MEMBRANES IN THE TWO-HIGGS STANDARD MODEL

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ABSTRACT

We present some non-topological static wall solutions in two-Higgs extensions of the standard model. They are classically-stable in a large region of parameter space, compatible with perturbative unitarity and with present phenomenological bounds.

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There are several reasons to consider extensions of the Higgs sector of the standard model, most notably the introduction of an extra source of soft CP violation [1], the possibility of generating sufficient baryon asymmetry at the electroweak transition [2], and low-energy supersymmetry. An extended Higgs sector allows for the possibility of discrete symmetries and of associated domain walls [3, 4]. In this letter we will present another class of membrane defects in the two-Higgs standard model that differ from domain walls in two important ways: (*a*) they are not tied to a discrete symmetry and are thus more generic, i.e. they exist in a codimension-zero region of parameter space, and (*b*) they are classically- but not topologically-stable and have a finite, though possibly cosmologically-long life time. They resemble in these respects the previously discussed Z strings [5], while contrary to these latter [6] they are as we will show stable in a realistic range of parameters. This range does not however include the minimal supersymmetric standard model. The new defects are embeddings of some recently found [7] solutions of the $2d$ Abelian-Higgs model with two or more complex scalars ‡ . They are characterized by the non-trivial winding of a relative U(1) phase of two Higgses in the direction (x) normal to the wall and are electrically neutral. Their energy per unit area and thickness are of order $m_W^2 m_A/\alpha$ and m_A^{-1} respectively, where α is the fine-structure constant and m_A the mass of the CP-odd Higgs scalar.

The nature of these defects is best illustrated by a complex-scalar field theory in four dimensions with potential $V(\Phi) = \frac{\lambda}{4}(\Phi^*\Phi - v^2)^2 - \mu^2 v Re\Phi$. This has a unique minimum at a real value of Φ, so that there are no topologically-stable domain walls. Nevertheless, it can be shown [7, 9] that for $\sqrt{2\lambda}v/\mu \geq 6.1$ there exists a classically-stable static wall solution characterized by the fact that the phase of Φ changes by 2π as x varies from $-\infty$ to ∞. In the $\lambda \to \infty$ limit the solution reduces to the well-known 2d sine-Gordon soliton, while for generic values of λ it can be analyzed numerically or via a $1/\lambda$ expansion. One can also study some features of the wall analytically [7] by trading the μ term in the potential with periodic conditions in the x direction. Notice indeed that the μ term lifts the U(1) vacuum degeneracy thereby forcing the field to come back to its minimum within a distance (wall thickness) $\Delta x \sim \mu^{-1}$. Alternatively we can achieve the same result by making space into a cylinder of radius $L \sim \mu^{-1}$. We will use this technical stratagem in the sequel, but we should stress that the existence and stability of the membranes is *not* tied to the existence of any accidental global symmetry.

The Lagrangian of the two-Higgs standard model is

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} + |D_\mu H_1|^2 + |D_\mu H_2|^2 - V(H_1, H_2) \tag{1}$$

where $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon^{abc}W^b_\mu W^c_\nu$ and $Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu$, the physical Z^0 and photon fields are $Z_\mu = W^3_\mu \cos\theta_W - Y_\mu \sin\theta_W$ and $A_\mu = W^3_\mu \sin\theta_W + Y_\mu \cos\theta_W$ and $\tan\theta_W = g'/g$. Both Higgs doublets have hypercharge equal to one, the covariant derivative is

$$D_\mu H_I = (\partial_\mu + \frac{i}{2}g\tau^a W^a_\mu + \frac{i}{2}g' Y_\mu)H_I$$

‡The 2d Abelian-Higgs model has a plethora of other sphaleron-like solutions [8] which can be embedded similarly in the two-Higgs standard model. Since these are classically unstable they will be of no concern to us in this letter.

for $I = 1, 2$ and the potential reads

$$V(H_1, H_2) = \lambda_1\Big(|H_1|^2 - \frac{v_1^2}{2}\Big)^2 + \lambda_2\Big(|H_2|^2 - \frac{v_2^2}{2}\Big)^2 + \lambda_3\Big(|H_1|^2 + |H_2|^2 - \frac{v_1^2 + v_2^2}{2}\Big)^2$$
$$+ \lambda_4\Big[|H_1|^2|H_2|^2 - (H_1^\dagger H_2)(H_2^\dagger H_1)\Big] + \lambda_5\Big[\mathrm{Re}(H_1^\dagger H_2) - \frac{v_1 v_2}{2} cos\xi\Big]^2 \qquad (2)$$
$$+ \lambda_6\Big[\mathrm{Im}(H_1^\dagger H_2) - \frac{v_1 v_2}{2} sin\xi\Big]^2$$

where $|H_I|^2 \equiv H_I^\dagger H_I$. This is the most general potential [1] subject to the condition that both CP invariance and a discrete Z_2 symmetry ($H_1 \to -H_1$) are only broken softly. The softly broken Z_2 symmetry is there to suppress unacceptably large flavor-changing neutral currents. Assuming all the λ_i are positive, the minimum of the potential up to a gauge transformation is at

$$< H_1 >= \; e^{-i\xi}\begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix} \quad \text{and} \quad < H_2 >= \begin{pmatrix} 0 \\ v_2/\sqrt{2} \end{pmatrix} . \qquad (3)$$

To reduce the large number of parameters we will restrict ourselves to $\lambda_1 = \lambda_2$, $\lambda_5 = \lambda_6$, $\xi = 0$ and $v_1 = v_2 = v$ (or $\tan\beta = 1$) in the sequel. Relaxing these conditions is straightforward but beyond the scope of the present letter. In addition to the electroweak gauge bosons with masses $m_W^2 = g^2v^2/2$ and $m_Z = m_W/cos\theta_W$, the perturbative spectrum contains a charged Higgs boson H^+ with mass $m_{H^+}^2 = \lambda_4 v^2$, a CP-odd neutral scalar A^0 with mass $m_A^2 = \lambda_5 v^2$, and two CP-even neutral scalars h^0 and H^0 with masses $m_h^2 = 2\lambda_1 v^2$ and $m_{H^0}^2 = (2\lambda_1 + 4\lambda_3 + \lambda_5)v^2$ respectively.

For $\lambda_5 = 0$ there is an accidental U(1) global symmetry with A^0 the associated Goldstone boson. The corresponding phase is the phase of interest that winds around non-trivially when crossing the membrane. Using the previously-mentioned stratagem we will first work in this $m_A^2 = 0$ limit but compactify space into a cylinder of period $2\pi L$ in the x direction. We will also work in the temporal $Y_0 = W_0^a = 0$ gauge. The following configuration is then the relevant static, $y-$ and $z-$independent solution of the classical equations of motion

$$H_1 = e^{ix/L}\begin{pmatrix} 0 \\ F \end{pmatrix} \;,\quad H_2 = \begin{pmatrix} 0 \\ F \end{pmatrix} \quad , \; Z_x = \frac{cos\theta_W}{gL} \;\; \text{and} \;\; A_x = \frac{a}{L} \qquad (4)$$

where

$$F^2 = \frac{v^2}{2}\Big(1 - \frac{1}{2m_{H^0}^2 L^2}\Big) , \qquad (5)$$

a is an arbitrary constant [§], and all other fields are equal to zero. The energy per unit area ($\mathcal{A}$) can be computed easily with the result

$$E/\mathcal{A} = \frac{m_W^2 sin^2\theta_W}{4\alpha L}\Big(1 - \frac{1}{4m_{H^0}^2 L^2}\Big) . \qquad (6)$$

[§] Note that a is an angular variable since large gauge transformations can change it by integers. The possibility of this Wilson-line background is an artifact of our stratagem and has no analog in the non-compact case. Its net effect is to weaken the stability under charged-field fluctuations as we will show shortly.

It vanishes in the limit ($L \to \infty$) of a very thick membrane. Note that since the charged upper components of the Higgs fields as well as the charged gauge bosons $W^{\pm}{}_{\mu} = (W^1_\mu \mp iW^2_\mu)/\sqrt{2}$ vanish, the membrane has no electromagnetic couplings.

To check for classical stability we may restrict ourselves to x-dependent fluctuations and vector fields in only the x direction. These are the fluctuations of the theory in one spatial dimension. Stability under such perturbations is of course a necessary requirement, but it is also a sufficient one for the following reason: first, because of y- and z-translational invariance we may diagonalize the fluctuations by going to the (k_y, k_z) Fourier space. Second we may break the energy density of any static field configuration as $E = \int(\mathcal{E} + \tilde{\mathcal{E}})$ where

$$\mathcal{E} = |D_x H_1|^2 + |D_x H_2|^2 + V(H_1, H_2) \qquad (7a)$$

and

$$\tilde{\mathcal{E}} = \frac{1}{4} Y_{ij} Y_{ij} + \frac{1}{4} W^a_{ij} W^a_{ij} + \sum_{I=1,2} \left(|D_y H_I|^2 + |D_z H_I|^2 \right) . \qquad (7b)$$

Now the quadratic fluctuations of $\mathcal{E}$ are independent of (k_y, k_z) and of the y- and z-components of the vector fields, while $\tilde{\mathcal{E}}$ is a positive semi-definite contribution which vanishes when $k_y = k_z = 0$ and when all vectors point in the x direction. This proves that stability of any static 2d soliton guarantees the stability of the corresponding 4d wall solution.

Next we note that the fluctuations of electrically-charged and neutral fields do not mix at the quadratic level, so we can study these two sets of fields separately. Using the residual invariance under x-dependent gauge transformations we can go to a gauge in which the upper component of the second Higgs doublet (H_2^+) is zero. The fluctuations of the remaining charged fields can be Fourier decomposed as follows: $gFW_x^+/\sqrt{2} = -\sum \alpha_n e^{-inx/L}$ and $H_1^+ = e^{ix/L} \sum \beta_n e^{-inx/L}$. Using the form of the covariant derivative

$$D_\mu H_I = \left[\partial_\mu + ig \begin{pmatrix} A_\mu sin\theta_W + Z_\mu sin\theta_W cot(2\theta_W) & W_\mu^+/\sqrt{2} \\ W_\mu^-/\sqrt{2} & -Z_\mu/2cos\theta_W \end{pmatrix} \right] \begin{pmatrix} H_I^+ \\ H_I^0 \end{pmatrix} \qquad (8)$$

one finds after some straightforward algebra the following variation of the energy per unit wall area

$$\delta E/\mathcal{A} = \sum_{n=-\infty}^{+\infty} \begin{pmatrix} \alpha_n^* & \beta_n^* \end{pmatrix} \begin{pmatrix} 2 & n - \tilde{a} \\ n - \tilde{a} & (n-\tilde{a})(n+1-\tilde{a}) + \lambda_4 F^2 \end{pmatrix} \begin{pmatrix} \alpha_n \\ \beta_n \end{pmatrix} + \text{cubic} \qquad (9)$$

with $\tilde{a} = 1 + cos^2\theta_W + agsin\theta_W$. This is manifestly positive-definite for any $\tilde{a}$ if and only if

$$2\lambda_4 F^2 L^2 = m^2_{H^+} L^2 (1 - \frac{1}{2m^2_{H^0} L^2}) > 1 . \qquad (10)$$

The strongest conditions are in fact obtained for $\tilde{a}$ = integer corresponding to a vanishing W_x^3 background. Considering next the electrically-neutral sector we note first that up to the constant Wilson-line background a, the photon field $A_x(x)$ is a pure gauge. The fluctuations of the remaining fields Z_x, H_1^0 and H_2^0 are those analyzed in ref.[7] in the context of the Abelian-Higgs model with two complex scalars. Stability under these fluctuations yields one extra condition on the parameters of the model

$$4\lambda_1 F^2 L^2 = m^2_h L^2 (1 - \frac{1}{2m^2_{H^0} L^2}) > 1 . \qquad (11)$$

Taken together inequalities (10) and (11) tell us that for the winding solution to exist and be classically stable, all but the CP-odd scalar must be sufficiently massive in units of the cutoff or "membrane thickness" L. Notice that since the problem is effectively two-dimensional the gauge couplings do not enter into the stability conditions.

Now in the realistic case of a non-compact space we expect m_A^{-1} to replace L in the above discussion. This can be seen explicitly in the $\lambda_1, \lambda_3, \lambda_4 \to \infty$ limit in which all but the CP-odd scalar A^0 are infinitely massive. Finiteness of the potential energy constrains in this limit the Higgs doublets to the form:

$$H_1 = e^{i\theta/2} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \quad \text{and} \quad H_2 = e^{-i\theta/2} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \ ,$$

modulo of course a gauge transformation. The energy of any static configuration reads in this case $E = \int [\frac{v^2}{4}(\nabla\theta)^2 - \frac{v^2}{2} m_A^2 cos\theta] +$ gauge where the gauge contribution is minimum when $W_\mu^a = Y_\mu^a = 0$. The above energy functional admits the well-known sine-Gordon soliton solution

$$\theta = 4 \ \mathrm{arctan}\Big(\exp(m_A x)\Big) \tag{12}$$

which describes in our context a topologically-stable wall of thickness m_A^{-1} and surface tension $E/\mathcal{A} = 4v^2 m_A \equiv 2m_W^2 m_A sin^2\theta_W/\pi\alpha$. These results agree qualitatively with eqs. (4-6), if one replaces L by m_A^{-1} and takes $m_{H^0} \to \infty$.

For finite values of λ_1, λ_3 and λ_4 we have performed the analysis numerically as follows: starting with an initial configuration close to the above sine-Gordon soliton we followed the direction of steepest descent until either we arrived at the vacuum, or the change in energy per step was less than one part in 10^{12}. One step amounted typically to a change of fields $\sim 10^{-2}$ times the gradient of energy. Stopping at a non-zero energy was interpreted as evidence for the existence of a stable solution. We verified that these candidate solutions obeyed the virial relation to better than one part in 10^3, and that they were insensitive to changes of the initial configuration or of the cutoff on the convergence rate. The entire numerical analysis was performed in the $W_x^a = Y_x = 0$ gauge. The profile of a typical solution, plotted in figures 1 and 2 , differs little from the sine-Gordon soliton. Likewise the energy per unit area stayed typically within 10% of $4v^2 m_A$. The conditions for classical stability were to within a few percent found to be

$$m_h/m_A \geq 2.0 \quad \text{and} \quad m_{H^+}/m_A \geq 2.2 \tag{13}$$

while the condition on m_{H^0}/m_A was sensitive to the precise value of the ratio m_h/m_A. The region of stability for three selected values of this ratio is depicted in fig. 3. As already anticipated stable walls exist provided A^0 is sufficiently light compared to all other scalars. Taking $m_A \simeq 50 GeV$, close to its experimental lower bound, we may satisfy these constraints with scalar self-couplings $\leq 1/2$. This is well within the region of perturbative unitarity in which the semi-classical approximation can be trusted. The minimal supersymmetric standard model, on the other hand, lies outside this region of stability, as can be seen for instance from the fact that A^0 is not the lightest neutral scalar[1]. It is however conceivable that loop corrections modify this conclusion.

The defects described in this letter would not interact electromagnetically, unless they happen to acquire charge by trapping fermions. Due, on the other hand, to their large energy density ($E/\mathcal{A} \sim 10^{10} gr/cm^2$ assuming $m_A \sim m_W$) they would manifest themselves through gravitational attraction. A single wall crossing today the entire universe would for example overclose it and can be excluded. Smaller membranes which either collapsed or were torn apart by quantum tunneling may have acted as seeds for the formation of galaxies. The mass of a typical galaxy is in fact comparable to that of a membrane a few light years in size.

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FIGURE CAPTIONS

Figure 1: The profile of a typical membrane, for $m_h = 2.5$, $m_{H^0} = 5.0$ and $m_{H^+} = 4.0$ in units of m_A. Both the gauge fields and the charged upper components of the doublets vanish. Plotted are the real and imaginary parts as well as the magnitude squared of the neutral components H_1^0 and H_2^0, as functions of the coordinate x normal to the membrane.

Figure 2: Plot of the neutral components of the two Higgs doublets in the complex plane. Their phases wind around in opposite directions as one crosses the wall, starting at zero and joining at $\pm\pi$.

Figure 3: The boundaries of classical stability in the (m_{H^0}, m_{H^+}) plane for three different values of m_h. The scale is chosen so that $m_A = 1$. Classically-stable membranes exist above the indicated lines. Also given is the energy density in units of $v^2 m_A$, for some selected points close to the boundary.