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On the Convergence of Perturbative QCD at High Temperature

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Abstract

The free energy for QCD at high temperature T is calculated to order g^5 using effective-field-theory methods to separate the contributions from the momentum scales T and gT . The effects of the scale T enter through the coefficients in the effective lagrangian for the 3-dimensional effective theory obtained by dimensional reduction. The perturbation series for these coefficients seem to be well-behaved if the running coupling constant is sufficiently small: $\alpha_s(2\pi T) \ll 1$. For the contribution to the free energy from the scale gT , the perturbation series is well-behaved only if $\alpha_s(2\pi T)$ is an order of magnitude smaller. The implication for applications of perturbative QCD to the quark-gluon plasma are briefly discussed.

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One of the most dramatic predictions of quantum chromodynamics (QCD) is that when hadronic matter is raised to a sufficiently high temperature or density, it will undergo a phase transition to a quark-gluon plasma. One of the major thrusts of nuclear physics in the next decade will be the effort to study the quark-gluon plasma through relativistic heavy-ion collisions. For this effort to be successful, it will be important to understand the properties of the plasma as accurately as possible. The two major theoretical tools for studying the quark-gluon plasma are lattice gauge theory and perturbative QCD. Lattice gauge theory has the advantage that it is a nonperturbative method and applies equally well to the hadronic phase. It is an effective method for calculating the static equilibrium properties of a plasma with 0 baryon density, but it can not be easily applied to problems involving dynamical properties or to a plasma that is away from equilibrium or has nonzero baryon density. These problems can however be studied using perturbative QCD, provided that the temperature T of the plasma is sufficiently high. As the temperature decreases, the running coupling constant g of QCD increases, causing perturbation theory to break down at some temperature above the critical temperature T_c for the phase transition. One of the basic questions in the theory of the quark-gluon plasma is how large must T be in order for perturbative QCD to be applicable. Is this method useful at temperatures that may be achievable in heavy-ion collisions, which are at most several times T_c ?

In order to answer this question, it is necessary to understand the structure of the perturbation series to all orders and also to carry out explicit higher-order calculations. The first step has been carried out for the free energy [1]. The structure of the perturbation series is nontrivial, because a strict perturbation expansion in g^2 has severe infrared divergences associated with the exchange of static gluons. Physically, these divergences are screened by plasma effects. The screening of electrostatic gluons can be taken into account by a resummation of perturbation theory, but the screening of magnetostatic gluons can only be treated using nonperturbative methods. Once the structure of the perturbation expansion is understood, it is still necessary to carry out explicit perturbative calculations to determine

quantitatively how high the temperature must be in order for perturbative QCD to be accurate. Only recently has the calculational technology of thermal field theory progressed to the point where it is possible to carry out perturbative calculations to a high enough order that the running of the coupling constant comes into play [2–8]. The first such calculation for the quark-gluon plasma was the calculation of the free energy to order g^4 by Arnold and Zhai in 1994 [4]. This calculation has recently been extended to order g^5 by Kastening and Zhai [5] and by the authors [6]. In this Letter, we summarize the calculations of Ref. [6] and discuss implications for the application of perturbative QCD to the quark-gluon plasma.

The static equilibrium properties of a quark-gluon plasma at temperature T are governed by the free energy density $F = -(T/V)\log \mathcal{Z}_{\text{QCD}}$, where V is the volume of space. The partition function \mathcal{Z}_{QCD} is given by a functional integral over quark and gluon fields on a 4-dimensional Euclidean space-time, with the Euclidean time τ taking its values on a circle with circumference $1/T$. In the limit in which the quarks are massless, the free energy is a function of T and $\alpha_s = g^2/(4\pi)$ only.

The structure of the weak-coupling expansion for the free energy to all orders was deduced in Ref. [1]. The free energy can be expressed as the sum of three contributions coming from the momentum scales T , gT , and g^2T :

$$F = [f_E(\Lambda_E) + f_M(\Lambda_E, \Lambda_M) + f_G(\Lambda_M)]T, \quad (1)$$

where Λ_E is an arbitrary factorization scale that separates the scales T and gT , while Λ_M separates the scales gT and g^2T . The contributions from the three momentum scales can be unraveled by constructing a sequence of two effective field theories. The first effective theory, electrostatic QCD (EQCD), is a 3-dimensional Euclidean field theory involving the electrostatic gauge field $A_0^a(\mathbf{x})$ and the magnetostatic gauge field $A_i^a(\mathbf{x})$. The lagrangian for EQCD is

$$\mathcal{L}_{\text{EQCD}} = \frac{1}{4}G_{ij}^a G_{ij}^a + \frac{1}{2}(D_i A_0)^a (D_i A_0)^a + \frac{1}{2}m_E^2 A_0^a A_0^a + \delta\mathcal{L}_{\text{EQCD}}, \quad (2)$$

where D_i is the covariant derivative for the adjoint representation with coupling constant g_E

and G_{ij}^a is the magnetostatic field strength. The term $\delta\mathcal{L}_{\text{EQCD}}$ in (2) includes all other local gauge-invariant operators that can be constructed out of A_0 and A_i , including nonrenormalizable interactions. Static gauge-invariant correlation functions in thermal QCD can be reproduced at long distances by tuning g_E , m_E^2 , and the parameters in $\delta\mathcal{L}_{\text{EQCD}}$ as functions of g , T , and the ultraviolet cutoff Λ_E of EQCD. In physical quantities, the Λ_E -dependence of the parameters cancels the Λ_E -dependence from loop integrals in EQCD. Since the parameters of EQCD take into account effects from the scale T , they can be calculated as perturbation series in the running coupling constant $\alpha_s(\mu)$ with μ of order T .

Since the free energy is a static quantity, it can be calculated using EQCD. The free energy can be written

$$F = T \left(f_E - \frac{\log \mathcal{Z}_{\text{EQCD}}}{V} \right), \quad (3)$$

where $\mathcal{Z}_{\text{EQCD}}$ is the partition function for EQCD and f_E is the coefficient of the unit operator, which was omitted from the effective lagrangian (2). This coefficient gives the contribution to the free energy (1) from the momentum scale T . The logarithm of $\mathcal{Z}_{\text{EQCD}}$ in (3) includes the contributions f_M and f_G from the scales gT and g^2T , respectively. These contributions can be separated by constructing a second effective field theory, magnetostatic QCD (MQCD), which involves only the magnetostatic gauge field $A_i^a(\mathbf{x})$. The term f_M in (1) is the coefficient of the unit operator in the lagrangian for MQCD. It can be computed using perturbative methods as an expansion in powers of g starting at order g^3 . The term f_G in (1) is proportional to the logarithm of the partition function of MQCD. It can only be calculated using nonperturbative methods. Surprisingly, it can be expanded in powers of g beginning at order g^6 , with coefficients that can be calculated using lattice simulations of MQCD [1]. Since we only calculate the free energy to order g^5 , we do not consider the term f_G any further.

To calculate the free energy to order g^5 , the only parameters of EQCD that are required are g_E^2 to leading order in g^2 and m_E^2 and f_E to order g^4 . The gauge coupling constant g_E for EQCD is determined at leading order simply by comparing the lagrangians for EQCD and full

QCD: $g_E^2 = g^2 T$. The other two parameters can be determined by computing static quantities in both full QCD and EQCD, and demanding that they match. It is convenient to carry out these matching calculations using a strict perturbation expansion in g^2 . This expansion is afflicted with infrared divergences due to long-range forces mediated by static gluons, and an infrared cutoff is therefore required. Physically, these divergences are screened by plasma effects, but screening is not taken into account in the strict perturbation expansion. Nevertheless, this expansion can be used as a device for determining the parameters of EQCD, since they depend only on short distances of order $1/T$.

The parameter m_E can be determined by matching the strict perturbation expansions for the electric screening mass m_{el} in full QCD and in EQCD. Beyond leading order in g , m_{el} becomes sensitive to magnetostatic screening and requires a nonperturbative definition [7]. However, in the presence of an infrared cutoff, the electric screening mass can be defined in perturbation theory by the location of the pole in the propagator for $A_0(\tau, \mathbf{x})$ at spacelike momentum ($k_0 = 0, \mathbf{k}$). Denoting the appropriate component of the gluon self-energy tensor by $\Pi(k^2)$ where $k^2 = \mathbf{k}^2$, we must solve the equation $k^2 + \Pi(k^2) = 0$ at $k^2 = -m_{\text{el}}^2$. Since the solution m_{el}^2 is of order g^2 , we can expand $\Pi(k^2)$ as a Taylor series around $k^2 = 0$. To determine m_{el}^2 to order g^4 , we must calculate $\Pi(0)$ to two-loop accuracy and $\Pi'(0)$ to one-loop accuracy. We use dimensional regularization with $3 - 2\epsilon$ spacial dimensions to cut off both infrared and ultraviolet divergences. The sums and integrals can be evaluated analytically using methods developed by Arnold and Zhai [4]. The resulting expression for m_{el}^2 is an expansion in integral powers of α_s . There is no $\alpha_s^{3/2}$ term, unlike in the expression for m_{el}^2 that correctly incorporates the effects of electrostatic screening [8]. This g^3 term arises because the g^4 correction includes a linear infrared divergence that is cut off at the scale gT . Since we use dimensional regularization as an infrared cutoff, this power infrared divergence is set equal to 0.

In EQCD with an infrared cutoff, the electric screening mass m_{el} can be defined in perturbation theory by the location of the pole in the propagator for the field $A_0(\mathbf{x})$.

Denoting the self-energy function for $A_0(\mathbf{x})$ by $\Pi_E(k^2)$, the screening mass m_{el} satisfies $k^2 + m_E^2 + \Pi_E(k^2) = 0$ at $k^2 = -m_{\text{el}}^2$. In the strict perturbation expansion for EQCD, we treat m_E^2 as a perturbation parameter of order g^2 . After Taylor-expanding $\Pi_E(k^2)$ around $k^2 = 0$, there is no scale in the loop integrals, so they all vanish with dimensional regularization. The solution for the screening mass is therefore trivial: $m_{\text{el}}^2 = m_E^2$. Matching this result with the strict perturbation expansion from the full theory and taking the limit $\epsilon \rightarrow 0$, we find

$$m_E^2 \Big|_{\epsilon=0} = 4\pi \alpha_s(\mu) T^2 \left\{ 1 + \frac{1}{6}n_F + \left[0.612 - 0.488n_F - 0.0428n_F^2 + \frac{11}{2} \left(1 + \frac{1}{6}n_F \right) \left(1 - \frac{2}{33}n_F \right) \log \frac{\mu}{2\pi T} \right] \frac{\alpha_s}{\pi} \right\}, \quad (4)$$

where n_F is the number of flavors of quarks and μ is the renormalization scale for the QCD coupling constant. The order- ϵ terms in m_E^2 are also required in the calculation of the free energy. These terms are given by

$$\frac{\partial m_E^2}{\partial \epsilon} \Big|_{\epsilon=0} = g^2 T^2 \left\{ 3.97 + 2 \log \frac{\Lambda_E}{4\pi T} + \left(0.597 + \frac{1}{3} \log \frac{\Lambda_E}{4\pi T} \right) n_F \right\}, \quad (5)$$

where the infrared cutoff Λ_E is the scale introduced by dimensional regularization.

The coefficient f_E can be determined by matching the strict perturbation expansions for the free energy in full QCD and in EQCD. In full QCD, the free energy F is calculated to order g^4 by evaluating the sum of vacuum diagrams through three-loop order, using dimensional regularization to cut off both infrared and ultraviolet divergences. The resulting expression for F is an expansion in integral powers of α_s . There is no $\alpha_s^{3/2}$ term, in contrast to the expression for the free energy that correctly includes the effects of electrostatic screening [9, 10]. This g^3 term arises because the g^4 correction includes a linear infrared divergence that is cut off at the scale gT . In the strict perturbation expansion, this term appears as a power infrared divergence that is set to zero in dimensional regularization.

In EQCD, the free energy is given by (3). All the loop diagrams in the strict perturbation expansion for $\log \mathcal{Z}_{\text{EQCD}}$ vanish with dimensional regularization, since there is no scale for

the integrals. The only contribution to $\log \mathcal{Z}_{\text{EQCD}}$ comes from the counterterm δf_E which cancels logarithmic ultraviolet divergences proportional to the unit operator. The resulting expression for the free energy is simply $F = (f_E + \delta f_E)T$. The counterterm is determined by calculating the ultraviolet divergent terms in $\log \mathcal{Z}_{\text{EQCD}}$. If we use dimensional regularization together with a minimal subtraction renormalization scheme in EQCD, then δf_E is a polynomial in g_E^2 , m_E^2 , and the other parameters in the EQCD lagrangian. The leading term in δf_E is proportional to $g_E^2 m_E^2$, and its coefficient can be determined by a simple 2-loop calculation:

$$\delta f_E = -\frac{3}{8\pi^2\epsilon} g_E^2 m_E^2. \quad (6)$$

When this counterterm is expressed in terms of the parameters g and T of the full theory, we must take into account the fact that m_E^2 in (6) multiplies a pole in ϵ . Thus, in addition to the expression for m_E^2 given in (4), we must also include the terms of order ϵ which are given by (5). Matching $F = (f_E + \delta f_E)T$ with the strict perturbation expansion for F in the full theory, we obtain

$$f_E(\Lambda_E) = -\frac{8\pi^2}{45} T^3 \left\{ 1 + \frac{21}{32} n_F - \frac{15}{4} \left(1 + \frac{5}{12} n_F \right) \frac{\alpha_s(\mu)}{\pi} + \left[244.9 - 17.24 n_F - 0.415 n_F^2 - \frac{165}{8} \left(1 + \frac{5}{12} n_F \right) \left(1 - \frac{2}{33} n_F \right) \log \frac{\mu}{2\pi T} - 135 \left(1 + \frac{1}{6} n_F \right) \log \frac{\Lambda_E}{2\pi T} \right] \left(\frac{\alpha_s}{\pi} \right)^2 \right\}. \quad (7)$$

This expression differs from that given in Ref. [1], where the counterterm (6) was not taken into account.

We have calculated two terms in the perturbation series for m_E^2 and three terms in the series for f_E . We can use these results to study the convergence of perturbation theory for the parameters of EQCD. We consider the case of $n_F = 3$ flavors of quarks, although our conclusions will not depend sensitively on n_F . The question of the convergence is complicated by the presence of the arbitrary renormalization and factorization scales μ and Λ_E . The next-to-leading-order (NLO) correction to f_E is independent of μ and Λ_E , and is small compared to the leading-order (LO) term provided that $\alpha_s(\mu) \ll 1.1$. The NLO correction to m_E^2 and the next-to-next-to-leading-order (NNLO) correction to f_E both depend on the renormalization

scale μ . One scale-setting scheme that is physically well-motivated is the BLM prescription [11], in which μ is adjusted to cancel the highest power of n_F in the correction term. This prescription gives $\mu = 0.93\pi T$ when applied to m_E^2 and $\mu = 4.4\pi T$ when applied to f_E . These values differ only by about a factor of 2 from $2\pi T$, which is the lowest Matsubara frequency for gluons. Below, we will consider the three values $\mu = \pi T$, $2\pi T$, and $4\pi T$. For the NLO correction to m_E^2 to be much smaller than the LO term, we must have $\alpha_s(\mu) \ll 0.8$, 3.8, and 1.4 if $\mu = \pi T$, $2\pi T$, and $4\pi T$, respectively. Based on these results, we conclude that the perturbation series for the parameters of EQCD are well-behaved provided that $\alpha_s(2\pi T) \ll 1$.

The NNLO correction for f_E depends not only on μ , but also on the factorization scale Λ_E . Because the coefficient of $\log(\Lambda_E/2\pi T)$ in (7) is so much larger than that of $\log(\mu/2\pi T)$, the NNLO correction for f_E is much more sensitive to Λ_E than to μ . It is useful intuitively to think of the infrared cutoff Λ_E as being much smaller than the ultraviolet cutoff μ . However, these scales can be identified with momentum cutoffs only up to multiplicative constants that may be different for μ and Λ_E . Both parameters are introduced through dimensional regularization, but μ arises from ultraviolet divergences of 4-dimensional integrals, while Λ_E arises from infrared divergences of 3-dimensional integrals. We might be tempted to set $\Lambda_E = \mu$, but then the NNLO coefficient in f_E is large. For the choice $\mu = 2\pi T$, the correction to the LO term is a multiplicative factor $1 - 0.9\alpha_s + 6.46\alpha_s^2$. The NNLO correction can be made small by adjusting Λ_E . It vanishes for $\Lambda_E = 5.8\pi T$, $5.1\pi T$, and $4.5\pi T$ if $\mu = \pi T$, $2\pi T$, and $4\pi T$, respectively. We conclude that the perturbation series for f_E is well-behaved if the factorization scale Λ_E is chosen to be approximately $5\pi T$. Whether this choice is reasonable can only be determined by calculating other EQCD parameters to higher order to see if the same choice leads to well-behaved perturbation series.

The choice of Λ_E that makes the perturbation series for the EQCD parameters well-behaved may be much larger than the largest mass scale m_E of EQCD. Perturbative corrections in EQCD will then include large logarithms of Λ_E/m_E . This problem can be avoided

by using renormalization group equations to evolve the parameters of EQCD from the initial scale Λ down to some scale Λ'_E of order m_E . The coefficient f_E satisfies the renormalization group equation

$$\Lambda_E \frac{d}{d\Lambda_E} f_E = -\frac{3}{2\pi^2} g_E^2 m_E^2. \quad (8)$$

The evolution of g_E^2 and m_E^2 occurs only at higher order in the coupling constant and therefore can be ignored. The solution to the renormalization group equation is therefore trivial:

$$f_E(\Lambda'_E) = f_E(\Lambda_E) - \frac{3}{2\pi^2} g_E^2 m_E^2 \log \frac{\Lambda'_E}{\Lambda_E}. \quad (9)$$

Having determined the parameters of EQCD to the necessary accuracy, we proceed to calculate the free energy using (3). The contribution from the scale T is given by the coefficient f_E in (7). The contribution from the scale gT is given by $f_M = -\log \mathcal{Z}_{\text{EQCD}}/T$. In order to calculate f_M using perturbation theory in EQCD, we must include the effects of the mass parameter m_E^2 to all orders, but the gauge coupling constant g_E can be treated as a perturbation parameter. The contributions to $\log \mathcal{Z}_{\text{EQCD}}$ of orders g^3 , g^4 , and g^5 are given by the 1-loop, 2-loop, and 3-loop vacuum diagrams in EQCD, respectively. The integrals can be calculated analytically using methods developed by Broadhurst [12]. The two-loop integrals include an ultraviolet pole in ϵ that is proportional to $g_E^2 m_E^2$. This divergence is cancelled by the counterterm δf_E for the coefficient of the unit operator, which is given in (6). Our final result for the coefficient f_M in (1) is

$$f_M(\Lambda_E) = -\frac{2}{3\pi} m_E^3 \left[1 - \left(0.256 - \frac{9}{4} \log \frac{\Lambda_E}{m_E} \right) \frac{g_E^2}{2\pi m_E} - 27.6 \left(\frac{g_E^2}{2\pi m_E} \right)^2 \right]. \quad (10)$$

Note that the dependence of f_M on Λ_E cancels that of f_E in (7). The expression (10) can be expanded in powers of g by setting $g_E^2 = g^2 T$ and using the expansion (4) for m_E^2 .

We now consider the convergence of the perturbation series (10) for f_M . The size of the NLO correction depends on the choice of the factorization scale Λ_E . It is small if Λ_E is chosen to be approximately m_E . The NNLO correction in (10) is independent of any arbitrary scales. If $n_F = 3$, it is small compared to the leading order term only if $\alpha_s \ll 0.17$.

Thus the perturbation series for f_M is well-behaved only for values of $\alpha_s(2\pi T)$ that are much smaller than those required for the parameters of EQCD to have well-behaved perturbation series.

Adding (7) and (10) and expanding in powers of $\sqrt{\alpha_s}$, the complete expression for the free energy F is

$$F = -\frac{8\pi^2}{45}T^4 \left[F_0 + F_2 \frac{\alpha_s(\mu)}{\pi} + F_3 \left(\frac{\alpha_s(\mu)}{\pi} \right)^{3/2} + F_4 \left(\frac{\alpha_s}{\pi} \right)^2 + F_5 \left(\frac{\alpha_s}{\pi} \right)^{5/2} \right], \quad (11)$$

where the truncation error is of order $\alpha_s^3 \log \alpha_s$. The coefficients in this expansion are

$$F_0 = 1 + \frac{21}{32}n_F, \quad (12)$$

$$F_2 = -\frac{15}{4} \left(1 + \frac{5}{12}n_F \right), \quad (13)$$

$$F_3 = 30 \left(1 + \frac{1}{6}n_F \right)^{3/2}, \quad (14)$$

$$F_4 = 237.2 + 15.97n_F - 0.413n_F^2 + \frac{135}{2} \left(1 + \frac{1}{6}n_F \right) \log \left(\frac{\alpha_s}{\pi} \left(1 + \frac{1}{6}n_F \right) \right) - \frac{165}{8} \left(1 + \frac{5}{12}n_F \right) \left(1 - \frac{2}{33}n_F \right) \log \frac{\mu}{2\pi T}, \quad (15)$$

$$F_5 = \left(1 + \frac{1}{6}n_F \right)^{1/2} \left[-799.2 - 21.96n_F - 1.926n_F^2 + \frac{495}{2} \left(1 + \frac{1}{6}n_F \right) \left(1 - \frac{2}{33}n_F \right) \log \frac{\mu}{2\pi T} \right]. \quad (16)$$

The coefficient F_2 was first given by Shuryak [9], while F_3 was first calculated correctly by Kapusta [10]. The coefficient F_4 was computed in 1994 by Arnold and Zhai [4]. The coefficient F_5 in (16) has been calculated independently by Kastening and Zhai using a different method [5].

We now ask how small α_s must be in order for the expansion (11) to be well-behaved. For simplicity, we consider the case $n_F = 3$, although our conclusions are not sensitive to n_F . If we choose the renormalization scale $\mu = 2\pi T$ motivated by the BLM criterion [11], the correction to the LO result is a multiplicative factor $1 - 0.9\alpha_s + 3.3\alpha_s^{3/2} + (7.1 + 3.5 \log \alpha_s)\alpha_s^2 - 20.8\alpha_s^{5/2}$. The $\alpha_s^{5/2}$ term is the largest correction unless $\alpha_s(2\pi T) < 0.12$. We can make the $\alpha_s^{5/2}$ term small only by choosing the renormalization scale to be near the value

$\mu = 36.5\pi T$ for which F_5 vanishes. This ridiculously large of μ arises because the scale μ has been adjusted to cancel the large g^5 correction to f_M in (10). This contribution arises from the momentum scale gT and has nothing to do with renormalization of α_s . We conclude that the expansion (11) for F in powers of $\sqrt{\alpha_s}$ is well-behaved only if $\alpha_s(2\pi T)$ is an order of magnitude smaller than the value required for the EQCD parameters to be well-behaved.

We now consider briefly the implications for theoretical studies of the quark-gluon plasma. We have found that the convergence of perturbation theory requires much smaller values of $\alpha_s(2\pi T)$ for quantities at the scale gT than for quantities at the scale T . The critical temperature T_c for formation of a quark-gluon plasma is approximately 200 MeV. It may be possible in heavy-ion collisions to produce a quark-gluon plasma with temperatures several times T_c . At $T = 350$ MeV, $\alpha_s(2\pi T) \approx 0.3$, which is small enough that perturbation theory may be reasonably convergent at the scale T , but it is certainly not convergent at the scale gT . We conclude that at the temperatures achievable in heavy-ion collisions, perturbative QCD may be accurate when applied to quantities that involve the scale T only. However nonperturbative methods are required to accurately calculate quantities that involve the scales gT and g^2T associated with screening in the plasma.

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