Remarks on the Coulomb and Covariant Gauges in Finite Temperature QED

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Abstract

We compare the use of the Coulomb gauge in finite temperature QED with a recently proposed prescription for covariant gauges, in which only the transverse photon degrees of freedom are thermalized. Using the Landau rule as a guide, we clarify the relation between the retarded electron self-energy and the elements of the self-energy matrix in the real-time formulation of . The general results are illustrated by means of the one-loop expressions for the electron self-energy in a QED plasma.

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In a theory with fermions and scalars only, the real-time formulation of Finite Temperature Field Theory (FTFT) [1, 2] is quite straightforward. However, the situation becomes more involved in gauge theories like QED. For covariant gauges, the traditional approach has been to assume that all the degrees of freedom of the gauge bosons are in thermal equilibrium[3]. Recently, Landshoff and Rebhan (LR)[4] showed that it is possible and even simpler to assume that only the physical transverse components of the gauge field are thermalized. The price is that the operator averages cannot be expressed as traces and therefore some formulas of the standard formalism do not apply anymore. Here, we elaborate this point by considering the relation between the physical (retarded) fermion self-energy and the elements of the self-energy matrix calculated with the Feynman rules of the theory. The formulas we write for the dispersive and absorptive parts of the effective self-energy satisfy the Landau rule and are the appropriate ones for those situations where statistical averages cannot be represented by a trace.

Within the real-time formalism, the self-energy of a fermion in a thermal background is a 2×2 matrix, whose elements are defined by

$$
\begin{aligned}
i\Sigma_{21}(z - y)_{\alpha\beta} &= -\langle \eta_\alpha(z)\overline{\eta}_\beta(y) \rangle \,, \\
i\Sigma_{12}(z - y)_{\alpha\beta} &= \langle \overline{\eta}_\beta(y)\eta_\alpha(z) \rangle \,, \\
-\Sigma_{11}(z - y) &= \Sigma_{21}(z - y)\theta(z^0 - y^0) + \Sigma_{12}(z - y)\theta(y^0 - z^0) \,, \\
-\Sigma_{22}(z - y) &= \Sigma_{21}(z - y)\theta(y^0 - z^0) + \Sigma_{12}(z - y)\theta(z^0 - y^0) \,,
\end{aligned}
\tag{1}
$$

where η and $\overline{\eta}$ are the fermion source fields. In terms of them the interaction Lagrangian is

$$
L_{\text{int}} = \overline{\psi}\eta + \overline{\eta}\psi \,,
\tag{2}
$$

and in particular, for QED $\eta = -e\slashed{A}\psi$. The angle brackets in Eq. (1) stand for the statistical average which, for any operator $\mathcal{O}$ is defined by

$$
\langle \mathcal{O} \rangle = \frac{\sum_i \langle i|\rho\mathcal{O}|i \rangle}{\sum_i \langle i|\rho|i \rangle} \,,
\tag{3}
$$

where

$$
\rho = e^{-\beta H + \sum_A \alpha_A Q_A} \,.
\tag{4}
$$

H is the Hamiltonian of the system, the quantities Q_A are the (conserved) charges that commute with H, $1/\beta$ is the temperature T, and the α_A are the chemical potentials that characterize the composition of the background.

In a theory without gauge fields the sums in Eq. (3) are over all the states of the system and are unambiguous. The same is true for QED in the Coulomb gauge. In this case, the Hilbert space contains only physical states and the unphysical photon degrees of freedom disappear, along with the associated question of wether they have a thermal distribution or not. Then, the photon propagator takes the form

$$\Delta_{ab}^{\mu\nu}(k) = (-S^{\mu\nu})[\Delta_{ab}^{(0)}(k) + \Delta_{ab}^{(T)}(k)]\,, \tag{5}$$

with $S_{\mu\nu}$ given by

$$S_{\mu\nu} = g_{\mu\nu} + \frac{1}{\kappa^2}k_\mu k_\nu - \frac{\omega}{\kappa^2}(u_\mu k_\nu + k_\mu u_\nu)\,, \tag{6}$$

where $\omega = k \cdot u$ and $\kappa = \sqrt{\omega^2 - k^2}$ are the energy and the magnitude of the 3-momentum $\vec{\kappa}$ of the photon in the frame where the medium is at rest. We have introduced the vector u^μ representing the velocity 4-vector of the background, with components $(1, \vec{0})$ in its own rest frame. In Eq. (5),

$$\Delta_{ab}^{(0)}(k) = \begin{pmatrix} \frac{1}{k^2 + i\epsilon} & -2\pi i \delta(k^2)\theta(-k \cdot u) \\ -2\pi i \delta(k^2)\theta(k \cdot u) & \frac{-1}{k^2 - i\epsilon} \end{pmatrix}\,, \tag{7}$$

and

$$\Delta_{ab}^{(T)}(k) = -2\pi i \delta(k^2)\frac{1}{e^{\beta|k \cdot u|} - 1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\,. \tag{8}$$

It is useful to observe that

$$S_{\mu\nu}\big|_{\omega=\kappa} = - \sum_{\lambda=1,2} \epsilon_\mu(k,\lambda)\epsilon_\nu(k,\lambda)\bigg|_{\omega=\kappa}\,, \tag{9}$$

where the polarization vectors are given by $\epsilon^\mu(k,\lambda) = (0, \vec{e}(k,\lambda))$, with $\vec{e}(k,\lambda) \cdot \vec{k} = 0$.

The situation is not so obvious in a covariant gauge, because the set of physical states does not span the whole space. In the traditional approach the sum is made over a complete set of states and the unphysical degrees of freedom acquire a thermal part. The covariant photon propagator has the same form as in Eq. (5), but with $S_{\mu\nu}$ replaced by the tensor $C_{\mu\nu}$ whose

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explicit expresion depends on the gauge in which the theory is quantized. For example, in the Feynman gauge $C_{\mu\nu} = g_{\mu\nu}$. On the other hand, according to the prescription of LR, the sums in Eq (3) involve only physical states, even when the theory is formulated in a covariant gauge. We will refer to this approach as the *mixed gauge*, and in it the thermal part of the photon propagator is the same as in the Coulomb gauge, while the zero-temperature term has the structure that corresponds to a covariant gauge:

$$\Delta_{ab}^{\mu\nu}(k)^{(mix)} = (-C^{\mu\nu})\Delta_{ab}^{(0)}(k) + (-S^{\mu\nu})\Delta_{ab}^{(T)}(k) \,. \tag{10}$$

In that manner, this approach attempts to combine the simplicity of covariant gauges for the vacuum part with the advantages of the (noncovariant) Coulomb gauge for the temperature-dependent terms and in principle, may be more convenient for calculational purposes. Nevertheless, as we discuss next, the fact that thermal averages are made by summing over a subset of states of the Hilbert space, has consequences that cannot be ignored.

In a medium, the effective field equation for a fermion with momentum $p^{\mu} = (\varepsilon, \vec{P})$ is [1]

$$(\not{p} - m - \Sigma_{eff})\psi = 0 \,, \tag{11}$$

where

$$\Sigma_{eff}(p) = \Sigma_{11}(p) + \Sigma_{12}(p) \,. \tag{12}$$

As seen from Eq. (1) Σ_{eff} corresponds to the retarded self-energy. Denoting by $\operatorname{Re}\Sigma_{11}$ and $\operatorname{Im}\Sigma_{11}$ the dispersive and absorptive parts of Σ_{11}:

$$
\begin{aligned}
\operatorname{Re}\Sigma_{11} &= \frac{1}{2}(\Sigma_{11} + \gamma_0\Sigma_{11}^{\dagger}\gamma_0) \,, \\
\operatorname{Im}\Sigma_{11} &= \frac{1}{2i}(\Sigma_{11} - \gamma_0\Sigma_{11}^{\dagger}\gamma_0) \,,
\end{aligned}
\tag{13}
$$

with a similar decomposition for Σ_{eff}, Eq. (12) is equivalent to

$$
\begin{aligned}
\Sigma_r(p) &= \operatorname{Re}\Sigma_{11}(p) \,, \\
\Sigma_i(p) &= \operatorname{Im}\Sigma_{11}(p) - i\Sigma_{12}(p) \,.
\end{aligned}
\tag{14}
$$

We have used the fact that Σ_{12} is purely absorptive, as follows from its definition in (1). Now we verify that these formulas are related correctly by the

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[1]Eq. (11) can be derived from the functional derivative of the effective action, by a procedure similar to the one described in Ref. [2] for a scalar particle [5].

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spectral representation as required on the basis of fundamental principles[6]. By using the integral representation of the step function, the following expressions are easily derived from Eq. (1):

$$\text{Re}\,\Sigma_{11}(p) \;=\; \frac{1}{2\pi i}\mathcal{P}\int d\varepsilon' \frac{\Sigma_{21}(\varepsilon',\vec{P}) - \Sigma_{12}(\varepsilon',\vec{P})}{\varepsilon - \varepsilon'}\,,$$

$$\text{Im}\,\Sigma_{11}(p) \;=\; \frac{i}{2}[\Sigma_{21}(p) + \Sigma_{12}(p)]\,, \tag{15}$$

where $\varepsilon = p\cdot u$. The second of these equations implies that Σ_i, determined according to Eq. (14), can be also computed by means of

$$\Sigma_i(p) = \frac{i}{2}[\Sigma_{21}(p) - \Sigma_{12}(p)]\,, \tag{16}$$

that substituted in the formula for $\text{Re}\,\Sigma_{11}(p)$, gives

$$\Sigma_r(p) = \frac{-1}{\pi}\mathcal{P}\int d\varepsilon' \frac{\Sigma_i(\varepsilon',\vec{P})}{\varepsilon - \varepsilon'}\,, \tag{17}$$

or, equivalently

$$\Sigma_i(p) = \text{Im}\,\Sigma_r(\varepsilon + i\epsilon, \vec{P})\,. \tag{18}$$

Eqs.(17) and (18) are just the statement of the Landau rule within the present context, and as the above reasoning shows, they will always be satisfied if the retarded self-energy is calculated from Eqs. (14) or (16).

The formulas for $\Sigma_{r,i}(p)$ in Eq. (14) have been obtained without reference to any specific gauge, and are valid independently of the particular choice used to compute the quantities Σ_{ab}. However, when the sum in Eq. (3) runs over all the states of the system, the thermal averages can be written as a trace and some simplification occurs. In those cases the cyclic property implies that

$$\Sigma_{21}(p) = -e^x \Sigma_{12}(p)\,, \tag{19}$$

and then, from Eqs. (15) and (16) the following familiar formula follows:

$$\Sigma_i(p) \;=\; \frac{\text{Im}\,\Sigma_{11}(p)}{1 - 2n_F(x)} = \frac{\Sigma_{12}(p)}{2in_F(x)}\,, \tag{20}$$

where

$$n_F(x) = \frac{1}{e^x + 1}\,, \tag{21}$$

is the fermion distribution written in terms of the variable $x = \beta\varepsilon - \alpha$, with α being the chemical potential.

In conclusion, the usual expressions given in Eq. (20) can be applied in the Coulomb and covariant gauges, but not in approaches like the one of LR, where thermal averages are not expressible as a trace [2]. In the last case we have to resort to Eq. (16), or equivalently to Eq. (14), to determine the absorptive part of Σ_{eff} On the contrary, if we insist in using (20), then the Landau relation as given by (17) or (18), is not satisfied.

It should be noticed that, when the relation of Eq. (19) is valid, then the elements Σ_{ab} can be parametrized in terms of a single quantity Σ. Then, Eq. (12) can be written as

$$\Sigma_{eff}(p) = \Sigma(p)\theta(\varepsilon) + \overline{\Sigma}(p)\theta(-\varepsilon),\qquad(22)$$

as is customarily done in the real-time formulation. As an specific illustration of the the general results established here, we have considered the the one-loop contributions to the self-energy of a (massless) electron in a QED plasma, calculated both the Coulomb and the mixed gauge [7]. In what follows we quote the main results of this calculation.

We begin with the Coulomb gauge. The 12 element of the self-energy matrix is given by

$$-i\Sigma_{12}(p) = (ie)(-ie)\int\frac{d^4k}{(2\pi)^4}i\Delta_{21}^{\mu\nu}(k)\gamma_\mu iS_{12}(p')\gamma_\nu,\qquad(23)$$

with $p' = p + k$. Using the expressions for the component $\Delta_{21}^{\mu\nu}(k)$ and $\Sigma_{12}(p)$ of the photon an the electron bare propagators, we find

$$\begin{aligned}
\Sigma_{12}(p) &= \left(\frac{-ie^2}{4\pi^2}\right)n_F(x)\int\frac{d^3k}{2\omega_k}\frac{d^3p'}{2E'}(-S^{\mu\nu}\gamma_\mu\not{p}'\gamma_\nu)\times\\
&\quad\left[\delta^{(4)}(p+k-p')(n_e+n_\gamma)+\delta^{(4)}(p-k-p')(1-n_e+n_\gamma)\right.\\
&\quad\left.+\delta^{(4)}(p-k+p')(\overline{n}_e+n_\gamma)+\delta^{(4)}(p+k+p')(1-\overline{n}_e+n_\gamma)\right],
\end{aligned}$$
$$\qquad(24)$$

where $p'^0 = E' = |\vec{p}'|$ and $k^0 = \omega_k = |\vec{k}|$. n_e and n_γ stand for the electron and photon density distributions, while $\overline{n}_e$ is the positron distribution, which

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[2] For gauge bosons, the observation that the relation equivalent to (19) does not hold in the mixed gauge is contained in the second paper of Ref. [4].

is obtained from n_e by changing the sign of α. Similarly,

$$-i\Sigma_{21}(p) = (ie)(-ie)\int \frac{d^4k}{(2\pi)^4}i\Delta_{12}^{\mu\nu}(k)\gamma_\mu iS_{21}(p')\gamma_\nu \,, \tag{25}$$

and with the help of the relations

$$\begin{aligned}
S_{21}(p') &= -e^{x'}S_{12}(p')\,, \\
\Delta_{12}^{\mu\nu}(k) &= e^{-x_\gamma}\Delta_{21}^{\mu\nu}(k)\,,
\end{aligned} \tag{26}$$

Eq. (19) is immediately verified at the one-loop level. We have introduced the variables $x' = \beta p' \cdot u - \alpha$ and $x_\gamma = \beta k \cdot u = x' - x$.

Turning now the attention to Σ_{11}, we have

$$-i\Sigma_{11}(p) = (-ie)^2\int \frac{d^4k}{(2\pi)^4}i\Delta_{11}^{\mu\nu}(k)\gamma_\mu iS_{11}(p')\gamma_\nu \,. \tag{27}$$

It is convenient to separate the background dependent part $\Sigma_{11}^{(T)}$ from the standard vacuum contribution

$$\Sigma_{11}^{(0)}(p) = ie^2\int \frac{d^4k}{(2\pi)^4}\frac{(-S^{\mu\nu}\gamma_\mu \not{p}'\gamma_\nu)}{(p'^2 + i\epsilon)(k^2 + i\epsilon)} \,. \tag{28}$$

The dispersive and absorptive parts of $\Sigma_{11}^{(T)}$ are given by

$$\begin{aligned}
\mathrm{Re}\,\Sigma_{11}^{(T)} &= e^2\int \frac{d^4k}{(2\pi)^3}\delta(k^2)\eta_B(k)\frac{(-S^{\mu\nu}\gamma_\mu \not{p}'\gamma_\nu)}{p'^2} \\
&\quad - e^2\int \frac{d^4p'}{(2\pi)^3}\delta(p'^2)\eta_F(p')\frac{(-S^{\mu\nu}\gamma_\mu \not{p}'\gamma_\nu)}{k^2} \,,
\end{aligned} \tag{29}$$

$$\begin{aligned}
\mathrm{Im}\,\Sigma_{11}^{(T)} &= \frac{e^2}{4\pi^2}\int d^4k\,\delta(k^2)\delta(p'^2)(-S^{\mu\nu}\gamma_\mu \not{p}'\gamma_\nu) \times \\
&\quad [\eta_B(k)\eta_F(p') - \frac{1}{2}\eta_B(k) + \frac{1}{2}\eta_F(p')] \,,
\end{aligned} \tag{30}$$

where

$$\begin{aligned}
\eta_B(k) &= n_B(x_\gamma)\theta(k \cdot u) + n_B(-x_\gamma)\theta(-k \cdot u)\,, \\
\eta_F(p') &= n_F(x')\theta(p' \cdot u) + n_F(-x')\theta(-p' \cdot u)\,,
\end{aligned} \tag{31}$$

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with n_F given by Eq. (21) and $n_B(x_\gamma) = (e^{x_\gamma} - 1)^{-1}$.

For $\text{Im}\,\Sigma_{11}^{(0)}$, the Cutkosky rules yield

$$
\text{Im}\,\Sigma_{11}^{(0)} = \frac{-e^2}{8\pi^2} \int d^4 k\, \delta(k^2)\delta(p'^2)(-S^{\mu\nu}\gamma_\mu \not{p}'\gamma_\nu) \times
$$
$$
[\theta(p'\cdot u)\theta(-k\cdot u) + \theta(-p'\cdot u)\theta(k\cdot u)]\,, \qquad (32)
$$

and using the identity

$$
2\eta_B(k)\eta_F(p') - \eta_B(k) + \eta_F(p') = \theta(k\cdot u)\theta(-p'\cdot u) + \theta(-k\cdot u)\theta(p'\cdot u)
$$
$$
+(e^{x_\gamma} - e^{x'})n_F(x')n_B(x_\gamma)\epsilon(k\cdot u)\epsilon(p'\cdot u), \quad (33)
$$

it follows that, in the combination $\text{Im}\,\Sigma_{11}^{(0)} + \text{Im}\,\Sigma_{11}^{(T)}$, the vacuum term is cancelled by an identical contribution coming from the temperature dependent part. The remaining terms can be rewritten by means of the relation $e^{x_\gamma} n_B(x_\gamma)n_F(x') = n_F(x)[n_F(x') + n_B(x_\gamma)]$, and comparing them with Eq. (24) it is seen that

$$
\text{Im}\,\Sigma_{11}(p) = \frac{i}{2}(1 - e^x)\Sigma_{12}(p)\,, \qquad (34)
$$

in agreement with Eq. (20).

Introducing $\Sigma_{11\,r} \equiv \text{Re}\,\Sigma_{11}$, from (29) it follows that

$$
\text{Im}\,\Sigma_{11r}^{(T)}(\varepsilon + i\epsilon, \vec{P}) = \frac{-e^2}{8\pi^2} \int \frac{d^3 k}{2\omega_k}\frac{d^3 p'}{2E'}(-S^{\mu\nu}\gamma_\mu \not{p}'\gamma_\nu) \times
$$
$$
\left[\delta^{(4)}(p + k - p')(n_\gamma + n_e) + \delta^{(4)}(p - k - p')(n_\gamma - n_e)\right.
$$
$$
\left. + \delta^{(4)}(p - k + p')(n_\gamma + \overline{n}_e) + \delta^{(4)}(p + k + p')(n_\gamma - \overline{n}_e)\right].
$$
$$
(35)
$$

In a similar fashion, from Eq. (28)

$$
\text{Im}\,\Sigma_{11r}^{(0)}(\varepsilon + i\epsilon, \vec{P}) = \frac{-e^2}{8\pi^2} \int \frac{d^3 k}{2\omega_k}\frac{d^3 p'}{2E'}(-S^{\mu\nu}\gamma_\mu \not{p}'\gamma_\nu) \times
$$
$$
[\delta^{(4)}(p - k - p') + \delta^{(4)}(p + k + p')]\,. \qquad (36)
$$

Adding both expressions and comparing with Eq. (24), we arrive at

$$
\text{Im}\,\Sigma_{11r}(\varepsilon + i\epsilon, \vec{P}) = \frac{\Sigma_{12}(p)}{2in_F(x)}\,. \qquad (37)
$$

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This last result explicitly shows that the physical self-energy determined by Eq. (20) satisfies the Landau condition given in Eq. (18). The same conclusion remains valid in a covariant gauge, with $-S^{\mu\nu}$ replaced by $-C^{\mu\nu}$.

Finally, we repeat the above analysis for the approach of LR. Our strategy is to decompose the electron self-energy into two pieces,

$$\Sigma_{ab}^{(mix)} = \Sigma_{ab} + \Sigma_{ab}', \tag{38}$$

with Σ_{ab} being the quantity we have calculated previously and Σ_{ab}' representing an additional contribution that arises from the term $(S^{\mu\nu} - C^{\mu\nu})\Delta_{ab}^{(0)}$ in the photon propagator. In this way, the 12 element of the self-energy in the mixed gauge is expressed as in Eq. (38), with Σ_{12} given by Eq. (24) and

$$
\begin{aligned}
\Sigma_{12}'(p) \;=\; & \frac{-ie^2}{4\pi^2} \int \frac{d^3k}{2\omega_k} \frac{d^3p'}{2E'} (S^{\mu\nu} - C^{\mu\nu}) \gamma_\mu \not{p}'\gamma_\nu \times \\
& \left[\delta^{(4)}(p + k - p') n_e + \delta^{(4)}(p + k + p')(1 - \overline{n}_e) \right]. \tag{39}
\end{aligned}
$$

By the same procedure,

$$
\begin{aligned}
\Sigma_{21}'(p) \;=\; & \frac{-ie^2}{4\pi^2} \int \frac{d^3k}{2\omega_k} \frac{d^3p'}{2E'} (S^{\mu\nu} - C^{\mu\nu}) \gamma_\mu \not{p}'\gamma_\nu \times \\
& \left[-\delta^{(4)}(p - k - p')(1 - n_e) - \delta^{(4)}(p - k + p')\overline{n}_e \right]. \tag{40}
\end{aligned}
$$

For the absorptive part of Σ_{11}', the vacuum and the temperature dependent contributions can be read from Eqs. (30) and (32) respectively, by replacing $-S^{\mu\nu}$ by $(S^{\mu\nu} - C^{\mu\nu})$ and putting $\eta_B(k) = 0$. Adding the corresponding results yields

$$
\begin{aligned}
\operatorname{Im}\Sigma_{11}' \;=\; & \frac{-e^2}{8\pi^2} \int \frac{d^3k}{2\omega_k} \frac{d^3p'}{2E'} (S^{\mu\nu} - C^{\mu\nu}) \gamma_\mu \not{p}'\gamma_\nu \times \\
& \Big[\delta^{(4)}(p - k - p')(1 - n_e) - \delta^{(4)}(p + k - p') n_e \\
& - \delta^{(4)}(p + k + p')(1 - \overline{n}_e) + \delta^{(4)}(p - k + p')\overline{n}_e \Big]. \tag{41}
\end{aligned}
$$

Following similar steps for the real part of Σ_{11}' we obtain

$$
\begin{aligned}
\operatorname{Im}\Sigma_{11\,r}'(\varepsilon + i\epsilon, \vec{P}) \;=\; & \frac{-e^2}{8\pi^2} \int \frac{d^3k}{2\omega_k} \frac{d^3p'}{2E'} (S^{\mu\nu} - C^{\mu\nu}) \gamma_\mu \not{p}'\gamma_\nu \times \\
& \Big[\delta^{(4)}(p - k - p')(1 - n_e) + \delta^{(4)}(p + k - p') n_e \\
& + \delta^{(4)}(p + k + p')(1 - \overline{n}_e) + \delta^{(4)}(p - k + p')\overline{n}_e \Big]. \tag{42}
\end{aligned}
$$

From the previous expressions for Σ'_{ab}, it is easy to verify that the $\Sigma_{ab}^{(mix)}$ satisfy Eq. (15), and that $\Sigma_{eff}^{(mix)}$ determined from Eq. (14) or (16), satisfies the Landau condition Eq. (18). In addition, by comparing the formulas in Eqs. (39) and (40) we see that there is no simple relation between Σ'_{12} and Σ'_{21}, and consequently $\Sigma_{21}^{(mix)}(p) \neq -e^x \Sigma_{12}^{(mix)}(p)$, which explicitly confirms that Eq. (19) is not applicable in the mixed gauge. In fact, if Σ_i were calculated by using the usual expressions in terms of Σ_{11} or Σ_{12} given in (20), which in the present case do not yield equivalent results, then the resulting formulas would not verify the Landau condition. As already explained in the paragraph above Eq. (19), this is due to the fact that the sum used to define the statistical averages is not carried over a complete set of states of the system.

References

[1] A comprehensive review, with an extensive list of references, is given by N. P. Landsman and Ch. G. van Weert, Phys. Rep. **145** (1987) 141.

[2] For a canonical approach based on the Keldysh formalism see J. F. Nieves, Phys. Rev. D**42** (1990) 4123.

[3] C. W. Bernard, Phys. Rev. D**9** (1974) 3312; H. Hata and T. Kugo, *ibid.***21** (1980) 3333.

[4] P. V. Landshoff and A. Rebhan, Nucl. Phys. B**383** (1992) 607; *ibid.***410** (1993) 23.

[5] See J. F. Nieves, in *Proceedings of the IV Mexican School of Particles and Fields*, edited by J. L. Lucio and A. Zepeda (World Sc., Singapore, 1992), pp 283-319.

[6] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill,1971, New York), p. 292.

[7] J. C. D'Olivo and J. F. Nieves, in *Proceedings of the VI Mexican School of Particles and Fields*, edited by J. C. D'Olivo, M. Moreno and M. Perez (World Sc., Singapore), to be published.