

THE NUCLEON-NUCLEON INTERACTION IN THE CHROMO-DIELECTRIC SOLITON MODEL: DYNAMICS

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Abstract

The present work is an extension of a previous study of the nucleon-nucleon interaction based on the chromo-dielectric soliton model. The former approach was static, leading to an adiabatic potential. Here we perform a dynamical study in the framework of the Generator Coordinate Method. In practice, we derive an approximate Hill-Wheeler differential equation and ob-

tain a local nucleon-nucleon potential as a function of a mean generator coordinate. This coordinate is related to an effective separation distance between the two nucleons by a Fujiwara transformation. This latter relationship is especially useful in studying the quark substructure of light nuclei. We investigate the explicit contribution of the one-gluon exchange part of the six-quark Hamiltonian to the nucleon-nucleon potential, and we find that the dynamics are responsible for a significant part of the short-range N-N repulsion.

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I. INTRODUCTION

In a previous investigation [1], we studied the N-N interaction in the framework of the chromo-dielectric soliton model from a static point of view: we used the Born-Oppenheimer approximation to derive an adiabatic N-N potential, which showed a soft core repulsion due essentially to the color-electrostatic part of the one-gluon exchange. Previous studies of the N-N interaction in terms of quark degrees of freedom [2] have pointed out the importance of dynamical methods (such as Generator Coordinate or Resonating Group) in the calculation of a realistic N-N potential. For example, in a preceding application of the non-topological soliton model to the N-N problem, Schuh et al. [3] showed that a significant part of the repulsion was due to dynamics; the absence of a repulsive core in some previous works was also interpreted as an artifact of the adiabatic approximation [2].

The Lagrangian of the chromo-dielectric model is defined as in Ref. [1]:

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_\sigma + \mathcal{L}_G , \quad (1)$$

with

$$\mathcal{L}_q = \bar{\psi} (i\gamma^\mu D_\mu - m_q) \psi , \quad (2)$$

$$\mathcal{L}_\sigma = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) , \quad (3)$$

$$\mathcal{L}_G = -\frac{1}{4} \kappa(\sigma) F_{\mu\nu}^a F_a^{\mu\nu} , \quad (4)$$

where ψ is the quark operator and m_q the current quark mass matrix, set here to $m_q = 0$. The quark Lagrangian \mathcal{L}_q is expressed in terms of the covariant derivative $D_\mu = \partial_\mu - ig_s T^a A_\mu^a$, and $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c$ is the $SU(3)$ -color tensor, where f^{abc} are the $SU(3)$ structure constants and T^a the $SU(3)$ generators. The quantity $U(\sigma)$ is the self-interaction of the scalar field, σ , taken to be of the form:

$$U(\sigma) = \frac{a}{2!} \sigma^2 + \frac{b}{3!} \sigma^3 + \frac{c}{4!} \sigma^4 + B , \quad (5)$$

and the dielectric function $\kappa(\sigma)$ is:

$$\kappa(\sigma) = 1 + \theta(\sigma) \left(\frac{\sigma}{\sigma_v} \right)^2 \left[2 \frac{\sigma}{\sigma_v} - 3 \right] , \quad (6)$$

where σ_v is the scalar field's vacuum expectation value and θ the usual step function.

The quark self-energy, due to interactions with confined gluons in the dielectric medium, generates an effective coupling between the quarks and the scalar field:

$$\mathcal{L}_{q\sigma} = -g_{eff}(\sigma) \bar{\psi} \psi ; \quad (7)$$

we choose $g_{eff}(\sigma)$ to be of the form:

$$g_{eff}(\sigma) = g_0 \sigma_v \left(\frac{1}{\kappa(\sigma)} - 1 \right) . \quad (8)$$

The expression in Eq. (8) is an approximation to what has been calculated in Ref. [4], and it is constructed to simulate spatial confinement already at the mean field level. Note that the coupling in Eq. (8) breaks the chiral invariance of the Lagrangian of Eq. (1). This is an example of dynamical symmetry breaking from which a massless Goldstone boson emerges naturally.

The parameters involved in our calculation are a , b , c , g_0 and $\alpha_s = g_s^2/4\pi$, as discussed in detail in Ref. [1]. By fitting the nucleon and the Δ masses and the proton's rms charge radius one remains with two free parameters, for which it is convenient to use the dimensionless quantities c and $f = b^2/ac$. In this paper, we have chosen the set $f = \infty$ and $c=10000$ taken from Table 1 of Ref. [1]. Contrary to Ref. [3], the quarks here are not only coupled to the σ -field but also interact among themselves through one-gluon exchange (OGE). The OGE is treated in Abelian approximation, and it can be separated into two terms: a self-interaction term (in addition to $g_{eff}(\sigma)$ of Eq. (8)), which is required for color confinement and which contributes to the one-body part of the Hamiltonian, and a term of mutual interactions, which gives rise to the two-body part of the Hamiltonian. As mentioned earlier, in the adiabatic approximation of Ref. [1], it was the color-electrostatic part of the OGE, which arises from the time-component of the gluonic quadrivector A_μ^c , and especially the corresponding self-energy diagrams, which were responsible for the soft-core repulsion.

In this work, we incorporate the dynamics of the N-N interaction by employing the Generator Coordinate Method (GCM); we derive an approximate differential equation for the N-N wave function describing the relative motion of the two nucleons. This equation contains a local N-N potential and an effective, coordinate dependent mass. By means of a Fujiwara transformation, we then define a N-N separation length, x , from the deformation parameter used previously in the adiabatic approximation. This allows us to introduce a constant mass and to rewrite the effective potential in terms of this coordinate x . One of our objectives is to study the explicit role of the one-gluon exchange effects on the local N-N potential, included for the first time in such type of calculations. Another aim is to establish a connection between our effective deformation parameter and the true internucleon separation. The latter will enable us to apply our six-quark wave functions to studies of the quark substructure of light nuclei, as has been carried out already, for instance, in Ref. [5]. The present numerical results correspond to the $(TS)=(10)$ sector, although the formalism at hand can easily be extended to other isospin-spin channels.

II. THE GENERATOR COORDINATE METHOD

The GCM was introduced in the fifties by Hill and Wheeler [6] to describe collective motion in nuclear systems, such as rotation, vibration or center of mass motion [7,8]. Starting from a many-body wave function $|\alpha\rangle$ depending on a collective coordinate α (the deformation parameter of the six-quark system in our case), a trial wave function is constructed by taking a linear combination of the states $|\alpha\rangle$ with some weight function $\Phi(\alpha)$,

$$|\Psi\rangle = \int \Phi(\alpha) |\alpha\rangle d\alpha , \quad (9)$$

where $\Phi(\alpha)$ is determined through the variational principle

$$\delta E = \frac{\delta \langle \Psi | H | \Psi \rangle}{\delta \Phi^*} \frac{1}{\langle \Psi | \Psi \rangle} = 0 , \quad (10)$$

which leads to the Hill-Wheeler integral equation:

$$\int \langle \alpha | H - E | \alpha' \rangle \Phi(\alpha') d\alpha' = 0 . \quad (11)$$

This is a homogeneous Fredholm-type equation of the first kind, notoriously unstable numerically. Although some methods exist to make it stable (such as regularization [9], removal of the zero normalization eigenmodes [10], Gaussian transform [11], etc.), we prefer to solve a differential equation approximately equivalent to the Hill-Wheeler equation, both for numerical stability and to facilitate comparison with analyses based on the Schrödinger equation. In general, α is a multidimensional parameter. It is at least three-dimensional when correspondence is made to \mathbf{r} . We here restrict the calculations to the zero-impact parameter case, which reduces the problem to a one-dimensional one, and leave consideration of the angles to a later study.

III. THE HILL-WHEELER DIFFERENTIAL EQUATION

To derive such a differential equation, it is more convenient to work with mean and relative deformation parameters, β and δ , defined as

$$\begin{aligned} \beta &= \frac{\alpha + \alpha'}{2} , \\ \delta &= \alpha - \alpha' . \end{aligned} \quad (12)$$

Expanding the weight function in a Taylor series around $\delta = 0$, one has:

$$\begin{aligned} \langle \Psi | H - E | \Psi \rangle &= \int d\beta \int d\delta \left[\Phi^*(\beta) + \frac{\delta}{2} \Phi'^*(\beta) + \frac{\delta^2}{8} \Phi^{*''}(\beta) + \dots \right] \\ &\quad \left\langle \beta + \frac{\delta}{2} | H - E | \beta - \frac{\delta}{2} \right\rangle \left[\Phi(\beta) - \frac{\delta}{2} \Phi'(\beta) + \frac{\delta^2}{8} \Phi''(\beta) + \dots \right] . \end{aligned} \quad (13)$$

It is convenient to introduce the moments:

$$H_n = \int d\delta \left\langle \beta + \frac{\delta}{2} | H | \beta - \frac{\delta}{2} \right\rangle \delta^n , \quad (14)$$

$$N_n = \int d\delta \left\langle \beta + \frac{\delta}{2} | \beta - \frac{\delta}{2} \right\rangle \delta^n . \quad (15)$$

Because $\langle \beta + \delta/2 | H - E | \beta - \delta/2 \rangle$ is an even function of δ , the odd moments are zero. Supposing, moreover, that $\langle \beta + \delta/2 | H - E | \beta - \delta/2 \rangle$ is a sharply peaked function of δ , one

can stop the expansion at second order in δ . Partial integration and variation by $\delta\Phi^*$ leads then to the Hill-Wheeler differential equation:

$$\frac{1}{2} \frac{d}{d\beta} \left((H_2 - EN_2) \frac{d\Phi}{d\beta} \right) + \left[H_0 + \frac{1}{8} \frac{d^2 H_2}{d\beta^2} \right] \Phi = E \left[N_0 + \frac{1}{8} \frac{d^2 N_2}{d\beta^2} \right] \Phi . \quad (16)$$

The introduction of a new function into the hermitian,

$$\tilde{\Phi}(\beta) = \sqrt{\tilde{N}_0(\beta)} \Phi(\beta) , \quad (17)$$

where

$$\tilde{N}_0 = N_0 + \frac{1}{8} \frac{d^2 N_2}{d\beta^2} , \quad (18)$$

allows us to transform Eq. (16) into hermitian form:

$$\left[-\frac{d}{d\beta} \frac{1}{2B(\beta)} \frac{d}{d\beta} + V(\beta) \right] \tilde{\Phi}(\beta) = E \tilde{\Phi}(\beta) , \quad (19)$$

where $V(\beta)$ is given by:

$$V(\beta) = \frac{\tilde{H}_0}{\tilde{N}_0} + \frac{1}{2\sqrt{\tilde{N}_0}} \frac{d}{d\beta} \left((H_2 - EN_2) \frac{d}{d\beta} \left(\frac{1}{\sqrt{\tilde{N}_0}} \right) \right) , \quad (20)$$

with

$$\tilde{H}_0 = H_0 + \frac{1}{8} \frac{d^2 H_2}{d\beta^2} . \quad (21)$$

The term $B(\beta)$ is the effective mass:

$$B(\beta) = -\frac{\tilde{N}_0}{H_2 - EN_2} . \quad (22)$$

The total energy E enters the definition of B ; its asymptotic form at threshold is:

$$E = 2m_N , \quad (23)$$

where m_N is the nucleon mass. Note that because we didn't incorporate center of mass corrections the asymptotic value of the potential in Eq. (20) is not equal to the experimental value of $2m_N$. We have indeed $V(\infty) = 2468$ MeV when gluons are not included and $V(\infty)$

= 2240 MeV when they are. In practice, we could obtain a value closer to the experimental value by subtracting recoil corrections from the asymptotic energy:

$$m_N^2 = \left(\frac{V(\infty)}{2} \right)^2 - \langle P^2 \rangle , \quad (24)$$

but we prefer to avoid this step. This simplification does not affect our conclusions. Following Brink and Banerjee [12], we replace E in the mass term by:

$$E \rightarrow \frac{H_0(\beta)}{N_0(\beta)} . \quad (25)$$

This approximation is consistent with neglecting higher order derivatives of the moments in the Hamiltonian.

The moments H_n ($n=0, 2$) and the corresponding quantities $B(\beta)$ and $V(\beta)$ have been calculated for three distinct cases:

$$\begin{aligned} (a) \quad H &= H_1^{bag} + H_{OGE} , \\ (b) \quad H &= H_1^{bag} + H_{OGE}^{mag} , \\ (c) \quad H &= H_1^{bag} , \end{aligned} \quad (26)$$

where H_1^{bag} , H_{OGE}^{mag} and H_{OGE} are, respectively, the non-gluonic one-body term of the Hamiltonian, the color-magnetic and the full one-gluon exchange contribution; they are given explicitly in Ref. [1]. In case (c), the one-gluon exchange was left out altogether. This is in the spirit of an earlier investigation where the Friedberg-Lee soliton model was applied to N-N scattering without considering gluonic degrees of freedom [3]. In case (b), the color-magnetic hyperfine interaction was accounted for, and in case (a) the full color-magnetic and color-electrostatic OGE was included. The reason to distinguish between cases (a) and (b) is that in the literature it was claimed that the color-magnetic part of the OGE itself is responsible for the repulsive core of the N-N interaction [13]. We shall return to this point at the end of Section V.

The plot of B as a function of β is given in Fig. 1 for the three cases (a), (b) and (c). B converges towards a constant value μ , which can be calculated from considering two well-separated non-interacting three-quark bags:

$$\mu = 763.6\text{MeV} \quad \text{in cases (a) and (b) ,} \quad (27a)$$

$$\mu = 502.3\text{MeV} \quad \text{in case (c) .} \quad (27b)$$

We would expect μ to be equal to the reduced mass, $m_N/2$. The discrepancy between the values of μ and $m_N/2$ – which is especially drastic if the OGE is included, i.e., in cases (a) and (b) – is related to the well-known Peierls-Yoccoz disease [8,10].

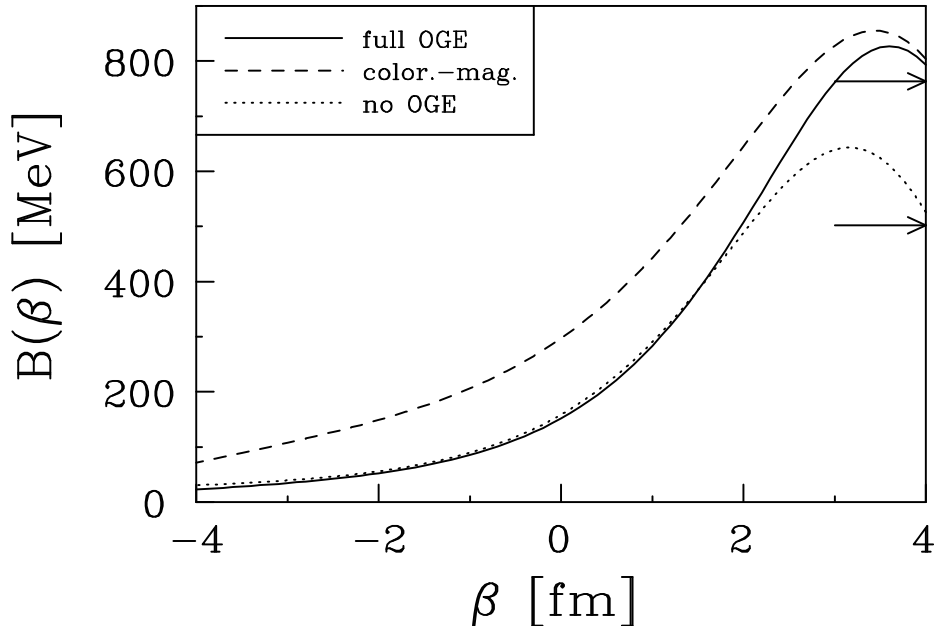


FIG. 1. The effective mass, $B(\beta)$ of Eq. (22), as a function of the deformation parameter β ; the solid, dashed and dotted lines correspond, respectively, to the cases (a), (b) and (c) introduced in Eq. (26). The asymptotic values of Eqs. (27a) and (27b) are indicated by the arrows.

IV. THE FUJIWARA TRANSFORMATION

The dependence of the effective mass on β prevents us from directly interpreting the potential in Eq. (20) as an ordinary N-N potential. Moreover, β doesn't correspond to the true N-N separation distance (except for large positive β when the two nucleons are well separated). Therefore, we wish to transform Eq. (19) into a Schrödinger-like equation with a constant, coordinate independent mass term. For this purpose, one can use a Fuji-

wara transformation [14,15], which relates the generator coordinate β to an effective N-N separation length:

$$x(\beta) = - \int_{\beta}^{\infty} \left[\sqrt{\frac{B(\beta')}{\mu}} - 1 \right] d\beta' + \beta . \quad (28)$$

If one now redefines the weight function in Eq. (19) as

$$\tilde{\Phi}(\beta) = \sqrt[4]{\frac{B(\beta)}{\mu}} \psi(x) , \quad (29)$$

Eq. (19) transforms into the familiar form

$$\left[-\frac{1}{2\mu} \frac{d^2}{dx^2} + V + V_F \right] \psi(x) = E\psi(x) , \quad (30)$$

with V given by Eq. (20) and

$$V_F(\beta) = \frac{7}{32B^3} \left(\frac{dB}{d\beta} \right)^2 - \frac{1}{8B^2} \frac{d^2B}{d\beta^2} . \quad (31)$$

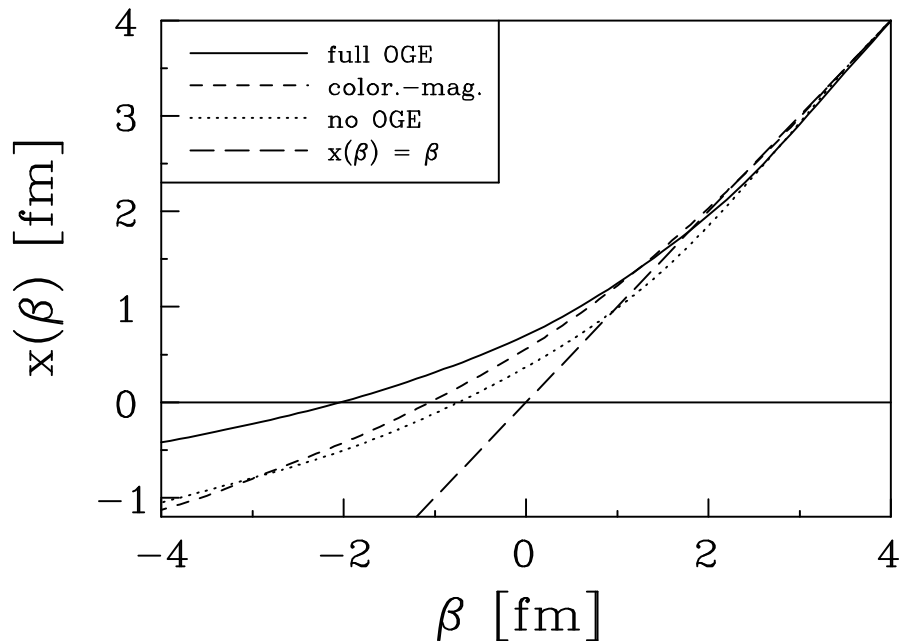


FIG. 2. The Fujiwara coordinate, $x(\beta)$ of Eq. (28), as a function of the deformation parameter β . The long-dashed line shows the asymptotic solution, $x(\beta) = \beta$, and the remaining labeling is the same as in Fig. 1.

Figure 2 displays the explicit relationship between x and β , as obtained from Eq. (28). As expected, the deformation parameter β converges asymptotically towards the effective

internucleon separation x . The correspondence $\beta \leftrightarrow x$ should be very useful in discussions of the quark substructure of nuclei or nuclear matter using Schrödinger-based many-nucleon calculations and employing our six-quark wave functions.

V. RESULTS FOR THE EFFECTIVE N-N POTENTIAL

We now wish to present detailed results for:

$$V_{loc}(\beta) = V(\beta) + V_F(\beta) - V(\infty) , \quad (32)$$

where $V(\beta)$ and $V_F(\beta)$ are given in Eqs. (20) and (31). The value of $V(\infty)$ corresponds to the asymptotic value of \tilde{H}_0/\tilde{N}_0 calculated from two well-separated non-interacting three-quark bags, and it is given in Section III. This asymptotic value is the same in cases (a) and (b) because the color-electrostatic mutual and self-energy terms cancel exactly due to color neutrality when the two nucleons are well separated.

It is convenient to rewrite $V_{loc}(\beta)$ in the following form:

$$V_{loc}(\beta) = V_0(\beta) + V_1(\beta) , \quad (33)$$

with:

$$\begin{aligned} V_0(\beta) &= \frac{\tilde{H}_0}{\tilde{N}_0} - V(\infty) , \\ V_1(\beta) &= \frac{1}{4B} \left[\frac{d^2 \ln \tilde{N}_0}{d\beta^2} + \frac{1}{2} \left(\frac{d \ln \tilde{N}_0}{d\beta} \right)^2 - \frac{d \ln \tilde{N}_0}{d\beta} \frac{d \ln B}{d\beta} \right] \\ &\quad + \frac{1}{8B} \left[\frac{3}{4} \left(\frac{d \ln B}{d\beta} \right)^2 - \frac{d^2 \ln B}{d\beta^2} \right] . \end{aligned} \quad (34)$$

In order to calculate these derivatives, $\ln B$ and $\ln \tilde{N}_0$ were fitted to polynomials. The two contributions $V_i(\beta)$ ($i=0, 1$) to $V_{loc}(\beta)$ are plotted in Fig. 3 as functions of the deformation parameter β for the three cases outlined previously. Fig. 4 shows V_{loc} as a function of β and of the Fujiwara coordinate x , respectively. Note that Eq. (35) was obtained from Eq. (20) by replacing $H_2 - EN_2$ with $-\tilde{N}_0/B$, as indicated in Eq. (22).

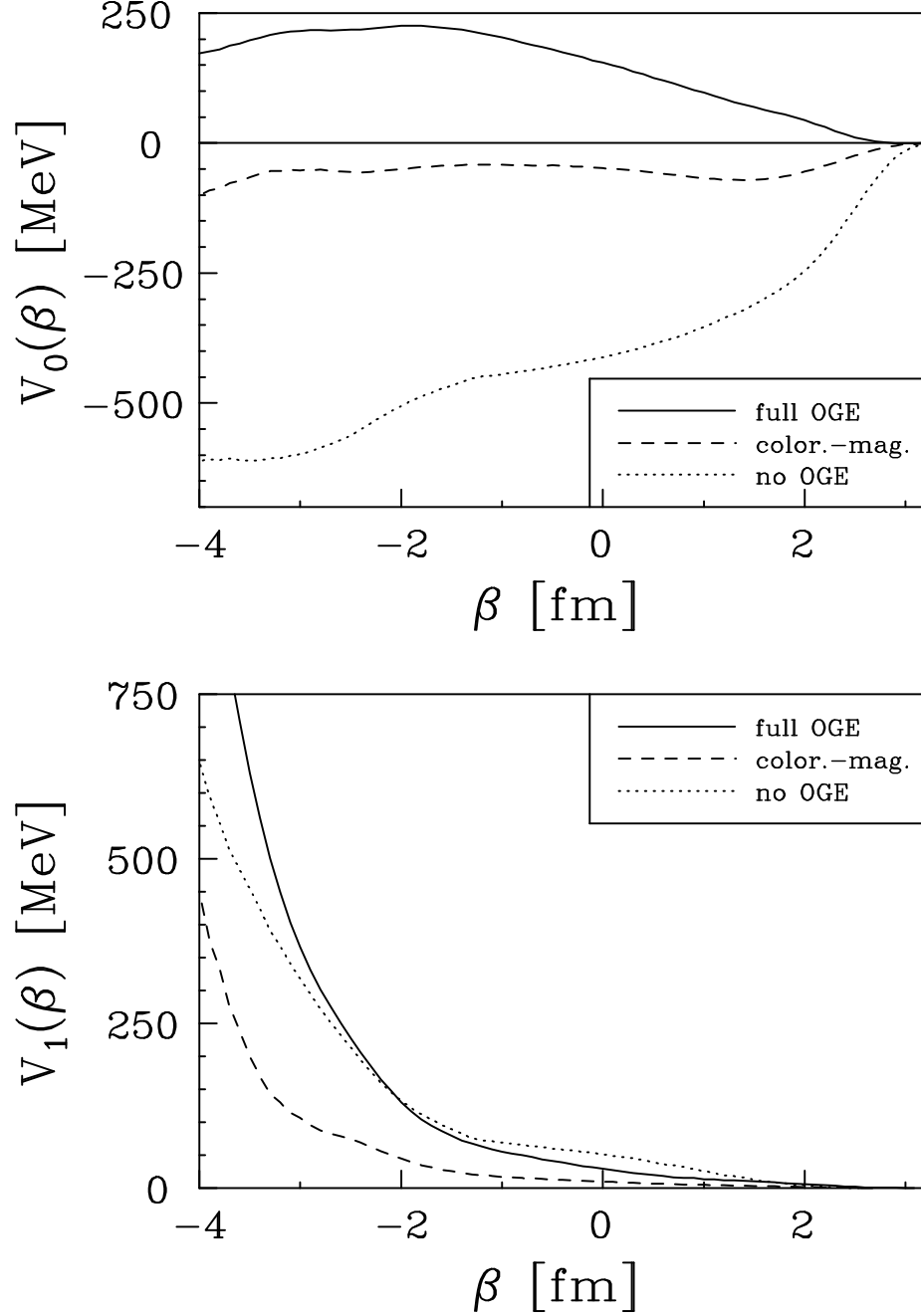


FIG. 3. The two contributions to the local potential, $V_0(\beta)$ of Eq. (34) and $V_1(\beta)$ of Eq. (35), as functions of the mean generator coordinate β . The solid, dashed and dotted lines correspond to the cases (a), (b) and (c) introduced in Eq. (26).

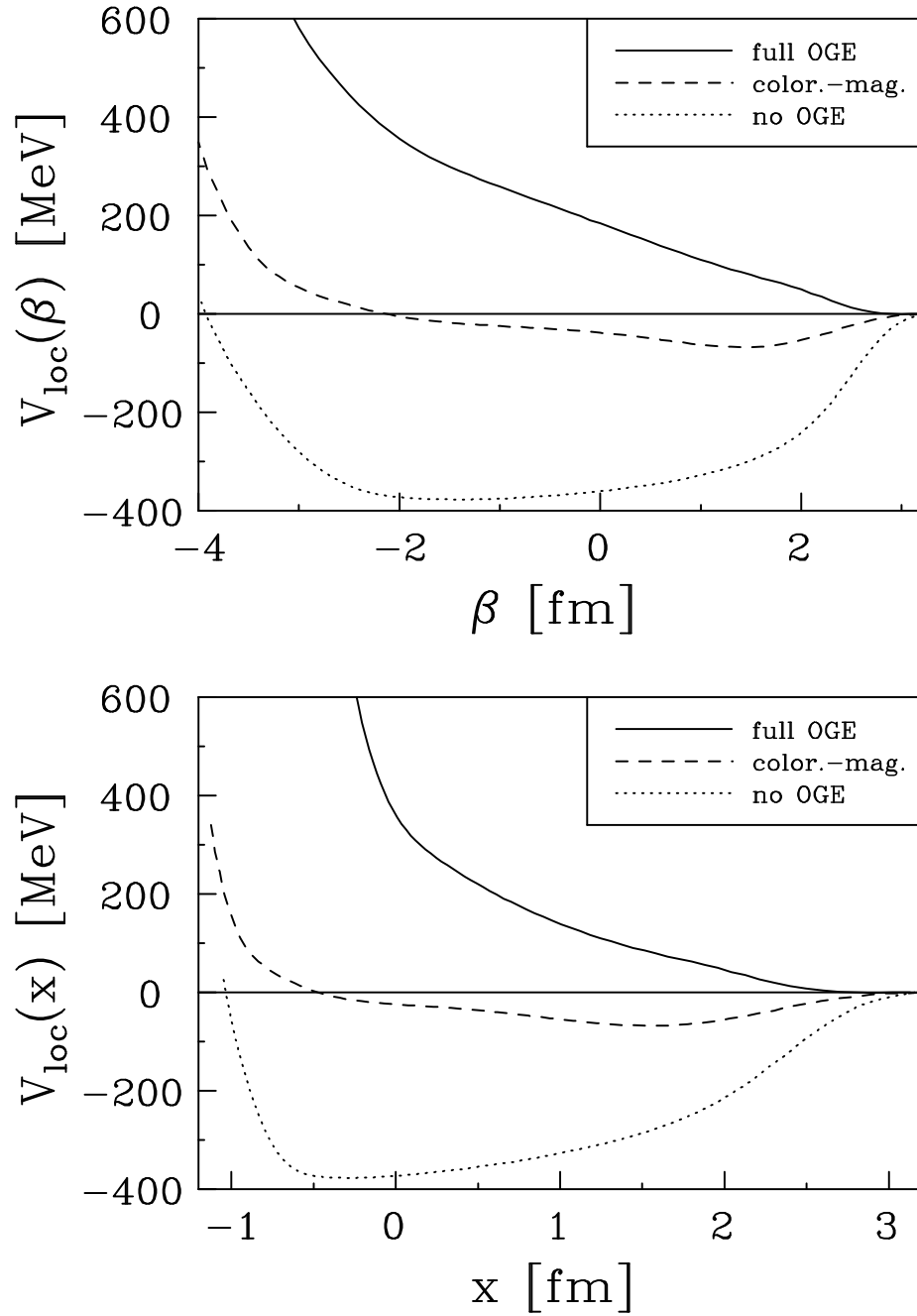


FIG. 4. The non-adiabatic, local potential, V_{loc} of Eq. (33), as a function of the deformation parameter β and the Fujiwara coordinate x , respectively. The labeling is the same as in Fig. 3.

The shape of $V_0(\beta)$ is quite similar to the adiabatic potentials displayed in Fig. 10 of Ref. [1], both for the “full OGE” and “no-OGE” cases. This tends to confirm our assumption that the matrix elements $\langle \beta + \delta/2 | H - E | \beta - \delta/2 \rangle$ are rather sharply peaked around $\delta = 0$. The term $V_1(\beta)$ corresponds to the contribution of non-adiabaticity. It grows important only for $\beta \lesssim -2$ fm, and yields in all cases a repulsion due to the dynamics. This is according to our expectation and in agreement with Ref. [3]. Note that in cases (b) and especially (c), we also obtain an intermediate range attraction in V_{loc} . The fact that our N-N potential extends to negative x should not be taken too literally. It simply reflects inadequacies in the relationship between the deformation parameter β and the N-N separation length x , which are connected to the Peierls-Yoccoz disease mentioned earlier.

We recall that one of the main objectives of this and our previous study [1] was to incorporate explicitly one-gluon exchange effects, in contrast to Ref. [3] where they were neglected. Comparing, for instance, cases (a) and (c), one can see that the OGE reinforces the repulsive core considerably. The existence of a repulsive core in all three cases makes us to attribute it to dynamics rather than to the color-magnetic interaction (case (b)), as was inferred in Ref. [13].

VI. CONCLUSIONS

In this investigation, we found that the dynamics are manifestly responsible for the hard-core repulsion of the short-range part of the N-N interaction, and we observed that we could obtain both short-range repulsion and some intermediate range attraction if the entire one-gluon exchange or at least its electrostatic part were neglected.

In the results containing the full OGE effects the lack of attraction is due to the omission of explicit meson exchanges. Then, to reproduce the experimental phase shifts or other two-body data one necessitates to attach a local OBE potential beyond a certain internuclear distance [16]. To obtain this potential in the framework of our model we could consider extending our calculations by either including quantum surface fluctuations and/or intro-

ducing configurations of the form $q^7\bar{q}$ in addition to the q^6 states. This would be a rather cumbersome procedure within the present model. The most convenient would be to either allow mesonic degrees of freedom and to consider, e.g., an explicit pion exchange between the individual quarks [17] or to simply choose a phenomenological potential.

Another important result of this work is the evaluation of the relationship between the deformation parameter β and the effective N-N separation length x through the Fujiwara transformation. This correspondence is very useful for applications of our model to the description of phenomena involving the quark substructure of light nuclei. It furthermore allows us to relate many-body correlation functions or N-N wave functions given in the literature to the GCM formalism presented here.

An attractive way to confirm our results would be to solve directly the Hill-Wheeler integral equation in order to obtain phase shifts. Projection on good angular momentum states should also improve our calculations.

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