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The Bekenstein bound,  
topological quantum field theory  
and pluralistic quantum cosmology

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# 1 Introduction

In this paper a new approach to the problem of constructing a quantum theory of gravity in the cosmological context is proposed. It is founded on results from four separate directions of investigation, which are:

1) A new point of view towards the interpretation problem in quantum cosmology[1, 2, 3, 4], which rejects the idea that a single quantum state, or a single Hilbert space, can provide a complete description of a closed system like the universe. Instead, the idea is to accept Bohr's original proposal that the quantum state requires for its interpretation a context in which we distinguish two subsystems of the universe -the quantum system and observer. However, we seek to relativize this split, so that the boundary between the part of the universe that is considered the system and that which might be considered the observer may be chosen arbitrarily. The idea is then that a quantum theory of cosmology is specified by giving an assignment of a Hilbert space and algebra of observables to every possible boundary that can be considered to split the universe into two such subsystems. A quantum state of the universe is then an assignment of a statistical state to every one of these Hilbert spaces, subject to certain conditions of consistency. Each of these states is interpreted to contain the information that an observer on one side of each boundary might have about the system of the other side.

This formulation then accepts the idea that each observer can only have incomplete information about the universe, so that the most complete description possible of the universe is given by the whole collection of incomplete, but mutually compatible quantum state descriptions of all the possible observers. At the same time, the information of different observers is, to some extent, different, so that there is no way, in principle, to combine the descriptions of each observer to construct a single quantum state that could give a complete description of the whole universe. This point of view, which has been developed in collaboration with Louis Crane[1] and Carlo Rovelli[2] and is discussed also in papers by them, may be called pluralistic quantum mechanics.

2) The Bekenstein bound[5], which requires, for reasons that will be reviewed in section 2, below, that the Hilbert spaces associated to timelike boundaries of fixed spatial area  $A$  must have finite dimension proportional to  $\exp(cA/l_{Planck}^2)$ , with  $c$  some fixed dimensionless constant. This has recently led 'tHooft[6] and Susskind[7, 8] to make the "holographic hypothesis", according to which quantum gravity must be understood to be constructed from field theories on two dimensional surfaces, that describe what knowl-

edge an observer looking through the surface at the world on the other side, might have. The question has then been raised as to what specifies the surfaces on which these “holographic field theories” are to be defined. The answer pluralistic quantum theory gives to this question is that the “holographic” description must be applicable to any possible surface, so that the task of the theory is not to pick out certain surfaces but to provide the conditions and relationships that hold between the theories on the different surfaces.

3) Topological quantum field theory[9, 10, 11] which, first of all, provides examples of pluralistic quantum theories[1], and, secondly, gives us a set of finite dimensional Hilbert spaces with which to realize the Bekenstein bound.

4) The loop representation approach to non-perturbative quantum gravity [12, 13, 14, 15, 16], and, in particular, the kinematical part of the theory, associated with the description of spatially diffeomorphism invariant states and observables in terms of spin networks [15, 17, 18, 19, 20, 21, 22, 23]. In particular, we may make use of the fact that the same mathematical structure- spin networks[24]- appears as labels of diffeomorphism invariant states in both the loop representation formulation of quantum gravity and in topological quantum field theory[9, 25, 26].

One of the motivations for the present work is a study recently carried out by the author of quantum general relativity in the presence of a particular set of boundary conditions[27]. In this context, the appearance of the spin networks as labels of states in these two theories was used to construct state spaces that represent the algebra of observables of general relativity on a timelike boundary out of the Hilbert spaces of certain Chern-Simons theories. It was then natural to contemplate the conjecture that the observables measured on the boundary are sufficient to determine the quantum state of the system. If this conjecture is true it means that the whole physical state space for quantum general relativity, in the presence of these boundary conditions, can be constructed from direct products of state spaces for Chern-Simons theory. Further, the correspondence between spin and area leads to the result that the state spaces for quantum gravity constructed in this way satisfy the Bekenstein bound. This is because they have the property that once the metric of the boundary is measured, the space of states accessible by measurements on the boundary is reduced to a finite dimensional subspace, whose dimension grew with the exponential of the area.

This work was carried out in a less general context than that contemplated here, in which state spaces are to be associated with any possible

timelike boundary dividing the universe into two parts, no matter what conditions are satisfied on it. However, its results are part of the motivation for this proposal. Indeed, I will argue below that general features of this construction, and in particular the decomposition of state spaces for quantum gravity in terms of the Hilbert spaces of topological quantum field theories, may be realized in this more general context.

Another motivation for this proposal is to try to reverse the usual strategy of constructing a quantum theory of gravity and to take the point of view that, rather than deriving that theory directly from the classical theory by following some more or less well defined quantization procedure, the goal is to formulate a set of postulates that define the theory directly[28]. The inspiration for these postulates is certain results and conjectures which have arisen in the course of investigations of the quantization of classical gravitational theories. The hope is that there may be a set of such postulates that are sufficient to found a theory which is well defined, has a meaningful physical interpretation and at the same time allows both classical general relativity and quantum field theory on fixed backgrounds emerge as approximations in particular limits<sup>1</sup>.

The reason for taking such an approach lies in the peculiar situation that research in quantum gravity has led to, in which several different approaches have yielded results that we may hope will be preserved in the final theory, while at the same time, no single approach has led to the formulation of a complete theory. Furthermore, there are, in each case, reasons to believe that the successful results of each approach may not easily be recovered in the others approaches. It may then be that what we have in each case is a partial theory that describes successfully some domain of quantum gravitational phenomena, but which does not tell the whole story.

Here, I am referring mainly to three approaches: string theory, quantum field theory in curved space-time and canonical quantization of general relativity based on the merging of the Ashtekar variables with the loop representation. For example the latter approach has yielded a kinematical description of quantum general relativity in terms of (three dimensional) diffeomorphism invariant states and observables that has many appealing features. These include the prediction of discrete spectra for areas and volumes[15, 17, 18, 19, 20, 23] the description of states in terms of spin

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<sup>1</sup>An important motivation for this idea is the recent work of Jacobson[43], where he does exactly this, by deriving classical general relativity from the connection between area and information (essentially the Bekenstein bound) and the laws of thermodynamics.

networks, and the suggestions of the existence of a linearized sector with a natural Planck scale cutoff[29]. Some of these results even turn out to be derivable in a completely rigorous mathematical framework[23]. At the same time, despite very exciting progress in the last year, this approach has yet to completely resolve the difficulties concerning physical observables, time and the inner product which plague it and, indeed, all canonical approaches to quantum gravity[30]. As a result of these difficulties, it is still not known whether the exact physical states[32, 12, 33], which were among the first results of this approach, are physically meaningful or useful.

This does not mean that this approach is not useful. It may lead to a completely well defined mathematical formalism which, however, suffers difficulties of interpretation coming from the problems of extending quantum theory to the cosmological case. It may be possible in certain contexts to overcome some of these difficulties by defining approximation procedures which yield physically meaningful predictions in restricted circumstances[36, 21, 34, 35]. Still, unless the much discussed fundamental difficulties facing such theories[30] are overcome, we still may not have a satisfactory quantum theory of gravity.

Another crucial issue for this kind of approach is the recovery of Lorentz invariance in the appropriate limit. In fact, the restoration of Lorentz invariance is an issue for any quantum field theory one of whose parameters is a length which serves as an invariant cutoff, for the following reason. An acceptable quantum theory of gravity must have a limit in which the physics can be described in terms of small excitations of a Lorentz invariant vacuum. But, the existence of a cutoff scale means that if an inertial observer, in the presence of this vacuum, measures the spectrum of graviton states he should see no gravitons with wavelengths smaller than  $l_{cutoff}$ . However, if the vacuum is Lorentz invariant, a second inertial observer, moving with a large  $\gamma$  with respect to the first, should see gravitons with all wavelengths longer than  $l_{cutoff}$  in their frame. But this means that they will see gravitons that have wavelength  $\gamma^{-1}l_{cutoff}$  in the first observers frame. We have an apparent contradiction.

Another way to see this problem is to ask whether Lorentz invariance can be consistent with the requirement that the Hilbert space describing the physics in any region of finite volume must be finite dimensional, as is required by the Bekenstein bound. At first sight it seems that the answer must be no. In conventional quantum field theory it is possible to construct wavepackets that describe quanta moving with any peak wavelength  $\lambda_0$ , with a spread  $\delta\lambda_0 < L$  where  $L \gg \lambda_0$  is the linear size of the region. Thus, the

existence of finite volume boundary conditions does not prevent the theory from having states of arbitrarily small wavelength. And, in a linear field theory, all these states are orthogonal to each other, which means that there are an infinite number of orthonormal states. Thus, it seems that Lorentz invariance cannot be consistent with a theory that has a finite number of degrees of freedom per fixed spatial region.

It is then very impressive that there is one context in which this problem has been definitely solved, which is perturbative string theory[38, 39, 40, 41, 7, 42]. The problem is solved there because the elementary excitations are extended one dimensional objects. As is explained in detail in [7, 42], string theory is consistent with Lorentz invariance in spite of having a finite number of degrees of freedom per fixed spatial region because the strings, representing the small excitations of the vacuum, can diffuse transversally as they are boosted longitudinally.

A theory of extended objects is different in this respect from a theory of pointlike objects, because any extended excitation of a theory that is both Lorentz invariant and finite must, if boosted sufficiently become effectively two dimensional, as its longitudinal extension is contracted below the scale of the cutoff. This means that extreme relativistic excitations may be naturally described as excitations of a two dimensional field theory. It is then very interesting that, as was shown by Klebanov and Susskind[42], continuum string theory can emerge from a lattice field theory in which there is a cutoff in the transverse directions by means of a limit in which the lengths of the strings diverge while the transverse cutoff remains fixed.

Given that string theory solves this problem and, more generally, that the only known perturbative quantum field theories that describe consistently the coupling of gravitons to themselves and other fields are perturbative string theories[38], it seems that any acceptable quantum theory of gravity, whatever its ultimate formulation, is likely to reduce to a perturbative string theory in the appropriate limit. It is, of course, possible to imagine that non-perturbative quantum general relativity will, for this reason, have some perturbative string theory as an appropriate limit[28]. However, while this is a possibility that must be explored, it's success is certainly not guaranteed by what we know about the theory so far.

At the same time, string theory cannot be itself the whole theory unless it has a nonperturbative formulation. While there have been recently some very interesting hints, coming from the very significant discovery of non-perturbative effects in string theory[31], it is still unclear to what extent string theory will have to be modified to arrive at a completely non-

perturbative formulation of the theory. Thus, counting also quantum field theory in curved space-time, our situation is that we have interesting results, and in some cases even physical predictions[28], coming from three different starting points, each of which, however, may not lead to a complete theory. In this circumstance it may be prudent to try to find a new starting point that combines what is useful about each of these directions.

Another way to see that a new starting point may be needed is that exactly to the extent that these three well explored approaches are successful, they cast doubt on the physical relevance of their starting points, which in each case involve the quantization of conventional classical field theories. The results of each of these three directions seem indeed to indicate that there is a natural Planck scale cutoff, below which physical degrees of freedom cannot be resolved. This suggests that, ultimately, a quantum theory of gravity will not be formulated most simply as a theory of fields on a differential manifold representing the idealized-and apparently nonexistent-“points” of space and time<sup>2</sup>.

To put this another way, the space of fields-the basic configuration space of classical field theory-has been replaced in the quantum theory by abstract Hilbert spaces. At the same time, ordinary space, in these formulations, remains classical, as it remains the label space for the field observables. This perpetuates the idealization of arbitrarily resolvable space-time points, that the results of string theory, non-perturbative quantum gravity and semiclassical quantum gravity (through the Bekenstein bound) suggest we must give up.

The final motivation for the present approach comes from the interpretational problem in quantum cosmology. Despite very interesting proposals [45, 46, 47, 48, 49, 50, 51, 52, 53] it may be said that no proposal to interpret the conventional Hilbert space formulation of quantum cosmology has so far convincingly succeeded[54]. (I will discuss this issue in some detail in section 4, below). This means that even if the problem of physical observables could be solved in the context of conventional Hamiltonian quantization, perhaps as envisioned by Rovelli[55] or recently by Ashtekar, Lewandowski, Marlof, Mourã and Thiemann[23], the resulting mathematical theory would still not have an acceptable physical interpretation. More precisely, as I will argue, while the theory might have a fanciful interpretation in terms of the observations made by some “God” outside of the universe, this would be of

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<sup>2</sup>Of course, that space and/or time must be fundamentally discrete is an old idea, for a good review, see [44]

no help to connect the mathematical framework to observations made by us observers who live in the universe. For this reason also, it may be useful to contemplate taking a new point of view according to which quantum cosmology might have an interpretation strictly in terms of information that might be held by observers inside the universe. This, as I will describe, is another goal of the present proposal.

This paper is divided roughly into two parts, the first of which is more general and motivational, while the second is more focused and mathematical. The next three sections motivate the idea of a pluralistic formulation of quantum cosmology. Sections 2 and 3 describe the argument for, and the implications of, the Bekenstein bound. Section 4 is devoted to the problems of the interpretation of quantum cosmology, and give an introduction to the main ideas of the approach of Crane[1], Rovelli[2] and the author[3, 4] for a pluralistic approach to quantum cosmology. The postulates of the theory are then presented at the end of section 5.

Section 6 is devoted to the problem of time. I show that this problem can be circumvented in pluralistic quantum cosmology by, as it were, standing on their heads certain proposals concerning the problem of time which are usually set in the conventional formulations in which a single state and Hilbert space describe the universe. These proposals, which fail in one way or another in that context, take on a somewhat different aspect in the context of pluralistic quantum cosmology. The problem of the classical limit, and the related question of the timelike initial data problem are the subject of section 7.

Sections 8 and 9 are more mathematical, and concern one attempt to realize the postulates of pluralistic quantum cosmology by building a theory on certain results of non-perturbative quantum gravity and topological quantum field theory. A representation for the connection and frame fields of the Ashtekar formalism of general relativity, pulled back into an arbitrary surface, is given in which the state space is constructed from certain direct sums of representation spaces of quantum Chern-Simons theory. This formalism incorporates a proposal for solving the constraints of the theory, in the form given by Reisenberger[56], quantum mechanically. One consequence of this formalism is that the Bekenstein bound is automatically satisfied.

The paper closes with a short summary of conclusions and open problems.



## 2 The Bekenstein argument

Consider a region of space,  $\Sigma$ , the quantum dynamics of which we wish to study. We will assume that the restriction of the system of interest to  $\Sigma$  is enforced by certain boundary conditions, defined as conditions the fields must satisfy on the spatial boundary of  $\Sigma$ , which we will denote  $\mathcal{S}$ . Given these, we will assume we can define a quantum theory which is described by a Hilbert space  $\mathcal{H}_\Sigma$ , and an algebra of physical observables  $\mathcal{A}_\Sigma$ . Among these there is an important subalgebra,  $\mathcal{A}_{boundary}$ , which consists of those physical observables which are functionals only of fields on the boundary. Among these we will assume are the Hamiltonian,  $H_\Sigma$  and the areas of regions  $\mathcal{R}$  of the boundary  $\mathcal{S}$ , which I will denote  $A[\mathcal{R}]$ . Recall that in general relativity the Hamiltonian is, up to terms proportional to constraints, defined as an integral on the boundary and is thus an element of  $\mathcal{A}_{boundary}$ .

Since the system contains gravitation, we may assume that among the spectrum of states are a set which correspond to black holes. These are semiclassical statistical states, and we will assume that their real statistical entropies are given by the usual formulas, at least in the semiclassical limit when their masses and areas are large in Planck units.

The argument is simplest in the case that we assume that the induced metric in  $\mathcal{S}$  is the two sphere metric. It proceeds by assuming that the region  $\Sigma$  can contain an object  $\mathcal{O}$  whose complete specification requires an amount of information  $I_{\mathcal{O}}$  which is larger than

$$I_{\mathcal{S}} = \frac{A[\mathcal{S}]}{4l_{Pl}^2} \tag{1}$$

which is of course the entropy of a black hole whose horizon just fits inside of  $\mathcal{S}$ .

Let us assume that initially we know nothing about  $\mathcal{O}$ , so that  $I_{\mathcal{O}}$  is a measure of the entropy of the system. However, with no other information we can conclude that  $\mathcal{O}$  is not a black hole, as the largest information that could be contained in any black hole in  $\Sigma$  is  $I_{\mathcal{S}}$ . We may then argue, using the Hoop theorem[57] that the energy contained within  $\Sigma$  (as measured either by a quasilocal energy on the surface or at infinity) must be less than that in a black hole whose horizon has area  $A[\mathcal{S}]$ . But this being the case we can now add energy to the system to bring it up to the mass of that black hole, which has the result of transforming  $\mathcal{O}$  into the black hole whose horizon just fits inside the sphere  $\mathcal{S}$ .

This can be done by dropping quanta slowly into the black hole, in a way that does not raise the entropy of its exterior. As a result, once the black hole has formed we know the entropy of the system, it is  $I_S$ . But we started with a system with entropy  $I_O$ , which we assumed is larger. Thus, we have violated the second law of thermodynamics. The only way to avoid this is if  $I_O < I_S$ .

We may remark that this argument employs a mixture of classical, statistical and semiclassical reasoning. For example, it assumes both that the hoop theorem from classical general relativity applies, at least in the case of black hole masses large in Planck units, to real, quantum black holes. One might attempt to make a detailed argument that this must be the case if the quantum theory is to have a good classical limit. However worthy of a task, this will not be pursued here, as it is unlikely that any such argument can be elevated to establish the necessity, rather than plausibility of the Bekenstein bound, in the absence of a complete theory of quantum gravity. Perhaps it is sufficient then just to note that in twenty years no plausible counter argument to the Bekenstein bound has survived.

### 3 Consequences of Bekenstein's bound

The Bekenstein bound has profound consequences for the question of how a quantum theory of gravity might be formulated. If the entropy of a quantum system is bounded above by  $S_{max}$ , it means that that system must have a finite dimensional state space, whose dimension is given by

$$\dim \mathcal{H} = e^{S_{max}} = e^{\frac{cA[S]}{4\ell_P^2}} \quad (2)$$

where  $c$  is a fixed dimensionless constant. This is true because a system whose entropy has an absolute upper limit can, in principle, be completely specified by giving the answers to a finite number of yes-no questions. Thus, not only can a quantum theory of gravity not be a conventional quantum field theory, with an infinite number of degrees of freedom per finite amount of spatial volume, it cannot even be a conventional cutoff quantum field theory with a fixed finite number of degrees of freedom per spatial volume. Instead, we have a fixed number of degrees of freedom per unit area of the surface of the region.

This has led 't Hooft[6] and Susskind[7, 8] to make the *holographic hypothesis*, which is that a theory of quantum gravity which describes a region

bounded by a spatial surface  $S$  may be described by a quantum theory with a finite number of degrees of freedom on the surface  $S$ .

There are, however, two difficulties with the holographic hypothesis which must be faced. The first is what kind of quantum theory will live on the surfaces. It seems challenging to imagine a kind of a field theory that could both provide a realistic description of interacting gravitational and matter fields, but is at the same time based on finite dimensional Hilbert spaces. Even the harmonic oscillator has an infinite dimensional Hilbert space. 't Hooft has explored the possibility that such a theory might be constructed from cellular automata. While this is an intriguing suggestion, in this paper a different proposal will be explored, which is that these theories are constructed, in a certain way to be described, from topological quantum field theories.

The second question that must be confronted is to what two dimensional surfaces are we to associate the hypothetical holographic field theories. Closely related to this is another worry. Most of the arguments given for the holographic hypothesis involve static situations. As a result, it is not completely clear whether the boundaries on which the field theories are to be defined should be in the general case two dimensional or three dimensional.

To address these issues, we must leave to one side for a moment the arguments of Bekenstein, 't Hooft and Susskind and consider a different set of problems, that seem at first sight to be completely independent. These are the interpretational problems of quantum cosmology.

## 4 Pluralistic quantum cosmology

In this section I motivate the main conceptual parts of the proposal of this paper. I begin with a review of the interpretational problems that the new proposal is intended to address.

### 4.1 Summary of the problem of the interpretation of quantum cosmology

The interpretational difficulties with quantum cosmology arise because the conventional interpretations of quantum theory require that the quantum state description be applied only to subsystems of the universe. The interpretation of the theory requires the existence of things which are in the universe but outside of the system described by the quantum state, including

the measuring instruments, the clocks that give meaning to the Schrodinger evolution and the observers.

There is, of course, a long history of attempts to modify the interpretation of the theory, while keeping the formalism fixed, in order to overcome the dependence of the quantum description on a split of the world into two parts, so that quantum theory may be applied to the universe as a whole. If I may briefly summarize the history of this story, it began with the many worlds interpretation of Everett[45], which may be put in several different forms, some more metaphysical[46] and some more operational[58]. However, all forms of the many worlds interpretation suffer from an ambiguity, which is called the preferred basis problem[52, 58]. The connection of the theory to actual observations requires the selection of a preferred basis, corresponding to observables we use to describe the real world. The theory itself seems to provide no such choice. A number of interesting ideas have been tried to solve this problem[52, 51], but none were completely successful.

The last few years attention shifted towards one kind of proposal to solve this problem, which is the consistent or decoherent histories program[47, 48, 49, 50]. The basic ideas here are two: first to interpret the theory in terms of sequences of projection operators, which pick out a history of quantum mechanical observations, and second, to give an interpretation only to measurements of consistent or decoherent sets of such histories, which are those for which the interference effects vanish, either absolutely or “for all practical purposes”. This program was based on the observation that sets of alternative histories that correspond to classical motions do have the property of being consistent, at least to an enormously good approximation.

However, this program is also facing a difficulty analogous to the preferred basis problem because, as shown in a recent paper by Dowker and Kent[54], there are an infinite number of equally consistent sets of histories, most of which do not correspond to anything that could be described in classical language. Thus, while it is true that there are sets of consistent histories that describe evolution in terms of quasi-classical variables, these are not distinguished by the formalism of the quantum theory. Thus, a selection principle, external to the formalism of the theory, seems to be needed to make a connection between the theory and what is actually observed.

Very interesting ideas which relate this selection problem to the existence of self-organized complexity have been proposed[49, 53], and are very much worth exploring. However, in light of this situation it may be interesting to examine other approaches, that require the formalism of quantum theory to be modified in order to extend it to a theory of the whole universe.

## 4.2 The basic idea of pluralistic quantum cosmology

I would like to describe here an approach to such a modification, that has evolved over a number of years through conversations with Louis Crane and Carlo Rovelli.

I would like to introduce this by stepping back for a moment and asking what the context of the problem is. The problem of extending our physics from a description of isolated systems to a description of the universe as a whole involves us in a number of profound issues, one of which is the question of what an observable in such a theory might be. Both abstract argument [59, 60, 61, 62], and the example of our one successful cosmological theory, which is general relativity [?, 64], tell us that the notion of what is observable must be different in certain aspects in a cosmological theory than it is in a theory of an isolated system. Indeed, general relativity differs from Newtonian physics profoundly in its formulation of observables, because it is based on a relational view which denies the existence of any nondynamical background structure that may give meaning to observables, such as coordinates or an a priori notion of time or spatial geometry. Instead, all observable quantities are defined in terms of relationships between dynamical degrees of freedom. This relational conception of observables, which has its origins in Leibniz's [59] and Mach's [60] criticisms of Newton, is realized mathematically in terms of invariance under active diffeomorphisms. As has been discussed in detail, the so called "problem of time" in classical and quantum cosmology is no more and no less than the expression of this situation with respect to the notion of evolution: the solution is that time evolution can only meaningfully be talked about in terms of relationships between physical degrees of freedom [55, 64, 65].

The only exceptions to this are the asymptotically flat case, or other cases in which boundary conditions are imposed, in which case the diffeomorphism invariance is broken by the conditions imposed on the boundary.

I would like to put this in the following way: the mathematical structure of general relativity in the cosmological case forbids any possibility of an interpretation in terms of operations or measurements made by an imaginary observer "outside of the universe." If we want to speak of an observer outside of the system we must modify the mathematical structure of the theory in a way that breaks the gauge invariance and so allows us to represent, in a kind of idealized way, the possibility of measurements made by an imaginary external observer.

We may then say that there should be a problem with extending quantum

mechanics unmodified to the cosmological case, because the theory does not satisfy the principle, just stated, that the mathematical structure of the theory should forbid the possibility of an interpretation in terms of an observer outside of the system. The mathematical structure of quantum mechanics in fact does implicitly refer to the existence of things outside the system. Among these are the clock, measuring instruments and observer that give meaning to the expectation values and their time evolution. It then follows that if we could find an interpretation of the theory in terms of measurements made by observers inside the quantum system, the theory would still be unsatisfactory, because the theory would allow us to formulate an interpretation in terms of an observer outside of the system. This is true by definition if we assume we can extend the formalism of quantum mechanics to the cosmological case without modification, because in that case the formal structures that are used in the standard Bohr interpretation are still present. Whatever other interpretation was offered, these structures could then be used as the basis of an interpretation of quantum cosmology in terms of an imaginary observer outside of the universe.

This is absurd, and it is my opinion that it is not enough just to refrain from doing this. As in the case of general relativity, we should require that *the mathematical structure of quantum theory must be modified in such a way that there is no possibility of an interpretation of the formalism in terms of an observer outside of the universe.*

I would like to raise this statement to the level of a fundamental principle, which we may call the *Principle of the absurdity of the possibility of an outside observer*. I have discussed its motivation and history in more detail elsewhere[3, 4, 62]. Here I would like to go on to sketch what modifications of quantum theory might lead to a theory that satisfies it.

If we follow the example of general relativity we have a clue, which is that the extension of the theory to the cosmological case should correspond to the restoration of a gauge symmetry, such that the standard theory, with fixed external observers, is obtained by breaking it in some way. What should this new gauge symmetry be? Given what has been said, it is clear that it should arise from treating the observer and the quantum system on an equal footing, so that the split between them can be made arbitrarily.

Thus, our goal is not to eliminate the observer, it is instead, to relativize him. We would like a formalism that allows us to divide the universe arbitrarily into two parts, and call one part of it the observer and the other the system. We would like there to be something like a gauge symmetry, that expresses the arbitrariness of the split. And, most importantly, to satisfy

the principle, we must do this in such a way that it is impossible to construct a single state space that would allow us the possibility of speaking in terms of a description of the whole system by an external observer.

There is a natural way to do this, which is to associate the operator algebras and their representations in terms of Hilbert spaces not to the universe as a whole, but to every possible splitting of the universe into subsystems<sup>3</sup>. Thus, for every way of splitting the universe into two parts we will have an associated state space and observable algebra. Unlike quantum mechanics, there will not be one preferred split, every possible split will be allowed. Instead of conditions that pick out which split is given an interpretation (analogous to the preferred basis problem) we seek consistency conditions that hold among the set of all state spaces and observable algebras. These conditions will specify the ways in which the observations made by each possible subsystem of the universe on the rest of it may be consistent with each other.

Thus, our slogan is “Not one state space and many worlds, but one world, described consistently by many state spaces.”

It is important for this point of view that we take the statistical interpretation of the quantum state, according to which the wavefunction is not something real, but is only a representation of the information that the observer has about the system[66]. We may note that the mixing of quantum and ordinary thermal statistics in gravitational fields, and the fact that they are mixed up by transforming between inertial and noninertial reference systems, strongly suggests that quantum statistics are really ordinary statistics which arise due to some non-local physics, and are distinguishable from conventional thermal effects only by special observers in special situations[67]. This point has been discussed at length elsewhere, for the present, it is only important to appreciate that there are reasons why the unification of quantum theory and relativity might force us to accept the statistical view of the quantum state<sup>4</sup>.

Further, we may note that there are clear arguments against the possibility of an observer, in either classical or quantum theory, making a complete measurement of the state of any system that includes them[68]. These arguments reply on the fact that a measurement requires a representation, within a subsystem of the observer, of the information gained about the

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<sup>3</sup>To my knowledge, this proposal was first made by Crane[1]

<sup>4</sup>Of course, there are claims of the “observability of the quantum state”. However, these seem to be in fact statements of the observability of non-classical or non-local quantum effects, which are undeniably real.

system. To make a measurement of something is then to gain some information, which is represented in the state of some degrees of freedom inside the observer. Thus, when an observer makes an observation, its state must change, to record the information gained. But if the observer is contained in the system, then the state of the system changes consequently, as a result of the measurement, from the measured state. The observer may attempt to again measure its state to ascertain the change, but again the state must change, so that the result is an infinite regress. Moreover, for a complete self-measurement he would need an infinite number of degrees of freedom, because there is a problem of infinite regress, each time he observes himself he needs a new place to put the information gained specifying his previous state.

Thus, as soon as we accept the point of view that the quantum state corresponds in some exact way with the information that an observer may gain about the system as the result of an observation of it, we must accept also that no such description can give a complete specification of the whole universe, assuming that it also contains the observer. The only way to have a description which is both complete and quantum mechanical is to use the fact that in fact the universe is complex enough that there are many subsystems that may be called observers, and to then postulate that the complete information for describing the state of the universe is represented in a number of Hilbert spaces, each of which represents the incomplete information that a particular observer may have about the universe.

We might also note that taking the Hilbert space to correspond to the interaction of an observer and a system brings the interpretation of the formalism into complete correspondence with what we actually do when we apply quantum theory to the real world. For in real practice, physicists describe different isolated systems with different Hilbert spaces and algebras of observables. One thing that this interpretation accomplishes is thus to make the theory correspond directly to actual practice. We no longer need to imagine that all the actual applications of quantum theory are to a greater or lesser extent approximations to some “real quantum description” in terms of some ideal “Hilbert space of the universe.”

### **4.3 The measurement problem and the consensus condition**

If we accept the basic idea of pluralistic quantum cosmology, we no longer have to answer questions about which split or which basis or which set of histories corresponds to reality. Instead, we have to answer a new kind of



question, which is what relationships may be expected to hold between any two possible Hilbert space descriptions, each giving an incomplete description of the world arising from a different division of the universe into two subsystems. This becomes the key question for the theory; we will see, indeed, that the whole of the theory, and indeed, even its dynamics, may be expressed in terms of these relationships.

The first questions that must be asked about these relationships is whether the existence of different incomplete descriptions of the same system can be consistent with the principles of quantum mechanics, and in particular with the uncertainty principle. Further, does this point of view shed any light on the measurement problem? An argument of Rovelli suggests that the answer is yes[2]. I would like now to review Rovelli's argument, as it suggests the need for a certain consistency condition, which I will call the *consensus condition*.

Suppose that there is a quantum system  $A$  surrounded by a two surface  $S_1$ , which we may informally call the "box" that contains  $A$ . Surrounding that box is second,  $S_2$ , which contains the system  $A$  and an observer,  $O_1$ . Outside of that is a second observer,  $O_2$ . The measurements the two observers,  $O_1$  and  $O_2$  make are recorded in the states in two Hilbert spaces we may call  $\mathcal{H}_{S_1}$  and  $\mathcal{H}_{S_2}$ .

Now, let us assume that  $O_1$  measures the state of  $A$ , using a measuring instrument that measures fields on the surface  $S_1$ . The result is described differently by the two observers, using the two Hilbert spaces. The first observer's instruments register a definite value,  $\lambda_1$  which means that she will project the state that describes  $A$  into a definite state  $|1\rangle \in \mathcal{H}_{S_1}$ . The second observer measures the combined state of  $A$  and the measuring instruments and observer  $O_1$ , and learns only that as a result of their interaction they are in some correlated state,  $\sum_i |i\rangle_1 \otimes |i\rangle_2 \in \mathcal{H}_{S_1}$  where the states  $|i\rangle_i$  are the quantum states of the first observer in which she has observed the system  $A$  to have eigenvalue  $\lambda_i$ .

The point is that there is no contradiction between these two records of the observation. The measurement of the second observer ascertains that the combined system of  $A$  and the first observer is in a correlated state, corresponding to a measurement having been made. But he is unable to ascertain what the result of the measurement was. In the meantime, the first observer, having made the measurement, has observed a definite value  $\lambda_1$ , and has reduced her description of the system  $A$  to the corresponding eigenstate.

This example shows that it may be possible to have different Hilbert

spaces which describe the results of observations made by different observers, in a way that allows the different states to be different from each other, but also consistent with each other. This example also shows us the way to a condition of consistency which must be imposed on the quantum states assigned to different boundaries. The state  $|i_o\rangle$  on the inner surface seen by the observer must not be precluded by the state  $\sum_i c_i |i\rangle_1 \otimes |i\rangle_2$  seen by the second observer, i.e. the amplitude  $c_{i_o}$  cannot be zero. We will call this the *consensus condition*. Stated formally, it means that

$$Tr_1 \left[ \rho^1 [Tr_2 \rho^{1+2}] \right] \neq 0 \quad (3)$$

where  $\rho^1$  is the statistical state the observer has that represents his information about the system,  $\rho^{1+2}$  is the statistical state describing the whole state of the observer and system and  $Tr_2$  means a trace over the observers degrees of freedom, while  $Tr_1$  means a trace over the system's state space.

#### 4.4 Other consistency conditions

There are two other consistency conditions that it is natural to impose at this stage. The first is that if the surface is composed of two disconnected parts,  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , we must have

$$\mathcal{H}_{\mathcal{S}_1 \cup \mathcal{S}_2} = \mathcal{H}_{\mathcal{S}_1} \otimes \mathcal{S}_2 \quad (4)$$

and, secondly, reversal of orientation is associated with Hermitian conjugation, so that, if  $\bar{\mathcal{S}}$  is the same surface, with orientation reversed,

$$\mathcal{H}_{\bar{\mathcal{S}}} = \mathcal{H}_{\mathcal{S}}^\dagger \quad (5)$$

The states associated with “the observer” then correspond to linear functionals of the states that correspond to “the system”, which is natural given that the state space is a description of their interface.

A simple objection may be raised to this, which concerns the case in which the area of the surface is small compared to the size of the whole universe. In this case the surface may contain a small observer looking at a large universe, but might it also not surround a small system being studied with all the resources available in a large universe? Thus, in the limit that the surface shrinks down, shouldn't one of its associated Hilbert spaces get very big, while the other becomes very small? The answer to this is that were this to happen it violates our *Principle of the impossibility of an interpretation of a cosmological theory by an observer outside of the universe*. We

must keep in mind that the state space associated to a boundary describes the information that an observer on one side may know about the part of the world in the other side. As such, when the area of the surface decreases, the dimension of the associated state space may decrease, representing either the limited information capacity of the observer, in the case that the “observer may be seen to be becoming small” or the decreasing number of degrees of freedom of the “system” in the case that it may be becoming small.

However, while it may help to think this way, we should also remember that there is no guarantee that the volume in one region or the other is a good physical observable in quantum gravity.

## 5 Postulates of pluralistic quantum cosmology

In the next section we will turn to the mathematical structure which is available to implement these ideas. For the present we may summarize the results of the reasoning up till this point in a list of postulates, that we would like a quantum theory of cosmology to satisfy.

**1) Definition of state spaces:** A quantum cosmological theory associates to every oriented three dimensional surface  $\Delta = S \times R$ , with  $S$  a compact two surface without boundary, a Hilbert space  $\mathcal{H}_\Delta^{QG}$ , which is a representation of an algebra of operators  $\mathcal{A}_\Delta^{QG}$ . These assignments of Hilbert spaces satisfy the conditions (4) and (5). Furthermore, for every cobordism  $\mathcal{M}$ , which is a four manifold with boundaries  $\partial\mathcal{M} = \Delta \cup \Delta'$  there is a linear map,  $\mathcal{M}_\Sigma : \mathcal{H}_\Delta^{QG} \rightarrow \mathcal{H}_{\Delta'}^{QG}$ .

**2) Consensus condition:** A *pluralistic quantum state of the universe* is an assignment of a statistical state  $\rho_{S \times R}$  into each  $\mathcal{H}_{S \times R}^{QG}$ , subject to the *consensus condition*, which in diffeomorphism invariant language is expressed as follows. Let the surface  $S$  be of the form of two disconnected pieces  $S = S_I \cup S_{II}$ . Then we know from (4) that

$$\mathcal{H}_{S \times R}^{QG} = \mathcal{H}_{S_I \times R}^{QG} \otimes \mathcal{H}_{S_{II} \times R}^{QG}. \quad (6)$$

The consensus condition is then the requirement that

$$Tr_I([Tr_{II} \rho_{I \cup II}] \rho_I) \neq 0 \quad (7)$$

The interpretations of these objects are the following. For every  $\Delta$  we may construct a four dimensional region of spacetime  $\mathcal{W}$  such that  $\Delta \subset$

$\partial\mathcal{W}$ . The region  $\mathcal{W}$  may contain an observer, together with clocks and measuring instruments that are able to measure the values of fields induced on  $\Delta$ . The algebra of observables  $\mathcal{A}_\Delta^{QG}$  then must have a classical limit which corresponds to an algebra of fields in a classical spacetime induced on a surface  $\Delta$  (see 4, below.) A state  $\rho_{S \times R}$  then corresponds to the information that an observer in  $\mathcal{W}$  may have about the fields induced on  $\Delta = S \times R$ . These, together with knowledge of the constraints and dynamics, may then be used to infer information about the values of fields in the region on “the other side of  $\Delta$ ”.

For example, if  $S = S^2 \times R$  whose interior is  $\mathcal{W} = S^3 \times R$ , then we may say that an observer who lives in the “world-tube”  $\mathcal{W}$  is able, by means of measurements of fields on the boundary  $S^2 \times R$ , to gain information about the universe external to the boundary. This information is then recorded in the value of a statistical state  $\rho_{S^2 \times R}$ .

**3) No interpretation in terms of external observers:**  $\mathcal{H}_{S \times R}^{QG} \rightarrow C$ , the complex numbers, in the limit that the two surface  $S$  shrinks to a point.

**4) Incorporation of quantum gravity:**  $\mathcal{A}_\Delta$  must have a subalgebra  $\mathcal{A}_S$ , which includes observables that measure the spacetime metric  $g_{ab}$  and left handed spin connection  $A_A^{AB}$  pulled back into the two surface  $S$ . (These will be denoted  $h_{\alpha\beta}$  and  $a_\alpha^{AB}$  respectively.)

**5) The Bekenstein condition:** Let  $\mathcal{H}_{S,h}$  be the subspace of  $\mathcal{H}_{S \times R}^{QG}$  containing eigenstates of the two metric on  $S$  with eigenvalues denoted by  $h$ . Then, if  $A(h)$  is the area of the two surface metric we require,

$$\dim(\mathcal{H}_{S,h}) = e^{cA(h)/l_{Pl}^2} \quad (8)$$

where  $c$  is a fixed dimensionless constant.

Finally, we require an axiom of correspondence with linearized quantum field theory, which may be formulated as follows,

**6) Correspondence with linearized QFT:** Let us choose a cosmological constant  $\Lambda$  (which is assumed one of the parameters of the theory), an area  $A_0$  and an ultraviolet cutoff  $r$  such that

$$l_{Pl}^2 \ll r^2 \ll A_0 \ll \Lambda^{-1/2} \quad (9)$$

Then, let  $\mathcal{H}_{\Lambda,A,r,\epsilon}^{lin}$  be the Hilbert space of linearized gravitons (and other massless fields, if desired) restricted to a region of DeSitter spacetime with cosmological constant  $\Lambda$  bounded by a three boundary  $S^2 \times R$ , with the metric induced on the two sphere being a two sphere metric with area  $A$  by the condition that the linearized fields vanish at the boundary. Further, in

the reference frame in which the boundary is static two conditions are put on the spectrum. First, the energy of the individual gravitons and other massless quanta must be less than the inverse cutoff  $r^{-1}$ . Second, the total energy (defined in terms of the quasi-local surface integral  $H = \int_{S^2} \kappa$ ) must be bounded by

$$E < \epsilon \sqrt{A} \quad (10)$$

with epsilon much less than one. Let  $\mathcal{A}_{\Lambda, A, r, \epsilon}^{lin}$  be an algebra of observables associated with this space.

The meaning of this space and algebra is they define the linearized limit of any quantum theory of gravity with small cosmological constant, restricted to the interior of a static surface. Two cutoffs are needed, one to limit the energy of any single graviton to less than the Planck scale, the other to limit the total energy to be much less than that at which gravitational binding energy, and the collapse to a black hole, would be relevant.

For the quantum theory of cosmology to have an acceptable linearized quantum field theory limit we require that here exists a family of maps,

$$\mathcal{N}_{\Lambda, A, r, \epsilon} : \mathcal{H}_{\Lambda, A, r, \epsilon}^{lin} \rightarrow \mathcal{H}_{S^2 \times R}^{QG} \quad (11)$$

$$\mathcal{N}_{\Lambda, A, r, \epsilon} : \mathcal{A}_{\Lambda, A, r, \epsilon}^{lin} \rightarrow \mathcal{A}_{S^2 \times R}^{QG} \quad (12)$$

such that

$$\langle \mathcal{N} \circ \Psi | \mathcal{N} \circ \mathcal{O} | \mathcal{N} \circ \Psi \rangle^{QG} = \langle \Psi | \mathcal{O} | \Psi \rangle^{lin} + O(r/l_{Pl}) + O(\epsilon) + O(l_{Pl}^2/A) \quad (13)$$

This guarantees that there is a subset of the physical states and observables whose physical expectation values reproduce the predictions of the linearized theory.

To proceed to investigate the reasonableness of these postulates we must ask if it is consistent with the physical interpretation of both general relativity and quantum theory, especially given the special problems associated with the notion of time. This is the subject of the next section.

## 6 The problem of time

One advantage of the proposal just made is that it may allow a new possibility for the resolution of the problem of time, which has plagued attempts so far to construct a quantum theory of cosmology.

To introduce the basic idea, let me recall some of the proposals to resolve the problem of time in quantum cosmology. One, due to DeWitt[69] and

developed and advocated by Rovelli[55], is to use the Heisenberg picture and construct physical operators that describe correlations between certain degrees of freedom, associated to clocks and other degrees of freedom. In the classical theory, it is certain that there are a large set of physical observables (that is functions on the kinematical phase space that commute with the constraints under the Poisson brackets) which are what Rovelli calls “evolving constants of motion.” It then may be that operators can be constructed in the quantum theory that have the same meaning (although, of course there are daunting technical problems in doing this for a realistic field theory rather than for an integrable system.)

However, even if these technical problems could be overcome, there is an important conceptual problem, which is how the expectation values of these operators are to be correlated with observers made by us observers inside the universe. If we could imagine that there are observers outside of the universe, who make observations of the universe, and are able to many times prepare and measure the state of the whole universe, they would be able to verify whether or not the expectation values of these “evolving constant of motion operators” in a particular physical state are in accord with their observations. But, this is something we, as observers in the universe, are unable to do.

If we insist on our principle that there should be no possibility of speaking in terms of an observer outside of the universe, then Rovelli’s proposal, even if it works technically, would not lead to predictions that could be checked by us observers inside the universe.

The same situation holds in other proposals concerning the problem of time in quantum cosmology such as the proposal of Page and Wootters[70], and the proposal of [65]. In the first, one imagines that the Hilbert space is a direct product

$$\mathcal{H} = \mathcal{H}^{clock} \otimes \mathcal{H}^{other} \tag{14}$$

of a space representing a clock degree of freedom and a space representing other degrees of freedom. Physical quantum states describe correlations between certain degrees of freedom considered to be clocks and the other degrees of freedom, coded in the entanglement of the state, as in

$$|\Psi \rangle = \sum_i c_i |t_i \rangle^{clock} \otimes |\chi_i \rangle^{other} . \tag{15}$$

These correlations are understood to be induced by the constraints, which impose the dynamics. The different possible physical states then correspond

to the different sets of correlations that are possible between the clock and the other degree of freedom, given the dynamics.

Again, we have the problem that the predictions of the existence of such correlations could only be detected by an observer outside the system. Even if we accept the many worlds point of view, which Page and Woiters seem to do, an observer inside the system can only verify the presence of one branch, but they cannot give meaning to the coefficients  $c_i$  nor they can tell if the theory gives the right relative weights to the different correlations.

Conversely, in the Barbour picture, the notion of a single clock degree of freedom is given up in favor of the more realistic picture that time is measured from the coincidences among all the degrees of freedom in a complex universe. Instead, the “wavefunction of the universe” is interpreted to give directly the relative frequency for the occurrence of instantaneous configurations in a great collection of such configurations. The collection of such “moments” is taken to constitute reality; any impression of time comes from the existence of memories, records and what Barbour calls “time capsules” in complex instantaneous configurations.

The problem is again that only an observer outside of the universe can give a meaning to these relative frequencies for “moments” in Barbour’s conjectured reality.

Of course, all of these proposals allow that classical general relativity coupled to quantum fields is recovered if the state is appropriately “semi-classical”. But while a necessary condition for an interpretation, this is not sufficient, as a good quantum theory of cosmology should in principle give predictions for all states, not only those that are semiclassical. Otherwise, its predictions may not differ from those of semiclassical gravity, perhaps together with the addition of some particular boundary condition.

From the present point of view, the problem which all these proposals have in common can be resolved in the following way. The physical quantum state is now associated to a three dimensional surface, which is understood to represent the evolution in time of the two dimensional boundary of a part of the universe. The description can be completely invariant, with respect to diffeomorphisms both in and inside of the surface. Thus, time must be understood in terms of correlations between degrees of freedom inside the surface, whether these are expressed in Barbour’s, Page and Woiters’ or Rovelli’s pictures. But there are in fact observers outside of the system, they live just outside the boundary. Furthermore, if we use the statistical interpretation of the quantum state, then the statistics predicted from the expectation values of operators are interpreted in terms of the relative fre-

quency probability for what correlations between clocks and observers they will see if they observe the system on their surface.

For example, if the state is of the form (15) then the  $|c_i|^2$  are the probability that if they observe the system they will find that the clock reads a time  $t_i$  while the rest of the system is in a state  $|\chi_i\rangle$  (assuming for simplicity the trivial inner product.) They can in principle check the predicted probabilities by preparing the part of the universe inside the boundary many times in the state  $|\Psi\rangle$  and repeating the experiment.

Suppose our observers do the experiment in sequence many times, will they record a sequence of times that are in chronological order? Will the times they measure for the clock in the system be synchronized with clocks they may carry outside of their boundary? To answer such questions we cannot just rely on the quantum description of the observers themselves. Such questions can only be answered in the quantum theory by enclosing the observers and the system inside a second boundary  $S' \times R$  and constructing a Hilbert space to represent the combined system of the first system and its observer. The state of this combined system then describes the information that a second observer, outside the second boundary can have about it. This observer can make observers of the combined system of the first observer plus the first system, and by doing so answer questions like these.

In this context as well, the consensus condition (3) is essential to guarantee that the probabilities actually seen by the first observer agree with the  $|c_i|^2$  in the expansion of the state seen by the second observer. For, by the theorem of Finkelstein[71] and Hartle[72] it guarantees that in the limit of a large number of observations, these two probabilities will agree. Thus, in the present context, this theorem plays an essential role, but one rather different than in the many worlds interpretation. Rather than giving a meaning to probability it guarantees that the probabilities for events seen by the different observers are consistent with each other.

Thus, it may be said that this interpretation is giving meaning to Bohr's statement that the split between the classical and quantum world can be made arbitrarily[73]. For it allows us to consider a description that is given by specifying simultaneously the states associated with all such possible splittings. It is very interesting that to use this interpretation to resolve problems in quantum cosmology such as the problem of time we must use some of the results associated with the many worlds interpretation. However, as in the case of the theorem of Finkelstein and Hartle, their role is transformed from giving a meaning to an interpretation in terms of one state and many worlds, to ensuring the consistency of an interpretation in which



many states describe many partial observations of one world.

## 7 The classical limit and the timelike initial data problem

If the picture we are describing is to work, it must have a classical limit which makes sense in terms of classical general relativity. We must then ask whether there is a formulation of classical general relativity in the spacially compact case in which the theory may be described in terms of fields induced on a timelike surface  $\Delta = S \times R$ . This is equivalent to posing the *timelike initial data problem* [74]. This is the question of whether there is data, subject to constraints, that may be given on such a surface  $\Delta$ , such that the three metric given on  $\Delta$  has signature  $(-, +, +)$  and such that the data determines a solution to Einstein's equations, at least in a neighborhood of the surface.

The timelike initial data problem, along with its cousin, the null initial data problem, is not as well studied as the conventional initial data problem in terms of spacelike surfaces. But, it has often been observed that the Hamiltonian formalism does not require that the surface on which the fields and momenta are defined be spacelike. Thus, at least formally, one can construct a formalism for first order evolution of data, subject to constraints, from a timelike initial data surface. In this formalism, momenta correspond to derivatives off of the surface, and there are constraints, which are of the same form as the usual constraints, with certain changes of sign due to the change in the signature of the induced three metric.

This formalism will be discussed in detail elsewhere[75]. For the present, we need only know that at the formal level the Hamiltonian formalism can be translated to a timelike surface. It is also interesting to note that in this case it is the diffeomorphism constraints that induces the correlations between degrees of freedom on the surface that might be used to measure time and other degrees of freedom. The role of the Hamiltonian constraint is then to develop the solution into the interior, and thus establish correlations between fields on the surface and fields in the interior.

## 8 Incorporating topological quantum field theory and quantum gravity

Now that we have seen that the problem of time, as well as the classical limit, seem to pose no problems of principle, we may turn to the question of trying to realize the postulates made in section 5 in terms of a concrete mathematical construction.

The proposal made above relies on an apparant coincidence, which is that both the Bekenstein bound and the point of view about quantum cosmology proposed in the section 4 lead to the same conclusion, which is that quantum gravity, in the cosmological case, may be defined in terms of state spaces attached to three dimensional “timelike” surfaces which represent the information that observers who measure fields induced on the surface may have about the degrees of freedom in its interior. Thus, it is very natural to synthesize the two lines of development, which is what we have done in formulating these postulates.

The question that must now be posed, however, is how these postulates are to be realized in a concrete theory. In the following sections I will present one proposal for how to do this, which is based on a further apparent coincidence.

In order to proceed the main question that must be answered is how the finite dimensional state spaces, which are the eigenspaces of the two metric operators are to be realized. The hope to realize this and the other conditions comes from an apparent coincidence of two results, one from non-perturbative quantum gravity and one from topological quantum field theory. The first is the discovery that in non-perturbative quantum gravity the operators that measure areas of regions,  $\mathcal{R}$  of any two dimensional surface  $\mathcal{S}$ , which may be denoted  $A[\mathcal{R}]$ , have discrete spectra. To say this more precisely, the basic result is that the quantum states of a diffeomorphism invariant quantum field theory in a region  $\Sigma$  are given by the diffeomorphism classes of the spin networks in  $\Sigma$  [18, 20]. The spin networks are the eigenstates of  $A[\mathcal{R}]$  and the corresponding eigenvalues are given by the spins of the edges of the network that intersect the surface at points in the region  $\mathcal{R}$ . Thus, if  $|\Gamma\rangle$  is the quantum state associated to the spin network  $\Gamma$  in  $\Sigma$  and  $y_\alpha^{\Gamma, \mathcal{R}}$  are the points at which the spin network meets the region  $\mathcal{R}$  of the boundary, and  $j_\alpha$  are the spins of the edges that meet boundary, we have

$$\hat{A}[\mathcal{R}]|\Gamma\rangle = l_{Pl}^2 \sum_{\alpha} \sqrt{j_\alpha(j_\alpha + 1)} |\Gamma\rangle \quad (16)$$

One way to measure the two metric, up to diffeomorphisms, is to measure the  $A[\mathcal{R}]$  for all the  $\mathcal{R} \subset \mathcal{S}$ . We see then that the two metric can be considered to have a discrete spectrum. The simultaneous eigenspaces of the  $A[\mathcal{R}]$  are labeled by punctures  $y_\alpha$  marked by spins  $j_\alpha$  which label representations of  $SU(2)$ . Thus, the eigenspaces  $\mathcal{H}_{\mathcal{S},h}^{QG}$ , which should have finite dimension, should more precisely be labeled  $\mathcal{H}_{\mathcal{S},y_\alpha,j_\alpha}^{QG}$ .

As a result, there are no problems with continuum measures in the statement of postulate **5**, so that we may write,

$$\mathcal{H}_{\mathcal{S} \times \mathcal{R}}^{QG} = \bigoplus_n \bigoplus_{j_\alpha} \mathcal{H}_{\mathcal{S},y_\alpha,j_\alpha}^{QG} \quad (17)$$

This is a step, but it does not yet explain to us how to construct state spaces that are eigenspaces of these operators that have finite dimension that grows exponentially with the area. To see how to do this, we must turn to some basic results of topological quantum field theory.

Before explaining how this problem is solved, though, we must divert to discuss a remarkable observation of Louis Crane, which is that the axioms of topological quantum field theory in three dimensions are exactly what is needed to guarantee the consistency of a many-Hilbert space reformulation of quantum theory of the type we have been discussing[1]. This is because by the axioms of Atiyah[10], TQFT's automatically assign state spaces to boundaries, so that the conditions (4) and (5) are satisfied. Moreover, the posulate **3**) is *automatically* realized as a result of the axioms. For these and other reasons, described in his papers, Crane has advocated the construction of a quantum theory of gravity as an extension of topological quantum field theory. The proposal I will shortly make is indeed an attempt to realize this program<sup>5</sup>.

Furthermore, topological quantum field theory naturally extends to the case of two dimensional surfaces with points marked by representations of groups[26] and spin networks play a natural role in this formulation. Among the axioms of *TQFT* one finds that a three dimensional *TQFT* associated to a quantum group  $G$  is a functor that assigns[10, 1]:

- to every two dimensional surface  $\mathcal{S}$  with  $n$  punctures  $y_\alpha$ ,  $\alpha = 1, \dots, n$  labeled with representations  $j_\alpha$  of  $G$  a finite dimensional Hilbert space  $\mathcal{H}_{\mathcal{S},j_\alpha}^G$ .

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<sup>5</sup>Other attempts to realize quantum gravity as an extension of TQFT are described in[1, 76, 77, 78, 79, 80].

- to every three manifold  $\Sigma$  with imbeded  $G$ -spin network  $\Gamma$ , such that  $\partial\Sigma = \mathcal{S}$  and  $\Gamma$  intersects  $\mathcal{S}$  in the  $n$  labeled points  $y_\alpha$ , with the line intersecting  $y_\alpha$  being labeled by the same representation  $j_\alpha$ , a state  $|\Gamma, \Sigma\rangle \in \mathcal{H}_{\mathcal{S}, j_\alpha}^G$ .

Note that this is defined to be a functor from the category of manifolds, with cobordims as maps to the category of Hilbert spaces, with linear maps as the maps. This guarantees that the conditions (4) and (5), as well as postulate **3**) are automatically satisfied.

Thus, it seems that topological quantum field theories are exactly the objects we need to construct the finite dimensional state spaces that represent eigenspaces of  $\mathcal{H}_{\mathcal{S}, h}$ . More precisely, if we use the coincidence that spin networks and points marked with representations of a group appear in both *TQFT* and quantum gravity we may then make the hypothesis that there is a topological quantum field theory associated with  $SU(2)$ , to which we may identify these spaces, so that

$$\mathcal{H}_{\mathcal{S}, y_\alpha, j_\alpha}^{QG} = \mathcal{H}_{\mathcal{S}, y_\alpha, j_\alpha}^{TQFT} \quad (18)$$

There is a natural candidate for this TQFT, which is the Chern-Simons theory associated with the group  $SU(2)$ . The variable there, which is an  $SU(2)$  connection, is exactly what we need to represent  $a_\alpha^{AB}$ , the pull back of the Ashtekar connection to  $\mathcal{S}$ . If we are to try to realize this hypothesis, however, there are three questions we must answer.

- Can we give an interpretation, in terms of the parameters of quantum gravity, to the coupling constant, or level,  $k$  of the Chern-Simons theory? This involves two related puzzles, first that it seems to be ordinary  $SU(2)$  spin networks that play a role in non-perturbative quantum gravity, while it is the q-deformed networks that are relevant for TQFT, and second that the commutation relations of the two theories are different.
- Can the constraints of quantum gravity be represented, or its solutions expressed, in terms of the states of the Chern-Simons theory?
- Is the Bekenstein bound satisfied?

It turns out, as I will describe in the next section, that there are natural answers to the first two questions and that, furthermore, the answer to the third question is yes.

## 9 A proposal

From the side of quantum gravity, we seek a theory that provides a representation for the observables that will be induced in a three dimensional timelike surface,  $\Delta = S \times R$ , where  $S$  may be considered an intersection of  $\Delta$  with a surface of constant “time”. We may then, to make a connection with the standard 3 + 1 canonical formalism consider the algebra of observables induced in the two surface  $S$ . These include the pull backs into  $S$  of the curvature two form  $F_{ab}^{AB}$  and the dual of the densitized form field  $E_{ab}^{AB} = \epsilon_{abc} \tilde{E}^{cAB}$  which I will denote by  $f^{AB}$  and  $e^{AB}$ , respectively<sup>6</sup>. We may note that these all commute with each other under the standard Poisson brackets of general relativity.

In ref. [27] I studied the case in which  $\Delta$  was a boundary of spacetime, to which we needed to associate particular boundary conditions as well as a boundary term in the action. There I studied a particular condition called the “self-dual” boundary condition, in which

$$f^{AB} = \frac{2\pi}{kG} e^{AB} \quad (19)$$

where  $k$ , was required by gauge invariance to be an integer, and was as well related to the parameters of general relativity by

$$k = \frac{6\pi}{G^2 \Lambda} + \alpha \quad (20)$$

where  $G$  and  $\Lambda$  are Newton’s constant and the cosmological constant, respectively and  $\alpha$  is a  $CP$  violating parameter coming from an  $\int F^{AB} \wedge F_{AB}$  term in the action.

This has the effect of inducing a topological quantum field theory in the boundary, which was in fact  $SU(2)_q$  Chern-Simon theory, with  $k$  the level, or quantum deformation parameter. This is possible because an additional effect of imposing the boundary condition (19) is that the algebra of observables in  $S$  was deformed, so that the connection one form  $a_\alpha^{AB}$ , which is the pull back of the Ashtekar connection  $A_a^{AB}$  into  $S$  satisfies,

$$\{a_\alpha^{AB}(\sigma), a_\beta^{CD}(\sigma')\} = \frac{2\pi}{k} \epsilon_{\alpha\beta} \delta^2(\sigma, \sigma') (\epsilon^{AC} \epsilon^{BD} + \epsilon^{AD} \epsilon^{BC}) \quad (21)$$

which are in fact the Poisson brackets of Chern-Simons theory.

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<sup>6</sup>  $A, B, \dots$  stand for two component spinor indices. We will also choose conventions in which  $f^{AB}$  and  $e^{AB}$  have dimensions of  $length^{-2}$  and  $length^0$ , respectively.

As a result, a Hilbert space that represents this algebra of observables, subject to the condition (19) is given, for each choice of the boundary  $S$  by the direct sum,

$$\mathcal{H}_S^{gr} = \bigoplus_{n=1}^{\infty} \bigoplus_{j_1, \dots, j_n} \int d^2 y_1 \dots d^2 y_n \mathcal{H}_{S, y_\alpha, j_\alpha}^{CS, k} \quad (22)$$

where  $y_\alpha$  with  $\alpha = 1, \dots, n$  are the positions of  $n$  punctures on the surface  $S$ , the  $j_\alpha$  are  $n$  labels of representations of  $SL(2)_q$  (or quantum spins) located at  $n$  punctures on the surface  $S$ . By standard constructions in TQFT there is a finite dimensional Hilbert space associated with each choice of  $S, n, j_\alpha$  and  $k$ .

A question that was left open in [27] was whether the diffeomorphisms of the two surface  $S$  can be imposed as a gauge symmetry. There seem to be technical obstacles to doing so, but there is also reason to believe they may be overcome. In this case the state space associated to the surface would simply be

$$\mathcal{H}_S^{gr} = \bigoplus_{n=1}^{\infty} \bigoplus_{j_1, \dots, j_n} \mathcal{H}_{S, j_\alpha}^{CS, k} \quad (23)$$

because, of course, the Hilbert space of the TQFT does not depend on the positions of the punctures.

In reference [27] the conjecture was then made that the algebra of observables on the boundary was sufficient to label the quantum states of the interior. While this is not yet shown, there is some evidence for it, which was discussed there. If this is the case, then the physical state space of quantum gravity, in the presence of the self-dual boundary conditions, would then have the form (22).

In the case we are discussing here, the two surface plays a somewhat different role. Rather than being a boundary of space, at which certain boundary conditions may be imposed, it is now to be thought of as simply an arbitrary two dimensional surface in space. The question is whether any of the structure discovered for this particular set of boundary conditions is at all relevant to this case.

To investigate this question, let us ask what replaces the self-dual condition (19) in the general case? We may recall that, in the case that the field equations (or at least their pullback, into  $S$ ) are satisfied, there is a relationship between the fields  $f^{AB}$  and  $e^{AB}$  in  $S$ , which is given by

$$f^{AB} = \left( \Psi^{ABCD} + \frac{G\Lambda}{3} \epsilon^{AB} \epsilon^{CD} \right) e_{CD} \quad (24)$$

where  $\Psi^{ABCD}$  is a totally symmetric spinor, representing the spin-two degrees of freedom of the gravitational field. This relationship can be seen both from the  $CDJ$  formalism[81] and the Newman-Penrose equations[82]. We may note that this equation constitutes in fact the general solution to the Hamiltonian and diffeomorphism constraints, or equivalently the frame field equations, pulled back into any three surface that contains  $S$ .

This equation can also be understood to be an expression of the hamiltonian and diffeomorphism constraints. As pointed out by Reisenberger[56], one can express the constraints coming from the CDJ action by the statement that there must exist a symmetric spinor  $\Psi^{ABCD}$  such that (23) is satisfied. This may have some advantages over the standard, Ashtekar, form of the constraints, as its derivation makes no assumption of the invertibility of the metric.

How are we to represent this relationship quantum mechanically? It is clear that what the equation is saying is that in general the two fields  $e^{AB}$  and  $f^{AB}$  are free to some extent to vary independently, as long as they are related by (24). This means that they can differ by the action of two terms, there is a contribution to  $f^{AB}$  which is proportional to  $\Lambda$  times  $e^{AB}$  and there is a second term which is proportional to the action of a spin two field.

To translate this quantum mechanically let us note that in the case that the spin two field is absent they are strictly proportional to each other. In this case, how is the equation (19) represented quantum mechanically? In the loop representation,  $e^{AB}$  will be represented by a set of punctures, which are the intersections of a spin network  $\Gamma$  with the two surface  $S$ . These punctures are labeled by representations of  $SU(2)_q$ , which are those carried by the lines of the spin network at the intersections. If  $f^{AB}$  is, in this limiting case, to be proportional to  $e^{AB}$ , then it must also be represented by the same set of punctures, labeled by the same representations. This is what happens when we impose the self-dual condition (19), as described in [27]. The result is that, under these conditions, when we measure the metric of the two surface (and hence  $e^{AB}$ ), we pick out an eigenstate of the operators that measure elements of area in the surface, and this picks out a set of punctures and representations. Because of the proportionality of  $e^{AB}$  and  $f^{AB}$ , the result is that this picks out a two dimensional quantum field theory, which is  $SU(2)_q$  Chern-Simons theory with that choice of punctures and representations. The states of this theory represent the freedom that the connection,  $a_\alpha^{AB}$ , still has, once the condition (19) is imposed.

Now let us consider the case that the spin two field  $\Psi^{ABCD}$  is non-zero, so that the self-dual condition (19) is relaxed to (24). In this case we may

represent the situation as follows. The representation of the metric observables  $e^{AB}$  should be the same, so these will have eigenstates, associated with the intersections of spin networks with  $S$ , which are hence labeled by choices of punctures,  $y_\alpha$  and representations  $j_\alpha$ . How are we to represent the degrees of freedom of the connection  $a_\alpha^{AB}$ , now subject to the constraint (24)? Let us represent this as before by the Hilbert space of  $SU(2)_q$  Chern-Simons theory, with the same set of punctures as before. This is in agreement with (24), which tells us that in regions over which the integral of  $e^{AB}$  vanishes, the integral of  $f^{AB}$  must vanish as well. However, when it is allowed to be nonvanishing, let us allow for the connection to vary independently by letting the spins, which we will call  $l_\alpha$  at these punctures vary.

Thus, the degrees of freedom of the quantum theory will be punctures,  $y_\alpha$ , each of which is labeled by two spins,  $j_\alpha$  and  $l_\alpha$  corresponding to the separate metric and connection degrees of freedom. However, the metric and connection are not completely free to vary independently, because they are constrained classically by (24). Since  $\Psi^{ABCD}$  is a spin two field, this means that quantum mechanically, each  $l_\alpha$  must either be proportional to the corresponding  $j_\alpha$ , or must be in the decomposition of the multiplication of  $j_\alpha$  with the spin-two representation. That is, the quantum mechanical version of (24) may be hypothesized to be,

$$l_\alpha \in (2 \otimes j_\alpha, j_\alpha) \quad (25)$$

(Of course  $j_\alpha$  is already contained in the decomposition of the product of itself with spin 2, but for clarity, because it comes from a separate term, we list it separately.)

The result is that the state space of quantum gravity, associated with the two surface  $S$  now becomes,

$$\mathcal{H}_S^{QG} = \bigoplus_n \bigoplus_{j_\alpha} \mathcal{H}_{S, j_\alpha}^{QG} \quad (26)$$

where the  $\mathcal{H}_{S, j_\alpha}^{QG}$  are the eigenspaces of  $\mathcal{A}[S]$ , the area of  $S$ , with eigenvalues  $\sum_\alpha \sqrt{j_\alpha(j_\alpha + 1)}$  (neglecting corrections in  $1/k$  [37].) Each of these eigenspaces is then further decomposed,

$$\mathcal{H}_{S, j_\alpha}^{QG} = \bigoplus_{l_\alpha \in (2 \otimes j_\alpha, j_\alpha)} \mathcal{H}_{S, l_\alpha}^{CS, k} \quad (27)$$

Having specified the state space of the theory we must now discuss the role of the parameter  $k$ . We may first note that  $k$  must be considered to be a parameter of the theory as it comes into the specification of the observables



algebra, in (21). We can see from that equation that  $k^{-1}$  not vanishing indicates that there is a kind of anomolie in the theory, so that the classical commutation relations have been modified so that all components of  $A_a^{AB}$  no longer commute. As is shown in [27] this turns out to be related to the necessity to frame the loops that go into the spin network constructions, so that a diffeomorphism in  $S$  that has the effect of twisting a line of a spin network (or rotating a puncture) has the effect of multiplying the states of the theory in the spin network basis by a phase.

We may note that there are reasons to believe that this is a real effect in the case of non-vanishing cosmological constant, which comes from the attempts to define states in the loop representation that would correspond to the loop transform of the Kodama state[83]. Since the anomolie is proportional to  $k^{-1}$ , it is consistent to take  $k \approx \Lambda^{-1}$ , as equation (20) asserts. To fix the value of  $k$ , we can use the arguments from [27], which apply, however, to the case that  $\Psi^{ABCD}$  is constrained to vanish, yielding eq. (20). Alternatively, we can use arguments coming from the attempts to define the path integral involved in taking the loop transform of the Kodama state, which lead to the same conclusion[85, 84]. It would, however, be better to have a more general argument, independent of either a restriction of the degrees of freedom at the surface, or the choice of a particular state. Thus, at present, it is perhaps best to take (20) over to the present case as a working hypothesis.

Further, we may note that as (24) is the general solution to the Hamiltonian and diffeomorphism constraints, what we have expressed quantum mechanically through (25) is in fact a parametrization of the space of solutions of the constraints. Further, the fact that the frame field has been expressed in terms of spin networks (leading to the punctures) means that the Gauss's law constraint has been solved. We may then take (27) to be a description of the physical state space of the theory.

Finally, we may note that the Bekenstein bound is in fact satisfied in this case, at least in the limit of large  $k$ , or small  $G^2\Lambda$ . The bound will hold (in the limit  $k \rightarrow \infty$ ) if there is a fixed constant  $c$  such that

$$\ln \dim \mathcal{H}_{S, j_\alpha}^{QG} \leq c \sum_{\alpha} \sqrt{j_\alpha(j_\alpha + 1)} \quad (28)$$

for all choices of punctures and labels. However, in the same limit the dimension of  $\mathcal{H}_{S, j_\alpha}^{QG}$  is just the product of all dimensions of the allowed rep-

representations of  $SU(2)$  at each puncture[9]. Thus, we have,

$$\sum_{\alpha} \sum_{l \in (2 \times j_{\alpha}, j_{\alpha})} (2l + 1) \leq c \sum_{\alpha} \sqrt{j_{\alpha}(j_{\alpha} + 1)} \quad (29)$$

It is not difficult to discover that this will be true for all  $c \geq 28/\sqrt{3}$ .

We have thus achieved our goal, which is to postulate a starting point for a quantum theory of gravity that is consistent with the results we want to keep from previous investigations, including the kinematical structure of the loop representation. Further, we have the theory in a language which is suitable for pluralistic quantum theory and, using the connection between spin networks, as they represent eigenstates of the area operator in quantum gravity, and spin networks as they label states in topological quantum field theory, we have found a way to naturally implement the Bekenstein bound. This means that we have a field theory, associated to two dimensional surfaces, that implements the holographic hypothesis, and implements naturally the condition that the subspaces of the state space that are eigenspaces of the metric of the boundary are finite dimensional, with the dimension of the boundary growing exponentially with the physical area.

## 10 Conclusions

It need hardly be said that the present theory is put forward with all due reserve. What I have described here is only a sketch of a research program. There are several directions in which the proposals outlined here might be pursued. The theory sketched in the previous section needs to be developed by the introduction of additional observables, corresponding to components of the metric, connections and curvatures which are not tangent to the surfaces. It is possible, following the results of section VII of [27], that this will enable the full tangle algebra of Baez [86] to be represented.

At the same time, one may try to sharpen the system of postulates proposed here to a system of axioms that could be taken as the foundations of a quantum theory of gravity. The question in this case is whether a system of axioms can be found which allows both classical general relativity and quantum field theory on fixed backgrounds to be derived in the appropriate limits.

In this respect, a very interesting possibility is that Jacobson's recent construction[43] can be used to derive the Einstein equations directly from thermodynamics together with a set of postulates about the fundamental

quantum theory, such as those proposed here. According to Jacobson's construction all that is needed to derive the Einstein's equations as the classical limit of a quantum theory of gravity are three assumptions: 1) the laws of thermodynamics 2) the recovery of linearized quantum field theory in restricted regions whose curvatures are small in Planck units and 3) the connection between entropy and surface area. The idea is that 2) may be guaranteed by the last postulate while 3) is guaranteed by the Bekenstein condition (8). We may expect that the laws of thermodynamics are to be recovered from the usual statistical assumptions, given that the use of ordinary, relative frequency statistics is guaranteed by the consensus condition. Whether this can be carried out in detail is a very interesting question; if it can then the conclusion would be that in a quantum theory of gravity the fundamental axiom that introduces gravitation is the connection between information and entropy, from which Newton's laws of gravity would follow as consequences of the recovery of general relativity in the classical limit. This reverses the conventional point of view in which the law of gravitation, expressed in terms of Einstein's equations is taken to be fundamental, with the connection between information and surface area a consequence.

In this respect, the key result from canonical quantum gravity that is being carried over into this context is the connection between spin, labeling the spin networks that are the kinematical quantum states, and area.

Finally, on the conceptual side, the proposal made here concerning the interpretation of quantum theory has certain implications that need to be investigated. As presented here, the quantum states are taken to describe information held by, or available to, certain observers. This certainly leaves open the possibility that the quantum theory might turn out to be the statistical mechanics of a more fundamental theory such as a non-local hidden variables theory[87]. For reasons that have been often discussed, any departure from the physical postulates of the quantum theory is likely to be cosmological in origin[3, 4, 87, 88]. It may thus be hoped that, even if quantum mechanics, expressed in terms of linear state spaces and operators, is not a fundamental theory, a formulation of quantum theory that sensibly extends to a theory of cosmology may be a stepping stone to a deeper theory.

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