions may become a geometrical effect, a consequence of the large size.

Limiting cases

Observe that both the thermal and the geometrical lenght-scales enter both the parameters of the Bose-Einstein correlation function and those of the invariant momentum distribution. Various limiting cases can be obtained as combinations of basically the relative size of the thermal and the geometrical scales in the transverse, longitudinal and temporal directions. These in turn are:

i) If $R_T(M_t) >> R_G$ in a certain M_t interval, we have also $T >> T_G(m_t)$ at the same transverse mass scale. In this region, the side radius parameter shall be determined by the geometrical size $R_{side} = R_* \approx R_G$, and this parameter becomes transverse mass independent.

The m_t distribution at mid-rapidity shall be proportional to $\exp(-m_t/T_0)$.

ii) If $\Delta\eta_T >> \Delta\eta$, we have $R_L = \tau_0 \Delta\eta$, and the rapidity-width of the IMD shall be dominated by the thermal scale, $\Delta^2 y(m_t) = \Delta\eta_T = T/m_t$.

iii) If $\Delta\tau_T >> \Delta\tau$, the temporal duration shall be measured by $R_{out}^2 - R_{side}^2 = \beta_t^2 \Delta\tau^2$. The invariant momentum distribution shall be influenced only through the $\Delta\tau_*/\Delta\tau \approx 1$ factor in V_*.

These cases are rather conventional limiting cases. An unconventional limit complements each:

iv) If $R_T(M_t) << R_G$ in a certain M_t interval, we have also $T << T_G(m_t)$ at the same transverse mass scale. In this region, the side radius parameter shall be determined by the thermal size $R_{side} = R_* \approx R_T(M_t)$, and this parameter becomes transverse mass dependent, $R_{side}^2 \propto \tau_0^2 T_0/M_t$.

The m_t distribution at mid-rapidity shall be proportional to $\exp(-m_t/T_*)$. If $a^2 << b^2$, we have $T_* \approx T_G(m)$ as follows from eq. (59).

v) If $\Delta\eta_T << \Delta\eta$, we have the leading order LCMS result $R_L = \tau_0 \Delta\eta_T = \tau_0^2 T/M_t$, and the rapidity-width of the IMD shall be dominated by the geometrical scale, $\Delta^2 y \approx \Delta^2\eta$.

vi) If $\Delta\tau_T << \Delta\tau$, the *thermal* duration shall be measured by $R_{out}^2 - R_{side}^2 = \beta_t^2 \Delta\tau_T^2 \approx \beta_t^2 \tau_0^2 T_0/(d^2 M_t)$. For large values of the transverse mass, the model thus shall feature a dynamically generated vanishing duration parameter, which has a specific transverse mass dependence. The invariant momentum distribution shall be influenced only through the $\Delta\tau_*/\Delta\tau \approx 1/\sqrt{m_t}$ factor in V_*.

Some combinations of cases *i) – vi),* are especially interesting, as:

vii) If all the finite geometrical source sizes, $R_G, \Delta\eta$ and $\Delta\tau$ are large compared to the corresponding thermal length-scales we have

$$\Delta\tau_*^2 = \Delta\tau_T^2 \frac{1}{1 + \frac{\Delta\tau_T^2}{\Delta\tau^2}} \approx \frac{\tau_0^2}{d^2} \frac{T_0}{M_t} \tag{66}$$

$$R_L^2 \approx \tau_0^2 \frac{T_0}{M_t} \tag{67}$$

$$R_{side}^2 \approx \frac{\tau_0^2}{a^2 + b^2} \frac{T_0}{M_t} \tag{68}$$

Thus if $d^2 >> a^2 + b^2 \approx 1$ the model features *a dynamically generated vanishing difference* between the side and out radii.

If the vanishing duration parameter is generated dynamically, the model *predicts an M_t - scaling for the duration parameter* as

$$\Delta\tau_*^2 = \frac{R_{out}^2 - R_{side}^2}{\beta_t^2} \simeq \Delta\tau_T^2 \propto \frac{1}{M_t}, \tag{69}$$

Note that this prediction could be checked experimentally if the error bars of the measured radius parameters were decreased to such a level that the difference between the out and the side radius parameters would be significant.

Alternatively, if the vanishing duration parameter of the BECF is generated due to a very fast hadronization process as discussed in ref. [21], then one has

$$\Delta\tau_*^2 \simeq \Delta\tau^2 \propto const, \tag{70}$$

i.e. in this case the duration parameter becomes independent of the transverse mass.

If the finite source sizes are large compared to the thermal length-scales and if we also have $a^2 + b^2 \approx 1$, one obtains an M_t *-scaling* for the parameters of the BECF,

$$R_{side}^2 \simeq R_{out}^2 \simeq R_L^2 \simeq \tau_0^2 \frac{T_0}{M_t}, \quad \text{valid for} \quad \beta_t << \frac{(a^2 + b^2)}{b^2} \simeq \frac{1}{b^2}. \tag{71}$$

Note that this relation is independent of the particle type and has been seen in the recent NA44 data [22]. This M_t-scaling may be valid to arbitrarily large transverse masses with $\beta_t \approx 1$ if $b^2 << 1$. Thus, to generate vanishing difference between the side and out radius and M_t-scaling simultaneously, the parameters have to satisfy $b^2 << a^2 + b^2 \approx 1 << d^2$, i.e. the fastest process is the cooling, the next dominant process within this phenomenological picture has to be the development of the transverse temperature profile and finally the transverse flow shall be relatively weak.

We would like to emphasize that there are a number of conditions in the model which need to be satisfied simultaneously to get the scaling behavior, which is supported by 9 high-precision NA44 data-points (3 for kaons and 6 for pions). One has to wait for future data points to learn more about the experimental status of the scaling. The model presented in this paper may describe more complex transverse momentum dependences of the parameters of the Bose-Einstein correlation function, too, the M_t-scaling is only one of its virtues in a specific limiting case. However, it is rather difficult to get a limiting case with $R_L \approx R_{side} \approx R_{out} \propto 1/\sqrt{M_t}$ in analytically solvable models. Such a behavior is related to the cylindrical symmetry of the emission function. In the saddle-point approximation this means that the saddle-point sits basically at $r_t = 0$ with next to leading order corrections, in the transverse momentum region where the scaling was observed.

Thus the symmetry of the BECF in LCMS is a strong indication for a three-dimensionally expanding source, possibly with a temperature profile. The LCMS frame is selected if the mean emission point or saddle-point is located not too far from the symmetry axis even for particles with a large transverse mass, $r_{x,s}(m_t) << \tau_0$ and if the finite longitudinal size introduces only small difference between the LCMS and LSPS frames, i.e. $\mid y - y_0 \mid << 1 + \Delta\eta^2(m_t/T - 1)$. In the considered case the emission function is cylindrically symmetric and so the BECF is symmetric in the LCMS of the pair (and *not* in the center of mass system of the pair). This picture is further supported by the similar M_t dependence of the side, out and longitudinal components [25].

vii) It is interesting to investigate the other limiting case when $R_T >> R_G, \Delta\eta_T >> \Delta\eta$ and $\Delta\tau_T >> \Delta\tau$. In this case we obtain

$$R_L^2 = \tau_0^2 \Delta\eta^2 = R_{L,G}^2, \qquad R_{side}^2 = R_G^2, \qquad R_{out}^2 = R_G^2 + \beta_T^2 \Delta\tau^2, \tag{72}$$

$$\Delta y^2 = \Delta\eta_T^2 = \frac{T_0}{m_t}, \qquad T_* = T_0. \tag{73}$$

Thus, if the thermal length scales are larger than the geometrical sizes in all directions, the BECF measurement determines the geometrical sizes properly, and the p_t and the dn/dy distribution will be determined by the temperature of the source. In this case the momentum distribution will be given by

$$N_1(\mathbf{p}) \propto \exp\left(-\frac{m_t}{T_0} - \frac{\mathrm{y}^2 m_t}{2T_0}\right) \approx \exp\left(-\frac{E}{T_0}\right), \tag{74}$$

which is just a thermal distribution for a static source.

Thus two length-scales are present in all the three principal directions of three-dimensionally expanding systems. The BECF radius parameters are dominated by the *shorter* of the thermal and geometrical length scales. However, the rapidity-width of the $d^2N/dy/dm_t^2$ distribution, $\Delta^2 y(m_t)$ is the quadratic sum of the geometrical and the thermal length scales, thus it is dominated by the *longer* of the two. Similarly, the effective temperature is dominated by the *higher* of the two temperature scales for $f \approx 1$ according to eq. (59). The effective temperature of the m_t distribution is decreasing in the target and projectile rapidity region.

In summary, we have presented model-independent formulation for the two-particle Bose-Einstein correlation function for cylindrically symmetric systems undergoing collective hydrodynamical expansion. We have expressed these in the LAB, LCMS and LSPS systems, where the functional form of the BECF becomes more and more simplified. We have identified the cross-term generating hyperbolic mixing angle with the value of the η variable of the saddle point in the considered frame.

We have also introduced a class of Gaussian models which in some regions of the model-parameters may obey an *M_t-scaling for the side, out and longitudinal radius parameters.* Vanishing effective duration of the particle emission may be generated by the temporal changes of the local temperature during the evaporation. The model predicts an *M_t-scaling also for the duration parameter* in this limiting case.

Finally we stress that both the invariant momentum distribution and the Bose-Einstein correlation function may carry only partial information about the phase-space distribution of particle emission. However, their simultaneous analysis may shed more light on the dynamics and may reveal e.g. large hidden geometrical source-sizes.

Acknowledgments: Cs. T. would like to thank M. Asakawa, G. Gustafson, M. Gyulassy, U. Heinz, S. Hegyi, Y. Pang and Yu. Sinyukov for stimulating discussions, B. Lörstad and M. Gyulassy for hospitality at Lund and Columbia University. This work was supported by the Human Capital and Mobility (COST) programme of the EEC under grant No. CIPA - CT - 92 - 0418 (DG 12 HSMU), by the Hungarian NSF under Grants No. OTKA-F4019, OTKA-

T2973 and OTKA-W01015107 and by the Hungarian - US Joint Fund MAKA 378/93.

I. APPENDIX

In this appendix we give a simple example when the Fourier-transformed emission function exists but the saddle-point method is not applicable. Let us consider the one-dimensional Lorentzian distribution function

$$S(r) = \frac{1}{\pi R} \, \frac{1}{(1 + r^2/R^2)}. \tag{75}$$

Here r is a real variable (in one dimension). The corresponding correlation function is

$$C(q) = 1+ \mid \tilde{S}(q) \mid^2, \tag{76}$$

with

$$\tilde{S}(q) = \int_{-\infty}^{\infty} dr \, S(r) \exp(-iqr), \tag{77}$$

which yields

$$C(q) = 1 + \exp(-2 \mid q \mid R). \tag{78}$$

This function is not analytic at $q = 0$ because it depends on the absolute value of q, and for positive values of q its Taylor expansion starts with a linear term. This is to be contrasted with the results for the saddle-point method. If the saddle-point method is applicable, then $\tilde{S}(q)$ can be expanded into a Taylor series around $q = 0$ as

$$\tilde{S}(q) = 1 + i\langle r \rangle q - \langle r^2 \rangle q^2/2 + ... \tag{79}$$

Here the average of a function of variable r is defined as

$$\langle f(r) \rangle = \int_{-\infty}^{\infty} dr \, f(r) S(r) \tag{80}$$

and the two-particle correlation function can be written as

$$C(q) \approx 2 - q^2 R_G^2 \approx 1 + \exp(-q^2 R_G^2) \tag{81}$$

with

$$R_G^2 = \langle r^2 \rangle - \langle r \rangle^2. \tag{82}$$

Since for the considered function $R_G^2 = \infty$, the saddle–point method is not applicable. Still, the Fourier-transformed emission function and the BECF exist, as given by eq. (78).

Cases similar to this are characterized by non-Gaussian correlation functions [14]. Similar examples can be found among multi-variate distributions, too.

REFERENCES

[1] R. Hanbury-Brown and R. Q. Twiss, Phyl. Mag. **45** 663 (1954); R. Hanbury-Brown and R. Q. Twiss, Nature (London) **177**, (1956) 27. and **178**, (1956) 1046.

[2] Proceedings of the Quark Matter conferences, especially Nucl. Phys. **A498**, (1989), Nucl. Phys. **A525**, (1991), Nucl. Phys. **A544**, (1992) and Nucl. Phys. **A566**, (1993).

[3] B. Lörstad, Int. J. Mod. Phys. **A12** (1989) 2861-2896; W. A. Zajc, in NATO ASI Series B303, p. 435. (Plenum Press, 1993, ed. by H. Gutbrod and J. Rafelski).

[4] A. Makhlin and Y. Sinyukov, Z. Phys. **C39**, 69 (1988).

[5] T. Csörgő and S. Pratt, **KFKI-1991-28/A**, p. 75; S. Pratt, Phys. Rev. **D 33**, 1314 (1986).

[6] T. Csörgő, Phys. Lett. **B347** (1995) 354-360

[7] Yu. Sinyukov, Nucl. Phys. A566 (1994) 589c-592c.

[8] Yu. Sinyukov, in Proc. HHM:TE (Divonne, June 1994, Plenum Press, J. Rafelski et al, eds.)

[9] S. Chapman, P. Scotto and U. Heinz, Regensburg preprint TPR-94-29, hep-ph/9409349; S. Chapman, P. Scotto and U. Heinz, hep-ph/9408207, Heavy ion Physics **1** (1995) 1.

[10] T. Csörgő, B. Lörstad and J. Zimányi, Phys. Lett. **B338** (1994) 134-140

[11] S. Pratt, T. Csörgő, J. Zimányi, Phys. Rev. **C42** (1990) 2646

[12] G. F. Bertsch, Nucl. Phys. **A498** (1989) 173c

[13] T. Csörgő and J. Zimányi, Nucl. Phys. A517 (1990) 588-598

[14] E. Lukacs, *Characteristic Functions*, (Charles Griffin and Co. Ltd, London, 1964); E. Lukacs and R. G. Laha, *Applications of Characteristic Functions*, (Charles Griffin and Co. Ltd, London, 1964)

[15] S. Chapman, P. Scotto and U. Heinz, Nucl. Phys. **A590** (1995) 449c

[16] S. Chapman, J. R. Nix and U. Heinz, nucl-th/9505032

[17] L. P. Csernai, J. Kapusta, Gy. Kluge and E. E. Zabrodin, Z. Phys. **C58** (1993) 453-460

[18] S. V. Akkelin and Yu. M. Sinyukov, Preprint ITP-63-94E

[19] V. A. Averchenkov, A. N. Makhlin and Yu. M. Sinyukov, Yad. Phys. **46** (1987) 1525-1234; Sov. J. Nucl. Phys. **46** (1987) p. 905

[20] T. Csörgő and B. Lörstad, CU-TP-684, LUNFD6/(NFFL-7106) 1995, hep-ph/9503494, Nucl. Phys. **A590** (1995) 465c

[21] T. Csörgő and L. P. Csernai, Phys. Lett. **B333** (1994) 494-499

[22] H. Beker et al, NA44 Collaboration, CERN-PPE/94-119, Phys. Rev. Lett. in press

[23] T. Csörgő, B. Lörstad and J. Zimányi, LUNFD6-NFFL-7088, hep-ph/9411307

[24] T. Csörgő, LUNFD6-NFFL-7092-1994, hep-ph/9412323

[25] B. Lörstad, NA44 Collaboration, Lund university preprint LUNFD6/(NFFL-7087) 1994, submitted to the 27-th Int. Conf. on High Energy Physics, Glasgow, July 20-27 1994.