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On the Corrections to Dashen's Theorem

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Abstract

The electromagnetic corrections to the masses of the pseudoscalar mesons π and K are considered. We calculate in chiral perturbation theory the contributions which arise from resonances within a photon loop at order $O(e^2 m_q)$. For the corrections to Dashen's theorem we find rather moderate deviations in discrepancy to the values found in the literature.

1 Introduction

Dashen's theorem [1] states that the squared mass differences between the charged pseudoscalar mesons $\pi^\pm, K^\pm$ and their corresponding neutral partner π^0, K^0 are equal in the chiral limit, i.e. $\Delta M_K^2 - \Delta M_\pi^2 = 0$, where $\Delta M_P^2 = M_{P^\pm}^2 - M_{P^0}^2$. In

recent years several groups have calculated the electromagnetic corrections to this relation from non-vanishing quark masses. The different conclusions are either that the violation is large [2, 3] or that it *may be* large [4, 5, 6].
The electromagnetic mass difference of the pions ΔM_π^2 has been determined in the chiral limit using current algebra by Das et al. [7]. Ecker et al. [8] have repeated the calculation in the framework of chiral perturbation theory (χPT) [9] by resonance exchange within a photon loop. The occurring divergences from these loops are absorbed by introducing an electromagnetic counterterm (with a coupling constant $\hat{C}$) in the chiral lagrangian. They find that the contribution from the loops is numerically very close to the experimental mass difference, and thus conclude that the finite part of $\hat{C}$ is almost zero.
In this article we proceed in an analogous manner for the case $m_q \neq 0$. We calculate in χPT the contributions of order $O(e^2 m_q)$ to the masses of the Goldstone bosons due to resonances. The divergences are absorbed in the corresponding electromagnetic counterterm lagrangian, associated with the couplings $\hat{K}_i$, where $i = 1, \ldots, 14$. The most general form of this lagrangian has been given in [5, 6, 10]. We find again that the contribution from the loops represents the measured mass difference ΔM_π^2 very well, and the finite part of the $\hat{K}_i$ may be considered small. Using this assumption also for the calculation of ΔM_K^2, we may finally read off the corrections to Dashen's theorem from one-loop resonance exchange. We find a (scale dependent) result that is controversial to the values found in the literature.
In [2] the authors have calculated the Compton scattering of the pseudoscalar mesons including the resonances and determined from this amplitude the mass differences. They concluded first of all by using three low-energy relations that the one-loop result is finite, i.e. there is no need of a counterterm lagrangian at order $O(e^2 m_q)$ in order to renormalize the contributions from the resonances, secondly they found a strong violation of Dashen's theorem. We are in disagreement with both of these results.
The article is organized as follows. In section 2 we present the ingredients from χPT and the resonances needed for the calculation. In section 3 we give the contributions to the masses and to Dashen's theorem and renormalize the counterterm lagrangian. The numerical results and a short conclusion are given in section 4.

2 The Lagrangians at lowest and next-to-leading Order

The chiral lagrangian can be expanded in derivatives of the Goldstone fields and in the masses of the three light quarks. The power counting is established in the following way: The Goldstone fields are of order $O(p^0)$, a derivative ∂_μ, the vector and axial vector currents v_μ, a_μ count as quantities of $O(p)$ and the scalar (incorporating the masses) and pseudoscalar currents s, p are of order $O(p^2)$. The effective lagrangian starts at $O(p^2)$, denoted by $\mathcal{L}_2$. It is the non-linear σ-model lagrangian coupled to external fields, respects chiral symmetry $SU(3)_R \times SU(3)_L$, and is invariant under P and C transformations [9],

$$
\begin{aligned}
\mathcal{L}_2 &= \frac{F_0^2}{4}\langle d^\mu U^\dagger d_\mu U + \chi U^\dagger + \chi^\dagger U\rangle \\
d_\mu U &= \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \\
v_\mu &= QA_\mu + \cdots \\
Q &= \frac{e}{3}\,\mathrm{diag}\,(2,-1,-1) \\
\chi &= 2B_0(s+ip) \\
s &= \mathrm{diag}\,(m_u, m_d, m_s) \\
F_\pi &= F_0\,[1 + O(m_q)] \\
B_0 &= -\frac{1}{F_0^2}\langle 0|\bar{u}u|0\rangle\,[1 + O(m_q)] \quad .
\end{aligned}
\tag{1}
$$

The brackets $\langle \cdots \rangle$ denote the trace in flavour space and U is a unitary 3×3 matrix that incorporates the fields of the eight pseudoscalar mesons,

$$
\begin{aligned}
U &= \exp\,(i\Phi/F_0) \\
\Phi &= \sqrt{2}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \quad .
\end{aligned}
\tag{2}
$$

Note that the photon field A_μ is incorporated in the vector current v_μ. The corresponding kinetic term has to be added to $\mathcal{L}_2$,

$$
\mathcal{L}^\gamma_{kin} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\left(\partial_\mu A^\mu\right)^2 \,, \tag{3}
$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the gauge fixing parameter chosen to be $\lambda = 1$. In order to maintain the usual chiral counting in $\mathcal{L}^\gamma_{kin}$, it is convenient to count the photon field as a quantity of order $O(p^0)$, and the electromagnetic coupling e of $O(p)$ [5].

The lowest order couplings of the pseudoscalar mesons to the resonances are linear in the resonance fields and start at order $O(p^2)$ [8, 11]. For the description of the fields we use the antisymmetric tensor notation for the vector and axialvector mesons, e.g.

the vector octet has the form

$$
V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0_{\mu\nu} + \frac{1}{\sqrt{6}}\omega_{8\,\mu\nu} & \rho^+_{\mu\nu} & K^{*+}_{\mu\nu} \\ \rho^-_{\mu\nu} & -\frac{1}{\sqrt{2}}\rho^0_{\mu\nu} + \frac{1}{\sqrt{6}}\omega_{8\,\mu\nu} & K^{*0}_{\mu\nu} \\ K^{*-}_{\mu\nu} & \overline{K^{*0}}_{\mu\nu} & -\frac{2}{\sqrt{6}}\omega_{8\,\mu\nu} \end{pmatrix} \quad . \tag{4}
$$

This method is discussed in detail in [8], we restrict ourselves on the formulae needed for the calculations in the following section. The relevant interaction lagrangian contains the octet fields only,

$$
\begin{aligned}
\mathcal{L}_2^V &= \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle \\
\mathcal{L}_2^A &= \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle \\
f_\pm^{\mu\nu} &= uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu} u \\
F_{R,L}^{\mu\nu} &= \partial^\mu(v^\nu \pm a^\nu) - \partial^\nu(v^\mu \pm a^\mu) - i[v^\mu \pm a^\mu, v^\nu \pm a^\nu] \\
u^\mu &= iu^\dagger d^\mu U u^\dagger = u^{\dagger\mu} \\
U &= u^2 \quad .
\end{aligned} \tag{5}
$$

The coupling constants are real and are not restricted by chiral symmetry [11], numerical estimates are given in [8]. In the kinetic lagrangian a covariant derivative acts on the vector and axialvector mesons,

$$
\begin{aligned}
\mathcal{L}_{kin}^R &= -\frac{1}{2}\langle \nabla^\mu R_{\mu\nu}\nabla_\sigma R^{\sigma\nu} - \frac{1}{2}M_R^2 R_{\mu\nu}R^{\mu\nu}\rangle \qquad\qquad R = V, A \\
\nabla^\mu R_{\mu\nu} &= \partial^\mu R_{\mu\nu} + [\Gamma^\mu, R_{\mu\nu}] \\
\Gamma^\mu &= \frac{1}{2}\left\{u^\dagger[\partial^\mu - i(v^\mu + a^\mu)]u + u[\partial^\mu - i(v^\mu - a^\mu)]u^\dagger\right\} \quad ,
\end{aligned} \tag{6}
$$

where M_R is the corresponding mass in the chiral limit. Finally we collect all the different terms together into one lagrangian,

$$
\mathcal{L}_2^{eff} = \mathcal{L}_2 + \mathcal{L}_2^R + \mathcal{L}_{kin}^\gamma + \mathcal{L}_{kin}^R \quad . \tag{7}
$$

The one-loop electromagnetic mass shifts of the pseudoscalar mesons calculated with this lagrangian (see section 3) contain divergences that can be absorbed in a counterterm lagrangian. In its general form, this lagrangian has one term of order $O(e^2)$ and 14 terms of $O(e^2p^2)$ [5, 6, 10],

$$
\begin{aligned}
\mathcal{L}_2^C &= \hat{C}\langle QUQU^\dagger\rangle \\
\mathcal{L}_4^C &= \hat{K}_1 F_0^2\langle d^\mu U^\dagger d_\mu U\rangle\langle Q^2\rangle + \hat{K}_2 F_0^2\langle d^\mu U^\dagger d_\mu U\rangle\langle QUQU^\dagger\rangle
\end{aligned}
$$

$$
\begin{aligned}
&+\hat{K}_3 F_0^2 \left(\langle d^\mu U^\dagger Q U\rangle\langle d_\mu U^\dagger Q U\rangle + \langle d^\mu U Q U^\dagger\rangle\langle d_\mu U Q U^\dagger\rangle\right)\\
&+\hat{K}_4 F_0^2 \langle d^\mu U^\dagger Q U\rangle\langle d_\mu U Q U^\dagger\rangle\\
&+\hat{K}_5 F_0^2 \langle\left(d^\mu U^\dagger d_\mu U + d^\mu U d_\mu U^\dagger\right) Q^2\rangle\\
&+\hat{K}_6 F_0^2 \langle d^\mu U^\dagger d_\mu U Q U^\dagger Q U + d^\mu U d_\mu U^\dagger Q U Q U^\dagger\rangle\\
&+\hat{K}_7 F_0^2 \langle \chi U^\dagger + \chi^\dagger U\rangle\langle Q^2\rangle + \hat{K}_8 F_0^2 \langle \chi U^\dagger + \chi^\dagger U\rangle\langle Q U Q U^\dagger\rangle\\
&+\hat{K}_9 F_0^2 \langle(\chi U^\dagger + \chi^\dagger U + U^\dagger\chi + U\chi^\dagger)Q^2\rangle \qquad (8)\\
&+\hat{K}_{10} F_0^2 \langle(\chi^\dagger U + U^\dagger\chi) Q U^\dagger Q U + (\chi U^\dagger + U\chi^\dagger) Q U Q U^\dagger\rangle\\
&+\hat{K}_{11} F_0^2 \langle(\chi^\dagger U - U^\dagger\chi) Q U^\dagger Q U + (\chi U^\dagger - U\chi^\dagger) Q U Q U^\dagger\rangle\\
&+\hat{K}_{12} F_0^2 \langle d^\mu U^\dagger \left[c_\mu^R Q, Q\right] U + d^\mu U \left[c_\mu^L Q, Q\right] U^\dagger\rangle\\
&+\hat{K}_{13} F_0^2 \langle c^{R\,\mu} Q U c_\mu^L Q U^\dagger\rangle + \hat{K}_{14} F_0^2 \langle c^{R\,\mu} Q c_\mu^R Q + c^{L\,\mu} c_\mu^L Q\rangle\\
&+O(p^4, e^4) \quad .
\end{aligned}
$$

with $c_\mu^{R,L} Q = -i\,[v_\mu \pm a_\mu, Q]$. The three last terms contribute only to matrix elements with external fields, we are therefore left with 12 relevant counterterms. Note that we have omitted terms which come either from the purely strong or the purely electromagnetic sector in $\mathcal{L}_4^C$.

3 Corrections to Dashen's Theorem

Using the lagrangian given in (7) it is a straightforward process to calculate the mass shift between the charged pseudoscalar mesons $\pi^\pm, K^\pm$ and their corresponding neutral partner π^0, K^0 at the one-loop level. The relevant diagrams for the mass of the charged pion are shown in Fig.1. Graph (a) contains the off-shell pion form factor, (b) vanishes in dimensional regularization and (c) is called "modified seagull graph". Graph (d) contains an a_1-pole. The mass of the neutral pion does not get contributions from the loops.
If we take the resonances to be in the $SU(3)$ limit according to (6), i.e. all vector resonances have the same mass M_V and all axialvector resonances the mass M_A, we get the contributions listed below. For the graphs with the pion form factor,

$$
\begin{aligned}
\Delta_{p.f.} M_\pi^2 &= -ie^2 \int \frac{d^4 q}{(2\pi)^4} \frac{q^2 + 4\nu + 4M_\pi^2}{q^2(q^2+2\nu)}\\
&\quad -i\frac{8e^2 F_V G_V}{F_0^2} \int \frac{d^4 q}{(2\pi)^4} \frac{q^2 M_\pi^2 - \nu^2}{q^2(q^2+2\nu)(M_V^2 - q^2)}\\
&\quad -i\frac{4e^2 F_V^2 G_V^2}{F_0^4} \int \frac{d^4 q}{(2\pi)^4} \frac{q^2[q^2 M_\pi^2 - \nu^2]}{q^2(q^2+2\nu)(M_V^2 - q^2)^2} \quad ,
\end{aligned} \qquad (9)
$$

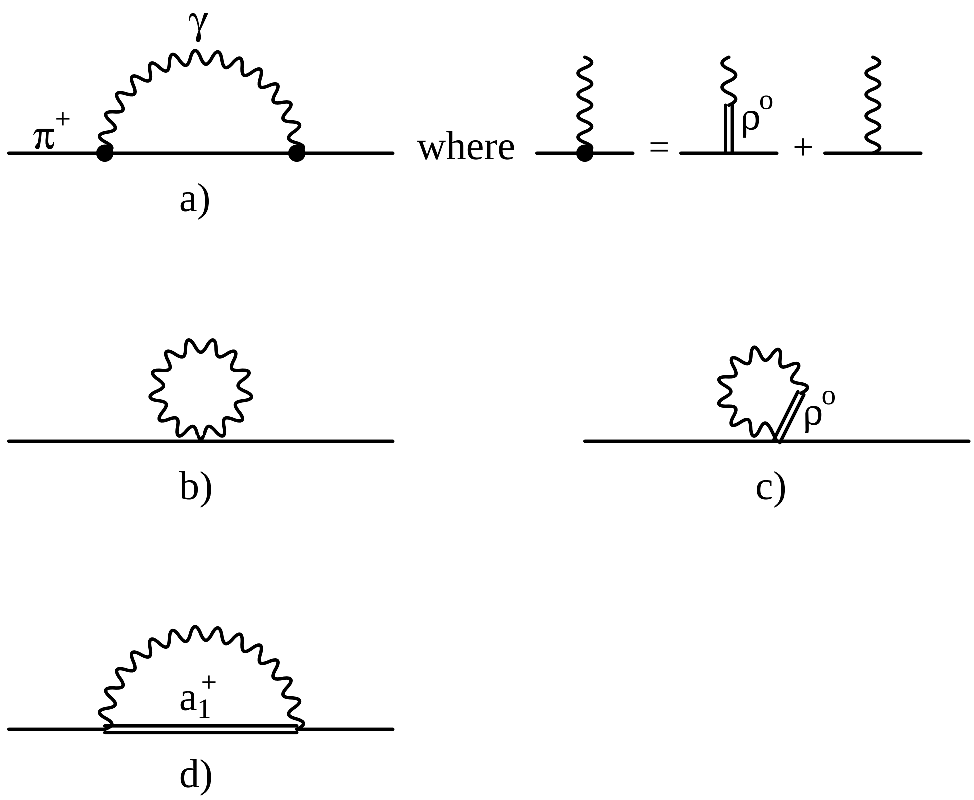

Figure 1: One-loop contributions to the electromagnetic mass shift of $\pi^{\pm}$.

where $\nu = pq$ and p is the momentum of the pion. Using the relation $F_V G_V = F_0^2$ [11] we obtain

$$\Delta_{p.f.} M_\pi^2 = -ie^2 M_V^4 \int \frac{d^4q}{(2\pi)^4} \frac{2\nu + 4M_\pi^2}{q^2(q^2+2\nu)(M_V^2-q^2)^2} \quad . \tag{10}$$

The modified seagull graph gives

$$\Delta_{s.g.} M_\pi^2 = i\frac{e^2 F_V^2}{F_0^2}(3-\epsilon)\int \frac{d^4q}{(2\pi)^4}\frac{1}{M_V^2-q^2} \tag{11}$$

with $\epsilon = 4-d$, and finally for the a_1-pole graph, where unlike [2] we get an additional second term,

$$\Delta_{a_1} M_\pi^2 \quad = \quad -i\frac{e^2 F_A^2}{F_0^2}(3-\epsilon)\int \frac{d^4q}{(2\pi)^4}\frac{1}{M_A^2-q^2}$$

$$- \quad i\frac{e^2F_A^2}{F_0^2}\int\frac{d^4q}{(2\pi)^4}\frac{q^2\left[M_\pi^2+(3-\epsilon)\nu\right]+(2-\epsilon)\nu^2}{q^2\left[M_A^2-(q+p)^2\right]} \quad . \tag{12}$$

We can now perform the appropriate integrals. The integrals, however, have divergences in $d=4$ dimensions proportional to $O(e^2)$ and to $O(e^2m_q)$. We will show that they can be absorbed in the counterterm lagrangians given in (8) . Their contribution to the mass shift of the pions has been evaluated in [5, 6]. Adding this result to the one-loop result above, we get up to and including order $O(e^2m_q)$,

$$\begin{aligned}
\Delta M_\pi^2 \quad = \quad & -\frac{3e^2}{F_0^2 16\pi^2}\left[F_V^2M_V^2\left(\ln\frac{M_V^2}{\mu^2}+\frac{2}{3}\right)-F_A^2M_A^2\left(\ln\frac{M_A^2}{\mu^2}+\frac{2}{3}\right)\right]\\
& -\frac{e^2F_A^2}{F_0^2 16\pi^2}M_\pi^2\left[\frac{7}{2}+\frac{3}{2}\ln\frac{M_A^2}{\mu^2}+I_1\left(\frac{M_\pi^2}{M_A^2}\right)\right]\\
& +\frac{2e^2}{16\pi^2}M_\pi^2\left[\frac{7}{2}-\frac{3}{2}\ln\frac{M_\pi^2}{M_V^2}+I_2\left(\frac{M_\pi^2}{M_V^2}\right)\right]\\
& +\frac{2e^2\hat{C}}{F_0^2}-\frac{6e^2}{F_0^2}(F_V^2M_V^2-F_A^2M_A^2)\lambda\\
& +8e^2M_K^2\hat{K}_8+2e^2M_\pi^2\hat{R}_\pi-\frac{3e^2F_A^2}{F_0^2}M_\pi^2\lambda
\end{aligned} \tag{13}$$

with

$$\begin{aligned}
I_1(z) &= \int_0^1 x\ln[x-x(1-x)z]\,dx\\
I_2(z) &= \int_0^1(1+x)\left\{\ln[x+(1-x)^2z]-\frac{x}{x+(1-x)^2z}\right\}dx\\
\hat{R}_\pi &= -2\hat{K}_3+\hat{K}_4+2\hat{K}_8+4\hat{K}_{10}+4\hat{K}_{11}\\
\lambda &= \frac{\mu^{d-4}}{16\pi^2}\left\{\frac{1}{d-4}-\frac{1}{2}[\ln 4\pi+\Gamma'(1)+1]\right\} \quad .
\end{aligned} \tag{14}$$

The divergences are now canceled by renormalizing the coupling constants $\hat{C}$ and $\hat{K}_i$,

$$\hat{C} = \hat{C}(\mu)+3(F_V^2M_V^2-F_A^2M_A^2)\lambda \tag{15}$$

$$\hat{R}_\pi = \hat{R}_\pi(\mu)+\frac{3F_A^2}{2F_0^2}\lambda \quad . \tag{16}$$

Using the second Weinberg sum rule [12]

$$F_V^2M_V^2-F_A^2M_A^2=0 \quad , \tag{17}$$

the divergence in (15) cancels, but the divergence in (16) does not. Even if we used an extension of this sum rule to order $O(m_q)$ [13],

$$F_\rho^2 M_\rho^2 - F_{a_1}^2 M_{a_1}^2 \simeq F_\pi^2 M_\pi^2 \tag{18}$$

and assumed $F_A = F_0$ [11], the divergence would not cancel, on the contrary, it would become larger.
We finally get the result

$$\begin{aligned}
\Delta M_\pi^2 \;&=\; -\frac{3e^2}{F_0^2 16\pi^2} F_V^2 M_V^2 \ln \frac{M_V^2}{M_A^2} \\
&\quad -\frac{e^2 F_A^2}{F_0^2 16\pi^2} M_\pi^2 \left[\frac{7}{2} + \frac{3}{2}\ln\frac{M_A^2}{\mu^2} + I_1\left(\frac{M_\pi^2}{M_A^2}\right)\right] \\
&\quad +\frac{2e^2}{16\pi^2} M_\pi^2 \left[\frac{7}{2} - \frac{3}{2}\ln\frac{M_\pi^2}{M_V^2} + I_2\left(\frac{M_\pi^2}{M_V^2}\right)\right] \\
&\quad +\frac{2e^2 \hat{C}(\mu)}{F_0^2} + 8e^2 M_K^2 \hat{K}_8 + 2e^2 M_\pi^2 \hat{R}_\pi(\mu) \quad ,
\end{aligned} \tag{19}$$

where we used (17) to simplify the first term. In the chiral limit ΔM_π^2 reduces to the expression given in [8].
The mass difference for the kaons is determined in an analogous way, in the contribution from the loops we merely have to replace M_π^2 by M_K^2. Finally the formula for the corrections to Dashen's theorem may be read off,

$$\begin{aligned}
\Delta M_K^2 - \Delta M_\pi^2 \;&=\; -\frac{e^2 F_A^2}{F_0^2 16\pi^2} \left\{ M_K^2 \left[\frac{7}{2} + \frac{3}{2}\ln\frac{M_A^2}{\mu^2} + I_1\left(\frac{M_K^2}{M_A^2}\right)\right] \right. \\
&\qquad\qquad \left. - M_\pi^2 \left[\frac{7}{2} + \frac{3}{2}\ln\frac{M_A^2}{\mu^2} + I_1\left(\frac{M_\pi^2}{M_A^2}\right)\right]\right\} \\
&\quad + \frac{2e^2}{16\pi^2} \left\{ M_K^2 \left[\frac{7}{2} - \frac{3}{2}\ln\frac{M_K^2}{M_V^2} + I_2\left(\frac{M_K^2}{M_V^2}\right)\right] \right. \\
&\qquad\qquad \left. - M_\pi^2 \left[\frac{7}{2} - \frac{3}{2}\ln\frac{M_\pi^2}{M_V^2} + I_2\left(\frac{M_\pi^2}{M_V^2}\right)\right]\right\} \\
&\quad -2e^2 M_K^2 \left[\frac{2}{3}\hat{S}_K(\mu) + 4\hat{K}_8\right] + 2e^2 M_\pi^2 \left[\frac{2}{3}\hat{S}_\pi - \hat{R}_\pi(\mu)\right] \quad ,
\end{aligned} \tag{20}$$

where $\hat{S}_{\pi,K}$ represent the contributions from the counterterm lagrangian to ΔM_K^2,

$$\begin{aligned}
\hat{S}_\pi &= 3\hat{K}_8 + \hat{K}_9 + \hat{K}_{10} \\
\hat{S}_K &= \hat{K}_5 + \hat{K}_6 - 6\hat{K}_8 - 6\hat{K}_{10} - 6\hat{K}_{11} \\
\hat{S}_K &= \hat{S}_K(\mu) + \frac{3F_A^2}{2F_0^2}\lambda \quad .
\end{aligned}$$

4 Numerical Results and Conclusion

We put F_0 equal to the physical pion decay constant, $F_\pi = 92.4$ MeV and the masses of the mesons to $M_\pi = 135$ MeV, $M_K = 495$ MeV. We take $F_V = 154$ MeV [8] and $M_V = M_\rho = 770$ MeV. To eliminate the parameters of the axialvector resonances we use Weinberg's sum rules [12],

$$F_V^2 - F_A^2 = F_0^2 \qquad\qquad F_V^2 M_V^2 - F_A^2 M_A^2 = 0 \quad . \tag{21}$$

Putting the numbers in (19) we get for the contribution from the loops to ΔM_π^2 at the scale points $\mu = (0.5, 0.77, 1)$ GeV (see Fig.2a)

$$\Delta M_\pi^2 = 2M_\pi \times (\ 4.9\ ,\ 5.0\ ,\ 5.0\ )\ \mathrm{MeV} \quad . \tag{22}$$

This remains close to the experimental value of $2M_\pi \times 4.6$ MeV [14]. We therefore may conclude that the (finite) contributions from the low energy constants $\hat{C}, \hat{K}_i$ are small, i.e.

$$\hat{C}(\mu) = \hat{C} \approx 0 \qquad\qquad \hat{K}_i(\mu) \approx 0 \quad . \tag{23}$$

Of course, in order to get a scale independent result, the counterterms are not allowed to vanish completely. We assume, however, that they do not shift the result essentially, at least in the case of ΔM_π^2. This statement may be expressed in a formal way: Consider χPT without resonances, then the contributions from the vector and axialvector mesons to electromagnetic matrix elements are absorbed in the counterterm lagrangian (8) and the coupling constants may be written as

$$\begin{aligned} C &= \sum_R C^R + \hat{C} \\ K_i &= \sum_R K_i^R + \hat{K}_i \qquad\qquad R = V, A \quad , \end{aligned} \tag{24}$$

where the terms with the superscript R represent the contributions from the resonances. Eq.(23) now means that the non-resonant part is assumed to be considerably smaller than the resonant contributions, in some analogy to the case of the coupling constants L_i in the strong sector [8]. However, in contrast to the latter case we have a scale dependence in the result (22) reminding us of an additional contribution beside the resonances, which we assume to be small numerically.
Using this assumption we obtain for the corrections to Dashen's theorem at the three scale points $\mu = (0.5, 0.77, 1)$ GeV

$$\Delta M_K^2 - \Delta M_\pi^2 = (\ -0.48\ ,\ -0.18\ ,\ 0.01\ ) \times 10^{-3}\ (\mathrm{GeV})^2 \quad , \tag{25}$$

which does not agree with the values found in the literature,

$$\Delta M_K^2 - \Delta M_\pi^2 = \left\{ \begin{array}{ccc} 1.23 & & [2] \\ 1.3 \pm 0.4 & \times 10^{-3}\ (\mathrm{GeV})^2 & [3] \\ 0.55 \pm 0.25 & & [4] \end{array} \right. \tag{26}$$

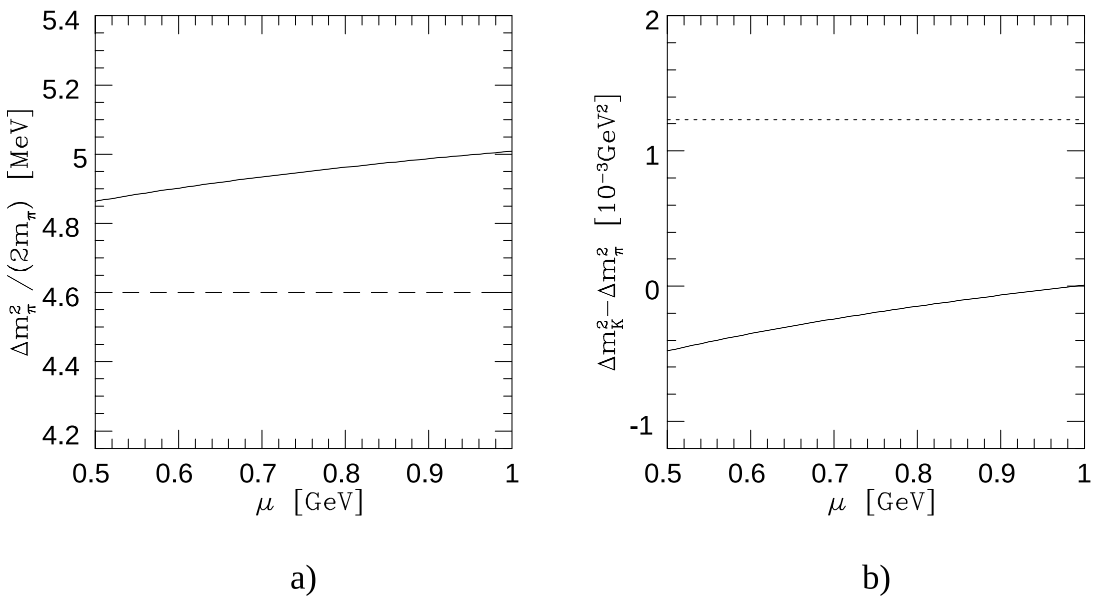

Figure 2: The solid lines show our results, the dashed and dotted curves represent in a) the experimental value [14], in b) the result of [2], respectively.

In [2] the authors have calculated the Compton scattering of the Goldstone bosons within the same model that we use in the present article and have determined the corrections to Dashen's theorem by closing the photon line. Their calculation is finite (without counterterms) and gives a considerably large value for $\Delta M_K^2 - \Delta M_\pi^2$. The difference to our result may be identified in (12), where we find an additional (singular) term that gives a large negative and scale dependent contribution. The two results are compared in Fig.2b. Note that in [2] the physical masses for the resonances are used in the calculation of ΔM_K^2, whereas we work in the $SU(3)$ limit throughout.
The other calculations are not strongly connected to our approach. The discrepancy to Bijnens' result [3] remains to be clarified, for a discussion of the value given in [4] we refer to [5].

We conclude with the observation that taking into account the resonances properly and working strictly in the $SU(3)$ limit for the resonances leads to moderate rather than large corrections to Dashen's theorem. But we have to keep in mind the uncertainties due to the missing part needed for the final and scale independent result.

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