MICRO–UNIVERSES AND "STRONG BLACK–HOLES": A PURELY GEOMETRIC APPROACH TO ELEMENTARY PARTICLES

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ABSTRACT

We present here a panoramic view of our unified, *bi–scale* theory of gravitational and strong interactions [which is mathematically analogous to the last version of N.Rosen's bi–metric theory; and yields physical results similar to *strong gravity*'s]. This theory, developed during the last 15 years, is purely geometrical in nature, adopting the methods of General Relativity for the description of hadron structure and strong interactions. In particular, hadrons are associated with "*strong* black–holes", from the external point of view, and with "micro–universes", from the internal point of view. Among the results herein presented, let us mention the derivation: (i) of confinement and (ii) asymptotic freedom for the hadron constituents; (iii) of the Yukawa behaviour for the strong potential at the static limit; (iv) of the strong coupling "constant", and (v) of mesonic mass spectra.

1. Premise

Probably each of us, at least when young, has sometimes imagined that every small particle of matter could be, at a suitably reduced scale, a whole cosmos. This idea has very ancient origins. It is already present, for example, in some works by Democritus of Abdera (about 400 B.C.). Democritus, simply inverting that analogy, spoke about huge atoms, as big as our cosmos. And, to be clearer, he added: if one of those super-atoms (which build up super-cosmoses) abandoned his "giant universe" to fall down on our world, our world would be destroyed...

Such kind of considerations are linked to the fantasies about the physical effects of a dilation or contraction of all the objects which surround us, or of the whole "world". Fantasies like these have also been exploited by several writers: from F. Rabelais (1565) to J. Swift, the narrator of Samuel Gulliver's travels (1727); or to

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I. Asimov. It is probably because of the great diffusion of such ideas that, when the planetary model of the atom was proposed, it achieved a great success among people.

Actually, we meet such intuitive ideas in the scientific arena too. Apart from the already quoted Democritus, let us remember the old conception of a *hierarchy* of universes —or rather of cosmoses— each of them endowed with a particular *scale factor* (let us think, for instance, of a series of russian dolls). Nowadays, we can really recognize that the microscopic analysis of matter has revealed grosso modo a series of "chinese boxes": so that we are entitled to suppose that something similar may be met also when studying the universe on a large scale, *i.e.*, in the direction of the *macro* besides of the *micro*. Hierarchical theories were formulated for example by J.H. Lambert (1761) and, later on, by V.L. Charlier (1908, 1922) and F. Selety (1922–24); followed more recently by O. Klein, H. Alfvén and G. de Vaucouleurs, up to the works of A. Salam and co-workers, K.P. Sinha and C. Sivaram, M.A. Markov, E. Recami and colleagues, D.D. Ivanenko and collaborators, M. Sachs, J.E. Charon, H. Treder, P. Roman, R.L. Oldershaw, and others.[1]

2. Introduction

In this paper we confine ourselves to examine the possibility of considering elementary particles as micro universes:[2] that is to say, the possibility that they be similar —in a sense to be specified— to our cosmos. More precisely, we shall refer ourselves to the thread followed by P. Caldirola, P. Castorina, A. Italiano, G.D. Maccarrone, M. Pavsic, and ourselves.[3]

Let us recall that Riemann, as well as Clifford and later Einstein,[4] believed that the fundamental particles of matter were the perceptible evidence of a strong local space curvature. A theory which stresses the role of space (or, rather, space-time) curvature already does exist for our whole cosmos: General Relativity, based on Einstein gravitational field equations; which are probably the most important equations of classical physical theories, together with Maxwell's electromagnetic field equations. Whilst much effort has already been made to generalize Maxwell equations, passing for example from the electromagnetic field to Yang–Mills fields (so that almost all modern gauge theories are modelled on Maxwell equations), on the contrary Einstein equations have never been applied to domains different from the gravitational one. Even if they, as any differential equations, *do not* contain any inbuilt fundamental length: so that they can be used a priori to describe cosmoses of any size.

Our first purpose is now to explore how far it is possible to apply successfully the methods of general relativity (GR), besides to the world of gravitational interactions, also to the domain of the so–called nuclear, or *strong*, interactions:[5] namely, to the world of the elementary particles called hadrons. A second purpose is linked to the fact that the standard theory (QCD) of strong interactions has not yet fully explained why the hadron constituents (quarks) seem to be permanently *confined* in the interior of those particles; in the sense that nobody has seen up to now an isolated

"free" quark, outside a hadron. So that, to explain that confinement, it has been necessary to invoke phenomenological models, such us the so–called "bag" models, in their MIT and SLAC versions for instance. The "confinement" could be explained, on the contrary, in a natural way and on the basis of a well–grounded theory like GR, if we associated with each hadron (proton, neutron, pion,...) a particular "cosmological model".

3.   The Model by Micro-Universes

Let us now try to justify the idea of considering the strong interacting particles (that is to say, hadrons) as micro-universes. We meet a first motivation if we think of the so–called "large number coincidences", already known since several decades and stressed by H. Weyl, A.I. Eddington, O. Klein, P. Jordan, P.A.M. Dirac, and by others.

The most famous among those empirical observations is that the ratio R/r between the radius $R \simeq 10^{26}$m of our cosmos (gravitational universe) and the typical radius $r \simeq 10^{-15}$m of elementary particles is grosso modo equal to the ratio S/s between the strength S of the nuclear ("strong") field and the strength s of the gravitational field (we will give later a definition of S, s):

$$\rho \equiv \frac{R}{r} \simeq \frac{S}{s}. \tag{1}$$

This does immediately suggest the existence of a *similarity*, in a geometrico–physical sense, between cosmos and hadrons. As a consequence of such similarity, the "theory of models" yields —by exploiting simple dimensional considerations— that, if we contract our cosmos of the quantity

$$\rho = R/r \approx 10^{41}$$

(that is to say, if we transform it in a hadronic micro-cosmos *similar* to the previous one), the field strength would increase in the same ratio: so to get the gravitational field transformed into the strong one.

If we observe, in addition, that the typical duration of a decay is inversely proportional to the strength of the interaction itself, we are also able to explain why the mean-life of our gravitational cosmos ($\Delta t \simeq 10^{18}$ s: duration —for example— of a complete expansion/contraction cycle, if we accept the theory of the cyclic *big bang*) is a multiple, with the same ratio, of the typical mean-life ($\Delta \tau \simeq 10^{-23}$s) of the "strong micro-universes", or hadrons:

$$\Delta t \simeq \rho \Delta \tau. \tag{2}$$

It is also interesting that, from the self-consistency of these deductions implies —as we shall show later— that the mass M of our cosmos should be equal to $\rho^2 \simeq$

$(10^{41})^2$ times the typical mass m of a hadron: a fact that seems to agree with reality, and constitutes a further "numerical coincidence", the so–called Eddington relation. Another numerical coincidence is shown and explained in ref.[6]

By making use of Mandelbrot's language[7] and of his general equation for self-similar structures, what precedes can be mathematically translated into the claim that cosmos and hadrons are systems, with scales N and $N-1$, respectively, whose "fractal dimension" is $D = 2$, where D is the auto-similarity exponent that characterizes the hierarchy. As a consequence of all that, we shall assume that cosmos and hadrons (both of them regarded of course as finite objects) be *similar* systems: that is, that they be governed by similar laws, differing only for a "global" scale transformation which transforms R into r and gravitational field into strong field. [To fix our ideas, we may *temporarily* adopt the naïve model of a "newtonian ball" in three–dimensional space for both cosmos and hadrons. Later on, we shall adopt more sensible models, for example Friedmann's]. Let us add, incidentally, that we should be ready *a priori* to accept the existence of other cosmoses besides ours: let us recall that man in every epoch has successively called *"universe"* his valley, the whole Earth, the solar system, the Milky Way and today (but with the same simple–mindedness) our cosmos, as we know it on the basis of our observational and theoretical instruments. . .[8]

Thus, we arrive at a *second* motivation for our theoretical approach: That physical laws should be covariant (= form invariant) under *global* dilations or contractions of space-time. We can easily realize this if we notice that: (i) when we dilate (or contract) our measure units of space and time, physical laws, of course, should *not* change their form; (ii) a dilation of the measure units is totally equivalent to a contraction (leaving now "meter" and "second" unaltered) of the observed world.

Actually, Maxwell equations of electromagnetism —the most important equations of classical physics, together with Einstein equations, as we already said— are by themselves covariant also under conformal transformations and, in particular, under dilations. In the case when electric charges are present, such a covariance holds provided that charges themselves are suitably "scaled".

Analogously, also Einstein *gravitational* equations are covariant[9] under dilations: provided that, again, when in the presence of matter and of a cosmological term Λ, they too are scaled according to correct dimensional considerations. The importance of this fact had been well realized by Einstein himself, who two weeks before his death wrote, in connection with his last unified theory: <<From the form of the field [*gravitational + electromagnetic*] equations it follows immediately that: if $g_{ik}(x)$ is a solution of the field equations, then also $g_{ik}(x/\alpha)$, where α is a positive constant, is a solution ("similar solutions"). Let us suppose, for example, that g_{ik} represents a finite crystal embedded in a flat space. It is then possible that a second 'universe' exists with another crystal, identical with the first one, but dilated α times with respect to the former. As far as we confine ourselves to consider a universe containing only one crystal, there are no difficulties: we just realize that the size of such a crystal (standard of length) is not determined by the field equations. . .>>.

These lines are taken from Einstein's preface to the Italian book *Cinquant'anni di Relatività*.[10] They have been written in Princeton on April 4th, 1955, and stress the fact, already mentioned by us, that differential equations —as all the fundamental equations of physics— do not contain any inbuilt "fundamental length". In fact, Einstein equations can describe the internal dynamics of our cosmos, as well as of much bigger super-cosmoses, or of much smaller micro-cosmoses (suitably "scaled").

4.   A Hierarchy of "Universes"

As a first step for better exploiting the symmetries of the fundamental equations of classical physics, let us therefore fix our attention on the space-time *dilations*

$$x'_\mu = \rho x_\mu \tag{3}$$

with $x_\mu \equiv (t; x, y, z)$ and $\mu = 0, 1, 2, 3$, and explicitly require physical laws to be covariant with respect to them: under the hypothesis, however, that only *discrete* values of ρ are realized in nature. As before, we are moreover supposing that ρ is constant as the space or time position varies (global, besides *discrete*, dilations).

Let us recall that natural objects interact essentially through four (at least) fundamental forces, or interactions: the gravitational, the "weak", the electromagnetic and the "strong" ones; here listed according to their (growing) strength. It is possible to express such strengths by pure numbers, so to be allowed to compare them each other. For instance, if one chooses to define each strength as the dimensionless *square* of a "vertex coupling constant", the electromagnetic strength results to be measured by the (dimensionless) coefficient $Ke^2/\hbar \equiv \alpha \simeq 1/137$, where e is the electron charge, $\hbar$ the reduced Planck constant, c is the light speed in vacuum and K is the electromagnetic interaction universal constant (in the International System of units, $K = (4\pi\varepsilon_0)^{-1}$, with ε_0 = vacuum dielectric constant). Here we are interested in particular in the gravitational and strong interaction strengths:

$$s \equiv Gm^2/\hbar c; \qquad S \equiv Ng^2/\hbar c,$$

where G and N are the gravitational and strong universal constants, respectively; quantities m e g representing the gravitational charge (=mass) and the strong charge[11, 12] (cf. Fig. 1), respectively, of one and the same hadron: for example of a nucleon $\mathcal{N}$ or of a pion π. More precisely, we shall often adopt in the following the convention of calling m and g "*gravitational mass*" and "*strong mass*", respectively.

Let us consider, therefore, two identical particles endowed with both gravitational (m) and strong (g) mass, *i.e.*, two identical hadrons, and the ratio between the strengths S and s of the corresponding strong and gravitational interactions. We find $S/s \equiv Ng^2/Gm^2 \simeq 10^{40\div41}$, so that one verifies that $\rho \equiv R/r \simeq S/s$. For example for $m = m_\pi$ one gets $Gm^2/\hbar c \simeq 1.3 \times 10^{-40}$, while the pp$\pi$ or $\pi\pi\rho$ (or quark-quark-gluon: see below) coupling constant squares are $Ng^2/\hbar c \simeq 14$ or 3 (or 0.2), respectively.

Fig. 1 – *"Coloured" quarks and their strong charge* – This scheme represents the complex plane[3, 12, 13] of the *sign* s of the quark strong–charges g_j in a hadron. These strong charges can have *three* signs, instead of two as in the case of the ordinary electric charge e. They can be represented, for instance, by $s_1 = (i - \sqrt{3})/2$; $s_2 = (i + \sqrt{3})/2$; $s_3 = -i$, which correspond to the arrows separated by 120° angles. The corresponding anti-quarks will be endowed with strong charges carrying the complex conjugate signs $\overline{s}_1$, $\overline{s}_2$, $\overline{s}_3$. The three quarks are represented by the "yellow" (Y), "red" (R) and "blue" (B) circles; the three anti-quarks by the "violet" (V), "green" (G) and "orange" (O) circles. The latter are complementary to the former corresponding colors. Since in real particles the inter–quark forces are saturated, hadrons are white. The white colour can be obtained either with three–quark structures, by the combinations YRB or VGO (as it happens in baryons and antibaryons, respectively), or with two–quark structures, by the combinations YV or RG or BO [which are actually quark–antiquark combinations], as it happens in mesons and their antiparticles. See also note[11].

Already at this point, we can make some simple remarks. First of all, let us notice that, *if we put* conventionally $m \equiv g$, then the strong universal constant N becomes

$$N \simeq \rho G \approx hc/m_\pi^2. \tag{4}$$

On the contrary, if *we choose* units such that $[N] = [G]$ and moreover $N = G = 1$, we obtain $g = m\sqrt{\rho}$ and, more precisely (with $n = 2$ or $n = 3$),

$$g_o = g/n \simeq \sqrt{\hbar c/G} \equiv \text{Planck mass},$$

which tells us that —in suitable units— the so–called "Planck mass" is nothing but the *magnitude* of the rest strong–mass [= strong charge] of a typical hadron, or rather of quarks.[11]

From this point of view, we should *not* expect the "micro black–holes" (with masses of the order of the Planck mass), predicted by various Authors, to exist; in fact, we *already* know of the existence of quarks, whose *strong charges* are of the order of the Planck mass (in suitable units). Moreover, the fact —well known in standard theories— that gravitational interactions become as strong as the "strong" ones for masses of the order of the Planck mass does simply mean in our opinion that the *strong gravity* field generated by quarks inside hadrons (strong micro-universes) is nothing but the strong nuclear field.

5. "Strong Gravity"

A consequence of what stated above is that inside a hadron (*i.e.*, when we want to describe strong interactions among hadron constituents) it must be possible to adopt the same Einstein equations which are used for the description of gravitational interactions inside our cosmos; with the only warning of scaling them down, that is, of suitably *scaling*, together with space distances and time durations, also the gravitational constant G (or the masses) and the cosmological constant Λ.

Let us now recall that Einstein's equations for gravity do essentially state the equality of two tensorial quantities: the first describing the geometry (curvature) of space-time, and the second —that we shall call "matter tensor", $GT^{\mu\nu}$— describing the distribution of matter:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R_\rho^\rho - \Lambda g_{\mu\nu} = -kGT_{\mu\nu}; \qquad [k \equiv \frac{8\pi}{c^4}]. \tag{5}$$

As well-known, $G \simeq 6.7 \times 10^{-11} \text{m}^3/(\text{kg} \times \text{s}^2)$, while $\Lambda \approx 10^{-52} \text{m}^{-2}$.

Inside a hadron, therefore, equations of the same form will hold, except that instead of G it will appear (as we already know) quantity $N \approx hc/m_\pi^2$ and instead of Λ it will appear the "strong cosmological constant" (or "hadronic constant") λ:

$$N \equiv \rho_1 G; \qquad \lambda \equiv \rho^2 \Lambda; \qquad \rho_1 \approx \rho, \tag{6}$$

so that $\lambda \simeq 10^{30} \mathrm{m}^{-2} = (1 \mathrm{~fm})^{-2}$, or $\lambda^{-1} \approx 0.1$ barn.

For brevity's sake, we shall call $S_{\mu\nu} \equiv N T_{\mu\nu}$ the "strong matter tensor".

What precedes can be directly applied, with a satisfactory degree of approximation, to the case —for example— of the pion: *i.e.*, to the case of the cosmos/pion similarity. Almost as if our cosmos were a super–pion, with a super–quark (or "meta-galaxy", adopting Ivanenko's terminology) of matter and one of anti-matter. Let us recall however that, as we already warned in Section **3**, the parameter ρ *can* vary according to the particular cosmos and hadron considered. Analogously Λ, and therefore λ, can vary too: with the further circumstance that a priori also their *sign* can change, when varying the object (cosmos or hadron) taken into examination.

As far as ρ_1 is concerned, an even more important remark has to be made. Let us notice that the gravitational coupling constant $Gm^2/\hbar c$ (experimentally measured in the case of the interaction of two "tiny components" of our particular cosmos) should be compared with the analogous constant for the interaction of two tiny *components* (partons? partinos?) of the corresponding hadron, or rather of a particular constituent quark of its. That constant is unknown to us. We know however, for the simplest hadrons, the quark-quark-gluon coupling constant: $Ng^2/\hbar c \simeq 0.2$. As a consequence, the best value for ρ_1 we can predict —up to now— for those hadrons is $\rho_1 \simeq 10^{38} \div 10^{39}$ [and, in fact, 10^{38} is the value which has provided the results most close to the experimental data]: a value that however will vary, let us repeat it, with the particular cosmos and the particular hadron chose for the comparison.

The already mentioned "large numbers" empirical relations, which link the micro- with the macro-cosmos, have been obtained by us as a *by-product* of our scaled–down equations for the interior of hadrons, and of the ordinary Einstein equations. Notice, once more, that our "numerology" connects the gravitational interactions with the strong ones, and *not* with the electromagnetic ones (as Dirac, instead, suggested). It is worthwhile noticing that strong interactions, as the gravitational —but differently from the electromagnetic ones,— are highly non-linear and then associable to *non-abelian* gauge theories. One of the purposes of our theoretical approach consists, incidentally, in proposing an *ante litteram* geometrical interpretation of those theories.

Before going on, let us specify that the present *geometrization* of the strong field is justified by the circumstance that the "Equivalence principle" (which recognizes the identity, inside our cosmos, of inertial and gravitational mass) can be extended to the hadronic universe in the following way. The usual Equivalence principle can be understood, according to Mach, thinking of the inertia m_I of a given body as due to its interaction with all the other masses of the universe: an interaction which in *our* cosmos is essentially gravitational; so that m_I coincides with the gravitational mass: $m_\mathrm{I} \equiv m_\mathrm{G}$. Inside a "hadronic cosmos", however, the predominant interaction among its constituents is the strong one; so that the inertia m_I of a constituent will coincide with its strong charge g (and not with m_G). We shall see that our generalization of the Equivalence principle will be useful for geometrizing the strong field not only inside a hadron, but also in its neighborhood.

Both for the cosmos and for hadrons, we shall adopt Friedmann–type models; taking advantage of the fact that they are compatible with the Mach Principle, and are embeddable in 5 dimensions.

6. In the Interior of a Hadron

Let us see some *consequences* of our Einstein–like equations, re-written for the strong field and therefore valid inside a hadron:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^{\rho}_{\rho} - \lambda g_{\mu\nu} = -kS_{\mu\nu}; \qquad [S_{\mu\nu} \equiv NT_{\mu\nu}]. \tag{7}$$

In the case of a spherical constituent, that is to say of a spherically symmetric distribution g' of "strong mass", and in the usual Schwarzschild-deSitter r,t coordinates, the known geodesic motion equations for a small test–particle (let us call it a *parton*, with strong mass g'') tell us that it will feel a "force" easy to calculate,[3, 13] which for low speeds [*static limit*: $v << c$] reduces to the (radial) force:

$$F = -\frac{1}{2}c^2g''(1 - \frac{2Ng'}{c^2r} + \frac{1}{3}\lambda r^2)(\frac{2Ng'}{c^2r^2} + \frac{2}{3}\lambda r). \tag{8}$$

Notice that, with proper care, also in the present case one can introduce a language in terms of "force" and "potential"; for example in Eq. 8 we defined $F \equiv g''\mathrm{d}^2r/\mathrm{d}t^2$. In Fig. 2 the form is depicted of two typical potentials yielded by the present theory [cf. Eq. 8'].

At "intermediate distances" —i.e., at the newtonian limit— this force simply reduces to $F \simeq -\frac{1}{2}c^2g''(2Ng'/c^2r^2 + 2\lambda r/3)$, that is, to the sum of a newtonian term and of an elastic term *à la* Hooke. Let us notice that, in such a limit, the last expression is valid even when the test particle g'' does *not* posses a small strong mass, but is —for example— a second quark. Otherwise, our expressions for F are valid only *approximately* when also g'' is a quark; nevertheless, they can explain some important features of the hadron constituent behaviour, both for small and for large values of r.

At very large distances, when r is of the same order of (or is greater than) the considered hadron radius $[r \geq \sim 10^{-13}$ cm $\equiv 1$ fm], whenever we confine ourselves to the simplest hadrons (and thus choose $\Lambda \simeq 10^{30}$ m^{-2}; $N \simeq 10^{38\div39}G$), we end with an *attractive* radial force which is proportional to r:

$$F \approx -g''c^2\lambda r/3. \tag{9}$$

In other words, one naturally obtains a confining force (and a confining potential $V \div r^2$) able a priori to explain the so–called *confinement* of the hadron constituents (in particular, of quarks). Because of this force, the motion of g'' can be regarded in a first approximation as a harmonic motion; so that our *theory* can

Fig. 2 – In this figure the shape is shown of two typical inter–quark potentials V_{eff} yielded by the present theoretical approach: cf. Eq. 8. We show also the theoretical energy–levels *calculated* for the $1-^3s_1$, $2-^3s_1$ e $3-^3s_1$ states of "Bottomonium" and "Charmonium", respectively [by adopting for the *bottom* and *charm* quark the masses m(b)=5.25 and m(c)=1.68 GeV/c^2]. The comparison with experience is satisfactory:[17] see Section **5**.

include the various and interesting results already found by different Authors for the hadronic properties —for instance, hadron mass spectra— just by *postulating* such a motion.

Up to now we supposed λ to be positive. But it is worthwhile noticing that confinement is obtained also for negative values of λ. In fact, with less drastic approximations, for $r \geq \sim 1$ fm one gets:

$$F \approx -\frac{1}{3}g"c^2\lambda(r + \lambda r^3/3 - Ng'/c^2), \qquad (9')$$

where, for r large enough, the λ^2 term is dominating. Let us warn however that, when considering "not simple" hadrons (so that λ, and moreover N, may change their values), other terms can become important, like the newtonian one, $-Ng'^2/r^2$, or even the *constant* term $+N\lambda g'^2/3$ which corresponds to a linear potential. Let us observe, finally, how this last equation predict that, for inter–quark distances of the

order of 1 fm, two quarks have to attract each other with a force of *some tons*: a quite huge force, especially when recalling that it should act between two extremely tiny particles (the *constituents* of mesons and baryons), whose magnitute would increase with the distance.

Let us pass to consider, now, *not* too big distances, always at the static limit. It is then important to add to the radial potential the usual "kinetic energy term" (or centripetal potential), $(J/g")^2/2r^2$, in order to account for the orbital angular momentum of $g"$ with respect to g'. The effective potential[13] between the two constituents g', $g"$ gets thus the following form

$$V_{\text{eff}} = \frac{1}{2}g"c^2[2(\frac{Ng'}{c^2})^2\frac{1}{r^2} - \frac{2Ng'}{c^2}\frac{1}{r} - \frac{2\lambda Ng'}{3c^2}r + \frac{\lambda}{3}r^2 + \frac{1}{2}(\frac{\lambda}{3})^2r^4] + \frac{(J/g")^2}{2r^2}, \qquad (8')$$

which, in the region where GR reduces essentially to the newtonian theory, simplifies into:

$$V_{\text{eff}} \approx -Ng'g"/r + (J/g")^2/2r^2.$$

In such a case the test particle $g"$ can set itself (performing a circular motion, for example: and in Section **7** we shall give more details) at a distance r_{e} from the source–constituent at which V is minimum; *i.e.*, at the distance $r_{\text{e}} = J^2/Ng'g"^2$. At this distance the "effective force" vanishes. Thus we meet, at short distances, the phenomenon known as *asymptotic freedom*: For not large distances (when the force terms proportional to r and to r^3 become negligible), the hadron constituents behave as if they were (almost) free. If we now extrapolated, somewhat arbitrarily, the expression for r_{e} to the case of two quarks [for example, $|g'| = |g"| = g_{\text{o}} \simeq \frac{1}{3}m_{\text{p}}$], we would obtain the preliminary estimate $r_{\text{e}} \approx \frac{1}{100}$ fm. Vice-versa, by supposing — for instance in the case of baryons, with $g \equiv m \simeq m_{\text{p}}$ and $N \simeq 10^{40}G$— that the equilibrium radius r_{e} be of the order of a hundredth of a fermi, one would get the Regge–like relation $J/\hbar \simeq m^2$ (where m is measured in GeV/c^2).

Let us perform these calculations again, however, by using the *complete* expression of V_{eff}. First of all, let us observe that it is possible to evaluate the radius at which the potential reaches its minimum also in the case $J = 0$. By extrapolation to the case of the simplest quarks [for which $Ng^2/\hbar c \simeq 0.2$], one finds always at least one solution, $r_{\text{e}} \approx 0.25$ fm, for λ positive and of the order of 10^{30} m^{-2}. Passing to the case $J = \hbar$ (which corresponds classically to a speed $v \simeq c$ for the moving quark), we obtain under the same hypothesis the value

$$r_{\text{e}} \simeq 0.9 \text{ fm}.$$

Actually, for positive λ it exists the above solution *only*. For negative values of λ, however, the situation is more complex; let us summarize it in the case of the N and $|\lambda|$ values adopted by us. One meets —again— at least one solution, which for $J = 0$ takes the simple analytic form $r_{\text{e}}^3 = 3Ng'/c^2|\lambda|$.

More precisely, for $\lambda = -10^{30}$m^{-2} one finds the values 0.7 and 1.7 fm, in correspondence to $J = 0$ and $J = 1$; values that however become 0.3 and 0.6 fm,

respectively, for $\lambda = -10^{29}\mathrm{m}^{-2}$. In the $J = 0$ case, at last, two *further* solutions are met, the smaller one [for $\lambda = -10^{30}$ m^{-2}] being once more $r_\mathrm{e} \simeq 0.25$ fm.

By recalling that *mesons* are made up of two quarks (q, q̄), our approach suggests for mesons in their ground state —when $J = 0$, at least— the model of two quarks *oscillating* around an equilibrium position. It is rather interesting to notice that for small oscillations (harmonic motions in space) the dynamical group would then be SU(3). It is interesting to notice, too, that the value $m_\mathrm{o} = h\nu/c^2$, corresponding to the frequency $\nu = 10^{23}$ Hz, yields the pion mass: $m_\mathrm{o} \simeq m_\pi$.

Analogous results have to hold, obviously, for our cosmos (or, rather, for the cosmoses which are "dual" to the hadrons considered).

7. The Strong Coupling Constant

Here we want to add just that, in the case of a spherically symmetric, static metric (and in the coordinates in which it is diagonal), the Lorentz factor is proportional to $\sqrt{g_{oo}}$, so that the *strong coupling constant* $\alpha_\mathrm{S} \equiv S$ in our theory[14] assumes the form:[15]

$$\alpha_\mathrm{S}(r) \simeq \frac{N}{\hbar c}\frac{g'^2_\mathrm{o}}{1 - 2Ng'_\mathrm{o}/c^2 r + \lambda r^2/3}, \tag{10}$$

since the strong mass g" depends on the speed:

$$g" = \frac{g"_\mathrm{o}}{\sqrt{g_{oo}}} = \frac{g"_\mathrm{o}}{\sqrt{1 - 2Ng'_\mathrm{o}/r + \lambda r^2/3}}, \tag{11}$$

so as the ordinary relativistic mass does. The behaviour of our "constant" $\alpha_\mathrm{S}(r)$ is analogous to that one of the perturbative coupling constant of the "standard theory" (QCD): that is to say, $\alpha_\mathrm{S}(r)$ decreases as the distance r decreases, and increases as it increases, once more justifying both confinement and "asymptotic freedom". Let us recall that, when[15] $g"_\mathrm{o} = g'_\mathrm{o}$, the definition of α_S is $\alpha_\mathrm{S} \equiv S = Ng'^2/\hbar c$.

Since the Schwarzschild–like coordinates $(t; r, \theta, \varphi)$ do not correspond, as is well known, to any real observer, it is interesting from the *physical* point of view to pass to the local coordinates $(T; R, \theta, \varphi)$ associated with observers who are *at rest* "with respect to the metric" at each point (r, θ, φ) of space: $\mathrm{d}T \equiv \sqrt{g_{tt}}\mathrm{d}t$; $\mathrm{d}R \equiv \sqrt{-g_{rr}}\mathrm{d}r$, where $g_{tt} \equiv g_{oo}$ and $g_{rr} \equiv g_{11}$. These *"local" observers* measure a speed $U \equiv \mathrm{d}R/\mathrm{d}T$ (and strong masses) such that $\sqrt{g_{tt}} = \sqrt{1 - U^2}$, so that Eq. 11 assumes the transparent form

$$g" = \frac{g"_\mathrm{o}}{\sqrt{1 - U^2}}. \tag{11'}$$

Once calculated (thanks to the geodesic equation) the speed U as a function of r, it is easy to find again, for example, that for negative λ the minimum value of U^2 corresponds to $r = [3Ng'_\mathrm{o}/|\lambda|]^{1/3}$. While for positive λ we get a similar expression, i.e., $r_\mathrm{o} \equiv [6Ng'_\mathrm{o}/\lambda]^{1/3}$, which furnishes a limiting (*confining*) value of r, which cannot be reached by any of the constituents.

Let us finally consider the case of a geodesic circular motion, as described by the "physical" observers, *i.e.*, by our local observers (even if we find it convenient to express everything as a function of the old Schwarzshild-deSitter coordinates). If a is the angular momentum per unity of strong rest-mass, in the case of a test–quark in motion around the source–quark, we meet the interesting relation $g" = g'_o\sqrt{1 + a^2/r^2}$, which allows us to write the strong coupling constant in the particularly simple form[14]

$$\alpha_S \simeq \frac{N}{\hbar c}g'_o(1 + \frac{a^2}{r^2}). \tag{10'}$$

We can now observe, for instance, that —if $\lambda < 0$— the specific angular momentum a vanishes in correspondence to the customary geodesic $r \equiv r_{qq} = [3Ng'_o/|\lambda|]^{1/3}$; in this case the test–quark can remain *at rest*, at a distance r_{qq} from the source–quark. With the "typical" values $\rho = 10^{41}$; $\rho_1 = 10^{38}$, and $g'_o = m_p/3 \simeq 313$ MeV/c^2, we obtain $r_{qq} \simeq 0.8$ fm.

8.   Outside a Hadron. Strong Interactions among Hadrons

From the "external" point of view, when describing the interactions among hadrons (as they appear to us in *our* space), we are in need of *new* field equations able to account for both the gravitational and strong field which surround a hadron. We need actually a *bi-scale* theory [Papapetrou], in order to study for example the motion in the vicinity of a hadron of a test–particle possessing both gravitational and strong mass.

What precedes suggests —as a first step— to represent the strong field around a source–hadron by means of a tensorial field, $s_{\mu\nu}$, so as it is tensorial (in GR) the gravitational field $e_{\mu\nu}$. Within our theory,[3, 2, 1] Einstein gravitational equations have been actually *modified* by introducing, in the neighborhood of a hadron, a strong deformation $s_{\mu\nu}$ of the metric, acting only on objects having a strong charge (*i.e.*, an intrinsic "scale factor" $f \simeq 10^{-41}$) and not on objects possessing only a gravitational charge (*i.e.*, an intrinsic scale factor $f \simeq 1$). Outside a hadron, and for a "test–particle" endowed with both the charges, the *new* field equations are:

$$R_{\mu\nu} + \lambda s_{\mu\nu} = -\frac{8\pi}{c^4}\,[S_{\mu\nu} - \frac{1}{2}g_{\mu\nu}S^\rho_\rho]. \tag{12}$$

They reduce to the usual Einstein equations far from the source–hadron, because they *imply* that the strong field exists only in the very neighborhood of the hadron: namely that (in suitable coordinates) $s_{\mu\nu} \to \eta_{\mu\nu}$ for $r >> 1$ fm.
Linear approximation: – For distances from the source–hadron $r \geq \sim 1$ fm, when our new field equations can be linearized, the total metric $g_{\mu\nu}$ can be written as the *sum* of the two metrics $s_{\mu\nu}$ and $e_{\mu\nu}$; or, more precisely (in suitable coordinates):

$$2g_{\mu\nu} = e_{\mu\nu} + s_{\mu\nu} \simeq \eta_{\mu\nu} + s_{\mu\nu}.$$

Quantity $s_{\mu\nu}$ can then be written as $s_{\mu\nu} \equiv \eta_{\mu\nu} + 2h_{\mu\nu}$, with $|h_{\mu\nu}| << 1$; so that $g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}$ (where, let us repeat, $h_{\mu\nu} \to 0$ per $r >> 1$ fm). For the sake of simplicity, we are in addition confining ourselves to the case of positive λ [on the contrary, if $\lambda < 0$, we should[13] put $s_{\mu\nu} \equiv \eta_{\mu\nu} - 2h_{\mu\nu}$].

One of the most interesting results is that, at the static limit (when only $s_{oo} \neq 0$ and the strong field becomes a scalar field), we get that $V \equiv h_{oo} \equiv \frac{1}{2}(s_{oo}-1) = g_{oo}-1$ is exactly the *Yukawa potential*:

$$V = -g\frac{\exp[-\sqrt{2|\lambda|}r]}{r} \simeq -\frac{g}{r}\exp[\frac{-m_\pi rc}{\hbar}], \tag{13}$$

with the correct coefficient —within a factor 2— also in the exponential.[3, 2, 1]

Intense field approximation: – Let us consider the source–quark as an *axially symmetric* distribution of strong charge g: the study of the metrics in its neighborhood will lead us to consider a Kerr-Newman-deSitter (KNdS)–like problem and to look for solutions of the type "*strong* KNdS black holes". We find that —from the "external" point of view— hadrons can be associated with the above mentioned "*strong black-holes*" (SBH), which result to have radii $r_S \approx 1$ fm.

For $r \to r_S$, that is, when the field is very intense, we can perform the approximation just "opposite" to the linear one, by assuming $g_{\mu\nu} \simeq s_{\mu\nu}$. We obtain, then, equations which are essentially identical with the "internal" ones [which is good for the matching of the hadron interior and exterior!]; a consequence being that what we are going to say can be valid also for *quarks*, and not only for hadrons. Before going on, let us observe that λ can *a priori* take a certain sign outside a hadron, and the opposite sign inside it. In the following we shall confine ourselves to the case $\lambda < 0$ for simplicity's sake.

In general for negative λ one meets[14] three "strong horizons", *i.e.*, three values of r_S, that we shall call r_1, r_2, r_3. If we are interested in hadrons which are *stable* with respect to the strong interactions, we have to look for those solutions for which the SBH Temperature[16] [= strong field strength at its surface] almost vanishes. It is worth noticing that the condition of a vanishing field at the SBH surface implies the coincidence of two, or more, strong horizons;[3, 14, 16] and that such coincidences imply in their turn some "Regge–like" relations among m, λ, N, q and J, if m, q, J are —now— mass, charge and *intrinsic angular momentum* of the considered hadron, respectively. More precisely, if we choose a priori the values of q, J, λ and N, then our theory yields *mass* and *radius* of the corresponding stable hadron. Our theoretical approach is, therefore, a rare example of a formalism which can yield —at least a priori— the *masses* of the stable particles (and of the quarks themselves).

9. Mass Spectra

We arrived at the point of checking whether and how our approach can yield the values of the hadron masses and radii: in particular for hadrons stable with

respect to strong interactions; one can guess a priori that such values will possess the correct order of magnitude. Several calculations have been performed by us, in particular for the meson mass spectra;[13, 14] although they —because of our laziness with respect to numerical elaborations— are still waiting for being reorganized.

Here we quickly outline just some of the results. At first, let us consider the case of the simultaneous coincidence of all the *three* horizons ($r_1 = r_2 = r_3 \equiv r_h$). We get a system of equations that —for example— rules out the possibility that intrinsic angular momentum (spin) J and electric charge q be simultaneously zero [*practically ruling out particles with $J = 0$*]; it also implies the interesting relation $\lambda^{-1} \simeq 2r_h{}^2$; and finally it admits (real and positive) solutions only for *low* values of J, the upper limit of the spin depending on the chosen parameters.

The values we obtained for the (small) radii and for the masses suggest that the "*triple coincidences*" represent the case of *quarks*. The basic formulae for the explicit calculations are the following.[14] First of all, let us put $N = \rho_1 G$, so that $g \equiv m$. Let us then define, as usual, $Q^2 \equiv Nq^2/Kc^4$; $a \equiv J/mc$; $M \equiv Nm/c^2$, and moreover $\delta \equiv 1 + \lambda a^2/3$. Then, the radii of the stable particles (quarks, in this case) are given by the *simple* equation $r = 3M/2\delta$; but the masses are given by the solution of a system of two Regge–like relations: $9M^2 = -2\delta^3/\lambda$; $9M^2 = 8\delta(a^2 + Q^2)$.

The cases of "*double coincidence*", that is, of the coincidence of two (out of three) horizons only, seem to be able to describe stable baryons and mesons. The fundamental formulae become, however, more complex.[14] Let us define $\eta \equiv a^2 + Q^2$; $\sigma \equiv \delta^2 + 4\lambda\eta$; $Z \equiv 3\delta^2 - 4\lambda\delta\eta + 18\lambda M^2$. The stable hadron's radii are then given by the relation $r \equiv 3M\sigma/Z$; while the masses are given by the non simple equation $9M^2\sigma(\delta\sigma - Z) + 2\eta Z^2 = 0$, which relates M with a, Q and λ. Of course, some simplifications are met in particular cases. For example, when $\lambda = 0$, we get the Regge–like relation:

$$M^2 = a^2 + Q^2, \tag{14}$$

which —when q is negligible— becomes $M^2 = cJ/G$, that is [with $c = G = 1$]:

$$m^2 = J. \tag{14'}$$

On the contrary, when $J = 0$, and q is still negligible, we obtain [always with $c = G = 1$]:

$$9m^2 = -\lambda^{-1}. \tag{15}$$

Also in the cases of "triple coincidence" simple expressions are found, when $|\lambda a^2| <<$ 1. Under such a condition, one meets the simple system of two equations:

$$9M^2 \simeq 8(a^2 + Q^2); \qquad 9m^2 \simeq -2\lambda^{-1}, \tag{16}$$

where the second relation is written with $c = G = 1$.

All the "geometric" evaluations of this Section **9** are referred —as we have seen— only to *stable* hadrons (*i.e.*, to hadrons corresponding to SBHs with "temperature" $T \simeq 0$), because we do not know of general rules associating a temperature T

with the many *resonances* experimentally discovered (which will correspond[1, 2, 3] to temperatures *of the order* of 10^{12}K, if they have to "evaporate" in times of the order of 10^{-23}s). Calculations apt at comparing our theoretical approach with experimental mass spectra (for mesons, for example) have been till now performed, therefore, by making recourse to the trick of inserting our inter–quark potential V_{eff}, found in Section **6**, into a Schroedinger equation. Also such (many) calculations —kindly performed by our colleagues Prof. J.A. Roversi e Dr. L.A. Brasca–Annes of the "Gleb Wataghin" Physics Institute of the State University at Campinas (S.P., Brazil)— have not yet been reordered! Here let us specify, nevertheless, that potential (8') has been inserted into the Schroedinger equation in spherical (polar) coordinates, which has been solved by a finite difference method.[13]

In the case of "Charmonium" and of "Bottomonium", for example, the results obtained (by adopting[17] for the quark masses the values $m(\text{charm}) = 1.69 \text{ GeV}/c^2$; $m(\text{bottom}) = 5.25 \text{ GeV}/c^2$) are the following (Fig. 2). For the states $1-^3s_1$, $2-^3s_1$ and $3-^3s_1$ of Charmonium, we obtained the energy levels 3.24, 3.68 and 4.13 GeV, respectively. Instead, for the corresponding quantum states of Bottomonium, we obtained the energy levels 9.48, 9.86 and 10.14 GeV, respectively. The radii for the two fundamental states resulted to be $r(\text{c})=0.42$ fm, and $r(\text{b})=0.35$ fm, with $r(\text{c})> r(\text{b})$ [as expected from "asymptotic freedom"]. Moreover, the values of the parameters obtained by our computer fit are actually those expected: $\rho = 10^{41}$ and $\rho_1 = 10^{38}$ (just the "standard" ones) for Charmonium; and $\rho = 0.5 \times 10^{41}$ and $\rho_1 = 0.5 \times 10^{38}$ for Bottomonium.

The correspondence between experimental and theoretical results[17] is satisfactory, especially when recalling the approximations adopted (in particular, the one of treating the second quark g" as a test–particle).

10.   Acknowledgements

11. References

1. See for example A. Salam and D. Strathdee: *Phys. Rev.* D**16** (1977) 2668; D**18** (1978) 4596; A. Salam: in *Proceed. 19th Int. Conf. High-Energy Physics (Tokio,1978)*, p.937; *Ann. N.Y. Acad. Sci.* **294** (1977) 12; C. Sivaram and K.P. Sinha: *Phys. Reports* **51** (1979) 111; M.A. Markov: *Zh. Eksp. Teor. Fiz.* **51** (1966) 878; E. Recami and P. Castorina: *Lett. Nuovo Cim.* **15** (1976) 347; R. Mignani: *ibidem* **16** (1976) 6; P. Caldirola, M. Pavsic and E. Recami: *Nuovo Cimento* B**48** (1978) 205; *Phys. Lett.* A**66** (1978) 9; P. Caldirola and E. Recami: *Lett. Nuovo Cim.* **24** (1979) 565; D.D. Ivanenko: in *Astrofisica e Cosmologia, Gravitazione, Quanti e Relatività – Centenario di Einstein*, edited by M. Pantaleo and F. de Finis (Giunti-Barbera; Florence, 1978), p.131; R.L. Oldershaw: *Int. J. General Systems* **12** (1986) 137. See also N. Rosen: Found. Phys. **10** (1980) 673.

2. See for example E. Recami: in *Old and New Questions in Physics, Cosmology,...*, by A. van der Merwe (Plenum; New York, 1983); *Found. Phys.* **13** (1983) 341. Cf. also P. Ammiraju, E. Recami and W.A. Rodrigues: *Nuovo Cimento* A**78** (l983) 172.

3. For an extended summary of that theory, see for example E. Recami: *Prog. Part. Nucl. Phys.* **8** (1982) 401, and refs. therein; E. Recami, J.M. Martínez and V. Tonin–Zanchin: *Prog. Part. Nucl. Phys.* **17** (1986) 143; and E. Recami and V. Tonin–Zanchin: *Il Nuovo Saggiatore* **8** (1992; issue no.2) 13. See also E. Recami and V. Tonin–Zanchin: *Phys. Lett.* B**177** (1986) 304; B**181** (1986) E416.

4. See for example A. Einstein: "Do gravitational fields play an essential role in the structure of elementary particles?", *Sitzungsber. d. Preuss. Akad. d. Wiss.*, 1919 (in German).

5. See also M. Sachs: *Found. Phys.* **11** (1981) 329; and *General Relativity and Matter* (Reidel; Dordrecht, 1982).

6. A. Italiano and E. Recami: *Lett. Nuovo Cim.* **40** (1984) 140.

7. See for example B.B. Mandelbrot: *The Fractal Geometry of Nature* (W.H.Freeman; San Francisco, 1983).

8. This clarifies that our geometrico–physical similarity holds between two classes of objects of different scale (hadrons and cosmoses), in the sense that the factor ρ will vary according to the particular cosmos and hadron considered. That will be important for the practical applications. At last, let us recall that in Mandelbrot's philosophy, analogous objects do exist at every hierarchical level, so that we can conceive a particular type of cosmos for each particular type of hadron, and vice-versa. As a consequence, we should expect ρ to change a little in each case (for example, according to the hadron type considered).

9. Le us notice that we do not refer here to the usual "general covariance" of the Einstein's equations (that are supposed to hold in our cosmos), but to their

covariance with respect to transformations (*dilations*) which bring –for example– from our cosmos to the hadronic micro-cosmos.

10. M. Pantaleo (editor): *Cinquant'anni di Relatività* (Giunti; Firenze, 1955).

11. Let us recall that the hadron constituents (2 for mesons and 3 for baryons) have named *quarks* by M. Gell-Mann. This Anglo-Saxon word, which usually means mush or also curd, is usually ennobled by literary quotations (for example, Gell-Mann was inspired —as it is well known— by a verse of J. Joyce's *Finnegans wake*, 1939). Let us here quote that Goethe had properly used such a word in his *Faust*, verse 292, where Mephistopheles referring to mankind exclaims: <<In Jeden Quark begräbt er seine Nase">>!
By considering quarks to be the real carriers of the strong charge (cf. Fig. 1), we can call "colour" the *sign* s_j of such strong charge;[12] namely, we can regard hadrons as endowed with a zero total strong charge, each quark possessing the strong charge $g_j = s_j|g_j|$ with $\Sigma s_j = 0$. Therefore, when passing from ordinary gravity to "strong gravity", we shall replace m by $g = ng_o$, quantity g_o being the average *magnitude* of the constituent quarks *rest*–strong-charge, and n their number.[12]

12. Cf. for example E. Recami: in *Annuario '79, Enciclopedia EST-Mondadori*, ed. by E. Macorini (Mondadori; Milan, 1979), p.59.

13. A. Italiano *et al.*: *Hadronic J.* **7** (1984) 1321; P. Ammiraju *et al.*: *Hadronic J.* **14** (1991) 441; V. Tonin–Zanchin: M.Sc. Thesis (UNICAMP; Campinas, S.P., 1987); E. Recami and V.T. Zanchin: (*in preparation*).

14. V. Tonin–Zanchin, E. Recami, J.A. Roversi and L.A. Brasca–Annes: "About some Regge–like relations for (stable) black holes", report IC/91/219 (ICTP; Trieste, 1991), to appear in *Comm. Theor. Phys.*; E. Recami and V. Tonin–Zanchin: "The strong coupling constant: Its theoretical derivation from a geometric approach to hadron structure", *Comm. Theor. Phys.* **1** (1991) 111.

15. Actually, if we considered a (light) test–particle g" in the field of a "heavy" constituent g' (a quark for instance), we would rather obtain only a square root at the denominator; namely $\alpha_S \simeq (Ng'_o g"_o/\hbar c)[\sqrt{1 - 2Ng"_o/c^2r + \lambda r^2/3}]^{-1}$. When we pass to consider two heavy constituents (two quarks) endowed with the same *rest* strong-mass $g"_o = g'_o$, we ought to tackle the two body problem in GR; however, in an approximate way, and looking at an *average* situation, one can propose a formula like Eq. 13, where r is the distance from the common "centre of mass".

16. See for example J.D. Bekenstein: *Phys. Rev.* D**9** (1974) 3292; S.W. Hawking: *Comm. Math. Phys.* **43** (1975) 199.

17. C. Quigg: report 85/126–T (Fermilab, Sept. 1985).

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