One Loop Anomalies and Wess-Zumino Terms for General Gauge Theories

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Abstract

One loop anomalies and their dependence on antifields for general gauge theories are investigated within a Pauli-Villars regularization scheme. For on-shell theories *i.e.*, with open algebras or on-shell reducible theories, the antifield dependence is cohomologically non trivial. The associated Wess-Zumino term depends also on antifields. In the classical basis the antifield independent part of the WZ term is expressed in terms of the anomaly and finite gauge transformations by introducing gauge degrees of freedom as the extra dynamical variables. The complete WZ term is reconstructed from the antifield independent part.

1   Introduction.

Most known fundamental theories are gauge theories. The deep knowledge of their gauge structure is crucial to understand the classical and the quantum natures completely. The recent work of strong-weak duality in supersymmetric gauge theories [1] suggests that our understanding of the gauge properties is incomplete and new ideas to understand these issues seem to be required. In this paper we address a more traditional aspect of gauge theories. We will study the structure of gauge anomalies and the form of the WZ term [2] for general gauge theories using the BRST symmetry [3][4]. In particular we will discuss a parametrization of the one loop anomaly and the associated WZ term using a PV regularization scheme[5]. We will see how the anomaly can be expressed in terms of PV regulator and the BRST transformation [6]. This form of the anomaly is very useful in order to study the antifield dependence. For theories with on-shell structure, i.e., with open algebras or on-shell reducible theories [7] [8], the anomalies will be dependent in a non-trivial way on the sources (antifields) of the BRST transformation. While for closed theories this dependence is cohomologically trivial. However, using the WZ consistency conditions and cohomological techniques it is possible to construct for closed theories antifield dependent candidate anomalies [9]. They cannot appear in any regularized field theory calculation. The regularization procedure selects a subset of the ghost number one nontrivial cocycles.

The presence of gauge anomalies implies that some classical gauge degrees of freedom become dynamical (propagating) at quantum level. The WZ term can be written using these new degrees of freedom. In general the WZ term also depends on the sources of the BRST transformation. The antifield independent part of the WZ term in the classical basis of the fields and the antifields can be expressed in terms of the anomaly and the finite gauge transformations associated with the on-shell structure as in the ordinary case, like Yang-Yills theory [10].

The organization of the paper is as follows: in section 2 we will introduce some basic concepts and the effective action regularized by PV scheme up to one loop. In section 3 we will discuss the structure of the anomaly. In section 4 the WZ term is analyzed and in section 5 we give some conclusions. In appendix we give some discussion of finite on-shell structures.

2   One loop PV regularized effective action

Let us consider a general gauge theory with classical action $\mathcal{S}_0(\phi)$. We assume its infinitesimal gauge transformations $\delta\phi = R^i{}_\alpha(\phi)\epsilon^\alpha$ have a reducible on-shell open algebra structure [1]. Some relations of the gauge structure functions are :

$$R^j{}_{[\alpha}(\phi)R^i{}_{\beta],j}(\phi) + T^\gamma_{\alpha\beta}(\phi)R^i{}_\gamma(\phi) + E^{ij}_{\alpha\beta}(\phi)\mathcal{S}_{0,j} = 0, \qquad (2.1)$$

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[1]To make notation simpler we will consider only bosonic fields and bosonic gauge transformations.

$$R^i_\alpha(\phi)Z^\alpha_a(\phi) \; + \; V^{ij}_a(\phi)\mathcal{S}_{0,j} \; = \; 0 \tag{2.2}$$

and

$$\sum_{P\in\mathrm{Perm}[\alpha\beta\gamma]} (-1)^P (\; T^\delta_{\beta\gamma,j}\; R^j_\alpha \;+ T^\delta_{\mu\gamma}\; T^\mu_{\alpha\beta} \;+\; Z^\delta_a\; F^a_{\alpha\beta\gamma} \;-\; D^{j\delta}_{\alpha\beta\gamma}\; S_{0,j}\;) \; = \; 0, \tag{2.3}$$

where $\mathcal{S}_{0,j} \approx 0$ is the classical equation of motion. The first one implies that the algebra closes on shell, the second one means the transformation is reducible on shell and the third one is the generalized Jacobi identity.

The whole set of such relations can be expressed in a compact way within the Field-Antifield formalism [8] [2]. We denote all the fields by Φ^A (classical fields ϕ^i, $i = 1,...,n$; ghosts c^α, $\alpha = 1,...,m_0$; antighosts $\bar{c}_\alpha$; ghost for ghosts η^a, $a = 1,...,m_1$; etc) and their corresponding antifields (Afs) by Φ^*_A. The proper solution $\mathcal{S}(\Phi,\Phi^*)$ of the Classical Master Equation (CME)

$$(\mathcal{S},\mathcal{S}) = 0 \tag{2.4}$$

admits the local expansion in Afs,

$$\mathcal{S}(\Phi,\Phi^*) = \mathcal{S}_0(\phi) + \Phi^*_A \mathcal{S}^A_1(\Phi) + \frac{1}{2}\Phi^*_A\Phi^*_B\mathcal{S}^{BA}_2(\Phi) + ... \tag{2.5}$$

The CME (2.4) encodes all the relations among these structure functions such as (2.1)-(2.3) and is determining the complete classical gauge structures [14][15]. The terms $\mathcal{O}(\Phi^{*2})$ in the proper solution appear for theories with open or reducible on-shell algebras. This basis reproducing all the classical gauge structure is called classical basis(ClB). The BRST transformation in the space of fields and antifields is generated by $\delta\cdot = (\;\cdot\;,\mathcal{S})$ and it is nilpotent off shell,

$$\delta^2 \; = \; 0. \tag{2.6}$$

Even for fields this transformation gives terms depending in general on Afs,

$$\delta\Phi^A = (\Phi^A,\mathcal{S}) = \mathcal{S}^A_1(\Phi) + \Phi^*_B\mathcal{S}^{BA}_2(\Phi) + ... \tag{2.7}$$

To perform perturbative calculations the basis is changed from the ClB to the gauge fixed basis (GFxB). This change is implemented by an antibracket canonical transformation [16] in general. Often discussed are those generated by gauge fixing fermions $\Psi(\Phi)$. Under such canonical transformation the fields are unchanged while Afs Φ^*_A are transformed to new Afs K_A in GFxB as

$$\Phi^*_A = K_A + \frac{\delta\Psi(\Phi)}{\delta\Phi^A}\,. \tag{2.8}$$

In GFxB the proper solution reads

$$
\begin{aligned}
\hat{\mathcal{S}}(\Phi,K) \quad &= \left[\mathcal{S}_0 + \frac{\delta\Psi}{\delta\Phi^A}\mathcal{S}^A_1 + \frac{1}{2}\frac{\delta\Psi}{\delta\Phi^A}\frac{\delta\Psi}{\delta\Phi^B}\mathcal{S}^{BA}_2 + ...\right] \\
&\quad + K_A\left[\mathcal{S}^A_1 + \frac{\delta\Psi}{\delta\Phi^B}\mathcal{S}^{BA}_2 + \frac{1}{2}\frac{\delta\Psi}{\delta\Phi^B}\frac{\delta\Psi}{\delta\Phi^C}\mathcal{S}^{CBA}_3 + ...\right] + ... \\
&=: \mathcal{S}_\Sigma(\Phi) + K_A\left[\delta_\Sigma\Phi^A\right] + ...
\end{aligned}
\tag{2.9}
$$

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[2]For reviews see [11][12][13].

The Afs independent part of the proper solution, $\mathcal{S}_\Sigma(\Phi)$, has no more gauge invariance and gives well defined propagators. It is invariant under the Afs independent gauge fixed BRST operator δ_Σ that acts on local functionals $F(\Phi)$ of the fields by

$$\delta_\Sigma \cdot = (\cdot, \hat{\mathcal{S}})|_{K=0}. \tag{2.10}$$

One can verify that δ_Σ is nilpotent on-shell,

$$\delta_\Sigma^2 \simeq 0. \tag{2.11}$$

Here the weak equality "$\simeq$" means that it holds up to gauge fixed equations of motion $\mathcal{S}_{\Sigma,A} \simeq 0$. Nilpotent operators δ and δ_Σ are associated with the antibracket cohomology and the gauge fixed weak cohomology respectively. Their relation has been studied in [17].

   The quantum aspects of the BRST formalism are most suitably studied in terms of the effective action Γ which is obtained from the (connected part of) the generating functional by a Legendre transformation with respect to the sources J_A. We will consider a PV regularization at one loop level. The generating functional is

$$Z_{\text{reg}}(J, K) = \int \mathcal{D}\Phi \mathcal{D}\chi \exp\left\{\frac{i}{\hbar}\left[\hat{\mathcal{S}}(\Phi, K) + \hbar\hat{\mathcal{M}}_1(\Phi, K) + S_{\text{PV}}(\chi, \chi^* = 0; \Phi, K) + J_A\Phi^A\right]\right\}, \tag{2.12}$$

where χ^A are the PV fields. Each PV field χ^A comes with its associated antifield χ_A^*, and they can collectively be denoted as $w^a = \{\chi^A, \chi_A^*\}$, $a = 1, \ldots, 2N$. PV antifields χ_A^* have no physical significance and are put to zero at the end. The local counter term $\hat{\mathcal{M}}_1(\Phi, K)$ should guarantee the finiteness of theory while preserve the BRST structure at quantum level if it is possible.

   The PV action S_{PV} is determined from two requirements: i) massless propagators and couplings for PV fields should coincide with those of their partners and ii) BRST transformations for PV fields should be such that the massless part of the PV action, $S_{\text{PV}}^{(0)}$, and the measure in (2.12) be BRST invariant up to one loop. A suitable prescription for S_{PV} is [5] [13]

$$S_{\text{PV}} = S_{\text{PV}}^{(0)} + S_M = \frac{1}{2}w^a S_{ab}w^b - \frac{1}{2}M\chi^A T_{AB}\chi^B, \tag{2.13}$$

with the mass matrix T_{AB} being invertible but otherwise arbitrary and S_{ab} is defined by

$$S_{ab} = \left(\frac{\partial_l}{\partial z^a}\frac{\partial_r}{\partial z^b}\hat{S}(\Phi, K)\right), \tag{2.14}$$

where $z^a = \{\Phi^A, K_A\}$ and l, r stand for left and right derivatives.

   Application of the semiclassical approximation to (2.12) yields the regularized effective action up to one loop [6]

$$\Gamma(\Phi, K) = \hat{\mathcal{S}}(\Phi, K) + \hbar\hat{\mathcal{M}}_1(\Phi, K) + \frac{i\hbar}{2}\text{Tr Ln}\left[\frac{(T\hat{\mathcal{R}})}{(T\hat{\mathcal{R}}) - (TM)}\right] \equiv \hat{\mathcal{S}}(\Phi, K) + \hbar\Gamma_1(\Phi, K). \tag{2.15}$$

Here Tr stands for the supertrace and $(T\hat{\mathcal{R}})_{AB}$ is defined from (2.14) as

$$(T\hat{\mathcal{R}})_{AB} = \left(\frac{\partial_l}{\partial\Phi^A}\frac{\partial_r}{\partial\Phi^B}\hat{S}(\Phi, K)\right). \tag{2.16}$$

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The denominator of the "Tr Ln" term in (2.15) is the contribution from the integrations over the PV fields and the numerator is the one from the quantum fluctuation of Φ. The PV regulator $\hat{\mathcal{R}}$ is obtained from (2.16) and the form of the mass matrix T_{AB} in the PV action (2.13).

3 Antifield Dependence of the Anomaly

Once we obtain the effective action we can calculate the anomalous Slavnov-Taylor identity [18][19][20]

$$(\Gamma, \Gamma) = -i\hbar \hat{\mathcal{A}}_1 \cdot \Gamma. \tag{3.1}$$

Up to one loop we have

$$(\Gamma_1, \hat{\mathcal{S}}) = -i\hat{\mathcal{A}}_1 \tag{3.2}$$

where

$$\hat{\mathcal{A}}_1 =: \Delta\hat{\mathcal{S}} + i(\hat{\mathcal{M}}_1, \hat{\mathcal{S}}) \tag{3.3}$$

is the potential anomaly. More explicitly using (2.15), (3.2) and (3.3) we obtain [6][22]

$$\Delta\hat{\mathcal{S}}(\Phi, K) = \delta \left\{ -\frac{1}{2} \text{TrLn} \left[\frac{\hat{\mathcal{R}}(\Phi, K)}{\hat{\mathcal{R}}(\Phi, K) - M} \right] \right\}. \tag{3.4}$$

$\hat{\mathcal{A}}_1$ measures the BRST non invariance of the effective action and $\Delta\hat{\mathcal{S}}$ reflects the non invariance of the path integral measure

$$\mathcal{D}\Phi \longrightarrow \mathcal{D}\Phi(1 + \Delta\hat{\mathcal{S}}) \tag{3.5}$$

under the classical BRST transformations $\delta\Phi^A = (\Phi^A, \hat{\mathcal{S}})$. The form of (3.4) shows $\Delta\hat{\mathcal{S}}(\Phi, K)$ is satisfying the Wess-Zumino consistency conditions (WZCC), $\delta(\Delta\hat{\mathcal{S}}) = 0$.

$\Delta\hat{\mathcal{S}}(\Phi, K)$ can be rewritten as

$$\Delta\hat{\mathcal{S}}(\Phi, K) = \text{Tr} \left[-\frac{1}{2}(\hat{\mathcal{R}}^{-1}\delta\hat{\mathcal{R}}) \frac{1}{(1 - \frac{\hat{\mathcal{R}}}{M})} \right]. \tag{3.6}$$

This parametrization of the anomaly is very important in order to study the antifield dependence. Furthermore this expression shows that (potentially) anomalous symmetries are directly related with the transformation properties of the regulator $\hat{\mathcal{R}}$. In particular, if $\hat{\mathcal{R}}$ is invariant under some subset of symmetries or it is transformed as $\delta\hat{\mathcal{R}} = [\hat{\mathcal{R}}, \mathcal{G}]$ by some operator $\mathcal{G}$, the Tr of (3.6) leads to a vanishing result. Notice that the anomaly depends on both fields and antifields. In order to study this dependence it is useful to expand the PV regulator $\hat{\mathcal{R}}(\Phi, K)$ with respect to the Afs K_A, [6]

$$\hat{\mathcal{R}}(\Phi, K) = \mathcal{R}_\Sigma(\Phi) + K_A \mathcal{R}_1^A(\Phi) + \frac{1}{2!} K_A K_B \mathcal{R}_2^{BA}(\Phi) + \dots. \tag{3.7}$$

It follows

$$
\begin{aligned}
\Delta \hat{\mathcal{S}}(\Phi, K) \quad &= \delta_\Sigma \left\{ -\frac{1}{2} \mathrm{TrLn} \left[\frac{\mathcal{R}_\Sigma}{\mathcal{R}_\Sigma - M} \right] \right\} \\
&\quad - \mathrm{Tr} \left\{ -\frac{1}{2} \mathcal{R}_\Sigma^{-1} \mathcal{R}_1^A \left[\frac{1}{1 - \frac{\mathcal{R}_\Sigma}{M}} \right] \right\} \frac{\delta_l \mathcal{S}_\Sigma}{\delta \Phi^A} + \mathcal{O}(K) \\
&=: (\Delta \mathcal{S})_\Sigma(\Phi) - P^A(\Phi) \frac{\delta_l \mathcal{S}_\Sigma}{\delta \Phi^A} + \mathcal{O}(K) \,.
\end{aligned}
\tag{3.8}
$$

Note that in general $\Delta \mathcal{S}_\Sigma(\Phi)$ is weakly invariant under gauge fixed BRST transformations i.e,
$\delta_\Sigma \Delta \mathcal{S}_\Sigma \simeq 0$. For integrals of local functions with ghost numbers greater or equal to zero we
have an isomorphism between the antibracket cohomology (2.6) and the gauge fixed BRST
cohomology (2.11)[17]. This fact enables us to reconstruct $\Delta \hat{\mathcal{S}}(\Phi, K)$ from the part of $\Delta \mathcal{S}_\Sigma(\Phi)$
depending only on the minimal fields since the non-minimal sector (antighost, extraghost,
B-fields) is cohomologically trivial. For theories with closed algebras and reducible off-shell
algebras, their actions are linear in antifields and thus $\delta = \delta_\Sigma$, $\Delta \mathcal{S}_\Sigma$ is off shell BRST invariant.
For these theories the antifield dependent part of the anomaly is trivial. However, using the
WZ consistency conditions and cohomological techniques it is possible to construct for closed
theories antifield dependent candidate anomalies [9]; these candidates cannot appear in any
regularized field theory calculation. The regularization procedure selects a subset of the ghost
number one nontrivial cocycles.

So far we have discussed the anomalies in GFxB. To write the WZ terms it is useful to
work in the classical basis as we will see in the next section. We can pass to the classical
basis by performing a canonical transformation; the inverse of (2.8). Since the Jacobian of the
transformation is 1 the expression $\Delta \mathcal{S}(\Phi, \Phi^*)$ in ClB is given by expressing $\Delta \hat{\mathcal{S}}(\Phi, K)$ in terms
of the variables Φ, Φ^* in ClB,

$$
\Delta \mathcal{S}(\Phi, \Phi^*) \equiv \Delta \hat{\mathcal{S}}(\Phi, \Phi_A^* - \frac{\delta \Psi(\Phi)}{\delta \Phi^A}) = \left(-\frac{1}{2} \mathrm{TrLn} \left[\frac{\mathcal{R}(\Phi, \Phi^*)}{\mathcal{R}(\Phi, \Phi^*) - M} \right], \mathcal{S}(\Phi, \Phi^*) \right)
\tag{3.9}
$$

Here $\mathcal{R}(\Phi, \Phi^*) = \hat{\mathcal{R}}(\Phi, \Phi_A^* - \frac{\delta \Psi(\Phi)}{\delta \Phi^A})$ is the regulator in the ClB.

In order to exhibit the antifield dependence it is useful to consider first the following BRST
operator in classical basis. It acts on functions depending only on fields as

$$
\delta_0 \cdot = (\cdot , \mathcal{S})|_{\Phi^* = 0}.
\tag{3.10}
$$

It is nilpotent on shell, i.e. on the classical equations of motion; $\mathcal{S}_{0,j} \approx 0$;

$$
\delta_0^2 \approx 0.
\tag{3.11}
$$

Next by expanding $\mathcal{R}(\Phi, \Phi^*)$ with respect to Φ^*

$$
\Delta \mathcal{S}(\Phi, \Phi^*) = \delta_0 \left(-\frac{1}{2} \mathrm{TrLn} \left[\frac{\mathcal{R}(\Phi)}{\mathcal{R}(\Phi) - M} \right] \right) - P^i(\phi) \frac{\delta_l \mathcal{S}_0}{\delta \phi^i} + \mathcal{O}(\Phi^*).
\tag{3.12}
$$

Using the fact that the non-minimal sector is cohomologically trivial it can be written as

$$
\Delta \mathcal{S}(\Phi, \Phi^*) = \mathcal{A}_\alpha(\phi) c^\alpha + (P^i \phi_i^* + N, \mathcal{S}) + \mathcal{O}(\Phi^*),
$$

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where
$$\mathcal{A}_\alpha(\phi)c^\alpha = \delta_0(-\frac{1}{2}\text{TrLn}\left[\frac{\mathcal{R}(\phi)}{\mathcal{R}(\phi) - M}\right]) \qquad (3.13)$$

and $\mathcal{R}(\phi)$ depends only on the classical fields. N is the local counter term that reproduces the contribution from non-minimal fields. Notice that $\delta_0\mathcal{A}_\alpha(\phi)c^\alpha \approx 0$. As in the gauge fixed basis the whole anomaly in the classical basis $\Delta\mathcal{S}(\Phi, \Phi^*)$ can be reconstructed once the antifield independent part is known [21][22]. Only for theories with on shell algebras the anomaly will depend in a nontrivial way on the antifields.

4 The WZ term

If $\Delta\hat{\mathcal{S}}(\Phi, K)$ found in GFxB is different from zero the path integral measure is not BRST invariant. In some cases it is possible to restore the BRST invariance by a suitable choice of the local counter term $\hat{\mathcal{M}}_1(\Phi, K)$ such that $\hat{\mathcal{A}}_1 = 0$. In case no local counter term exists we have a genuine anomalous theory. Physically it means that some classical gauge degrees of freedom turn to be dynamical at quantum level. We can consider the gauge parameters as these extra degrees of freedom. A natural question arises, when we enlarge the space of fields to Φ^4 and these extra degrees of freedom θ^α [23][24], whether it is possible to find a local counter term $\hat{\mathcal{M}}_1(\Phi, \theta^\alpha, K)$ such that its BRST variation $\tilde{\delta}$ in the extended space gives the anomaly,

$$\tilde{\delta}\hat{\mathcal{M}}_1(\Phi, \theta^\alpha, K) = i\hat{\mathcal{A}}_1(\Phi, K). \qquad (4.1)$$

This local counter term becomes the WZ term in the extended formalism. Due to the cohomological reconstruction procedure it is sufficient to find its antifield independent part $\hat{\mathcal{M}}_1(\Phi, \theta^\alpha)$ verifying

$$\tilde{\delta}_\Sigma\hat{\mathcal{M}}_1(\Phi, \theta^\alpha) \simeq i\hat{\mathcal{A}}_1(\Phi), \qquad (4.2)$$

where $\hat{\mathcal{A}}_1(\Phi)$ is the antifield independent part of the anomaly in gauge fixed basis. A solution of this equation is given by a sum of general solution of the homogeneous equation and a particular solution. The latter is given by the antifield independent part of the effective action itself; $-\Gamma_1(\Phi)$.

There remains to find the general solution of the homogeneous equation. From the Yang-Mills case [10] we know the relevance of the finite gauge transformation. Here we will use the finite on-shell structure, see appendix. In contrast to the infinitesimal gauge or the associated BRST transformations, the finite action of the gauge group in the space of all fields Φ^4 is not known. We know the finite transformations for the classical fields ϕ^i. The composition of two of these transformations closes only on the classical equations of motion. This is the reason why we can give the general solution of the homogeneous equation in a compact form only in ClB and not in GFxB.

Therefore to find the WZ term we first consider an equation in ClB which is corresponding to (4.2) in GFxB

$$\tilde{\delta}_0\mathcal{M}_{1,0}(\Phi, \theta) \approx i\mathcal{A}_{0,\alpha}(\phi)c^\alpha, \qquad (4.3)$$

where $\mathcal{A}_{0,\alpha}(\phi)c^\alpha = \mathcal{A}_1(\Phi, \Phi^* = 0)$ and $\mathcal{M}_{1,0}(\Phi, \theta)$ are the Afs independent part of the anomaly and the WZ term, respectively, in ClB. The interesting property of the classical basis is that we

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can find a solution of $\mathcal{M}_{1,0}(\Phi,\theta)$ depending only on ϕ and θ. A particular non-local solution of (4.3) have been obtained in (3.13).

We can write the general solution of the homogeneous equation as an arbitrary function of variable $F^i(\phi,\theta)$, which is the finite transformation of ϕ^i, because we can introduce transformation properties of the extra variables (gauge parameters) such that $\tilde{\delta}_0 F^i(\phi,\theta) \approx 0$. In fact using a relation obtained from the on-shell composition law (A.13) and

$$0 \approx \tilde{\delta}_0 F^i(\phi,\theta) = \frac{\partial F^i(\phi,\theta)}{\partial \phi^j} \, R^j_\alpha(\phi) \, \epsilon^\alpha + \frac{\partial F^i(\phi,\theta)}{\partial \theta^\beta} \, \delta\theta^\alpha \tag{4.4}$$

we find

$$\delta\theta^\alpha = -\tilde{\mu}^\alpha{}_\beta(\theta,\phi)\epsilon^\beta + Z^\alpha_a(\theta,\phi)\epsilon^a, \tag{4.5}$$

where $\tilde{\mu}^\beta{}_\alpha(\theta,\phi) := \left.\frac{\partial\varphi^\beta(\theta',\theta;\phi)}{\partial\theta'^\alpha}\right|_{\theta'=0}$ and $Z^\alpha_a(\theta,\phi) := \left.\frac{\partial f^\alpha(\theta,\epsilon,\phi)}{\partial\epsilon^a}\right|_{\epsilon=0}$ is a nullvector of $\frac{\partial F^i(\phi,\theta)}{\partial\theta^\beta}$, see (A.15). The algebra of transformations of ϕ and θ remains to be open and on-shell reducible. Therefore we have a new realization of the on-shell structure.

Now we can write a solution of (4.3) as a sum of the particular solution and the general solution of the homogeneous equation,

$$\mathcal{M}_{1,0}(\phi,\theta) = \mathcal{M}^{non}_{1,0}(\phi) + G(F(\phi,\theta)), \tag{4.6}$$

where $\mathcal{M}^{non}_{1,0}(\phi)$ is obtained from (3.13). The function $G(F)$ in (4.6) is fixed if we impose a 1-cocycle condition for the on shell structure

$$\mathcal{M}_{1,0}(\phi,\varphi(\theta,\theta';\phi)) \approx \mathcal{M}_{1,0}(F(\phi,\theta),\theta') + \mathcal{M}_{1,0}(\phi,\theta), \tag{4.7}$$

we get

$$\mathcal{M}_{1,0}(\phi,\theta) \quad \approx \mathcal{M}^{non}_{1,0}(\phi) - \mathcal{M}^{non}_{1,0}(F^i(\phi,\theta)). \tag{4.8}$$

If we write (4.8) as a surface integral in a variable t and we use the on-shell Lie equations (A.11) we have

$$\mathcal{M}_{1,0}(\phi,\theta) \quad \approx -\int_0^1 dt \frac{d}{dt}\mathcal{M}^{non}_{1,0}(F(\phi,t\theta)) = -\int_0^1 dt \frac{\partial\mathcal{M}^{non}_{1,0}(F(\phi,t\theta))}{\partial F^i} \frac{\partial F^i(\phi,t\theta)}{\partial(t\theta^\beta)}\theta^\beta$$

$$\approx -\int_0^1 dt \left.\frac{\partial\mathcal{M}^{non}_{1,0}}{\partial\phi^i}\right|_{\phi=F(\phi,t\theta)} R^i_\alpha(F(\phi,t\theta))\lambda^\alpha_\beta(t\theta,\phi)\theta^\beta , \tag{4.9}$$

which can be written using (3.13) as

$$\mathcal{M}_{1,0}(\phi,\theta) = -i\int_0^1 dt\, \mathcal{A}_{0,\alpha}(F(\phi,t\theta))\lambda^\alpha_\beta(t\theta,\phi)\theta^\beta. \tag{4.10}$$

Notice that this antifield independent part of the WZ term in the classical basis has the same form as that for closed theories and off-shell reducible theories [10][24][6][26]. Now applying the Afs perturbative method we can find the full WZ term

$$\mathcal{M}_1(\tilde{\Phi},\tilde{\Phi}^*) = \mathcal{M}_{1,0}(\phi,\theta) + \tilde{\Phi}^*_A \mathcal{M}^A_{1,1}(\tilde{\Phi}) + \frac{1}{2}\tilde{\Phi}^*_A\tilde{\Phi}^*_B\mathcal{M}^{BA}_{1,2}(\tilde{\Phi}) + ..., \tag{4.11}$$

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where $\tilde{\Phi}$ is a collective notation of Φ and θ. It is important to emphasize that the higher terms such $\mathcal{M}_{1,1}^A(\tilde{\Phi})$ can not be written in a closed form in terms of the finite transformation.

The WZ term in the gauge fixed basis is obtained by means of the canonical transformation (2.8). The antifield independent part will have the form

$$\hat{\mathcal{M}}_1(\tilde{\Phi}) = \mathcal{M}_{1,0}(\phi, \theta) + \mathcal{M}_{1,1}^A(\tilde{\Phi})\left(\frac{\delta\Psi(\tilde{\Phi})}{\delta\tilde{\Phi}^A}\right) + ... \tag{4.12}$$

and is a solution of (4.2). The reconstruction procedure enables us to find the whole WZ term $\hat{\mathcal{M}}_1(\tilde{\Phi}, K)$ in the gauge fixed basis from the knowledge of $\hat{\mathcal{M}}_1(\tilde{\Phi})$. Note that these expressions are not written in terms of the finite transformation.

5 Conclusions

We have constructed the one loop effective action for general gauge theories in a PV regularization scheme. The non-invariance of the effective action gives the anomaly. The anomaly is parametrized in terms of the PV regulator and the BRST transformation. This parametrization is very useful in order to study the antifield dependence. The anomaly depends on fields and antifields in a non-trivial way for theories with an on-shell structure, *i.e.* theories with an open algebra or reducible on-shell algebra.

Introducing extra degrees of freedom corresponding to the classical gauge degrees of freedom they become dynamical at quantum level. We have constructed the local WZ term which is depending in general on the antifields. The antifield independent part, in the classical basis, has the usual form in terms of the anomaly and the finite gauge transformations. The full WZ term is obtained using the Afs perturbation methods. The WZ term in gauge fixed basis is obtained by the straightforward canonical transformation.

In this paper we did not discuss the issue of quantization of the extra variables, which will be discussed in a future work where we will also study in detail the finite form of the gauge structure and the corresponding infinitesimal structure functions.

A On-shell Quasigroup Structure

Let us consider the action of an on-shell quasigroup G which is locally described by a set of parameters θ^α on a manifold $\mathcal{M}$ parametrized by the classical fields ϕ^i

$$\begin{aligned} F : \mathcal{M} \times \mathcal{G} &\to \mathcal{M} \\ (\phi^i, \; \theta^\alpha) &\mapsto F^i(\phi, \theta) \end{aligned} \tag{A.1}$$

with the following properties [3].

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[3]Here we only write the relevant properties of the on-shell structure useful for the discussion below.

i)   Invariance of the action;

$$\mathcal{S}_0(F(\phi,\theta)) = \mathcal{S}_0(\phi). \tag{A.2}$$

As a consequence we have the following relation between the equations of motion

$$S_{0,j}(F(\phi,\theta)) \;=\; \tilde{S}_j{}^k(\phi,\theta)S_{0,k}(\phi). \tag{A.3}$$

ii)   On-shell redundancy in the parametrization;

$$F^i(\phi, f(\theta,\epsilon;\phi)) = F^i(\phi,\theta) + \Psi^{ij}(\theta,\epsilon;\phi)\mathcal{S}_{0,j}(\phi) \tag{A.4}$$

where $f(\theta,\epsilon;\phi)$ and $\Psi^{ij}(\theta,\epsilon;\phi)$ represents the on-shell reducibility.

iii)   Composition law;

$$F^i(F(\phi,\theta),\theta') = F^i(\phi,\varphi(\theta,\theta';\phi)) + M^{ij}(\theta,\theta';\phi)\mathcal{S}_{0,j}(\phi), \tag{A.5}$$

where $\varphi^\alpha(\theta,\theta';\phi)$ represents the composition function of the parameters of quasigroup G and $M^{ij}(\theta,\theta';\phi)$ represents the open character of the finite gauge transformations.

iv)   Associativity law;

$$\varphi^\alpha\left(\varphi\left(\theta,\theta';\phi\right),\theta'';\phi\right) = f^\alpha\left(\varphi\left(\theta,\varphi\left(\theta',\theta'';F(\phi,\theta)\right);\phi\right),\eta\left(\theta,\theta',\theta'';\phi\right),\phi\right) + M^{\alpha i}(\theta,\theta',\theta'';\phi)\mathcal{S}_{0,i}(\phi), \tag{A.6}$$

where $M^{\alpha i}(\theta,\theta',\theta'';\phi)$ and $\eta\left(\theta,\theta',\theta'';\phi\right)$ represent the modified on-shell associativity law for the parameters.

v)   On-shell structure of the on-shell functions M^{ij}

$$\{M^{ij}\left(\theta,\varphi(\theta',\theta'';F(\phi,\theta));\phi\right) + M^{ik}(\theta',\theta'';F(\phi,\theta))\tilde{S}_k{}^j(\phi,\theta)\,\}$$

$$-\,\{M^{ij}(\varphi(\theta,\theta',\phi),\theta'';\phi) \;+\; \frac{\partial F^i(F(\phi,\varphi(\theta,\theta';\phi)),\theta'')}{\partial F^k}M^{kj}(\theta,\theta';\phi)\;+$$

$$\frac{\partial F^i\left(\phi,f(\varphi(\theta,\varphi(\theta',\theta'';F(\phi,\theta));\phi)),\eta(\theta,\theta',\theta'';\phi);\phi\right)}{\partial f^\alpha}M^{\alpha j}(\theta,\theta',\theta'';\phi)\;+$$

$$+\,\Psi^{ij}\left(\varphi(\theta,\varphi(\theta',\theta'';F(\phi,\theta));\phi),\eta(\theta,\theta',\theta'';\phi);\phi\right)\,\} \;+\; \frac{\partial F^k(\phi,\theta)}{\partial\theta^\alpha}M^{\alpha i}(\theta,\theta',\theta'';\phi)$$

$$=\; -\,M^{ijk}(\theta,\theta',\theta'';\phi)S_{0,k}(\phi), \tag{A.7}$$

where $M^{ijk}(\theta,\theta',\theta'',\phi)$ represents the non-closure of the on-shell structure.

In a general situation the functions M^{ijk} are not close on-shell. New structure functions appear when we perform the composition of four or more transformations.

The structure functions appearing in the solution of the classical master equation are directly related to these functions *only in the classical basis*. For example those appearing in the algebra (2.1) and (2.2) are given as

$$R^i{}_\alpha(\phi) \;:=\; \left.\frac{\partial F^i(\phi,\theta)}{\partial\theta^\alpha}\right|_{\theta=0}, \qquad T^\gamma_{\alpha\beta}(\phi) := -\left(\frac{\partial^2\varphi^\gamma(\theta,\theta';\phi)}{\partial\theta^\alpha\partial\theta'^\beta} - (\beta\leftrightarrow\alpha)\right)_{\theta=\theta'=0},$$

$$E^{ij}_{\alpha\beta}(\phi) := -\left(\frac{\partial M^{ij}(\theta,\theta';\phi)}{\partial\theta^{\alpha}\theta'^{\beta}} - (\beta\leftrightarrow\alpha)\right)\Bigg|_{\theta=\theta'=0},$$

$$Z^{\alpha}_{a}(\phi) := \frac{\partial f^{\alpha}(\theta,\epsilon,\phi)}{\partial\epsilon^{a}}\Bigg|_{\theta=\epsilon=0}, \qquad V^{ij}_{a}(\phi) := -\frac{\partial\Psi^{ij}(\theta,\epsilon,\phi)}{\partial\epsilon^{a}}\Bigg|_{\theta=\epsilon=0} \qquad (A.8)$$

The ones appearing in the generalized Jacobi identity (2.3) are :

$$F^{a}_{\alpha\beta\gamma}(\phi) := -\frac{1}{3}\sum_{P\in\mathrm{Perm}[\alpha\beta\gamma]}(-1)^{P}\left(\frac{\partial^{3}\eta^{a}(\theta,\theta',\theta'',\phi)}{\partial\theta^{\alpha}\partial\theta'^{\beta}\partial\theta''^{\gamma}}\right)_{\theta=\theta'=\theta''=0} \qquad (A.9)$$

and

$$D^{\nu i}_{\alpha\beta\gamma}(\phi) := \frac{1}{3}\sum_{P\in\mathrm{Perm}[\alpha\beta\gamma]}(-1)^{P}\left(\frac{\partial^{3}M^{\nu i}(\theta,\theta',\theta'',\phi)}{\partial\theta^{\alpha}\partial\theta'^{\beta}\partial\theta''^{\gamma}}\right)_{\theta=\theta'=\theta''=0}. \qquad (A.10)$$

A detail analysis of the on-shell structure and the relation with the work of [25] will be published elsewhere.

From the on-shell composition law (A.5) we can obtain the on-shell Lie equation by multiplying an operator $\frac{\partial}{\partial\theta'}\big|_{\theta'=0}$ on it. It is explicitly

$$\frac{\partial F^{i}(\phi,\theta)}{\partial\theta^{\alpha}} = R^{i}_{\beta}(F(\phi,\theta))\lambda^{\beta}_{\alpha}(\theta,\phi) - \lambda^{\beta}_{\alpha}(\theta,\phi)\frac{\partial M^{ij}(\theta,\theta';\phi)}{\partial\theta'^{\beta}}\Bigg|_{\theta'=0}\mathcal{S}_{0,j}(\phi), \qquad (A.11)$$

where $\lambda^{\beta}_{\alpha}(\theta,\phi)$ is the inverse matrix of

$$\mu^{\alpha}_{\beta}(\theta,\phi) = \frac{\partial\varphi^{\alpha}(\theta,\theta',\phi)}{\partial\theta'^{\beta}}\Bigg|_{\theta'=0}. \qquad (A.12)$$

On the other hand if we operate $\frac{\partial}{\partial\theta}\big|_{\theta=0}$ on (A.5) we have

$$\frac{\partial F^{i}(\phi,\theta)}{\partial\phi^{k}}R^{k}_{\alpha}(\phi) = \frac{\partial F^{i}(\phi,\theta)}{\partial\theta^{\beta}}\tilde{\mu}^{\beta}_{\alpha}(\theta,\phi) + \frac{\partial M^{ij}(\theta',\theta;\phi)}{\partial\theta'^{\beta}}\Bigg|_{\theta'=0}\mathcal{S}_{0,j}(\phi), \qquad (A.13)$$

where

$$\tilde{\mu}^{\beta}_{\alpha}(\theta,\phi) := \frac{\partial\varphi^{\beta}(\theta',\theta;\phi)}{\partial\theta'^{\alpha}}\Bigg|_{\theta'=0}. \qquad (A.14)$$

Finally applying operator $\frac{\partial}{\partial\epsilon}\big|_{\epsilon=0}$ on (A.4) we get

$$\frac{\partial F^{i}(\phi,\theta)}{\partial\theta^{\beta}}Z^{\beta}_{a}(\theta,\phi) = \frac{\partial\Psi^{ij}(\theta,\epsilon;\phi)}{\partial\epsilon^{a}}\Bigg|_{\epsilon=0}\mathcal{S}_{0,j}(\phi), \qquad (A.15)$$

where

$$Z^{\alpha}_{a}(\theta,\phi) := \frac{\partial f^{\alpha}(\theta,\epsilon,\phi)}{\partial\epsilon^{a}}\Bigg|_{\epsilon=0}. \qquad (A.16)$$

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