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QUANTUM MECHANICAL BREAKING OF LOCAL GL(4) INVARIANCE

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Abstract

We consider the gravitational coupling of a scalar field, in a reformulation of General Relativity exhibiting local $GL(4)$ invariance at the classical level. We compute the one–loop contribution of the scalar to the quantum effective potential of the vierbein and find that it does not have $GL(4)$ invariance. The minima of the effective potential occur for a vierbein which is proportional to the unit matrix.

1. Introduction

In recent papers [1-3] we have discussed the quantum mechanical breaking of scale invariance in quantum gravity. One begins from a classical action which depends on the metric $g_{\mu\nu}$ only through the combination

$$\tilde{g}_{\mu\nu} = \rho^2 g_{\mu\nu} \ , \tag{1.1}$$

where ρ is a scalar field in the gravitational sector of the theory, called the dilaton. The classical action is then invariant under the local scale transformations

$$g_{\mu\nu} \to g'_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad , \qquad\qquad \rho \to \rho' = \Omega^{-1}\rho \ . \tag{1.2}$$

If in the functional integral the measure is defined with the metric g, rather than $\tilde{g}$, the functional integral, and hence in particular the effective potential, will be a function of g and ρ separately and therefore invariance under (1.2) will be broken.

Using traditional field theoretic methods we have computed the renormalized effective potential for ρ and found it to be of the Coleman–Weinberg type [1, 2]. Using the so-called average effective action we have also computed the renormalization group flow of various quantities of interest, in particular of the v.e.v. of the dilaton [2, 3]. This is relevant to the quantization of gravity, since the v.e.v. of the dilaton is proportional to Newton's constant.

Here we report on a generalization of these results, where local scale invariance is enlarged to local $GL(4)$ invariance. Instead of the dilaton field ρ one has a matrix–valued field $\theta^\mu{}_\nu$, and the classical action depends only on the combination

$$\tilde{g}_{\mu\nu} = \ \theta^\rho{}_\mu\, \theta^\sigma{}_\nu\, g_{\rho\sigma} \ . \tag{1.3}$$

The theory is invariant under the local $GL(4)$ transformations

$$g_{\mu\nu}(x) \mapsto g'{}_{\mu\nu}(x) = \Lambda^\rho{}_\mu(x)\, \Lambda^\sigma{}_\nu(x)\, g_{\rho\sigma}(x) \ , \tag{1.4a}$$

$$\theta^\rho{}_\mu(x) \mapsto \theta'^\rho{}_\mu(x) = \Lambda^{-1\rho}{}_\sigma(x)\, \theta^\sigma{}_\nu(x) \ , \tag{1.4b}$$

which leave $\tilde{g}_{\mu\nu}$ invariant. The dilatonic theory is recovered if we assume that $\theta^\mu{}_\nu = \rho\, \delta^\mu_\nu$. The local $GL(4)$ transformations specialize to the scale transformations (1.2) when $\Lambda^\rho{}_\mu = \Omega\, \delta^\rho_\nu$.

The reformulation of General Relativity or any other theory of gravity in this $GL(4)$–invariant way has been discussed in [4] and independently in [5], where the connection with Weyl's geometry was emphasized.

In addition to local $GL(4)$ invariance one also considers diffeomorphisms. There are two ways of realizing the diffeomorphism group on the fields [1]. The first is given by

$$g_{\mu\nu}(x) \mapsto g'{}_{\mu\nu}(x') = \frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu} g_{\rho\sigma}(x) \ , \tag{1.5a}$$

$$\theta^\rho{}_\mu(x) \mapsto \theta'^\rho{}_\mu(x') = \frac{\partial x'^\rho}{\partial x^\sigma}\theta^\sigma{}_\nu(x)\frac{\partial x^\nu}{\partial x'^\mu} \ . \tag{1.5b}$$

In this realization all fields are transformed as tensors on all indices.

The second realization consists of the first, followed by a local $GL(4)$ transformation with parameter $\Lambda^\mu{}_\nu = \frac{\partial x'^\mu}{\partial x^\nu}$:

$$g_{\mu\nu}(x) \mapsto g'_{\mu\nu}(x') = g_{\mu\nu}(x) \ , \tag{1.6a}$$

$$\theta^\rho{}_\mu(x) \mapsto \theta'^\rho{}_\mu(x') = \theta^\rho{}_\nu(x) \frac{\partial x^\nu}{\partial x'^\mu} \ . \tag{1.6b}$$

In both cases the metric field $\tilde{g}_{\mu\nu}$ transforms as usual.

There are two possible geometrical interpretations of this theory. In [4] we regarded the first index of θ and the indices on g as internal indices. In this interpretation, (1.6) describes the effect of a diffeomorphism on the fields. In the present paper we will adopt another interpretation, namely we treat all indices as coordinate indices in the tangent bundle. In this interpretation the action of a diffeomorphism is described by (1.5).

Just as in the case of scale invariance, local $GL(4)$ invariance will be broken in the quantum theory if the functional measure is constructed with the metric g rather than $\tilde{g}$ [1]. In particular, the effective potential will be a function of g and θ separately, rather than just of $\tilde{g}$. It can be regarded as an effective potential for the field θ.

The question is now: is the resulting effective potential a sensible one? If so, where are its minima? We know that in the case $\theta^\mu{}_\nu = \rho\,\delta^\mu_\nu$ the potential for ρ is bounded from below and has a minimum for some nonzero value of ρ. This is helpful information, but it does not guarantee that the potential for θ will have the same minimum. In this paper we will compute the contribution to the effective potential for θ coming from the quantum fluctuations of a scalar field. We show that the minimum occurs when θ is a multiple of the identity, and therefore coincides with the minimum that was found in the dilatonic theory.

The calculation of this potential in a full theory of gravity, taking into account the quantum fluctuations of fermions, photons and gravitons, is technically a much more complicated problem, but we expect that the final results will not be qualitatively different from the ones that we find here.

Throughout this paper we will work in the Euclidean theory. This is just to simplify the notation: there is no obstacle in doing the same calculations in the Minkowskian theory.

2. The contribution of a scalar field

In this paper we consider a scalar field ϕ coupled to gravity. Our basic assumption is that in the classical action (which in a quantum context describes the physics at the cutoff scale) the scalar is minimally coupled to the metric $\tilde{g}$ given in (1.1):

$$\begin{aligned} S(\phi, g, \theta) =& \frac{1}{2}\int d^4x \sqrt{\det \tilde{g}}\, \left[\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + c\,\phi^2\right] \\ =& \frac{1}{2}\int d^4x \sqrt{\det g}\, \det\theta\, \left[g^{\mu\nu}\theta_\mu^{-1\rho}\theta_\nu^{-1\sigma}\partial_\rho\phi\partial_\sigma\phi + c\,\phi^2\right] \ . \end{aligned} \tag{2.1}$$

Note that the scalar is coupled in a complicated nonminimal way to the metric $g_{\mu\nu}$ and the field $\theta^\mu{}_\nu$. We assume that $g_{\mu\nu}$ is dimensionless, $\theta^\mu{}_\nu$ has dimension of mass, ϕ is dimensionless and c is dimensionless. The physical picture underlying these choices has been discussed extensively, in the context of the dilatonic theory, in [2, 3].

The action (2.1) is invariant under the local $GL(4)$ transformations (1.4) and under the diffeomorphisms (1.5). Let us assume that the background metric $g_{\mu\nu}$ is flat. We can then choose the gauge so that $g_{\mu\nu} = \delta_{\mu\nu}$. This breaks local $GL(4)$ to local $O(4)$ invariance and diffeomorphisms to global $O(4)$ transformations. From (1.3) we see that in this case the field $\theta^\mu{}_\nu$ can be interpreted formally as a vierbein. We are interested in the effective potential for θ, so we shall assume that the matrix θ is constant. Under these circumstances the residue of the initial local $GL(4)$ invariance consists of global $O(4)$ transformations acting only on the first index of θ, while the residue of the diffeomorphism invariance consists of global $O(4)$ transformations acting on the matrix $\theta^\mu{}_\nu$ by similarity transformations. These two invariances are equivalent to left and right $O(4)$ invariance, the right $O(4)$ being the residue of the transformations (1.6).

Since the metric is flat we go to momentum space. The action (2.1) then becomes:

$$S(\phi,\theta) = \frac{1}{2}\det\theta \int \frac{d^4q}{(2\pi)^4}\,\phi(q)\,\mathcal{O}(q)\,\phi(-q)\ . \tag{2.2}$$

The operator $\mathcal{O}$ is given by

$$\mathcal{O}(q) = \mathrm{tr}\left(q^T\theta^{-1T}\theta^{-1}q\right) + c\ , \tag{2.3}$$

where q is thought of as a column vector.

There is an issue of how to treat the factor $\det\theta$ appearing in front of the integral, which is related to the choice of functional measure in the path integral. Let us assume first that the measure is given (formally) by $d\mu(\phi) = \prod_k d\phi(k)$. The one–loop effective action for θ induced by quantum fluctuations of the scalar field is the determinant of the quadratic operator appearing in (2.2):

$$\Gamma_{\rm eff}(\theta) = \int d^4x V_{\rm eff}(\theta) = \frac{1}{2}\ln\mathrm{Det}(\det\theta\cdot\mathcal{O}) = \frac{1}{2}\mathrm{Tr}\ln(\det\theta\cdot\mathbf{1}) + \frac{1}{2}\mathrm{Tr}\ln\mathcal{O}\ , \tag{2.4}$$

where $V_{\rm eff}$ is the effective potential, Det and Tr denote functional determinant and trace.

We have separated the contribution of the operator $\det\theta\cdot\mathbf{1}$, which is proportional to the unit matrix in the function space. This term is equal to

$$\frac{\mathcal{V}}{2}\ln(\det\theta)\,\delta(0)\ , \tag{2.5}$$

where $\mathcal{V} = \int d^4x$ is the spacetime volume. If we evaluate the functional trace in Fourier space and introduce an UV cutoff Λ we have $\delta(0) = \int \frac{d^4q}{(2\pi)^4} = \frac{1}{32\pi^2}\Lambda^4$.

On the other hand suppose we define the measure as $d\mu(\phi) = \prod_k d\phi(k)\,(\det\theta)^K$. This is equivalent to saying that the quantum field is not ϕ but rather φ, where

$$\varphi = \phi\,(\det\theta)^K\ . \tag{2.6}$$

The effective action changes by the addition of a term

$$-K\,\mathcal{V}\,\ln(\det\theta)\,\delta(0)\ . \tag{2.7}$$

In this paper we will assume that $K = 1/2$, so as to eliminate the first term on the r.h.s. of (2.4). This choice gives a dimension of squared mass to the quantum field φ. Instead, the canonical dimension of mass for φ is obtained when $K = 1/4$. This is the natural choice when $\theta^\mu{}_\nu = \rho^2\delta^\mu_\nu$ [1-3], since it simply gives: $\Gamma_{\rm eff} = \frac{1}{2}\ln{\rm Det}(q^2 + c\,\rho^2)$. However, when the eigenvalues of $\theta^\mu{}_\nu$ are all different, the choice $K = 1/4$ would in general produce, after renormalization, additional terms in $\Gamma_{\rm eff}$ proportional to $\ln\det\theta$; these can be eliminated with a further finite, but non-polynomial, renormalization.

The effective potential is then given by the formula

$$V_{\rm eff}(\theta) = \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}\ln\mathcal{O}\ . \tag{2.8}$$

This integral is divergent, so we regulate it with an UV cutoff Λ. Since the integration measure is defined with the background metric $g_{\mu\nu} = \delta_{\mu\nu}$, the cutoff is defined by $\sum_\mu q_\mu^2 < \Lambda^2$. As we have emphasized in [1, 2] there is in principle also the possibility of using the metric $\tilde g$ to define the cutoff, leading to very different results. It is the physical interpretation that dictates the choice we are making here.

In order to evaluate (2.8) we proceed as follows. We observe that using the residual global left and right $O(4)$ invariances, the matrix θ can be brought to diagonal form. Let M_L and M_R be such that

$$(M_L\cdot\theta\cdot M_R)_{\mu\nu} = \theta_\mu\delta_{\mu\nu}\ . \tag{2.9}$$

(Note that θ_μ are not the eigenvalues of the matrix $\theta^\mu{}_\nu$.) We then perform a change of variables $q' = M_L q$ in the integral (2.8). The cutoff condition is clearly unchanged. Inserting $M_R M_R^T = \mathbf{1}$ in the operator $\mathcal{O}$, the matrix θ is diagonalized and the effective potential becomes

$$V_{\rm eff}(\theta) = \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}\ln\left[\sum_{\mu=1}^{4}\frac{q_\mu^2}{\theta_\mu^2} + c\right]\ . \tag{2.10}$$

The integrand is now a function whose level surfaces are three dimensional ellipsoids. We have to evaluate the integral of this function over a spherical ball of radius Λ.

Note that $V_{\rm eff}$ depends on the eigenvalues θ_μ only through their squares and therefore must be an even function of these variables.

3. Two dimensions

In order to gain some insight in a simpler setting we shall evaluate first the effective potential for θ induced by a scalar field in two dimensions. It is given by the expression

$$V_{\rm eff}(\theta) = \frac{1}{2}\int\frac{d^2q}{(2\pi)^2}\ln\left[\frac{q_1^2}{\theta_1^2} + \frac{q_2^2}{\theta_2^2} + c\right] = \frac{1}{8\pi^2}\int_0^{2\pi}d\varphi\int_0^{\Lambda}dr\,r\ln(\omega r^2 + c)\ , \tag{3.1}$$

where we have used polar coordinates, defined by $q_1 = r\cos\varphi$, $q_2 = r\sin\varphi$, and $\omega(\varphi) = \frac{\cos^2\varphi}{\theta_1^2} + \frac{\sin^2\varphi}{\theta_2^2}$. One can perform first the radial integral, leading to

$$V_{\rm eff} = \frac{1}{16\pi^2}\int_0^{2\pi} d\varphi \left[(\Lambda^2 + \frac{c}{\omega})\ln(\Lambda^2\omega + c) - \Lambda^2 - \frac{c\ln c}{\omega}\right] , \tag{3.2}$$

For $\Lambda^2\omega \gg 1$ one can expand the logarithm and discard terms of order $\frac{1}{\Lambda^2}$. Integrating over the angle we then get:

$$V_{\rm eff} = \frac{1}{4\pi}\left[\Lambda^2\left(\ln\frac{\Lambda(|\theta_1| + |\theta_2|)}{2|\theta_1\theta_2|} - \frac{1}{2}\right) + c|\theta_1\theta_2|\left(\ln\frac{2\Lambda}{\sqrt{c}\,(|\theta_1| + |\theta_2|)} + \frac{1}{2}\right)\right] . \tag{3.3}$$

Note that this is indeed an even function of θ_μ, as expected. If we introduce a mass parameter $\mu \ll \Lambda$, playing the role of renormalization point, we can rewrite (3.3) as

$$V_{\rm eff} = \frac{1}{4\pi}\left[\Lambda^2\ln\frac{\mu(|\theta_1| + |\theta_2|)}{2|\theta_1\theta_2|} + c|\theta_1\theta_2|\ln\frac{\Lambda}{\mu} + c|\theta_1\theta_2|\left(\ln\frac{2\mu}{\sqrt{c}\,(|\theta_1| + |\theta_2|)} + \frac{1}{2}\right)\right] , \tag{3.4}$$

where some θ–independent terms have been dropped. This way of writing reveals the presence of quadratic and logarithmic divergences. The unusual feature of this result is that the divergent terms depend in a nonpolynomial way on θ_1 and θ_2. This feature of quantum gravity had been noted long ago [6].

To cancel these divergences one needs therefore nonpolynomial counterterms. Let us choose a renormalization scheme in which the divergences are exactly cancelled. The effective potential is then given by the finite term in (3.4). The renormalized, finite, effective action can be written

$$S_{\rm fin}(\theta) = \int d^2x V_{\rm fin}(\theta) = -\frac{c}{8\pi}\int d^2x|\det\theta|\left\{\ln\frac{c}{4\mu^2}\left[{\rm tr}\theta^T\theta + 2\det\theta(1 - {\rm sign}\det\theta)\right] - 1\right\} . \tag{3.5}$$

Note that the effective potential is invariant under the left and right $O(2)$ transformations.

The effective potential (3.5) has an absolute maximum for $\theta_1 = \theta_2 = \mu/\sqrt{c}$. The fact that it is unbounded from below is a standard problem with two dimensional theories and will not concern us here. The purpose of this calculation was simply to acquire some experience with the evaluation of integrals of the type (2.10).

4. Four dimensions

Introducing polar coordinates

$$\begin{aligned} q_1 \equiv r\hat{q}_1 &= r\sin\chi\sin\alpha\cos\varphi \\ q_2 \equiv r\hat{q}_2 &= r\sin\chi\sin\alpha\sin\varphi \\ q_3 \equiv r\hat{q}_3 &= r\sin\chi\cos\alpha \\ q_4 \equiv r\hat{q}_4 &= r\cos\chi \end{aligned} \tag{4.1}$$

the integral (2.10) becomes

$$V_{\text{eff}}(\theta) = \frac{1}{32\pi^4} \int d\Omega \int_0^{\Lambda} dr r^3 \ln \left[r^2 \omega + c \right] \ , \tag{4.2}$$

where now

$$\omega(\chi, \alpha, \varphi) = \sum_{\mu=1}^{4} \frac{\hat{q}_\mu^2}{\theta_\mu^2} = \frac{\cos^2 \chi}{\theta_4^2} + \sin^2 \chi \left(\frac{\cos^2 \alpha}{\theta_3^2} + \sin^2 \alpha \left(\frac{\cos^2 \varphi}{\theta_1^2} + \frac{\sin^2 \varphi}{\theta_2^2} \right) \right) \tag{4.3}$$

and

$$\int d\Omega = \int_0^{2\pi} d\varphi \int_0^{\pi} d\alpha \sin \alpha \int_0^{\pi} d\chi \sin^2 \chi \ .$$

One can perform the radial integration, leading to

$$V_{\text{eff}}(\theta) = \frac{1}{128\pi^4} \int d\Omega \left[\left(\Lambda^4 - \frac{c^2}{\omega^2} \right) \ln(\Lambda^2 \omega + c) - \frac{\Lambda^4}{2} + \frac{\Lambda^2 c}{\omega} + \frac{c^2}{\omega^2} \ln c \right] \ . \tag{4.4}$$

Unlike the two-dimensional case, the angular integration is too complicated to be performed exactly. It can be done explicitly in the case when two eigenvalues are equal (for example θ_1 and θ_2), since then one angular integration becomes trivial. This calculation is not very illuminating and will not be reported here.

When the cutoff Λ becomes very large, expanding the logarithms as in the previous section we arrive at the expression

$$\begin{aligned} V_{\text{eff}}(\theta) = \frac{1}{128\pi^4} \int d\Omega \Bigg[& \Lambda^4 \left(\ln \frac{\Lambda^2}{\mu^2} - \frac{1}{2} \right) + \Lambda^4 \ln(\mu^2 \omega) \\ & + 2c \frac{\Lambda^2}{\omega} + \frac{c^2}{\omega^2} \ln \frac{\Lambda^2}{\mu^2} + \frac{c^2}{\omega^2} \left(\ln \frac{c}{\mu^2 \omega} - \frac{1}{2} \right) \Bigg] \ . \end{aligned} \tag{4.5}$$

The first term is field–independent and will not be considered further; the next three terms are quartically, quadratically or logarithmically divergent. These divergent terms are all nonpolynomial in θ_μ. Finally, the last term is finite.

We will adopt a renormalization prescription which amounts simply to discarding all the divergent terms. The renormalized effective potential is therefore

$$V_{\text{fin}}(\theta) = \frac{1}{128\pi^4} \int d\Omega \left[\frac{c^2}{\omega^2} \left(\ln \frac{c}{\mu^2 \omega} - \frac{1}{2} \right) \right] \ . \tag{4.6}$$

Even though we cannot write a closed expression for the potential, we can study it by first taking derivatives with respect to θ_μ's, setting the θ_μ's to some desired value and then evaluating the integrals. In this way we can compute the Taylor expansion of $V_{\rm eff}$ around some point.

The point we choose is $\theta_1 = \theta_2 = \theta_3 = \theta_4 = \mu/\sqrt{c}$, which is known to be the minimum of the potential when all the θ_μ's are equal. (Because of the even parity of the potential, one could also individually change the signs of the θ_μ's without affecting the following arguments).

Taking the first derivative of (4.6) with respect to θ_μ^2, we find after a little algebra

$$\frac{\partial V_{\rm fin}}{\partial \theta_\mu^2} = \frac{1}{64\pi^2}\frac{c^2}{\theta_\mu^4}\int d\Omega\, \frac{\hat{q}_\mu^2}{\omega^3}\ln\frac{c}{\omega}\ . \tag{4.7}$$

Evaluating at the specified point gives zero. Therefore the chosen point is a stationary point of the effective potential. To see what kind of stationary point, we evaluate the second derivatives at the point

$$\left.\frac{\partial^2 V_{\rm fin}}{\partial\theta_\mu^2\partial\theta_\nu^2}\right|_{\theta_\nu=\mu/\sqrt{c}} = \frac{c^2}{64\pi^4}\int d\Omega\, \hat{q}_\mu^2\hat{q}_\nu^2 = \frac{c^2}{768\pi^2}\left(2\delta_{\mu\nu}+1\right)\ . \tag{4.8}$$

The matrix in parentheses has eigenvalues 6, 2, 2 and 2 and therefore the stationary point is a local minimum. One can check that it is the absolute minimum of the effective potential, which therefore coincides with the minimum found in the dilatonic theory [1, 2], where the field θ was taken to be proportional to the unit matrix from the beginning.

The effective potential can be approximated in the neighbourhood of the minimum by

$$V_{\rm fin} = \frac{1}{64\pi^2}\left[-2\mu^4 - \frac{c\mu^2}{2}{\rm tr}\theta^T\theta + \frac{c^2}{24}\left(({\rm tr}\theta^T\theta)^2 + 2{\rm tr}(\theta^T\theta)^2\right)\right]\ , \tag{4.9}$$

where we have used a matrix notation for $\theta^\mu{}_\nu$. This is exactly the type of potential that was studied in [7, 8]. There, this kind of potential was constructed to give a nonzero v.e.v. of the metric $\tilde{g}$. We have presented here a possible quantum mechanical origin for such a potential.

Notice that the contribution of massless scalar fields to $V_{\rm fin}$ is a θ-independent constant. One can check that this holds also in the case of higher spin bosons, like the photon or the graviton. Finally, the contribution of spinor fields to $V_{\rm fin}$, is also expected to be of the form (4.9). Therefore, even in presence of gauge fields and matter, the minimum of $V_{\rm fin}$ will occur for θ multiple of the identity.

The next step is the study of the renormalization group flow of this minimum, as the characteristic IR scale of the theory changes; this is of great relevance both in the quantization of gravity and in cosmological problems, as we pointed out in [3, 9]. We plan to discuss this point in a separate publication.

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