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Two-Dimensional Dilaton Black Holes

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Abstract. The two-dimensional CGHS model provides an interesting toy-model for the study of black hole evaporation. For this model, a quantum effective action, which incorporates Hawking radiation and backreaction, can be explicitly constructed. In this paper, we study a generalization of this effective action. In our extended model, it is possible to remove certain curvature singularities arising for the original theory. We also find that the flux of Hawking radiation is identical to that encountered in other two-dimensional models.

1 Introduction

In 1992, Callan, Giddings, Harvey and Strominger (CGHS) presented an interesting two-dimensional toy-model (Callan et al. [1992]) for the study of black hole evaporation (Hawking [1975]). Much greater analytic progress can be made in studying such a two-dimensional model because of the fewer number of degrees of freedom and reduced complexity as compared to gravity in four dimensions. However in two dimensions, one cannot use the Einstein action because it leads to trivial equations of motion. In the CGHS model, an extra scalar field, the dilaton, is included to produce a nontrivial theory of gravity. The "classical" CGHS model is based on the string-inspired action:

$$S_0 = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left\{e^{-2\phi}[R+4(\nabla\phi)^2+4\lambda^2]-\frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right\} . \tag{1.1}$$

This theory couples the two-dimensional metric, g_{ab}, and the dilaton, ϕ, to N massless scalar fields, f_i. It also includes a cosmological constant, λ^2.

In fact, the general solution for this theory may be constructed analytically (Callan et al. [1992]). Usually, the solutions are written in the conformal gauge where the metric is given in diagonal form as:

$$g_{\pm\pm} = 0 \qquad g_{+-} = -\frac{1}{2}e^{2\rho}$$

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where we use the null coordinates defined by $x^{\pm} = t \pm x$. The general solution includes many black hole solutions. The simplest of these is the eternal black hole, in which all of the matter fields are set to zero. In Kruskal gauge with $\rho = \phi$ (Callan et al. [1992]), one has:

$$e^{-2\phi} = e^{-2\rho} = \frac{M}{\lambda} - \lambda^2 x^+ x^-$$

where M is the mass of the black hole as it can be seen by a computation of the Bondi mass (Callan et al. [1992]). This solution has the same causal structure as the extended Schwarzschild black hole, with future and past spacelike singularities. With $M = 0$, one is left with the linear dilaton vacuum. In this case, the metric describes flat two-dimensional Minkowski space while (in appropriate coordinates) the dilaton increases linearly in the spatial direction.

Other solutions describe the formation of a black hole from the collapse of a shell of matter. One example, which contains an infinitely thin shell (or shock wave) collapsing along $x^+ = x_0^+$, has:

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^- - m\,(x^+ - x_0^+)\,\Theta(x^+ - x_0^+)\ .$$

This solution is divided in two different regions as we see from the step function $\Theta(x^+ - x_0^+)$. In the first region below the infall line $(x^+ < x_0^+)$, the solution corresponds to the linear dilaton vacuum. The region above the infall line $(x^+ > x_0^+)$ is a portion of the eternal black hole with an event horizon located at $x^- = -m/\lambda^2$. Thus it contains the future spacelike singularity.

1.1 One-loop effective action The previous black hole solutions are classical, and so they do not include Hawking radiation. In order to study the latter, we must include quantum effects. One approach is to define the quantum theory with the following functional integral:

$$\mathcal{Z} = \int \mathcal{D}g\,\mathcal{D}\phi\,\mathcal{D}f_i\ e^{iS_{DG}(g,\phi)+iS_M(g,f_i)} \tag{1.2}$$

where S_{DG} and S_M denote the pure dilaton-gravity and the matter contributions to the action S_0, respectively. Actually, we cannot perform this functional integral completely, but we can compute the matter functional integral which is a simple Gaussian. One finds:

$$\int \mathcal{D}f_i\ e^{iS_M(g,f_i)} = e^{iS_1(g)}$$

where $S_1(g)$ is the Polyakov action, which may be written in a covariant non-local form (Polyakov [1981]):

$$S_1 = -\frac{\kappa}{8\pi}\int d^2x\sqrt{-g}\,R\frac{1}{\nabla^2}R \tag{1.3}$$

where $\kappa \equiv N/12$, and $1/\nabla^2$ is the Green's function for the scalar d'Alembertian ∇^2. This action can also be written in a local form in conformal gauge. This one-loop contribution combined with S_0 yields a quantum effective action for the CGHS theory. Solutions of this effective action incorporate Hawking radiation and backreaction at least to leading order in a $1/N$ expansion for large N. Unfortunately, it is apparently not possible to solve the resulting equations of motion exactly, although some qualitative results have been produced.

2 Generalized model

The measure used in the functional integral (1.2) is not uniquely defined, and one can define alternative theories with different measures. Explicitly, this means that we can build new theories by adding local, covariant counterterms to the Polyakov action S_1. Here, we exploit this freedom to build a generalization of the previous quantum effective action, and look at the effects of the new interactions on black hole physics. Our extended action will be $S = S_0 + S_1 + S_2 + S_3$, where S_0 is the classical CGHS action (1.1) and S_1 is the Polyakov action (1.3). The remaining terms are:

$$S_2 = -\frac{\kappa}{8\pi}\int d^2x\sqrt{-g}\left[\alpha\phi R + \beta(\nabla\phi)^2\right]$$

$$S_3 = -\frac{\kappa}{8\pi}\int d^2x\sqrt{-g}\sum_{n=2}^{K}\left[a_n\phi^n R + b_n\phi^{n-1}(\nabla\phi)^2\right]$$

where α, β, a_n and b_n are coupling constants, and $K \geq 2$ is some integer. One of our objectives in selecting the coupling constants will be to produce an exactly soluble theory. A condition which will achieve this result is requiring the simple current equation (Russo et al. [1992], Kazama et al. [1995]):

$$\partial_+\partial_-(\rho - \phi) = 0 \ .$$

The difference between T_{+-} and the dilaton equation of motion yields the above if we set:

$$\begin{aligned} \beta &= 4 - 2\alpha \\ b_n &= -2n\, a_n \ . \end{aligned}$$

Note that further setting $\alpha = 2$ and $a_n = 0$ (and hence $\beta = 0 = b_n$) produces the model of Russo, Susskind and Thorlacius (Russo et al.[1992]).

2.1 Liouville theory With the above simple current conditions and in conformal gauge, our action S is simplified by performing the field redefinition:

$$\chi = \rho - \frac{\alpha}{4}\phi + \frac{1}{\kappa}e^{-2\phi} - \frac{1}{4}\sum_{n=2}^{K}a_n\phi^n$$

$$\Omega = \left(1 - \frac{\alpha}{4}\right)\phi + \frac{1}{\kappa}e^{-2\phi} - \frac{1}{4}\sum_{n=2}^{K}a_n\phi^n \ .$$

With these new fields, the action takes a Liouville form:

$$S = \frac{1}{\pi}\int d^2x\left\{-\kappa\,\partial_+\chi\partial_-\chi + \kappa\,\partial_+\Omega\partial_-\Omega + \lambda^2 e^{2(\chi-\Omega)} + \frac{1}{2}\sum_{i=1}^{N}\partial_+ f_i\partial_- f_i\right\} \ .$$

In terms of the fields χ and Ω, we can write the Ricci curvature scalar as:

$$\begin{aligned} R &= 8e^{-2\rho}\partial_+\partial_-\rho \\ &= 8e^{-2\rho(\chi,\Omega)}\frac{1}{\Omega'}\left[\partial_+\partial_-\chi - \frac{\Omega''}{\Omega'}\partial_+\Omega\partial_-\Omega\right] \end{aligned}$$

where $\rho(\chi,\Omega)$ is the conformal factor as an implicit function of the new fields and the prime ($'$) denotes a derivative with respect to the field ϕ. Hence the curvature diverges whenever Ω' vanishes. Such extrema of the function $\Omega(\phi)$ lead to timelike singularities, even for the vacuum solution in which the dilaton always increases along the spatial direction. However, in the extended model, we can avoid these singularities by properly constraining the coupling constants. For example, $\Omega(\phi)$ has no extrema if $\alpha > 4$, $a_n > 0$ for odd n, and $a_n = 0$ for even n.

2.2 Solution We can further simplify the solutions with another field redefinition:

$$U = \frac{\kappa}{2}(\chi + \Omega)$$
$$V = \chi - \Omega \ .$$

Then the metric equations of motion become:

$$\begin{aligned} T_{\pm\pm} &= -2\partial_\pm U \partial_\pm V + \frac{\kappa}{2}\partial_\pm^2 V + \partial_\pm^2 U + \frac{1}{2}\sum_{i=1}^{N}(\partial_\pm f_i)^2 + t_\pm(\sigma^\pm) = 0 \\ T_{+-} &= -\frac{\kappa}{2}\partial_+\partial_- V - \partial_+\partial_- U - \lambda^2 e^{2V} = 0 \end{aligned}$$

where the functions $t_\pm$ arise because of zero-mode ambiguities in defining the matter Green's function in the Polyakov action (1.3). We consider the solution for a collapsing shock wave for which the matter configuration is:

$$T^f_{++} = \frac{1}{2}\sum_{i=1}^{N}(\partial_+ f_i)^2 = m\,\delta(\sigma^+ - \sigma_0^+) \qquad T^f_{--} = \frac{1}{2}\sum_{i=1}^{N}(\partial_- f_i)^2 = 0$$

where m is the amplitude of the shock wave. We also choose $t_\pm$-functions to have the form (Russo et al. [1992]):

$$t_\pm(x^\pm) = -\frac{\kappa}{4}\frac{1}{(x^\pm)^2} \ .$$

Now before presenting the solution, we transform to asymptotically Minkowskian coordinates for which at large radius $ds^2 \simeq -d\sigma^+ d\sigma^-$. With these choices, the solution is:

$$\begin{aligned} U &= e^{\lambda(\sigma^+ - \sigma^-)} - \frac{m}{\lambda}\left(e^{\lambda(\sigma^+ - \sigma_0^+)} - 1\right)\Theta(\sigma^+ - \sigma_0^+) - \frac{\kappa}{4}\ln\left[1 + \frac{m}{\lambda}e^{\lambda\sigma^-}\right] \\ V &= \frac{\lambda}{2}(\sigma^+ - \sigma^-) \ . \end{aligned} \tag{2.1}$$

This solution is the analogue of the classical shock wave. However, the above solution now includes the effects of Hawking radiation. The black hole evaporation can be examined by looking at the evolution of the Bondi mass.

3 Bondi mass

The Bondi mass is defined on the future null infinity $\mathcal{J}_R^+$ and in the asymptotically Minkowskian coordinates, $\sigma^\pm$. It is expressed in terms of the linear variations δT_{ab} of the stress-energy tensor around some reference solution. As a reference, we take the "vacuum solution" which is given by setting $m = 0$ in the shock wave solution (2.1). Thus, the Bondi mass (de Alwis [1992]):

$$M(\sigma^-) = -\int^{\mathcal{J}_R^+} d\sigma^+ (\delta T_{++} + \delta T_{+-})$$

for the shock wave solution is:

$$M(\sigma^-) = m - \frac{\lambda}{4}\kappa \left\{ \ln\left[1 + \frac{m}{\lambda}e^{\lambda\sigma^-}\right] + \frac{m}{\lambda e^{-\lambda\sigma^-} + m} \right\} .$$

The flux of Hawking radiation may be obtained by differentiating this expression with respect to σ^-. Note that our results are completely independent of any of the coupling constants α and a_n. Hence the radiation in these solutions is essentially the same as in other two-dimensional models (Callan et al. [1992], Russo et al. [1992]). The radiation goes to zero in the far past $\sigma^- \to -\infty$, while it approaches a constant in the far future $\sigma^- \to \infty$. The latter constant flux means that the black holes never stop Hawking radiating even when their mass reaches zero! Thus our generalized model does not avoid this unphysical behavior found in other models.

Conclusion

We have presented a new generalization for the effective quantum action derived by CGHS for the study of Hawking radiation in two dimensions. We have shown that our generalization gives the possibility of removing the timelike curvature singularity arising in other models. Hence we may avoid the related boundary problems (Das and Mukherji [1994], Russo et al. [1993], Strominger and Thorlacius [1994]). Also, this model is exactly solvable in the sense that we can find the general solution analytically. By studying the solution for the collapsing shock wave, we found that black hole evaporation proceeds in essentially the same way as in other dilaton gravity models. In particular, the Hawking radiation in our generalized model never stops! Thus we must conclude that the physics of this generalized theory remains incomplete, and we are still unable to determine the end-state of the black hole evaporation.

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