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A CANONICAL APPROACH TO S-DUALITY IN ABELIAN GAUGE THEORY

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Abstract

We examine the electric-magnetic duality for a U(1) gauge theory on a general four manifold which generates the $SL(2,\mathbf{Z})$ group. The partition functions for such a theory transforms as a modular form of specific weight. However, in the canonical approach, we show that S-duality for the abelian theory, like T-duality, is generated by a canonical transformation leading to a modular invariant partition function.

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According to the S-duality assertion, certain four-dimensional gauge theories with a θ-term are invariant under $SL(2,\mathbf{Z})$ modular transformations on the complex coupling

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}\,, \tag{1}$$

where θ, g are the theta angle and the coupling constant, respectively [1]. This assertion has been tested for N=4 supersymmetric Yang-Mills theory by calculating the corresponding partition function [2] and the result was in favour of S-duality if the gauge group is replaced by its dual. For an abelian $U(1)$ gauge theory, S-duality extends the electric-magnetic duality [3],[4] of electromagnetism on $\mathbf{R}^4$ to the full action of $SL(2,\mathbf{Z})$ for a general four-manifold. The corresponding partition function fails to be modular invariant but rather it transforms as a modular form of specific weight [5]–[7]. This can be seen by compactifing M over a torus. The reduced two-dimensional theory looks like a linear sigma-model where the internal components of the photon are scalar fields. S-duality is then reduced to T-duality [8] and the modular anomaly appears because there is no dilaton field to compensate it [9],[10]. However, when the four manifold is of the form $\mathbf{R}\times X$ and a time direction can be chosen, by employing the canonical approach we will see that S-duality, like T-duality [11],[12], is just a canonical transformation leading to a modular invariant partition function.

Let us consider a $U(1)$ gauge field A_μ, i.e., a connection on a line bundle L over a four dimensional manifold M with corresponding field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This theory is described by the action

$$S[A] = \frac{1}{e^2}\int_M F\wedge *F - i\frac{\theta}{8\pi^2}\int_M F\wedge F\,, \tag{2}$$

where $F = \frac{1}{2}F_{\mu\nu}dx^\mu\wedge dx^\nu$ and $*F = \frac{1}{4}F^{\mu\nu}\epsilon_{\mu\nu\rho\kappa}dx^\rho\wedge dx^\kappa$. We assume that M is Euclidean so that, the saddle points of (2) turns out to be the Hodge-de Rham equations

$$dF = 0 \quad , \quad d*F = 0\,, \tag{3}$$

for harmonic two-forms. In general, the number of harmonic n-forms on M is b_n where b_n is the n^{th} Betti number, i.e., the dimension of the n^{th} de Rham cohomology group $H_2(M,\mathbf{R})$ while the Euler number of M is given by the algebraic sum $\chi = \sum_n(-1)^n b_n$.

The partition function of the $U(1)$ gauge theory defined on the four-manifold M is given by

$$Z = C\frac{1}{|\mathcal{G}|}\int_M DAe^{-S[A]}\,, \tag{4}$$

where C is a regularization constant, the integration is over all the $U(1)$ gauge fields on M and we divide by the volume $|\mathcal{G}|$ of the gauge group as usual. Moreover, a sum over isomorphic classes of the line bundle L over M is understood. By employing the Faddeev-Popov procedure to factor out this volume, the partition function may easily be evaluated to be

$$Z = C \frac{det' e^{-1} \Delta_{FP}}{(det' e^{-2} \Delta_A)^{1/2}} Z_{cl} \,, \tag{5}$$

where

$$Z_{cl} = \sum_{saddle\ points} e^{-S[A_{cl}]} \,, \tag{6}$$

is the sum of all saddle-point configurations of the action (2). $det' e^{-1} \Delta_{FP}$, $det' e^{-2} \Delta_A$ denote the Faddeev-Popov determinant and the determinant of the kinetic term for the gauge field (without counting the zero modes). The former is the determinant of the scalar Laplacian Δ_0 while the latter of the Laplacian Δ_1 for one-forms. One may easily verify that

$$det' e^{-2} \Delta_A = (\frac{1}{e^2})^{\zeta_{\Delta_A}(0)} det' \Delta_A \,, \tag{7}$$

where $\zeta_\Delta(s)$ is the generalised zeta function for the operator Δ. Since $\zeta_\Delta(0)$ is just the dimension of Δ (infinite if not properly regularized) without counting the zero modes, we may write

$$\zeta_{\Delta_A}(0) = dim \Delta_1 - b_1 \,, \tag{8}$$

Similarly, we have

$$det' e^{-1} \Delta_{FP} = (\frac{1}{e^2})^{\frac{1}{2}\zeta_{\Delta_{FP}}(0)} det' \Delta_{FP} \,, \tag{9}$$

with

$$\zeta_{\Delta_{FP}}(0) = dim \Delta_0 - b_0 \,. \tag{10}$$

Thus, by choosing the constant C in eq.(4) to be

$$C = (Im\tau)^{\frac{1}{2}(dim\Delta_1 - \dim \Delta_0)} (4\pi)^{\frac{1}{2}(dim\Delta_0 - dim\Delta_1 + b_1 - b_0)} \,, \tag{11}$$

the partition function is properly regularized and may be written as

$$Z = (Im\tau)^{\frac{1}{2}(b_1 - b_0)} \frac{det' \Delta_{FP}}{(det' \Delta_A)^{1/2}} Z_{cl} \,, \tag{12}$$

where numerical factors have been omitted. Note that we may express the action (2) in terms of self-dual ($F^+ = F + *F$), anti-self-dual ($F^- = F - *F$) fields and the complex coupling τ defined in eq.(1) as

$$S = \frac{i}{16\pi} \int_M (\bar{\tau} F^- \wedge F^- - \tau F^+ \wedge F^+). \tag{13}$$

Let us suppose that in M there exist non-trivial two-cycles, that is closed surfaces which do not bound any 3-dimensional sub-manifold of M. One may then consider the flux of the Maxwell field strength $F_{\mu\nu}$, through these cycles which satisfies the Dirac quantization condition

$$\int_{S_I} F = 2\pi n^I \,, \tag{14}$$

where $n^I \in \mathbf{Z}$ and $I = 1, ..., b_2$. We may define a basis α_I of harmonic two-forms ($d\alpha_I = d * \alpha_I = 0$) normalised as $\int_{S_I} \alpha_J = \delta_{IJ}$ which generates the harmonic representatives in $H^2(M, \mathbf{R})$ and allows us to express the field strength F as

$$F = 2\pi \sum_I n^I \alpha_I \,. \tag{15}$$

We may also define a basis in the space of self-dual/anti-self-dual harmonic two-forms by

$$\alpha_I^{\pm} = \alpha_I \pm *\alpha_I \,. \tag{16}$$

By employing eqs.(15,16), we may express the action (13) as

$$S[A_{cl}] = i\frac{\pi}{4}\bar{\tau} n^I H^+_{IJ} n^J - i\frac{\pi}{4}\tau n^I H^-_{IJ} n^J \,, \tag{17}$$

where

$$H^{\pm}_{IJ} = \int_{M^4} \alpha^{\pm} \wedge \alpha^{\pm} \,, \tag{18}$$

is the intersection form for harmonic self-dual and anti-self-dual two-forms. The partition function in eq.(12) may then be written as

$$Z = (Im\tau)^{\frac{1}{2}(b_1 - b_0)} \frac{det' \Delta_{FP}}{(det' \Delta_A)^{1/2}} \sum_{n^I} e^{i\frac{\pi}{4}\tau n^I H^-_{IJ} n^J - i\frac{\pi}{4}\bar{\tau} n^I H^+_{IJ} n^J} \,. \tag{19}$$

To examine S-duality transformations which are generated by $T : \tau \to \tau + 1$ and $S : \tau \to -1/\tau$, let us recall one of the basic invariants of a 4-manifold M, the intersection form ω. If M has a smooth structure, it is defined by using the de Rham cohomology $H^*(M)$ as

$$\omega(\alpha, \beta) = \int_M \alpha \wedge \beta \,, \tag{20}$$

for α, $\beta \in H^2(M, \mathbf{R})$ [13]. The dimensionality of ω is b_2 and the number of its positive (negative) eigenvalues is $b_2^+ (b_2^-)$. The signature σ of ω is defined as the number of positive eigenvalues minus the negative ones, i.e., $\sigma = b_2^+ - b_2^-$ and it is a topological invariant. For a simply-connected spin manifold, $\omega(\alpha, \alpha)$ is an even integer, otherwise is odd. Thus, for a spin manifold by taking $\alpha = F/2\pi$ one may verify that the action (2) is invariant under $\tau \to \tau + 1$ while for a non-spin manifold (2) is invariant under $\tau \to \tau + 2$ [2].

To examine the transformation properties of the partition function under $\tau \to -1/\tau$, let us consider the dual theory which can be found from the first-order action

$$\tilde{S} = \frac{1}{g^2} \int G \wedge *G - i\frac{\theta}{8\pi^2} \int G \wedge G + \frac{i}{2\pi} dG \wedge B \,, \tag{21}$$

where $G = \frac{1}{2} G_{\mu\nu} dx^\mu \wedge dx^\nu$ and $B = B_\mu dx^\mu$ are a 2-form and a 1-form, respectively. It can be constructed by requiring invariance under gauge transformations of the "third kind":$A \to A + B$ [14]. The partition function for this theory is given by

$$Z = C \int_M \frac{DB}{|\tilde{\mathcal{G}}|} DG e^{-\tilde{S}}, \tag{22}$$

where we have divide by the volume $|\tilde{\mathcal{G}}|$ of the gauge group which transforms B according to $B \to B + d\Lambda$. By integrating out the Lagrange multiplier field B on a topologically trivial manifold M, we get $dG = 0$ which implies that $G = d\tilde{A}$ and the original $U(1)$ gauge theory described by (2) is recovered. However, in a non-trivial manifold M, in order G to be the curvature (field strength) of a $U(1)$ connection (gauge field), the Dirac quantization condition must be taken into account, i.e., eq.(22) must be implemented by the sum

$$\sum_{n^I \in \mathcal{Z}} \delta \left(n^I - \int_{S_I} \frac{G}{2\pi} \right) = \sum_{m^I \in \mathcal{Z}} e^{-im^I \int_{S_I} G} \,, \tag{23}$$

where the sum in the righthand side in the above equation is over the dual lattice. We may also integrate the G-field and for this it is more convinient to express (21) as

$$\tilde{S} = \frac{i}{16\pi} \int_M \left(\bar{\tau} G^- \wedge G^- - \tau G^+ \wedge G^+ \right) - \frac{i}{8\pi} \int_M \left(G^+ \wedge W^+ + G^- \wedge W^- \right) \,, \tag{24}$$

where $W = dB$ and $G^\pm = G \pm *G$, $W^\pm = W \pm *W$ are the self-dual and anti-self-dual parts of G and W. Since self-dual and anti-self-dual forms are orthogonal, we may integrate them separately. The result is, ignoring numerical factors,

$$Z(\tau) = C \bar{\tau}^{-\frac{1}{2}B_2^-} \tau^{\frac{1}{2}B_2^+} \int \frac{DB}{|\tilde{\mathcal{G}}|} e^{-\tilde{S}} \,, \tag{25}$$

where

$$\tilde{S} = \frac{i}{16\pi} \int_M \left((-\frac{1}{\bar{\tau}}) W^- \wedge W^- - (-\frac{1}{\tau}) W^+ \wedge W^+ \right) \,, \tag{26}$$

and $B_2^\pm$ is the number of self-dual, anti-self-dual 2-forms on M in a lattice regulatization. Note that (26) does describe a $U(1)$ gauge theory since by integrating out G we also get

$$\int_{S_I} \frac{W}{2\pi} = m^I \in \mathcal{Z} \,,$$

for the fluxes of the dual theory. Thus, eq.(25) is indeed the partition function for a $U(1)$ gauge theory. As follows now from eqs. (4,11),

$$Z(-\frac{1}{\tau}) = C(\tau\bar{\tau})^{-\frac{1}{2}(B_1-B_0)}\int\frac{DA}{|\mathcal{G}|}e^{-\frac{i}{16\pi}\int_M\left((-\frac{1}{\bar{\tau}})F^-\wedge F^- -(-\frac{1}{\tau})F^+\wedge F^+\right)}\,, \tag{27}$$

where $B_1 = dim\Delta_1, B_0 = dim\Delta_0$. By comparing eqs.(25,27) we get

$$\begin{aligned} Z(-\frac{1}{\tau}) &= \tau^{\frac{1}{2}(B_0-B_1+B_2^+)}\bar{\tau}^{\frac{1}{2}(B_0-B_1+B_2^-)}Z(\tau) & (28)\\ &= \tau^{\frac{1}{4}(\chi-\sigma)}\bar{\tau}^{\frac{1}{4}(\chi+\sigma)}Z(\tau)\,, & (29)\end{aligned}$$

and therefore $Z(\tau)$ transforms as a modular form of weight $(\frac{1}{4}(\chi-\sigma), \frac{1}{4}(\chi+\sigma))$.

Let us now examine S-duality in the Hamiltonian approach. We will assume that the four-manifold M is Lorentzian and by separating space and time, M turns out to be of the form $\mathbf{R}\times X$ where X is a three-manifold. We will also assume that M is endowed with the product metric

$$ds^2 = -dt^2 + g_{ij}dx^i dx^j\,, \tag{30}$$

where g_{ij} is the metric on X. In this case the action (2) turns out to be

$$S = \int dt d^3x\sqrt{g}\left(-\frac{1}{2e^2}F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{32\pi^2}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}\right)\,, \tag{31}$$

which is of course real. The canonical momenta are easily found to be

$$\pi^i = \frac{2}{e^2}F_{0j}g^{ij} + \frac{\theta}{8\pi^2}\epsilon^{ijk}F_{ij}\,, \tag{32}$$

and by performing a Legendre transform, the Hamiltonian is given by

$$H = \frac{e^2}{4}\pi_i\pi^i + \pi^i\partial_i A_0 - \frac{e^2\theta}{16\pi^2}\pi_i\epsilon^{ijk}F_{jk} + (\frac{1}{2e^2} + \frac{e^2\theta^2}{128\pi^4})F_{ij}F^{ij}\,, \tag{33}$$

while the symplectic structure is provided by the equal-time Poisson bracket

$$\{A_i(x), \pi^j(y)\} = \delta_i^j\delta^{(3)}(x-y)\,. \tag{34}$$

The partition function is given by

$$Z = N\int DAD\pi\frac{1}{|\mathcal{G}|}e^{i\int dt d^3x\sqrt{g}(\pi^i\dot{A}_i - H)}\,, \tag{35}$$

where N is a regularization constant and we have divide as usual by the volume of the $U(1)$ gauge group. It contains a sum over the isomorphic classes of the line bundle L over X

and in each class we have to integrate over the momenta π^i and A_μ. Every π^i integration produces a factor $(1/e^2)^{1/2} \sim Im\tau^{1/2}$ while every A_i one produces a factor $(\frac{1}{2e^2} + \frac{e^2\theta^2}{128\pi^4})^{-1/2} \sim (Im\tau/\tau\bar{\tau})^{1/2}$ (A_0 integrations do not produce any factors but simply a delta function). By using a lattice regularization, the number of A_i integrations is $B_1 - B_0$ where B_1, B_0 is the number of 1-forms and 0-forms on X, respectively. Since the number of π^i integrations equals the number of A_i ones, the cut-off dependent term coming from the integration is

$$Im\tau^{\frac{1}{2}(B_1-B_0)} Im\tau^{\frac{1}{2}(B_1-B_0)} (\tau\bar{\tau})^{-\frac{1}{2}(B_1-B_0)} . \tag{36}$$

Thus, a properly regularized, cut-off independent partition function is given by

$$Z(\tau) = Im\tau^{B_0-B_1} (\tau\bar{\tau})^{\frac{1}{2}(B_1-B_0)} \int DAD\pi \frac{1}{|\mathcal{G}|} e^{i\int dtd^3x\sqrt{g}(\pi^i \dot{A}_i - H)} . \tag{37}$$

As usual, A_0 has no kinetic term and it is just a Lagrange multiplier leading to the constraint

$$\nabla_i \pi^i = 0 , \tag{38}$$

where ∇ is the covariant derivative on M. Let us now perform a canonical transformation generated by

$$G = \frac{1}{4\pi} \int_X d^3x \sqrt{g} (\tilde{A}_i \epsilon^{ijk} F_{jk} + A_i \epsilon^{ijk} \tilde{F}_{jk}) , \tag{39}$$

so that

$$\pi^i = \frac{\delta G}{\delta A_i} = \frac{1}{4\pi} \epsilon^{ijk} \tilde{F}_{jk}, \tag{40}$$

$$\tilde{\pi}^i = -\frac{\delta G}{\delta \tilde{A}_i} = -\frac{1}{4\pi} \epsilon^{ijk} F_{jk} . \tag{41}$$

It follows from (41) that the dual momenta $\tilde{\pi}^i$ satisfy the constraint

$$\nabla_i \tilde{\pi}^i = 0 , \tag{42}$$

which can be incorporated in the dual Hamiltonian by means of a Langrange multiplier $\tilde{A}_0$. Therefore, the dual Hamiltonian is

$$\begin{aligned} \tilde{H} &= 16\pi^2 (\frac{1}{2e^2} + \frac{e^2\theta^2}{128\pi^2}) \tilde{\pi}_i \tilde{\pi}^i + \frac{e^2\theta}{16\pi^2} \tilde{\pi}^i \epsilon^{ijk} \tilde{F}_{jk} \\ &\quad + \frac{e^2}{32\pi^2} \tilde{F}_{ij} \tilde{F}^{ij} + \frac{1}{4\pi} \epsilon^{ijk} \tilde{F}_{jk} \partial_i A_0 + \tilde{A}_0 \nabla_i \tilde{\pi}^i . \end{aligned} \tag{43}$$

The integration of A_0 gives the constraint

$$\epsilon^{ijk} \nabla_i \tilde{F}_{ijk} = 0 , \tag{44}$$

which is just the Bianchi identity and allows us to express $\tilde{F}_{ij}$ locally as

$$\tilde{F}_{ij} = \partial_i \tilde{A}_j - \partial_j \tilde{A}_i \,. \tag{45}$$

However, such expression will fail in general to hold globally in view of possible non-zero magnetic fluxes

$$\int_{S_I} \epsilon^{ijk} \tilde{F}_{ij} dS_k = 4\pi n^I \,, \tag{46}$$

through non-trivial two-cycles of X according to the Dirac condition. By performing an inverse Legendre transform in (43), the dual Lagrangian is found to be

$$\tilde{L} = -\frac{1}{2\tilde{e}^2} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\tilde{\theta}}{32\pi^2} \epsilon^{\mu\nu\rho\lambda} \tilde{F}_{\mu\nu} \tilde{F}_{\rho\lambda} \,, \tag{47}$$

where

$$\begin{aligned} \tilde{e}^2 &= e^2 \left(\frac{\theta}{4\pi^2} + \frac{16\pi^2}{e^4}\right) \,, \\ \tilde{\theta} &= -\theta \left(\frac{\theta}{4\pi^2} + \frac{16\pi^2}{e^4}\right)^{-1} \,. \end{aligned} \tag{48}$$

Thus, the classical dual theory is related to the original one by the transformation $\tau \to -1/\tau$.

Since the partition function is invariant under canonical transformations we get

$$Z(\tau) = Im\tau^{B_0 - B_1} (\tau\bar{\tau})^{\frac{1}{2}(B_1 - B_0)} \int D\tilde{A} D\tilde{\pi} \frac{1}{VolG} e^{i \int dt d^3x \sqrt{g} (\tilde{\pi}^i \tilde{A}_i - \tilde{H})} \,. \tag{49}$$

By employing the transformation $\tau \to -1/\tau$ in eq. (37) we get the righthand side of (49) so that

$$Z(-1/\tau) = Z(\tau) \,, \tag{50}$$

and therefore the partition function for the U(1)-gauge theory in the canonical approach is modular invariant.

Let us also note that there are no quantum corrections to the generating functional since it is linear to A_i and $\tilde{A}_i$ [15]. Moreover, physical states $\phi_k[A_i]$ and $\psi_k[\tilde{A}_i]$ in the original and in the dual theory are related through

$$\psi_k[\tilde{A}_i] = \int DA e^{iG(A,\tilde{A})} \phi[A_i] \,. \tag{51}$$

These states are invariant under gauge transformations, i.e. $\phi_k[A_i + \partial_i \epsilon] = \phi_k[A_i]$ so that

$$G(A - \partial\epsilon, \tilde{A}) - G(A, \tilde{A}) = 2\pi n \,, \; n \in \mathbf{Z} \,. \tag{52}$$

This condition is satisfied as long as the Dirac quantization condition (46) is satisfied.

As we have seen above, the partition function in the canonical approach is modular invariant. On the other hand, in T-duality the modular anomaly is compensated by a change of the dilaton field leading to a modular invariant theory. Similarly here, it seems that the anomaly is compensated by a change of the generalised momenta π^i. In this respect, π^i integrations imitates metric integrations in the string case. Finally, one should expect that the above discussion would also be carried out for the non-abelian case as well. However, in this case, one cannot non-trivially satisfy the Gauss' law constraint $D_i\pi^{\alpha i} = 0$ and the dual theory is identical to the original one. The non-abelian case will be discussed elsewhere.

While the present work was being proof-reading, we became aware of Ref[16] where S-duality is also confronted as a canonical transformation.

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