HYPERON POLARIZABILITIES IN THE BOUND STATE SOLITON MODEL

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ABSTRACT

A detailed calculation of electric and magnetic static polarizabilities of octet hyperons is presented in the framework of the bound state soliton model. Both seagull and dispersive contributions are considered, and the results are compared with different model predictions.

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1 Introduction

The electromagnetic polarizabilities are quantities of fundamental interest in the understanding of hadron structure [1]. They characterize the dynamical response of the hadron to external electromagnetic fields. While a rather large amount of work has been devoted, both theoretically and experimentally, to the study of the proton and neutron polarizabilities (see e.g. Refs.[2, 3] for recent experimental and theoretical reviews, respectively) very little is known about the hyperon polarizabilities. However, with the advent of hyperon beams at FNAL and CERN, the experimental situation is likely to change. In particular, Σ hyperon polarizabilities will be soon measured in the Fermilab E781 SELEX experiment [4, 5]. This has triggered a number of theoretical investigations in different hadron models. In fact, predictions have been made in the framework of the non-relativistic quark model (NRQM) [6] and heavy baryon chiral perturbation theory (HBCPT) [7]. As it is well-known the above-mentioned models have a few problems in describing baryon magnetic polarizabilities. Within the NRQM the large diamagnetic contribution to the nucleon magnetic polarizability is rather difficult to understand. In the case of HBCPT predictions are not expected to be very accurate unless the contributions due to P-wave excitations (Δ-like), which are of higher order in the chiral expansion, are included.

It is, therefore, interesting to attempt a description based on a completely different point of view, like the one given by the topological (Skyrme) soliton model. Within the chiral soliton model only electric hyperon polarizabilities have been so far studied [8]. In the present work we will explore the static electric and magnetic polarizabilities using the bound-state soliton model[9, 10], which has already given good results for hyperon magnetic moments and mean square radii[11, 12].

This article is organized as follows: In Sec.2 we introduce the soliton model effective action in the presence of e.m. fields. In Sec.3 we briefly discuss how hyperons are described in the bound state approach and in Sec.4 we calculate the static electric and magnetic polarizabilities. Numerical results are reported in Sec.5, while Sec.6 contains the conclusions. In Appendix A we estimate the dispersive contributions to the hyperon electric polarizability. In Appendix B and in Appendix C we give the explicit expressions of the (elementary) polarizabilities and magnetic moments respectively.

2 The effective action in the presence of electromagnetic fields

Our starting point is a gauged effective chiral action with an appropriate symmetry breaking term. It has the form

$$\Gamma = \Gamma_{SK} + \Gamma_{an} + \Gamma_{sb} \tag{1}$$

where Γ_{SK} is the gauged Skyrme action

$$\Gamma_{SK} = \int d^4x \Big\{ \frac{f_\pi^2}{4} \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger \right] + \frac{1}{32\epsilon^2} \mathrm{Tr}\left[[U^\dagger D_\mu U, U^\dagger D_\nu U]^2 \right] \Big\} . \tag{2}$$

Here f_π is the pion decay constant ($= 93\ MeV$ empirically), ϵ is a dimensionless constant (the so-called Skyrme parameter) and U is the $SU(3)$ valued chiral field. The covariant derivative is defined as

$$D_\mu U = \partial_\mu U + ie\ A_\mu\ [Q, U] \tag{3}$$

where A_μ is the electromagnetic field and Q the electric charge operator

$$Q = \frac{1}{2}\left[\lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \right] . \tag{4}$$

Moreover, e represents the elementary electric charge. Since we are using Gaussian units throughout the paper, in the following we adopt $e^2 = 1/137$.

Γ_{an} is the gauged Wess-Zumino action which for the electromagnetic case we are interested in reads[13]:

$$\begin{aligned} \Gamma_{an} &= -\frac{iN_c}{240\pi^2} \int \mathrm{Tr}[(U^\dagger dU)^5] \\ &\quad -\frac{N_c}{48\pi^2} \int d^4x\ \epsilon^{\mu\nu\rho\sigma} \Big\{ eA_\mu\ \mathrm{Tr}\left[Q \left(L_\nu L_\rho L_\sigma - R_\nu R_\rho R_\sigma \right) \right] \\ &\qquad -ie^2 A_\mu \partial_\nu A_\rho\ \mathrm{Tr}\left[2\ Q^2 (L_\sigma - R_\sigma) + Q U^\dagger Q U L_\sigma - Q U Q U^\dagger R_\sigma \right] \Big\} , \end{aligned} \tag{5}$$

where $L_\mu = U^\dagger \partial_\mu U$, $R_\mu = U \partial_\mu U^\dagger$ and N_c is the number of colors. Finally, $\Gamma_{\rm sb}$ is the symmetry breaking term [14]:

$$\begin{aligned} \Gamma_{sb} &= \int d^4x \Big\{ \frac{f_\pi^2 m_\pi^2 + 2 f_K^2 m_K^2}{12} \mathrm{Tr}\left[U + U^\dagger - 2 \right] \\ &\quad + \sqrt{3} \frac{f_\pi^2 m_\pi^2 - f_K^2 m_K^2}{6} \mathrm{Tr}\left[\lambda_8 \left(U + U^\dagger \right) \right] \\ &\quad + \frac{f_K^2 - f_\pi^2}{12} \mathrm{Tr}\left[(1 - \sqrt{3}\lambda_8) \left(U (D_\mu U)^\dagger D^\mu U + U^\dagger D_\mu U (D^\mu U)^\dagger \right) \right] \Big\} , \end{aligned} \tag{6}$$

where f_K is the kaon decay constant and m_π and m_K are the pion and kaon masses respectively.

For our purposes, the effective action can be more conveniently written as

$$\Gamma = \Gamma^{strong} + \Gamma^{lin} + \Gamma^{quad} \,, \tag{7}$$

where we have singled out the contributions linear and quadratic in the e.m. field:

$$\begin{aligned} \Gamma^{lin} &= \int d^4x \; e \; A_\mu J^\mu \,, & (8) \\ \Gamma^{quad} &= -\int d^4x \; e^2 \; A_\mu \; G^{\mu\nu} \; A_\nu \,. & (9) \end{aligned}$$

Here:

$$\begin{aligned} J^\mu \;=\; & i\frac{f_\pi^2}{2}\mathrm{Tr}\left\{Q(L^\mu + R^\mu)\right\} \\ & + i\frac{f_K^2 - f_\pi^2}{12}\mathrm{Tr}\left\{(1-\sqrt{3}\lambda_8)\left([U,Q]L^\mu - L^\mu[U^\dagger,Q] + [U^\dagger,Q]R^\mu - R^\mu[U,Q]\right)\right\} \\ & - \frac{i}{8\epsilon^2}\mathrm{Tr}\left\{Q\left([L_\nu,[L^\mu,L^\nu]] + [R_\nu,[R^\mu,R^\nu]]\right)\right\} \\ & - \frac{N_c}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left\{Q\left(L_\nu L_\rho L_\sigma - R_\nu R_\rho R_\sigma\right)\right\} \,, \end{aligned} \tag{10}$$

and

$$\begin{aligned} G^{\mu\nu} \;=\; & -g^{\mu\nu}\left[\frac{f_\pi^2}{4}\mathrm{Tr}P^2 + \frac{f_K^2 - f_\pi^2}{12}\mathrm{Tr}\left\{(1-\sqrt{3}\lambda_8)(P^2U^\dagger + UP^2)\right\}\right] \\ & + \frac{1}{8\epsilon^2}\left[g^{\mu\nu}h^\alpha_\alpha - h^{\mu\nu}\right] \\ & + \frac{iN_c}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left[(2Q^2 + QU^\dagger QU)L_\sigma - (2Q^2 + QUQU^\dagger)R_\sigma\right]\partial_\rho \,, \end{aligned} \tag{11}$$

where the following definitions have been used:

$$\begin{aligned} P &= Q - U^\dagger \; Q \; U \,, & (12) \\ h_{\mu\nu} &= \mathrm{Tr}\left[PL_\mu PL_\nu - P^2 L_\nu L_\mu\right] \,. & (13) \end{aligned}$$

In Eq.(7) Γ^{strong} is the action in the absence of the electromagnetic field. It describes the strong interactions that give rise to the hyperon. In the next section it will be treated following the usual steps of the bound state model.

3 Hyperons in the bound state soliton model

The bound state soliton model has been discussed in great detail in the literature (see e.g. Refs.[9, 10]). Therefore, in this section we will only present the main features of the model. Following Ref.[9] we introduce the Callan–Klebanov ansatz

$$U = \sqrt{U_\pi}\, U_K \, \sqrt{U_\pi} \;, \tag{14}$$

where

$$U_K = \exp\left[i\frac{\sqrt{2}}{f_K} \begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix} \right] , \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} , \tag{15}$$

and U_π is the soliton background field written as a direct extension to $SU(3)$ of the $SU(2)$ field u_π, i.e.,

$$U_\pi = \begin{pmatrix} u_\pi & 0 \\ 0 & 1 \end{pmatrix} , \tag{16}$$

with u_π being the conventional hedgehog solution $u_\pi = \exp[i\boldsymbol{\tau} \cdot \hat{\boldsymbol{r}} F(r)]$.

According to the usual procedure, one replaces the ansatz (14) in the effective action Γ^{strong} and expands up to the second order in the kaon field. The resulting Lagrangian density can then be written as the sum of a pure $SU(2)$ Lagrangian depending on the chiral field only and an effective Lagrangian, describing the interaction between the soliton and the kaon fields. The minimization of the first term determines the static soliton profile $F(r)$ (chiral angle). The minimization of the second one leads to an eigenvalue equation for the time-dependent meson field K in the static potential field of the $SU(2)$ soliton. The bound state solutions to this wave equation represent stable hyperon states. Due to the spin–isospin structure of the soliton this eigenvalue equation becomes separable if a mode decomposition of the kaon field in terms of the grand spin $\boldsymbol{\Lambda} = \boldsymbol{L} + \boldsymbol{T}$ (where $\boldsymbol{L}$ represents the angular momentum operator and $\boldsymbol{T}$ is the isospin operator) is performed. As shown in Refs.[9, 10] the lowest bound state is found in the $(\Lambda, l) = (1/2, 1)$ channel. The different octet and decuplet baryons are obtained by putting the corresponding number of kaons in this bound state. However, by naively adding $|\mathcal{S}|$ times the bound state energy ω to the soliton mass one obtains only the centroid mass of the hyperons with strangeness $\mathcal{S}$. The splittings among hyperons with different spin and/or isospin are given by the rotational corrections, introduced according to the time–dependent rotations:

$$\begin{aligned} u_\pi &\to A u_\pi A^\dagger , \\ K &\to A K , \end{aligned} \tag{17}$$

with $A = A(t)$ being the SU(2) rotation matrix. This transformation adds an extra term to the Lagrangian which is of order $1/N_c$. Therefore, within our approximations the strong hamiltonian reads

$$H^{strong} = M_{sol} + |\mathcal{S}|\omega + \frac{1}{2\Theta}(\vec{J}^c + c\vec{J}^K)^2 . \tag{18}$$

Here, Θ is the soliton moment of inertia, and c is the hyperfine splitting constant (its explicit form for the cases of interest in this paper can be easily obtained from the general form given in Ref. [15]). Moreover, $\vec{J}^c$ and $\vec{J}^K$ are the collective and bound kaons angular momentum operators, respectively. Taking matrix elements of these operators between the different octet and decuplet hyperon states we obtain their corresponding masses in the absence of e.m. fields

$$M_{I,J,\mathcal{S}} = M_{sol} + \omega|\mathcal{S}| + \frac{1}{2\Theta}\Big[cJ(J+1) + (1-c)I(I+1) + \frac{c(c-1)}{4}|\mathcal{S}|(|\mathcal{S}|+2)\Big] . \quad (19)$$

Here, I, J and $\mathcal{S}$ are the isospin, spin and strangeness hyperon quantum numbers respectively.

4 The hyperon static polarizabilities

In this paper we will be concerned with the static polarizabilities only, defined through the shift in the particle's energy due to the presence of an external constant electric and magnetic field as:

$$\delta M \;=\; -\frac{1}{2}\,\alpha\, E^2 - \frac{1}{2}\,\beta\, B^2 . \quad (20)$$

The electric α and magnetic β polarizabilities characterize the dynamical response to the external electromagnetic fields. In the following we will study the shifts in E^2 and B^2 separately, by a proper choice for the electromagnetic potential A^μ. As it is clear from the form of the interaction (7), there will be in principle two contributions to the static polarizabilities, one coming from the term quadratic in A^μ, known as the *seagull contribution* and one coming from second order perturbation theory applied to the term linear in A^μ, the so called *dispersive contribution*.

4.1 The static electric polarizability

The energy shift proportional to E^2 can easily be obtained from (7) by adopting a potential A^μ of the form

$$A^\mu = (A_0, 0)\,, \qquad\qquad A_0 = -zE \quad (21)$$

which corresponds to a constant electric field E along the z-axis. Using the definitions (12,13), the seagull contribution can be expressed as

$$\alpha_s \;=\; \frac{e^2}{2}\int d^3r\left\{z^2\left[f_\pi^2 \mathrm{Tr}(P^2) + \frac{1}{2\epsilon^2}h_{ii} + \frac{f_K^2 - f_\pi^2}{3}\mathrm{Tr}\left\{(1-\sqrt{3}\lambda_8)(P^2U^\dagger + UP^2)\right\}\right]\right\} . \quad (22)$$

It should be noticed that in deriving Eq.(22) we have assumed that the seagull contributions to the Hamiltonian are simply equal to the seagull contributions to the

Lagrangian, with the opposite sign. There has been recently some controversy about this point. In Refs.[3, 16] it has been argued that on general grounds in a field theory electric seagull contributions to the hamiltonian should vanish. However, as discussed in Ref.[17] this is not the case when the degrees of freedom are restricted to be in some collective subspace. In fact, in that reference it has been explicitly shown that the procedure above is completely valid when the Skyrme model in the presence of a constant electric field is treated by introducing collective coordinates.

The dispersive contribution α_d is determined by matrix elements describing transitions between the particular octet state under investigation and negative parity excited states. In general, α_d is believed to be much smaller than α_s[18]. For this reason we will not consider it further in our discussion. An estimate of the approximation introduced in this way is discussed in Appendix A.

Finally, we note that α_s contains no contributions coming from the anomaly term (5), because of the antisymmetric tensor $\epsilon^{\mu\nu\rho\sigma}$.

Introducing the adiabatically rotated bound state ansatz in α_s, one obtains the following operator form

$$\begin{aligned} \alpha_s &= \Big[\gamma_1^{(e)} + \gamma_2^{(e)} \left(R_{33}\right)^2 + \gamma_3^{(e)} |\mathcal{S}| \\ &\qquad + \gamma_4^{(e)} |\mathcal{S}| \left(R_{33}\right)^2 + \gamma_5^{(e)} \; J_a^K R_{3a} + \gamma_6^{(e)} \; J_3^K R_{33}\Big] \end{aligned} \tag{23}$$

where we have absorbed e^2 in the elementary polarizabilities $\gamma_i^{(e)}$ $(i = 1, \ldots, 6)$, which depend on the radial part of the soliton and bound kaon wavefunctions only. Their explicit expressions are listed in Appendix B. R_{ab} are the rotation matrices defined by

$$R_{ab} = \frac{1}{2} \mathrm{Tr} \left[\tau_a \; A \; \tau_b \; A^\dagger\right] . \tag{24}$$

In order to get the expressions for hyperon polarizabilities we have to evaluate the matrix elements of the operators appearing in (23) between hyperon states. This is done using standard angular momentum techniques. For the ground state octet baryons we obtain

$$\alpha_s(\Lambda) = \gamma_1^{(e)} + \gamma_3^{(e)} + \frac{1}{3}\left(\gamma_2^{(e)} + \gamma_4^{(e)}\right) , \tag{25}$$

$$\alpha_s(\Sigma_0) = \gamma_1^{(e)} + \gamma_3^{(e)} + \frac{1}{3}\left(\gamma_2^{(e)} + \gamma_4^{(e)}\right) , \tag{26}$$

$$\alpha_s(\Sigma_\pm) = \gamma_1^{(e)} + \gamma_3^{(e)} + \frac{1}{3}\left(\gamma_2^{(e)} + \gamma_4^{(e)}\right) \pm \frac{1}{2}\left(\gamma_5^{(e)} + \frac{1}{3}\gamma_6^{(e)}\right) , \tag{27}$$

$$\alpha_s(\Xi_{\underline{0}}) = \gamma_1^{(e)} + 2\gamma_3^{(e)} + \frac{1}{3}\left(\gamma_2^{(e)} + 2\gamma_4^{(e)}\right) \pm \frac{2}{3}\left(\gamma_5^{(e)} + \frac{1}{3}\gamma_6^{(e)}\right) . \tag{28}$$

4.2 The static magnetic polarizability

We proceed along similar lines to derive the static magnetic polarizability. In this case we adopt the vector potential

$$A^\mu = (0, -\frac{1}{2}\boldsymbol{r} \times \boldsymbol{B}) \tag{29}$$

appropriate for a constant magnetic field B along the z-axis, $\boldsymbol{B} = B\hat{z}$. Now we have to take into account both seagull and dispersive contributions. In fact, the Hamiltonian form of (7) reads

$$H = H^{strong} + H^{lin} + H^{quad}\,, \tag{30}$$

where, again, all the contributions from L^{an} vanish for symmetry reasons.
The quadratic part yields the seagull contribution as for the electric polarizability. Its explicit form, in terms of the operators P and $h_{\mu\nu}$ is

$$\beta_s = -\frac{e^2}{8}\int d^3r \Big\{ (r^2 - z^2)\left[f_\pi^2 \mathrm{Tr}(P^2) + \frac{f_K^2 - f_\pi^2}{3} \mathrm{Tr}\left\{ (1-\sqrt{3}\lambda_8)(P^2U^\dagger + UP^2) \right\} \right] + \\ + \frac{1}{2\epsilon^2}\left[r_i r_j h_{ij} + r^2 h_{33} - r_i r_3 (h_{i3} + h_{3i}) - (r^2 - z^2) h_{00} \right] \Big\}. \tag{31}$$

A lengthy calculation shows that β_s has the same operatorial form of α_s:

$$\begin{aligned} \beta_s \;=\; & \Big[\gamma_1^{(m)} + \gamma_2^{(m)} (R_{33})^2 + \gamma_3^{(m)} |\mathcal{S}| \\ & + \gamma_4^{(m)} |\mathcal{S}| (R_{33})^2 + \gamma_5^{(m)}\, J_a^K R_{3a} + \gamma_6^{(m)}\, J_3^K R_{33} \Big]\,, \end{aligned} \tag{32}$$

where the operators involved have to be evaluated again between the same hyperon states. Therefore for the seagull part of the magnetic polarizability we obtain again formal expressions similar to Eqs.(25)–(28), with the elementary polarizabilities $\gamma_i^{(e)}$ replaced by $\gamma_i^{(m)}$ $(i = 1, \ldots, 6)$. Their explicit form is reported in Appendix B.

We must stress that in deriving Eq.(31) we have again implicitly assumed $H^{quad} = -L^{quad}$. This is correct up to a small contribution coming from L^{lin}, proportional to the ratio $(\mu_s/M_N)^2$, where μ_s and M_N are the isoscalar magnetic moment operator and nucleon mass, respectively. For a more complete discussion, see for instance Ref.[19].

The dispersive contribution arises from the term H^{lin} in (30). Using second order perturbation theory we get

$$\beta_d^H = \frac{e^2}{2M_N^2} \sum_{H' \neq H} \frac{|\langle H|\mu_3|H'\rangle|^2}{m_{H'} - m_H} \tag{33}$$

where the indices H and H' refer to different hyperon states. In writing Eq.(33) we have used the explicit form of H^{lin} for the particular case of a constant magnetic field B along the z-axis

$$H^{lin} = -\frac{e}{2M_N}\ B\ \mu_3 \tag{34}$$

where μ_3 is the magnetic moment operator. It can be written as a sum of an isoscalar and isovector part as follows[20, 11]

$$\begin{aligned} \mu^3 &= \mu_s^3 + \mu_v^3\,, && (35)\\ \mu_s^3 &= \mu_{s,0} J_3^c + \mu_{s,1} J_3^K\,, && (36)\\ \mu_v^3 &= -2\ (\mu_{v,0} + \mu_{v,1}|\mathcal{S}|)\, R_{33}\,. && (37) \end{aligned}$$

The explicit expressions of the elementary magnetic moment operators $\mu_{s,i}$ and $\mu_{v,i}$ are reported in Appendix C. In terms of them the relevant matrix elements are

$$\begin{aligned} <\Lambda|\mu_3|\Sigma_0> &= -\frac{2}{3}\left[\mu_{v,0} + \mu_{v,1}\right]\,, && (38)\\ <\Lambda|\mu_3|\Sigma_0^*> &= \frac{2\sqrt{2}}{3}\left[\mu_{v,0} + \mu_{v,1}\right]\,, && (39)\\ <\Sigma_0|\mu_3|\Sigma_0^*> &= \frac{\sqrt{2}}{3}\left[\mu_{s,0} - \mu_{s,1}\right]\,, && (40)\\ <\Sigma_\pm|\mu_3|\Sigma_\pm^*> &= \frac{\sqrt{2}}{3}\left[\mu_{s,0} - \mu_{s,1} \pm (\mu_{v,0} + \mu_{v,1})\right]\,, && (41)\\ <\Xi_{\underline{0}}|\mu_3|\Xi_{\underline{0}}^*> &= \frac{\sqrt{2}}{3}\left[\mu_{s,0} - \mu_{s,1} \pm \frac{4}{3}(\mu_{v,0} + 2\mu_{v,1})\right]\,. && (42) \end{aligned}$$

Note that for each octet hyperon only a few matrix elements are non-vanishing.

5 Numerical results and discussion

In order to estimate the uncertainties intrinsic to our approach we have performed numerical calculations adopting two different sets of parameters, namely

$$\begin{aligned} \text{SET I}: &\quad f_\pi = 93 MeV,\ \epsilon = 4.26\,,\\ \text{SET II}: &\quad f_\pi = 54 MeV,\ \epsilon = 4.84\,. \end{aligned}$$

In both cases we use the empirical values $m_\pi = 138\ MeV$, $m_K = 495\ MeV$ and $f_K/f_\pi = 1.22$. In the first set of parameters we have taken the empirical value of f_π. In the second set we have taken the value of f_π that fits the empirical nucleon mass. In both sets ϵ is adjusted to reproduce the empirical $\Delta - N$ mass difference.

The results obtained with the sets of parameters given above are summarized in Tables I-V. In Tables I and II, we list the elementary polarizabilities γ_i for the electric and magnetic cases, respectively. In Table III we give the elementary magnetic moments needed to calculate the dispersive contributions to the magnetic polarizabilities. These values have already been given in Refs.[11, 21] and are included here for the sake of completeness. Finally, in Table IV and V we report our results for the electric and magnetic static hyperon polarizabilities, respectively. In the case of the magnetic polarizabilities we also list the dispersive and seagull contributions separately.

Let us first discuss the values of the elementary polarizabilities γ_i. We see that for both sets of parameters the purely solitonic contributions γ_1 and γ_2 are much larger than the others. This holds for both the electric and magnetic cases. As a result of this behaviour, we expect a rather small splitting between the seagull contributions to the polarizabilities of the different baryons. This can be in fact observed in Tables IV and V. We also note that the values of γ_i are rather strongly dependent on the values of the input parameters used. At least, in the case of γ_1 and γ_2 (which are, as already mentioned, the dominant terms) this is to be expected. As well-known within the Skyrme model these magnitudes are basically proportional to the square of the nucleon isovector radius[1] which in turn is quite sensible to the choice of parameters. SET I leads to the value $< r_v^2 >= 0.70\ fm^2$ while the value obtained with SET II is $< r_v^2 >= 1.08\ fm^2$ as compared with the empirical value $< r_v^2 >_{emp}= 0.81\ fm^2$. This dependence on the parameters reflects, of course, on the values of all the electric and diamagnetic hyperon polarizabilities. On the other hand, the dispersive contributions to the magnetic polarizabilities are much more stable under change of parameters. This comes as a result of the compensation between the parameter dependence of the numerator and denominator in Eq.(33).

It is interesting to compare our predictions with those obtained in other models. Our results indicate a rather large Σ^+ electric polarizability. This is in agreement with the quark model prediction of Ref.[6], $\bar{\alpha}_{\Sigma^+}^{NRQM} = 20.8 \times 10^{-4} fm^3$. However, such a model predicts a rather small value for the case of the Σ^-, namely $\bar{\alpha}_{\Sigma^-}^{NRQM} = 5.7 \times 10^{-4} fm^3$. Although we also predict a smaller value for the Σ^- as compared with that of the Σ^+, in our case the splitting between both values is much smaller. As mentioned above this is a direct consequence of the fact that in our model the electric polarizabilities are completely dominated by the purely solitonic contributions. Small splittings have been also found in the Skyrme model within the framework of the

[1]This relation holds strictly for the electric seagull term. In the magnetic case there is a (numerically) small correction.

perturbative approximation to the $SU(3)$ collective coordinate approach [8][2]. The use of an exact diagonalization procedure[22] does not change the overall behaviour [23]. Only by the introduction of a feedback from the collective $SU(3)$ rotation on the soliton, that is using the so-called slow rotator approach (SRA), large splittings between the electric polarizabilities of the different hyperons could be obtained. In such case, the electric polarizabilities decrease with increasing (absolute) values of strangeness. This behaviour is similar to the one obtained in chiral perturbation theory[7]. It should be noticed, however, that in the SRA calculation of Ref.[8] this is obtained at expenses of a rather small isovector radius $< r_v^2 >= 0.49\ fm^2$.

To complete our discussion, it is important to mention that for the case of the nucleon, where the empirical value of the electric polarizability $(\alpha_N)_{emp} = 12 \times 10^{-4} fm^3$ is rather well established (see Ref.[2] for a very recent determination and update of the experimental situation), the Skyrme model predicts a somewhat large value for SET I and too large for SET II. As in the case of the soliton mass, however, there are indications that this might be cured by next-to-leading order corrections[24]

We turn now to the magnetic polarizabilities. Due basically to the dispersive contributions our results indicate rather large splittings between the values corresponding to the different hyperons. We also observe that since seagull contributions are overestimated for Set II we obtain all negative values in that case. For Set I our predicted Σ^+ magnetic polarizability agrees well with the one obtained in the non-relativistic quark model[6]. On the other hand, in the case of the Σ^- although we also obtain a diamagnetic behaviour, our value is larger (in absolute value). The results obtained by using baryon chiral perturbation are rather different from ours. In should be noticed that such a calculation does not include P-wave excitations (Δ-like) since they are of higher order in the chiral expansion. Therefore, predictions are not expected to be as accurate as in the electric case.

6 Conclusions

In this paper we have presented a complete description of static electric and magnetic polarizabilities of octet hyperons in the framework of the bound state soliton model.

[2]In Ref.[8] it has been incorrectly stated that one of the non-minimal photon coupling terms does not contribute to the electric polarizability. As a matter of fact it does, and almost completely compensates the contribution from the other non-minimal term [23]. As a consequence of this cancellation the results of Ref.[8] have to be modified. In fact, the addition of the missing contributions amounts to a roughly 20% increase of all the polarizabilities corresponding to the perturbative calculation (PT) and a 40% increase of those corresponding to the slow rotator approach (SRA). In both cases the ratios taken with respect to the proton polarizability remain almost unchanged.

In the electric case the seagull contribution is dominant, while in the magnetic case both seagull and dispersive contributions are relevant.

As shown by numerical calculation, the seagull contributions are always dominated by the purely solitonic terms, γ_1 and γ_2. These pieces determine the general pattern for electric polarizabilities, where we obtain small splittings within the same set of parameters. The structure is richer in the magnetic polarizability case because of the interplay between a large (negative) seagull part with the relevant dispersive contribution.

Finally, we note that although some of our results are in agreement with those of the non-relativistic quark model, in general this is not the case. In addition, the calculations performed in the framework of heavy baryon chiral perturbation theory lead to still different predictions. In this situation, it is clear that the future experimental data from FNAL and CERN could be of great help to discriminate among the different existing models of hyperons.

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Appendix A

In this Appendix we give an estimate of the value of the dispersive contributions to the hyperon electric polarizability. They are due to dipole transitions to the negative parity excited states. As in the magnetic case, these contributions are obtained using second order perturbation theory with the linear terms H^{lin} in the hamiltonian. In the case of a static electric field the corresponding expression is

$$\alpha_d^H = 2e^2 \sum_{H'} \frac{| < H'|d_3|H > |^2}{m_{H'} - m_H} \,, \tag{43}$$

where d_3 is the third component of the dipole operator

$$d_3 = \int dV \; z \; \rho^{em} \,. \tag{44}$$

We consider here the particular example of the Λ electric polarizability, where the largest contribution is expected to be the one in which the intermediate state is the $\Lambda(1405)$. Then, $H = \Lambda$ and the sum over H' is restricted to $H' = \Lambda(1405)$. In this case we only need to consider the isoscalar kaon contributions to ρ^{em}

$$\rho^{em}(kaon) = \frac{i}{2} f \left[K^\dagger \dot{K} - \dot{K}^\dagger K \right] - \lambda K^\dagger K \,, \tag{45}$$

where

$$f = 1 + \frac{1}{4\epsilon^2 f_K^2} \left[F'^2 + 2\frac{\sin^2 F}{r^2} \right] \,, \tag{46}$$

$$\lambda = -\frac{N_c}{8\pi^2 f_K^2} \frac{\sin^2 F}{r^2} \; F' \,. \tag{47}$$

Taking matrix elements of ρ^{em} between $\Lambda(1405)$ and Λ we get

$$< \Lambda(1405)|\rho^{em}|\Lambda >= - \left[f(\tilde\omega + \omega) + 2\lambda \right] \; \frac{\tilde k \; k}{4\pi} \; \hat r \cdot < \vec J > \tag{48}$$

where $< \vec J >$ indicates the matrix elements of the spin operator between the $\Lambda(1405)$ and Λ spin states and $(\tilde\omega, \, \tilde k)$ and $(\omega, \, k)$ are the kaon eigenenergies and bound state radial wavefunctions in the $(1/2, 0)$ and $(1/2, 1)$ channels, respectively. Therefore

$$< \Lambda(1405)|d_3|\Lambda >= -\gamma \tag{49}$$

where

$$\gamma = \frac{1}{6} \int dr \; r^3 \left[f(\tilde\omega + \omega) + 2\lambda \right] \; \tilde k \; k \,. \tag{50}$$

To derive this expression the angular integral has been performed and $< J_3 >= 1/2$ has been used. Replacing in the expression for α we get that the contribution to the Λ electric polarizability due to dipole electric transition to $\Lambda(1405)$ is

$$\alpha_d^\Lambda = \frac{2\ e^2 \gamma^2}{m_{\Lambda(1405)} - m_\Lambda} \tag{51}$$

Numerically, we find

$$\alpha_d^\Lambda = 0.54 \times 10^{-4}\ fm^3 \tag{52}$$

for SET I and

$$\alpha_d^\Lambda = 1.08 \times 10^{-4}\ fm^3 \tag{53}$$

for SET II. As we see these values are much smaller than the seagull contributions given in Table IV. Of course, it should be kept in mind that this is just an estimation of the order of magnitude since a full calculation should include all possible intermediate states. Although here we have discussed only the case of the Λ similar results are expected for the other hyperons.

Appendix B

The elementary electric polarizabilities are given by:

$$\gamma_1^{(e)} = \frac{16}{15}\pi\ e^2 \int dr\ r^4 \sin^2 F \left[f_\pi^2 + \frac{1}{\epsilon^2}\left(F'^2 + \frac{\sin^2 F}{r^2} \right) \right], \tag{54}$$

$$\gamma_2^{(e)} = -\frac{8}{15}\pi\ e^2 \int dr\ r^4 \sin^2 F \left[f_\pi^2 + \frac{1}{\epsilon^2}\left(F'^2 + \frac{\sin^2 F}{r^2} \right) \right], \tag{55}$$

$$\begin{aligned} \gamma_3^{(e)} = \frac{1}{15}\ e^2 \int dr\ r^4 \Bigg\{ & k^2(1 + 4\cos^2 F) \\ & - \frac{1}{\epsilon^2 f_K^2} \left[\frac{9}{2} k^2 F'^2 \sin^2 F + 5k^2 \frac{\sin^4 F}{r^2} - \frac{5}{4} k^2 \left(F'^2 + 2\frac{\sin^2 F}{r^2} \right) \right. \\ & \quad -2k'^2 \sin^2 F - 2\frac{k^2}{r^2} \sin^2 F \cos^2 \frac{F}{2}(1 + 3\cos F) \\ & \quad \left. -3kk'F' \sin 2F \right] \Bigg\}, \end{aligned} \tag{56}$$

$$\begin{aligned} \gamma_4^{(e)} = \frac{2}{15}\ e^2 \int dr\ r^4 \Bigg\{ & k^2 \sin^2 F \\ & + \frac{1}{4\epsilon^2 f_K^2} \left[\frac{9}{2} k^2 F'^2 \sin^2 F + 5k^2 \frac{\sin^4 F}{r^2} - 3kk'F' \sin 2F \right. \end{aligned}$$

$$-2k'^2 \sin^2 F - 2\frac{k^2}{r^2} \sin^2 F \cos^2 \frac{F}{2}(1 + 3\cos F)\Big]\Big\}, \qquad (57)$$

$$\begin{aligned}
\gamma_5^{(e)} &= -\frac{2}{15}\, e^2 \int dr\, r^4 \Big\{ k^2(1 - 4\cos F) \\
&\quad -\frac{1}{4\epsilon^2 f_K^2}\left[16k^2 \cos^2 \frac{F}{2}\frac{\sin^2 F}{r^2} - k^2\left(F'^2 + 2\frac{\sin^2 F}{r^2}\right)(1 - 4\cos F)\right. \\
&\quad \left. +24kk'F' \sin F\right]\Big\}, \qquad (58)
\end{aligned}$$

$$\begin{aligned}
\gamma_6^{(e)} &= -\frac{8}{15}\, e^2 \int dr\, r^4 \Big\{ k^2 \cos^2 \frac{F}{2} \\
&\quad +\frac{1}{4\epsilon^2 f_K^2}\left[k^2 \cos^2 \frac{F}{2}\left(F'^2 + 4\frac{\sin^2 F}{r^2}\right) + 3kk'F' \sin F\right]\Big\}. \qquad (59)
\end{aligned}$$

For the magnetic polarizability[3] we have:

$$\gamma_1^{(m)} = -\frac{2}{5}\pi\, e^2 \int dr\, r^4 \sin^2 F \left[f_\pi^2 + \frac{1}{\epsilon^2}\left(F'^2 + \frac{1}{6}\frac{\sin^2 F}{r^2}\right)\right], \qquad (60)$$

$$\gamma_2^{(m)} = -\frac{2}{15}\pi\, e^2 \int dr\, r^4 \sin^2 F \left[f_\pi^2 + \frac{1}{\epsilon^2}\left(F'^2 + \frac{7}{2}\frac{\sin^2 F}{r^2}\right)\right], \qquad (61)$$

$$\begin{aligned}
\gamma_3^{(m)} &= -\frac{1}{30}\, e^2 \int dr\, r^4 \Big\{ k^2(2 + 3\cos^2 F) \\
&\quad +\frac{1}{4\epsilon^2 f_K^2}\left[5k^2 F'^2\left(1 - \frac{27}{10}\sin^2 F\right) + 6\sin^2 F(k'^2 - \omega^2 k^2)\right. \\
&\quad +5k^2 \frac{\sin^2 F}{r^2}\left(1 - \frac{7}{10}\sin^2 F\right) + 9kk'F' \sin 2F \\
&\quad \left. +3\frac{k^2}{r^2} \sin^2 F \cos^2 \frac{F}{2}(1 + \frac{1}{3}\cos F)\right]\Big\}, \qquad (62)
\end{aligned}$$

$$\begin{aligned}
\gamma_4^{(m)} &= \frac{1}{30}\, e^2 \int dr\, r^4 \Big\{ k^2 \sin^2 F \\
&\quad +\frac{1}{4\epsilon^2 f_K^2}\left[\frac{9}{2}k^2 \sin^2 F\left(F'^2 + \frac{29}{9}\frac{\sin^2 F}{r^2}\right) - 2\sin^2 F(k'^2 - \omega^2 k^2)\right. \\
&\quad -3kk'F' \sin 2F \\
&\quad \left. -\frac{k^2}{r^2} \sin^2 F \cos^2 \frac{F}{2}(1 + 27\cos F)\right]\Big\}, \qquad (63)
\end{aligned}$$

[3]Note that the expressions of $\gamma_1^{(m)}$ and $\gamma_2^{(m)}$ together with Eq.(32) do not lead to Eqs.(45-46) of Ref.[25] which are in error. This affects only the corresponding expressions for the Δ magnetic polarizabilities. The correct numerical values are, however, very close to those quoted in such reference.

$$
\begin{aligned}
\gamma_5^{(m)} &= \frac{1}{15}\, e^2 \int dr\; r^4 \Bigg\{ k^2 (2 - 3\cos F) \\
&\qquad - \frac{1}{2\epsilon^2 f_K^2} \left[k^2 F'^2 \left(\frac{3}{2}\cos F - 1 \right) + \frac{k^2}{r^2} \sin^2 F \left(\cos F - \frac{3}{2} \right) \right. \\
&\qquad \left. + 9 k k' F' \sin F \right] \Bigg\}, \qquad (64)
\end{aligned}
$$

$$
\begin{aligned}
\gamma_6^{(m)} &= -\frac{2}{15}\, e^2 \int dr\; r^4 \Bigg\{ k^2 \cos^2 \frac{F}{2} \\
&\qquad + \frac{1}{4\epsilon^2 f_K^2} \left[k^2 \cos^2 \frac{F}{2} \left(F'^2 + 14 \frac{\sin^2 F}{r^2} \right) + 3 k k' F' \sin F \right] \Bigg\}. \qquad (65)
\end{aligned}
$$

$\gamma_1^{(e,m)}$ and $\gamma_2^{(e,m)}$ depend on the chiral angle only, while the remaining integrals take into account the interplay between rotating soliton and bound kaon wavefunction.

Appendix C

For the sake of completeness we give in this Appendix the explicit expressions for the elementary magnetic moment operators needed to calculate the dispersive contribution to the magnetic polarizability. The pure soliton contribution is given by:

$$
\mu_{s,0} = -\frac{2M_N}{3\pi\Theta} \int \mathrm{d}r\, r^2 \sin^2 F\, F' , \qquad (66)
$$

$$
\mu_{v,0} = \frac{1}{2} M_N \Theta . \qquad (67)
$$

The part describing the interplay between soliton field and bound kaon reads

$$
\begin{aligned}
\mu_{s,1} &= c\, \mu_{s,0} - \frac{4}{3} M_N \int dr\; r^2 \left\{ k^2 \cos^2 \frac{F}{2} \right. \\
&\qquad \left. + \frac{1}{4\epsilon^2 f_K^2} \left[4 \frac{k^2}{r^2} \sin^2 F \cos^2 \frac{F}{2} + k^2 F'^2 \cos^2 \frac{F}{2} + 3 k k' F' \sin F \right] \right\}, \qquad (68) \\
\mu_{v,1} &= \frac{M_N}{3} \int dr\; r^2 \left\{ k^2 \cos^2 \frac{F}{2} \left(1 - 4\sin^2 \frac{F}{2} \right) \right. \\
&\qquad + \frac{1}{4\epsilon^2 f_K^2} \left[4 \frac{k^2}{r^2} \sin^2 F \cos^2 \frac{F}{2} \left(3 - 8 \sin^2 \frac{F}{2} \right) \right. \\
&\qquad + k^2 F'^2 \cos^2 \frac{F}{2} \left(1 - 18 \sin^2 \frac{F}{2} \right) - 2 k^2 \omega^2 \sin^2 F \\
&\qquad \left. \left. + 2 k'^2 \sin^2 F + 3 k k' F' \sin F \left(3 - 4 \sin^2 \frac{F}{2} \right) \right] \right\} \\
&\quad + \frac{N_c M_N}{36} \frac{\omega}{f_K^2 \pi^2} \int dr\; r^2 \left(k^2 \sin^2 F F' + k k' \sin 2F \right) . \qquad (69)
\end{aligned}
$$

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	Set I	Set II
$\gamma_1^{(e)}$	20.7	32.1
$\gamma_2^{(e)}$	-10.4	-16.0
$\gamma_3^{(e)}$	0.78	1.11
$\gamma_4^{(e)}$	0.19	0.34
$\gamma_5^{(e)}$	2.14	4.11
$\gamma_6^{(e)}$	-2.23	-3.83

Table I: The elementary electric polarizabilities (in $10^{-4} fm^3$), as defined in Appendix B, for Set I and Set II parameters.

	Set I	Set II
$\gamma_1^{(m)}$	-7.35	-11.1
$\gamma_2^{(m)}$	-3.00	-4.91
$\gamma_3^{(m)}$	-0.42	-0.62
$\gamma_4^{(m)}$	0.10	0.22
$\gamma_5^{(m)}$	-0.42	-0.93
$\gamma_6^{(m)}$	-0.72	-1.30

Table II: The elementary magnetic polarizabilities in $10^{-4} fm^3$ (seagull contribution).

	Set I	Set II
$\mu_{s,0}$	0.37	0.74
$\mu_{v,0}$	2.39	2.40
$\mu_{s,1}$	-1.11	-1.07
$\mu_{v,1}$	-0.10	-0.16

Table III: The elementary magnetic moments expressed in nuclear magnetons (for more details, see Ref.[11]).

	Set I	Set II
N	17.3	26.7
Λ	18.1	28.0
Σ^0	18.1	28.0
Σ^+	18.8	29.4
Σ^-	17.4	26.5
Ξ^0	19.9	31.1
Ξ^-	18.0	27.3

Table IV: Electric polarizabilities (in $10^{-4} fm^3$) for the low lying octet hyperons. Only the seagull contributions are here taken into account.

	Set I			Set II		
	β_s	β_d	β_{tot}	β_s	β_d	β_{tot}
N	-8.3	5.6	-2.7	-12.8	5.6	-7.2
Λ	-8.7	12.1	3.4	-13.3	12.0	-1.3
Σ^0	-8.7	-4.0	-12.7	-13.3	-4.0	-17.3
Σ^+	-9.1	10.4	1.3	-14.0	10.1	-3.9
Σ^-	-8.4	0.48	-7.9	-12.6	0.12	-12.5
Ξ^0	-9.6	14.0	4.4	-14.8	13.0	-1.8
Ξ^-	-8.7	1.5	-7.2	-13.0	0.59	-12.4

Table V: Magnetic polarizabilities (in $10^{-4} fm^3$) of octet hyperons. In this case, both seagull and dispersive parts contribute to the total polarizability.