Relation between space-time inversion and particle-antiparticle symmetry and the microscopic essence of special relativity

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Abstract

After analyzing the implication of investigations on the C, P and T transformations since 1956, we propose that there is a basic symmetry in particle physics. The combined space-time inversion is equivalent to particle-antiparticle transformation, denoted by $\mathcal{PT} = \mathcal{C}$. It is shown that the relativistic quantum mechanics and quantum field theory do contain this invariance explicitly or implicitly. In particular, (a) the appearance of negative energy or negative probability density in single particle theory – corresponding to the fact of existence of antiparticle, (b) spin- statistics connection, (c) CPT theorem, (d) the Feynman propagator are linked together via this symmetry. Furthermore, we try to derive the main results of special relativity, especially, (e) the mass-energy relation, (f) the Lorentz transformation by this one "relativistic" postulate and some "nonrelativistic" knowledge.

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I. INTRODUCTION

Since the historic discovery of parity violation in week interactions by Lee and Yang [1] and the experimental verification by Wu et al. [2], the investigation on C,P and T problems has been attracting serious attention in physics community. In 1964, Christensen et al. discovered the CP violation in the decay process of neutral K meson [3]. The further analysis showed that the T inversion is certainly violated whereas the CPT theorem still remains valid [4].

The purpose of this paper is trying to examine this problem from an alternative point of view [5]. In section II we suggest that it is the time to propose a new principle (postulate) as the replacement of CPT theorem. Then in sec.III the Dirac equation is analyzed in detail from the new point of view, the Klein-Gordon equation and Maxwell equation are also discussed. The sec. IV is devoting to discussing the connection between spin and statistics. In sec. V we make an observation on the Feynman propagator and the arrow of time in physics which are intimately related to this basic symmetry under consideration. Then in sec. VI we will be able to derive the main results of special relativity by means of this symmetry with some other "nonrelativistic" knowledge. The final section VII contains brief summary and discussions. Some other details are given in three appendices.

II. WHERE IS THE PROBLEM?

The discovery of parity violation led directly to the establishment of two component neutrino theory [6]

$$|\bar{\nu}>= \mathtt{CP}|\nu> \tag{2.1}$$

The success of this theory implies the ineffectiveness of the original definition of space reflection P and C transformation respectively. After the discovery of CP violation, Lee and Wu proposed a unified definition for particle-antiparticle transformation [7]:

$$|\bar{a}>= \mathtt{CPT}|a> \tag{2.2}$$

This is really a very important process of evolution in concept. The physicists have been correcting a long existing mistake in physics, the latter conceives that the matter is only located in the space-time and can be detached from it.

Here two remarks are important to us. (a). The difference between a theorem and a law should be treated carefully. Every quantity in a theorem must be defined unambiguously and separately before the theorem can be proved. Actually, the conclusion of a deduction method is already contained implicitly in the premise. On the other hand, since a law is verified by experiments, it is often (not always) capable of accommodating a definition of physical quantity, which is not defined independently before the law is established. (b). As we learn from the development of physics in this century, the definition of any observable in physics must depends on some invariance or dynamical law. Once it fails to do so, it will cease to be an observable. For example, the energy E (momentum $\vec{p}$) can be defined because of the existence of law of energy (momentum) conservation. The definition of inertial mass m in Newtonian mechanics is contained in the law $\vec{F}= m\ \vec{a}$. However, in the theory of special relativity, m should be defined as $m = d\ \vec{p}^{\,2}\ /2dE$ for taking the changeableness of mass into account (see Eq. (6.14) below). Therefore, some time the change of definition is necessary and important. To some extent, this is also true for the definition of a transformation, e.g. ,the particle- antiparticle transformation.

The ineffectiveness of individual definitions of P, T and C together with the validity of CPT theorem [8, 9] enlightened us that one should introduce new definition of space-time inversion and replace the CPT theorem by a fundamental principle (postulate) which can be stated as follows:

Under the combined space-time inversion, all particles with mutual interactions turn to their antiparticles respectively.

Here the meaning of inversion needs to be clarified. First consider the single particle quantum mechanics. The space reflection operator is denoted by $\mathcal{P}$. There are two equivalent statements:

(A) If there is a physical state described by wave function $\psi(\vec{x},t)$ in the coordinate system $\{x\}$, then after substitution $\vec{x}= -\ \vec{x}'$, the wave function changes to that in reversed system $\{x'\}$ as follows:

$$\psi(\vec{x},t) \longrightarrow \psi(-\ \vec{x}',t) = \psi'(\vec{x}',t'),\ (t=t') \tag{2.3}$$

The substitution $\vec{x}= -\ \vec{x}'$ also has to be made in the equations.

(B) Instead of introducing reversed system $\{x'\}$, one may introduce the space reflected state in the same $\{x\}$ system according to the following rule:

$$\psi(\vec{x},t) \longrightarrow \psi(-\vec{x},t) = \psi'(\vec{x},t), \tag{2.4}$$

The corresponding change $\vec{x} \rightarrow -\vec{x}$ also has to be made in the equations.

We shall adopt (A) or (B) statement freely in the later discussion.

Similarly, if adopting (B) statement, the time reversal operator $\mathcal{T}$ means that

$$t \longrightarrow -t,\ \psi(\vec{x},t) \longrightarrow \psi(\vec{x},-t)\ . \tag{2.5}$$

Notice that, however, we do not demand that the physical law is invariant under the $\mathcal{P}$ or $\mathcal{T}$ inversion individually. In other words, whether the space or time reflected state in the right hand side of (2.4) or (2.5) exists or not, a concrete analysis for different situation is needed.

But the fundamental postulate just mentioned before claims that: During

$$\vec{x} \longrightarrow -\vec{x},\ t \longrightarrow -t\ ,$$

$$\psi(\vec{x},t) \longrightarrow \psi(-\vec{x},-t) = \psi_c(\vec{x},t), \tag{2.6}$$

where the space-time reflected state in the right hand side, $\psi_c(\vec{x},t)$, is just the antiparticle state corresponding to $\psi(\vec{x},\ t)$, i. e. ,

$$\psi_c(\vec{x},t) = \mathcal{C}\psi(\vec{x},t)\ . \tag{2.7}$$

Here we introduce a new particle- antiparticle conjugate operator $\mathcal{C}$, which is not defined independently because the postulate (2.6) implies that

$$\mathcal{P}\mathcal{T} = \mathcal{C}\ . \tag{2.8}$$

In quantum mechanics, the momentum and energy operators for a particle read:

$$\hat{\vec{p}} = -i\hbar\nabla\ ,\ \hat{E} = i\hbar\frac{\partial}{\partial t}\ . \tag{2.9}$$

Note that after the space-time inversion, the components of four momenta of the antiparticle remain unaltered, i.e., (We adopt the Pauli metric. For notation see Ref. [10])

$$p_\mu^c = p_\mu\ \ (\mu = 1,\ 2,\ 3,\ 4)\ ,$$

while

$$p^c_\mu =< \psi_c|\hat{p}_{c\mu}|\psi_c >, \; p_\mu =< \psi|\hat{p}_\mu|\psi > \; . \tag{2.10}$$

Then from (2.6), we find that the momentum and energy operators for an antiparticle should be:

$$\hat{\vec{p}}_c = i\hbar\nabla \; , \; \hat{E}_c = -i\hbar\frac{\partial}{\partial t} \; . \tag{2.11}$$

For example, the plane waves

$$\psi(\vec{x}, t) = \exp\{i(\vec{p}\cdot\vec{x} - Et)/\hbar\} \tag{2.12}$$

and

$$\psi_c(\vec{x}, t) = \exp\{-i(\vec{p}\cdot\vec{x} - Et)/\hbar\} \tag{2.13}$$

describe a particle and an antiparticle with the same momentum $\vec{p}$ and positive energy E respectively. thus the phase of wave function is important. From now on, if we meet a "negative energy" ($\psi \sim \exp\{(iEt/\hbar)\}, \; E > 0$) in the exponential, we should recognize it describing an antiparticle. This point had been emphasized by Schwinger[11] and the expression (2.11) had also been written down by Konopinski and Mahmaud[12]. But here (2.11) and (2.13) are all the direct corollaries of our fundamental postulate. (From now on the superscript of an operator, $\hat{}$, will be omitted).

Comparing (2.2) and (2.7), we find that

$$\mathcal{C} = \mathcal{PT} = \texttt{CPT}. \tag{2.14}$$

In the following sections we will see that the present particle theory does have such a corresponding relation. This is not accidental and is not merely a change in definition. (See the discussion after Eq.(3.4)).

When considering the many-body problem, according to the quantum field theory, the wave function of different kind of field is promoted into the field operator in Hilbert space. Because the time inversion not only renders the individual field operator to change its argument from $t \to -t$, but also reverses the order of field operators in a product. Hence under the combined space-time inversion (denoted by $x \to -x$),we have

$$\psi_1(x_1)\psi_2(x_2)\cdots\psi_n(x_n) \longrightarrow \psi'_n(-x_n)\cdots\psi'_2(-x_2)\psi'_1(-x_1) \; , \tag{2.15}$$

where the superscript prime "," is added for taking care the change of annihilation and creation operators, (see sec. IV).

According to this new definition, the configuration space and Hilbert space are mixed together, so we do not use two kinds of notation of space-time inversion for single particle and many particle theory separately, but simply denote them by $\mathcal{P}$, $\mathcal{T}$, and $\mathcal{C} = \mathcal{PT}$.

The CPT theorem was proved by Pauli and Lüders [8] according to the original definitions and making use of so called strong reflection invariance. Thanks to CPT theorem and the researches over ten years, the physicists arrive at the correct cognition (2.2). However, in our point of view, once the relation (2.2)is reached, the historic mission of CPT theorem is coming to an end. It should be replaced by a fundamental postulate as shown in (2.8) [i.e. , (2.14)]. In other words, the relation between space-time inversion and particle- antiparticle conjugation is not a problem of definition and a proof of theorem, but a natural law which anyhow must be tested by experiments. In this sense, the present particle theory which is verified by numerous experimental facts will be the basis of this postulate. On the other hand, one may get some new insight from it.

III. THE RELATIVISTIC PARTICLE EQUATIONS

A. The particle with spin 1/2

The single particle theory basing on the Dirac equation is asymmetric with respect to electron and positron. One has to overcome the negative energy difficulty by means of the hole concept, then throw away the hole by the method of redefinition when performing the second quantization in order to obtain the formal symmetry between electron and positron [10,13]. In our point of view, as the equal existence of electron and positron is a fact in nature beyond any doubt, our theory should reflect this symmetry at every step. Let us start from the new postulate and view the negative energy solution directly as the wave function of positron. Hence the Dirac equation [10]

$$\left(\frac{\partial}{\partial x_\mu} - \frac{ie}{\hbar c}A_\mu(\vec{x},\ t)\right)\gamma_\mu\psi(\vec{x},\ t) + \frac{mc}{\hbar}\psi(\vec{x},\ t) = 0 \tag{3.1}$$

($e < 0$, $\gamma_k = -i\beta\alpha_k$, $\gamma_4 = \beta$) not only describes the electron, but also describes the positron. In the "positive energy solution" which describes the electron, the first and second components of spinor are large components whereas the third and fourth components are small ones. On the other hand, in the "negative energy solution", i.e., the positron wave function, the large components of spinor go down to the third and fourth position

(As to the problem of spin orientation, see Appendix A). Hence we may say that Dirac equation is mainly for describing the electron. Is there an equivalent equation mainly for describing the position?

Let us perform a $\mathcal{PT}$ inversion for Eq. (3.1):

$$x_\mu \longrightarrow -x_\mu \ , \ \psi(x_\mu) \longrightarrow \psi(-x_\mu) = \psi_c(x_\mu) \tag{3.2}$$

and note that at the same time electromagnetic potential will transform as follows:

$$A_\mu(\vec{x},\ t) \longrightarrow A_\mu(-\ \vec{x},\ -t) = -A_\mu(\vec{x},\ t) \tag{3.3}$$

This is because when we adhere to the fundamental postulate, not only the electron under observation, but all the charges which create the electromagnetic fields change their sign of charge as well. Thus we have

$$\left(\frac{\partial}{\partial x_\mu} - \frac{ie}{\hbar c}A_\mu(\vec{x},\ t)\right)\gamma_\mu\psi_c(\vec{x},\ t) - \frac{mc}{\hbar}\psi_c(\vec{x},\ t) = 0 \tag{3.4}$$

which corresponds to the transformation $m \to -m$ in (3.1) without the change of e, also corresponds to the representation transform $\psi = \gamma_5\psi_c$, (chirality transformation). In the past, a combined CPT transformation will lead to CPT$= -\gamma_5$ if suitable phase is chosen [13]. Therefore, for a particle with spin 1/2 we have illustrated Eq. (2.14).

To our knowledge, Tiomno first noticed the equivalence of the Dirac equation under γ_5 transformation, Sakurai had written down the Eq. (3.4) in 1958, Nambu and Jona-Lasinio in 1961 had derived a more general form: [14]

$$\left(\gamma_\mu\frac{\partial}{\partial x_\mu} + \frac{mc}{\hbar}(\cos 2\alpha + i\gamma_5 \sin 2\alpha)\right)\psi(x_\mu) = 0 \tag{3.5}$$

However, the meaning of either (3.4) or (3.5) had not been clarified before, so they did not get enough emphasis. In our point of view, being the space-time inversed equation of Dirac equation, Eq. (3.4) describes mainly the positron, but the electron as well. There is a simple relation under $\mathcal{PT}$ inversion between its solution and the solution of Dirac equation. This relation has nothing to do with the γ matrices (see Appendix B). For this reason we suggested to name Eq. (3.4) as the Carid equation.

Comparing the $\psi_{III}(x)$ and $\psi_{cIII}(x)$ in Appendix B, we see that the relation under space-time inversion is precisely: $\psi_{III}(x)$=$\psi_{cIII}(x')$ $(x' = -x)$. In other words, a positron in $\{x\}$ system is just equivalent to an electron (not a negative-energy electron) in inversed

$\{x'\}$ system. This implies a modification to the Stüeckelberg-Feynman rule [15] and is also a special statement of fundamental postulate in this paper.

Therefore, it seems to us that the original positron equation obtained via $e \to -e$ transformation is incorrect. Positron and electron obey the same equation—Dirac equation or Carid equation, the latter is obtained from the former via $m \to -m$ transformation. It is interesting to compare with the classical theory, where the motion of equation for electron

$$e\left(\vec{E} + \frac{1}{c}\,\vec{v} \times \vec{B}\right) = m\frac{d\,\vec{v}}{dt} \tag{3.6}$$

will lead to the motion equation for positron via either $e \to -e$ or $m \to -m$ transformation. However, we stress here that the difference between particle and antiparticle is not due to the difference in some "charge", but due to their opposite space-time phases in their wave functions. See also the Foldy-Wouthuysen transformation[13].

As in Ref. [13], let us write

$$\psi(x) = \begin{pmatrix} \theta(x) \\ \chi(x) \end{pmatrix},\ \psi_c(x) = \begin{pmatrix} \chi_c(x) \\ \theta_c(x) \end{pmatrix} \tag{3.7}$$

such that the Dirac equation is resolved into two simultaneous equations of two component spinors $\theta(x)$ and $\chi(x)$:

$$\left\{ \begin{array}{l} i\hbar\dfrac{\partial\theta}{\partial t} = c\,\vec{\sigma}\cdot\left(\dfrac{\hbar}{i}\nabla - \dfrac{e}{c}\vec{A}\right)\chi(x) + (eV + mc^2)\theta(x) \\ i\hbar\dfrac{\partial\chi}{\partial t} = c\,\vec{\sigma}\cdot\left(\dfrac{\hbar}{i}\nabla - \dfrac{e}{c}\vec{A}\right)\theta(x) + (eV - mc^2)\chi(x) \end{array} \right. . \tag{3.8}$$

Things become more symmetric in new point of view. For an electron, $|\theta| > |\chi|$, the $\theta(x)$ characterizing electron determines the phase of space-time, i.e., $\chi \sim \theta \sim \exp(-iEt/\hbar)$, $(E > 0)$. For a positron, the situation is just in the opposite, $|\chi| > |\theta|$, $\theta \sim \chi \sim \exp(iEt/\hbar)$, $(E > 0)$. When performing a space-time inversion to a electron wave function, $\theta(\vec{x},\ t) \to \theta(-\ \vec{x},\ -t) = \chi_c(\vec{x},\ t)$ remains as large component, whereas $\chi(\vec{x},\ t) \to \chi(-\ \vec{x},\ -t) = \theta_c(\vec{x},\ t)$ remains small. Hence an electron changes into a positron. Therefore, in some sense we can say that an electron contains some ingredient of positron implicitly and coherently. On the contrary, a positron state contains some ingredient of electron implicitly and coherently too. Any discrimination between particle from antiparticle is relative. There is no existence of either pure particle or pure antiparticle even at the single particle level. For further discussion see Appendix C.

B. The charged particle without spin

The Klein-Gordon equation describes a particle with spin zero:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\phi(\vec{x},\ t) = 0\ . \tag{3.9}$$

Obviously, K-G equation remains unaltered under $\mathcal{PT}$ inversion, whereas its complex wave solution does undergo the following change,

$$\phi(\vec{x},\ t) \longrightarrow \phi(-\ \vec{x},\ -t) = \phi_c(\vec{x},\ t) \tag{3.10}$$

which denotes a meson (say π^-) changes into antimeson (say π^+).

Furthermore, the K-G equation with external electromagnetic potential can be recast into two simultaneous equations of one order [16]. Denote

$$D_\mu = \frac{\partial}{\partial x_\mu} - \frac{ie}{\hbar c}A_\mu \tag{3.11}$$

and

$$\begin{cases} -\frac{1}{\kappa}D_4\phi = \theta - \chi\ ,\ (\kappa = \frac{mc}{\hbar}) \\ \phi = \theta + \chi \end{cases} \tag{3.12}$$

Instead of ϕ, now one has two parts, θ and χ, in coupling. Then K-G equation reads

$$\begin{cases} i\hbar\dfrac{\partial\theta}{\partial t} = \dfrac{1}{2m}\left(\dfrac{\hbar}{i}\nabla - \dfrac{e}{c}\vec{A}\right)^2(\theta+\chi) + (eV + mc^2)\theta \\ i\hbar\dfrac{\partial\chi}{\partial t} = -\dfrac{1}{2m}\left(\dfrac{\hbar}{i}\nabla - \dfrac{e}{c}\vec{A}\right)^2(\theta+\chi) + (eV - mc^2)\chi \end{cases} . \tag{3.13}$$

Similar to Eq. (3.8), this time we also see that under the $\mathcal{PT}$ inversion:

$$\theta(\vec{x},\ t) \longrightarrow \theta(-\ \vec{x},\ -t) = \chi_c(\vec{x},\ t),\ \chi(\vec{x},\ t) \longrightarrow \chi(-\ \vec{x},\ -t) = \theta_c(\vec{x},\ t)\ .$$

If the former, θ, is the larger one and so dominates the latter, χ, then a particle in turn changes into antiparticle. Moreover, because the probability density [3, 17]

$$\rho = \frac{i\hbar}{2mc^2}\left(\phi^*\frac{\partial\phi}{\partial t} - \phi\frac{\partial\phi^*}{\partial t}\right) \tag{3.14}$$

changes its sign under $\mathcal{PT}$ inversion ($\phi \to \phi_c$), as long as we interpret ρ as a "charge density" [18], the so called "negative probability difficulty" does not exist even at the level of single particle theory.

C. The photon

The Maxwell equation in vacuum can be recast in the following form [19]:

$$i\hbar\frac{\partial}{\partial t}\Phi = -i\hbar c\,\vec{S}\cdot\nabla\Phi\ , \tag{3.15}$$

where the vector operator $\vec{S}$ is 3×3 hermitian matrices with three components:

$$S_1 = i\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\ ,\ S_2 = i\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}\ ,\ S_3 = i\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\ , \tag{3.16}$$

while

$$\Phi(\vec{x},\ t) = \begin{pmatrix} E_1 + iB_1 \\ E_2 + iB_2 \\ E_3 + iB_3 \end{pmatrix}\ , \tag{3.17}$$

being the electromagnetic field "wave function". As before, it is composed of two parts, $\vec{E}$ and $\vec{B}$. If introducing the "orbital angular momentum" $\vec{L}$, and using the similar method in App. A, one can easily prove that:

$$\frac{d}{dt}(\vec{L} + \hbar\,\vec{S}) = 0 \tag{3.18}$$

This implies that a "photon" has a spin angular momentum $\hbar\,\vec{S}$ with spin quantum number $S = 1$. Eq. (3.15) can also be written as

$$H\Phi = c\,\vec{S}\cdot\vec{p}\ \Phi \tag{3.19}$$

which bears a close resemblance to the Weyl equation (see Eqs. (6.18), (6.19)). By means of the unified method in this paper, we see that there are only two basic states of electromagnetic wave, i.e., the right and left circular polarization states, (similar conclusion was arrived at by Hestenes via another method [20]). The corresponding photons are denoted as γ_R and γ_L:

$$|\gamma_L> = \mathcal{PT}|\gamma_R> = \mathcal{C}|\gamma_R>\ \ . \tag{3.20}$$

Note that, however, we have

$$\begin{cases} \mathcal{P}|\gamma_R>=|\gamma_L> , \\ \mathcal{T}|\gamma_R>=|\gamma_R> , \mathcal{T}|\gamma_L>=|\gamma_L> \ . \end{cases} \tag{3.21}$$

IV. CONNECTION BETWEEN SPIN AND STATISTICS

In quantum field theory, the wave function becomes a field operator. In this case we further assume that under $\mathcal{PT}$ inversion an annihilation operator $a(\vec{k})$ of particle with momentum $\vec{k}$ and energy $E>0$ will change to a creation operator $b^\dagger(\vec{k})$ of antiparticle with momentum $\vec{k}$ and energy $E>0$. Thus we see that the complex scalar field operator

$$\phi(x)=\frac{1}{\sqrt{V}}\sum_{\vec{k}}\frac{1}{\sqrt{2\omega}}\left\{a(\vec{k})e^{ik\cdot x}+b^\dagger(\vec{k})e^{-ik\cdot x}\right\} \tag{4.1}$$

has the invariance of $\mathcal{PT}=\mathcal{C}$ evidently, i.e., when $x\to -x$, $e^{ik\cdot x}\to e^{-ik\cdot x}$, $a(\vec{k})\rightleftarrows b^\dagger(\vec{k})$, one has

$$\phi(x)\longrightarrow \phi_c(x)=\phi(x) \ . \tag{4.2}$$

The situation for Dirac field is a little bit complicated:

$$\psi(x)=\frac{1}{\sqrt{V}}\sum_{\vec{p}}\sum_{r=1,2}\left\{c_r(\vec{p})u^{(r)}(\vec{p})e^{ip\cdot x}+d_r^\dagger(\vec{p})v^{(r)}(\vec{p})e^{-ip\cdot x}\right\} \ , \tag{4.3}$$

where $c_r(\vec{p})$ and $d^\dagger(\vec{p})$ are the annihilation operator of electron and the creation operator of positron respectively. For checking that (4.3) also has the invariance of space-time inversion, we must also expand the solution of Carid equation as a field operator:

$$\psi_c(x)=\frac{1}{\sqrt{V}}\sum_{\vec{p}}\sum_{r=1,2}\left\{d_r^\dagger(\vec{p})u^{(r)}(\vec{p})e^{-ip\cdot x}+c_r(\vec{p})v^{(r)}(\vec{p})e^{ip\cdot x}\right\} \ , \tag{4.4}$$

Then when $x\to -x$, $c_r(\vec{p})\rightleftarrows d_r^\dagger(\vec{p})$, one has

$$\psi(x)\longrightarrow \psi_c(x)=-\gamma_5\psi(x) \ . \tag{4.5}$$

So the $\mathcal{PT}=\mathcal{C}$ invariance exhibit itself as the transformation between Dirac representation and Carid representation. From now on, we try to propose an algorithm in quantum field theory: all field operators and the Lagrangian density constructed from them, all the operator algebra, should respect the invariance of $\mathcal{PT} = \mathcal{C}$. In the following we try to discuss the relation between spin and statistics by means of this invariance.

The spin-statistics connection was proved by Pauli [21] and discussed by Schwinger and other authors [22, 23]. However, different kinds of proof often carry some argument with negative character, among them the violation of microcausality seems the strongest. For instance, the complex K-G field is quantized via

$$[\phi(x),\ \phi^\dagger(y)] = i\hbar c\Delta(x-y) \tag{4.6}$$

with $\Delta(-x) = -\Delta(x)$ and $\Delta(x) = 0$ for $x^2 > 0$ (For notation $\Delta(x)$, see Ref.[17]). The Dirac field is quantized via

$$\{\psi(x),\ \bar{\psi}(y)\} = -i\left(\gamma_\mu \frac{\partial}{\partial x_\mu} - \frac{mc}{\hbar}\right)\Delta(x-y)\ . \tag{4.7}$$

The other function $\Delta_1(x)$ with property $\Delta_1(-x) = \Delta_1(x)$ is rejected because $\Delta_1(x) \neq 0$ for $x^2 > 0$. Thus one obtains the correct statistics and rejects the wrong ones.

Note that the $\mathcal{PT}$ inversion does keep the Eq. (4.6) invariant. It also keep (4.7) invariant in the sense of transforming the latter into the Carid representation:

$$\{\psi_c(x),\ \bar{\psi}_c(y)\} = -i\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + \frac{mc}{\hbar}\right)\Delta(x-y)\ . \tag{4.8}$$

On the very general ground we may replace the left hand side of (4.6) or (4.7) by general bracket $[\ ,\ ]_\omega$ with $\omega = -1$ corresponding to commutation while $\omega = +1$ to anticommutation. Then a strong statement could be that: " Under the condition of microcausality, $\mathcal{PT} = \mathcal{C}$ invariance determine the correct statistics uniquely". At least we have a weak statement that: "In determining the spin statistics connection, the microcausality is in conformity with the $\mathcal{PT} = \mathcal{C}$ invariance or vice versa". This consistency seems to us not quite a coincidence but has a deep implication as we will show further in the sec. VI.

We would like to point out that the antisymmetrical current which is equivalent to the normal ordered current [17]:

$$j_\mu = \frac{1}{2}i[\bar{\psi}(x),\ \gamma_\mu\psi(x)] = i : \bar{\psi}(x)\gamma_\mu\psi(x) : \tag{4.9}$$

also has the transformation property

$$\mathcal{C}j_\mu\mathcal{C}^{-1} = -j_\mu^c = -j_\mu\ , \tag{4.10}$$

where the current operator at the right hand side is written in Carid representation which in turn is equivalent to that in Dirac representation. By this way, the special demand that all Lagrangians are antisymmetrized in the fermion fields and symmetrized in the boson fields can also be substituted by the unified requirement of $\mathcal{PT} = \mathcal{C}$ invariance.

V. FEYNMAN PROPAGATOR AND THE ARROW OF TIME

Carrying on the concept in Newtonian mechanics, in quantum mechanics there is a time reversal (T) transformation. However, because the Schrödinger equation has the time derivative of first order, it changes sign under $t \to -t$ transformation. So a further complex conjugate transformation has to be made to cancel the sign change, which in turn implies some equivalence between ψ and ψ^*. The whole theory of time reversal in quantum mechanics seems to us rather artificial and deserves to be doubted.

Actually, there is time asymmetry rather than symmetry in quantum mechanics. This can obviously be seen from the Feynman path integral formalism:

$$\psi(\vec{x},\ t) = \int K(\vec{x},\ t|\ \vec{x}',\ t')\psi(\vec{x}',\ t')d\,\vec{x}' \ , \tag{5.1}$$

$$K(\vec{x},\ t|\ \vec{x}',\ t') = \int_\Gamma \mathcal{D}\ \vec{x}\ e^{iS/\hbar} \ . \tag{5.2}$$

In the expression of kernel K, the path Γ takes every zigzag way leading from point $(\vec{x}',\ t')$ to $(\vec{x},\ t)$ but without the reversal in time direction. Alternatively, we may look at the corresponding Green function for Schrödinger Equation:

$$\left(i\hbar\frac{\partial}{\partial t} - H\right) G(\vec{x},\ t|\ \vec{x}',\ t') = \delta(\vec{x} - \vec{x}')\delta(t - t') \ . \tag{5.3}$$

Then

$$G(\vec{x},\ t|\ \vec{x}',\ t') = -\frac{i}{\hbar}K(\vec{x},\ t|\ \vec{x}',\ t')\theta(t - t')$$

can be expanded by eigenfunctions of H as follows:

$$G(\vec{x},\ t|\ \vec{x}',\ t') = -\frac{i}{\hbar}\sum_n \phi_n(\vec{x})\phi_n^*(\vec{x}')\exp\{-\frac{i}{\hbar}E_n(t - t')\}\theta(t - t') \ . \tag{5.4}$$

The existence of $\theta(t - t')$ reflects the time asymmetry.

In relativistic quantum mechanics, let us look at the Feynman propagator K_F for Dirac equation: [10]

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m\right) K_F(x,\ \vec{x}) = -i\delta^{(4)}(x - x')\ , \tag{5.5}$$

$$K_F(x,\ x') = \sum_{\vec{p},\ s} \frac{m}{EV} \left\{ u^{(s)}(\vec{p})\bar{u}^{(s)}(\vec{p}) e^{ip\cdot(x-x')}\theta(t - t') - v^{(s)}(\vec{p})\bar{v}^{(s)}(\vec{p}) e^{-ip\cdot(x-x')}\theta(t' - t) \right\} . \tag{5.6}$$

It is still time asymmetric. But what new symmetry is there? Let us perform a $\mathcal{PT}$ inversion on (5.5) and (5.6), we get

$$\left(-\gamma_\mu \frac{\partial}{\partial x_\mu} + m\right) K_F^c(x,\ \vec{x}) = -i\delta^{(4)}(x - x')\ , \tag{5.7}$$

and

$$K_F^c(x,\ x') = \sum_{\vec{p},\ s} \frac{m}{EV} \left\{ u^{(s)}(\vec{p})\bar{u}^{(s)}(\vec{p}) e^{-ip\cdot(x-x')}\theta(t' - t) - v^{(s)}(\vec{p})\bar{v}^{(s)}(\vec{p}) e^{ip\cdot(x-x')}\theta(t - t') \right\} . \tag{5.8}$$

respectively. Notice that $K_F^c = \gamma_5 K_F(x,\ x')\gamma_5$. So except for the representation transformation, (Dirac $\to$ Carid), $K_F^c(x,\ x')$ is essentially the same as $K_F(x,\ x')$. In other words, it is the $\mathcal{PT} = \mathcal{C}$ invariance that forms the basis of selecting the Feynman propagator instead of simple advanced or retarded one. This observation was put forward in Ref.[24], where the relation between $\mathcal{PT}$ (i.e., CPT)invariance and the arrow of time is discussed.

At the microscopic level, the t asymmetry, i.e., the arrow of time is dictated by a larger symmetry, the $\mathcal{PT} = \mathcal{C}$ invariance. While at the level of classical electrodynamics, $\mathcal{PT} = \mathcal{C}$ =CPT invariance can still play the role of excluding the symmetric potential and selecting the correct retarded potential in the sense of calculating the energy radiation rate [24]. But the problem of how the macroscopic arrow of time, in the sense of thermodynamics, will be related to the microscopic arrow of time, is still controversial. It is well known that the H-theorem (i.e., the entropy increase principle) can only be proved in statistical mechanics after a coarse grain density is defined in the phase space. Van Hove generalized this theorem for quantum statistical case by defining a coarse density matrix [25]. But what is the meaning of performing an averaging procedure for defining the coarse grain density (matrix)? We conjecture that the averaging procedure corresponds to some operation which washes out the information of quantum phase and then destroys the quantum coherence. Some preliminary discussion was made via a simple system composed of two level atoms, radiations and thermal reservoirs [26].

VI. THE SPECIAL RELATIVITY

A. Where is the crucial point?

We are now in a position to search for a new derivation of the theory of special relativity. In his classical paper, Einstein introduced two postulates: (A). the principle of the constancy of the speed of light; (B). the principle of relativity. Since 1905 till recent years, some authors felt that there is some logic cycle in these two relativistic postulates, so they tried to derive the theory of special relativity without the postulate (A). At final they all failed to do so. In fact, Einstein had considered the problem from all aspects. While talking about the principle of relativity, one needs the definition of coordinates $\{x\}$ and $\{x'\}$ of two inertial systems, say S and S', moving with relative velocity $\vec{v}$. The Lorentz transformation is no more than the definition of $\{x'\}$ with respect to $\{x\}$ or vice versa. So the constant c in Lorentz transformation must be fixed in advance not only in meaning but also in magnitude, otherwise one will have no real physics in the principle of relativity. Einstein did the best work in his time. He was the first physicist who emphasized the necessity of distinguishing the observables from nonobservables.

Now we are living after the quantum theory and particle physics all have been well established. Can we derive the theory of special relativity by only one "relativistic" postulate rather than two? Then it is clear that we need a basic postulate stated only in one inertial system (S). In our opinion, this postulate is nothing but the $\mathcal{PT} = \mathcal{C}$ symmetry discussed in previous sections.

B. The relativistic wave equation for spin zero particle and the mass- energy relation

We will approach this problem by considering the special cases individually to see the role played by the $\mathcal{PT} = \mathcal{C}$ symmetry. In some sense, we will go along the opposite way in sec. III. Of course, in each case the special postulate of nonrelativistic nature should be supplemented.

Consider spin zero case first. A particle is resting in an inertial system S. Assume its energy E_0 is proportional to its rest mass m_0, $E_0 = m_0 c_1^2$, where c_1 being merely a constant with the dimension of velocity. Notice that, however, this is not an independent postulate (input), because we will soon derive in general $E = mc_1^2$, with mass m defined as $m = \frac{1}{2}(\frac{dp^2}{dE})$. Indeed, when particle is in slow motion, its energy reads:

$$E = m_0 c_1^2 + \frac{p^2}{2m_0}, \ (\vec{p} \to 0) \ . \tag{6.1}$$

The rest mass does obey the relation:

$$m_0 = \frac{1}{2}(\frac{dp^2}{dE}) \ , \ (\vec{p} \to 0) \ . \tag{6.2}$$

The particle velocity v equals to the group velocity of de Broglie wave associated with it:

$$v = v_g = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{p}{m_0}, \ (v \to 0) \ , \tag{6.3}$$

where the general quantum relations $E = \hbar\omega$, $p = \hbar k$ and (6.1) have been used.

Consider that the wave is described by $\theta(\vec{x},\ t)$ and obeys the following "nonrelativistic quantum equation"

$$i\hbar\frac{\partial}{\partial t}\theta(\vec{x},\ t) = m_0 c_1^2 \theta(\vec{x},\ t) - \frac{\hbar^2}{2m_0}\nabla^2\theta(\vec{x},\ t) \ , \tag{6.4}$$

where the quantum rules

$$E \longrightarrow i\hbar\frac{\partial}{\partial t}, \ \vec{p} \longrightarrow -i\hbar\nabla \tag{6.5}$$

have been used. It is just at this moment we introduce the "relativistic principle" into the formula. That is $\mathcal{PT} = \mathcal{C}$ symmetry. Corresponding to the particle state $\theta(\vec{x},\ t)$, there is an antiparticle state $\chi(\vec{x}, t)$ hiding inside a particle. and will couple to each other via the motion (kinetic energy). So instead of (6.4), we should have a couple of equations:

$$\begin{cases} i\hbar\dfrac{\partial}{\partial t}\theta = m_0 c_1^2\theta - \dfrac{\hbar^2}{2m_0}\nabla^2\theta - \dfrac{\hbar^2}{2m_0}\nabla^2\chi \ . \\ i\hbar\dfrac{\partial}{\partial t}\chi = -m_0 c_1^2\chi + \dfrac{\hbar^2}{2m_0}\nabla^2\chi + \dfrac{\hbar^2}{2m_0}\nabla^2\theta \ . \end{cases} \tag{6.6}$$

Now Eqs. (6.6) respect the $\mathcal{PT} = \mathcal{C}$ symmetry because

$$\chi(\vec{x},\ t) = \theta(-\ \vec{x},\ -t) \ . \tag{6.7}$$

Let $\theta = \frac{1}{2}(\phi + i\xi)$ with ϕ and ξ being real functions of $(\vec{x},\ t)$ and notice from (6.6) that $\chi(\vec{x},\ t) = \theta^*(\vec{x},\ t) = \frac{1}{2}(\phi - i\xi)$ we have

$$\begin{cases} \hbar\dot{\phi} = m_0 c_1^2 \xi \ , \\ \hbar\dot{\xi} = -m_0 c_1^2 \phi + \frac{\hbar^2}{m_0}\nabla^2\phi \ . \end{cases} \tag{6.8}$$

Then the Klein-Gordon equation follows immediately:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m_0^2 c_1^2}{\hbar^2}\right)\phi(\vec{x},\ t) = 0 \ . \tag{6.9}$$

Generalizing to the case of complex ϕ field, we set from the beginning that

$$\begin{cases} \theta = \frac{1}{2}(\phi + i\frac{\hbar}{m_0 c_1^2}\dot{\phi}) \\ \chi = \frac{1}{2}(\phi - i\frac{\hbar}{m_0 c_1^2}\dot{\phi}) \end{cases} \tag{6.10}$$

and still get the K-G Eq. (6.9). Note also that θ , χ and ξ all satisfy K-G equation. $|\chi|$ will increase from zero to $|\theta|$ when the particle energy E increases from $m_0 c_1^2$ to infinity.

Substituting the plane wave solution

$$\phi(\vec{x}, t) = \exp[i(\vec{p}\cdot\vec{x} - Et)/\hbar] \tag{6.11}$$

into (6.9), one obtains easily

$$E^2 = \vec{p}^{\,2} c_1^2 + m_0^2 c_1^4 \ . \tag{6.12}$$

Now the implication of constant c_1 has not been explored yet. The velocity of particle corresponds to the group velocity as in (6.3):

$$v = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{pc_1^2}{E} \ , \ (|\vec{p}| = p) \ . \tag{6.13}$$

Define the inertial mass as in (6.2):

$$m = \frac{p}{v} = p/\frac{dE}{dp} = \frac{1}{2}\frac{dp^2}{dE} \ . \tag{6.14}$$

Combining (6.13), (6.14) and (6.12), one obtains

$$E = mc_1^2 \tag{6.15}$$

and

$$m = m_0\left(1 - \frac{v^2}{c_1^2}\right)^{-\frac{1}{2}} \tag{6.16}$$

as expected. Evidently, c_1 is nothing but the limiting speed of particle. According to the experiments on π meson beam, we know that c_1 equals to the speed of light, i.e.,

$$c_1 = c = 3 \times 10^{10}\ cm/sec\ . \tag{6.17}$$

In some sense, from $E|_{v\to 0} = m_0 c_1^2$ to $E = mc_1^2$, we are using a trick similar to the inductive method in mathematics. But the "relativistic hormone" is just the symmetry $\mathcal{PT} = \mathcal{C}$.

C. The relativistic wave equations for spin 1/2

The present experiment and theory reveal that the neutrino is likely a particle with spin 1/2 and zero rest mass. So its wave equation may be

$$E\phi_R = i\hbar\frac{\partial}{\partial t}\phi_R = c_2\ \vec{p}\cdot\vec{\sigma}\ \phi_R = -ic_2\ \vec{\sigma}\cdot\nabla\phi_R \tag{6.18}$$

or

$$E\phi_L = i\hbar\frac{\partial}{\partial t}\phi_L = -c_2\ \vec{p}\cdot\vec{\sigma}\ \phi_L = ic_2\ \vec{\sigma}\cdot\nabla\phi_L \tag{6.19}$$

As before, here c_2 is still an unfixed constant with dimension of velocity. Both these two Weyl equations respect the $\mathcal{PT} = \mathcal{C}$ symmetry, but the nature seems to favor (6.19) and discard (6.18), as we learn from the two-component neutrino theory.

Now consider a particle with spin 1/2 and rest mass m_0. Then two-component spinors ϕ_L and ϕ_R will couple each other via m_0 (rather than via the "kinetic energy $c_2\ \vec{p}\cdot\vec{\sigma}$ "):

$$\begin{cases} i\hbar\frac{\partial}{\partial t}\phi_R = -ic_2\ \vec{\sigma}\cdot\nabla\phi_R + m_0c_2^2\phi_L\ , \\ i\hbar\frac{\partial}{\partial t}\phi_L = ic_2\ \vec{\sigma}\cdot\nabla\phi_L + m_0c_2^2\phi_R\ . \end{cases} \tag{6.20}$$

Defining four-component spinor

$$\psi = \begin{pmatrix} \theta \\ \chi \end{pmatrix} = \begin{pmatrix} \phi_R + \phi_L \\ \phi_R - \phi_L \end{pmatrix}\ . \tag{6.21}$$

one recovers the Dirac equation again[10]:

$$\left(\gamma_\mu\frac{\partial}{\partial x_\mu} + \frac{m_0c_2}{\hbar}\right)\psi = 0\ . \tag{6.22}$$

The equation obeyed by θ and χ in external fields had been written in (3.8). The experiments on electron beam verify that the limiting speed $c_2 = c = 3\times 10^{10}\ cm/sec$.

The important thing here is never looking at a wave function (like ψ) as a whole entity but an object composed of two parts in contradiction (θ and χ, or ϕ_R and ϕ_L), while respecting the $\mathcal{PT} = \mathcal{C}$ symmetry at the same time.

The similar experience used to spin 1 particle, photon, had been written in Eqs.(3.15)-(3.21). This time the wave function $\Phi = (\vec{E} + i\,\vec{B})$ is composed of the real and imaginary parts with $\vec{E}$ and $\vec{B}$ being observables.

D.The Lorentz transformation.

As explained at the beginning of this section, in Einstein's theory, the principle of the constancy of light speed must be put ahead of the principle of relativity. Now we are going to derive the Lorentz transformation after some knowledge about the dynamics of particles is known. Nevertheless, we still need an invariance to define the coordinates $\{x\}$ and $\{x'\}$ in S and S' systems. Once again, we resort to the brilliant idea of de Broglie, who named it as "the law of phase harmony" and was explained by Lochak as follows [27]. For any Galilean observer, the phase of the "internal clock" of the particle is, at each instant, equal to the value of the phase of the wave calculated at the same point at which the particle lies. (de Broglie considered this law to be the fundamental achievement in his life and it was appreciated very much by Einstein. However, it was nearly forgotten in most text books.).

The phase of wave reads $(\vec{p} \cdot \vec{x} - Et)/\hbar = (\vec{k} \cdot \vec{x} - \omega t)$, while the phase of "internal clock" (a stationary wave associated with the particle) will be $-Et'/\hbar = -\omega_0 t'$. The equality means:

$$(\vec{p} \cdot \vec{x} - Et) = -E_0 t' \ . \tag{6.23}$$

Consider that the momentum $p = mv$ is along the x axis. Substituting the expressions for $\vec{p}$, E and E_0 into (6.23), one finds

$$t' = \left(t - \frac{v}{c^2}x\right)\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \ . \tag{6.24}$$

Now our laboratory is S system, while S' system resting on particle is moving with velocity v. The clock is located at the origin of S' system, so $x' = 0$ and a generic point at S' system will have the coordinate

$$x' = a(x - vt) \ , \tag{6.25}$$

where a is a constant. Because the difference between S and S' is relative in direction, we anticipate

$$t=\left(t'+\frac{v}{c^2}x\right)\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad . \tag{6.26}$$

Substituting (6.24) and (6.25) into (6.26), one finds

$$a=\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad . \tag{6.27}$$

Alternatively, one may use more abstract language. The phase, i.e. ,the right hand side of (6.23) is an invariant in the sense of being a constant with respect to the change of $\vec{v}$. On the other hand, we have already

$$\vec{p}^{\,2}+\left(\frac{iE}{c}\right)^2=-m_0^2c^2=const.\ . \tag{6.28}$$

Rewriting (6.23)in the form as

$$\vec{p}\cdot\vec{x}+\left(\frac{iE}{c}\right)(ict)=const.\ , \tag{6.29}$$

we see that there is a four dimensional vector $(\vec{p},\ iE/c)$ in Minkowski space with its length fixed. This vector and $(\vec{x},\ ict)$ construct a scalar product which is also invariant with respect to the transformation of coordinates between S and S'. This means that $(\vec{x},\ ict)$ is also a four vector in Minkowski space with its length fixed. To be precise, the four dimensional interval between two space-time points is an invariant:

$$(\Delta\,\vec{x})^2+(ic\Delta t)^2=(\vec{x}_1-\vec{x}_2)^2+[ic(t_1-t_2)]^2=(\vec{x'}_1-\vec{x'}_2)^2+[ic(t_1'-t_2')]^2=const..\tag{6.30}$$

This equation together with (6.24) uniquely determines the Lorentz transformation:

$$\left\{\begin{array}{l} x'=\dfrac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \\ t'=\dfrac{t-\frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}} \\ y'=y,\ z'=z \end{array}\right. \qquad \left\{\begin{array}{l} x=\dfrac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}} \\ t=\dfrac{t'+\frac{v}{c^2}x'}{\sqrt{1-\frac{v^2}{c^2}}} \\ y=y',\ z=z' \end{array}\right. \tag{6.31}$$

Note that the constant c has the meaning of limiting speed of the particle, it equals to the speed of light experimentally. Moreover, the time dilatation effect can now be understood as follows. A particle state characterizing by $\theta(\vec{x},\ t)$ is always accompanying with some antiparticle ingredient characterizing by $\chi(\vec{x},\ t)$ which has the opposite space-time phase dependence essentially(implicitly). The observer in S system looks at the moving "internal clock" with its phase change as the time record. It seems slower and slower as the magnitude of χ increases larger and larger together with the increase of velocity v. In the limiting case, $v \to c$, $|\chi| \to |\theta|$, the moving "clock" tends to stop and its mass approaches to infinity. So we find that the kinetic effect in special relativity does have its universal dynamical origin. In some sense, the special relativity is more quantum than quantum mechanics.

VII. SUMMARY AND DISCUSSION.

(1) It is shown from sec. II to sec. V that the relativistic quantum mechanics and quantum field theory all contain a basic invariance– $\mathcal{PT} = \mathcal{C}$ symmetry – explicitly or implicitly. In particular, the following four things are linked together via this postulate quite naturally:

(a) The appearance of negative energy or negative probability density, which corresponds to the fact of existence of antiparticle in single particle equations;

(b) Spin-statistics theorem in many particle theory;

(c) CPT theorem;

(d) The Feynman propagator.

(2) Now a question arises. If the new postulate is used to replace the CPT theorem, the number of input to our foundation of physics would be increased. This could make the situation even worse in view of the criterion−−less input, more output−−long established in theoretical physics. So in sec. VI we devote to deriving the special relativity via the only "relativistic principle" in microscopic sense, i.e. , the $\mathcal{PT} = \mathcal{C}$ postulate. Especially, we obtain:

(e) The mass-energy relation;

(f) The Lorentz transformation.

Of course, we stress again, some well established "nonrelativistic" knowledge (postulate) and/or special prescription for individual case are needed.

Therefore, as a whole, the number of input to our foundation of physics is less (at least no more) than before.

(3) Evidently, we are treating the wave function, especially its phase, much more seriously than before. This is relevant to the basic explanation of quantum mechanics

and deserves further investigation.

(4) What we have done could be depicted by a diagram, see FIG.1 [28]. The lower part of this diagram is unobservable. Once when a particle is excited from the vacuum, it becomes observable. How can one detects a particle in experiments? One is relying on its apparent momentum or energy. So the momentum or energy is the existence form of a particle. In some sense we would also say that the space- time is the existence form of vacuum. There are two dotted lines connecting the lower part to upper part. The left one implies the quantum operator rule:

$$\vec{p} \longrightarrow -i\hbar\nabla \ , \ \vec{p}_c \longrightarrow i\hbar\nabla \ , \tag{7.1}$$

while the right one implies:

$$E \longrightarrow i\hbar\frac{\partial}{\partial t} \ , \ E_c \longrightarrow -i\hbar\frac{\partial}{\partial t} \ , \tag{7.2}$$

Now a universal constant, the Planck constant ($\hbar = h/2\pi$), emerges as a vertical link. On the other hand, the horizontal link on this diagram is provided by an another universal constant, the light speed c. Historically, Einstein discovered the horizontal link, first in the lower part, then in the upper part. Only after the knowledge about the content– how the particles exhibit themselves as the excitation states of vacuum– together with the quantum theory (the vertical link) have been accumulating in the past ninety years, can we try to examine this diagram via some what different way from that of Einstein.

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Dirac Equation

$H\psi = i\hbar\frac{\partial\psi}{\partial t}$

$H = c\,\vec{\alpha}\cdot\vec{p} + \beta mc^2,\ \vec{p} = -i\hbar\nabla$

A physical observable F will change with time as

$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{i}{\hbar}[H,\ F]$

From the orbital angular momentum of an electron

$\vec{L} = \vec{x}\times\vec{p}$

and spin

$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$

one has

$\frac{d}{dt}(\vec{L} + \frac{\hbar}{2}\vec{\Sigma}) = 0$

This means the conservation of total angular momentum. The first and third components of spinor correspond to spin along the z axis (up, $\uparrow$) while the second and fourth ones to spin down ($\downarrow$) states of electron.

For positron, the first and third components imply spin down ($\downarrow$) states while the second and fourth ones being up ($\uparrow$) states.

Carid Equation

$H_c\psi_c = -i\hbar\frac{\partial\psi_c}{\partial t}$

$H_c = c\,\vec{\alpha}\cdot\vec{p}_c + \beta mc^2,\ \vec{p}_c = i\hbar\nabla$

A physical observable F will change with time as

$\frac{dF}{dt} = \frac{\partial F}{\partial t} - \frac{i}{\hbar}[H_c,\ F]$

From the orbital angular momentum of a positron

$\vec{L}_c = \vec{x}\times\vec{p}_c$

and spin

$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$

one has

$\frac{d}{dt}(\vec{L}_c - \frac{\hbar}{2}\vec{\Sigma}) = 0$

This means the conservation of total angular momentum. The first and third components of spinor correspond to spin along $(-z)$ axis (down, $\downarrow$) while the second and fourth ones to spin up ($\uparrow$) states of positron.

For electron, the first and third state components imply spin up ($\uparrow$) while the second and fourth ones being down ($\downarrow$) states.

APPENDIX B.THE SOLUTIONS OF DIRAC EQUATION AND CARID EQUATION

Dirac equation

$\gamma_\mu \frac{\partial}{\partial x_\mu}\psi(x) + \frac{mc}{\hbar}\psi(x) = 0$

There are four independent solutions

$\psi_I(x) = u^{(1)}(\vec{p})\exp\{i(\vec{p}\cdot\vec{x} - Et)/\hbar\}$

$\psi_{II}(x) = u^{(2)}(\vec{p})\exp\{i(\vec{p}\cdot\vec{x} - Et)/\hbar\}$

$\psi_{III}(x) = v^{(1)}(\vec{p})\exp\{-i(\vec{p}\cdot\vec{x} - Et)/\hbar\}$

$\psi_{IV}(x) = v^{(2)}(\vec{p})\exp\{-i(\vec{p}\cdot\vec{x} - Et)/\hbar\}$

The former two solutions describe the electron, while latter two describe positron.

Carid equation

$\gamma_\mu \frac{\partial}{\partial x_\mu}\psi_c(x) - \frac{mc}{\hbar}\psi_c(x) = 0$

There are four independent solutions

$\psi_{cI}(x) = u^{(1)}(\vec{p})\exp\{-i(\vec{p}\cdot\vec{x} - Et)/\hbar\}$

$\psi_{cII}(x) = u^{(2)}(\vec{p})\exp\{-i(\vec{p}\cdot\vec{x} - Et)/\hbar\}$

$\psi_{cIII}(x) = v^{(1)}(\vec{p})\exp\{i(\vec{p}\cdot\vec{x} - Et)/\hbar\}$

$\psi_{cIV}(x) = v^{(2)}(\vec{p})\exp\{i(\vec{p}\cdot\vec{x} - Et)/\hbar\}$

2 The former two solutions describe the positron, while latter two describe electron.

$$u^{(1)}(\vec{p}) = N\begin{pmatrix} 1 \\ 0 \\ \frac{p_3 c}{E+mc^2} \\ \frac{(p_1+ip_2)c}{E+mc^2} \end{pmatrix}, \quad u^{(2)}(\vec{p}) = N\begin{pmatrix} 0 \\ 1 \\ \frac{(p_1-ip_2)c}{E+mc^2} \\ \frac{-p_3 c}{E+mc^2} \end{pmatrix},$$

$$v^{(1)}(\vec{p}) = N\begin{pmatrix} \frac{p_3 c}{E+mc^2} \\ \frac{(p_1+ip_2)c}{E+mc^2} \\ 1 \\ 0 \end{pmatrix}, \quad v^{(2)}(\vec{p}) = N\begin{pmatrix} \frac{(p_1-ip_2)c}{E+mc^2} \\ \frac{-p_3 c}{E+mc^2} \\ 0 \\ 1 \end{pmatrix},$$

$$E = \sqrt{p^2c^2 + m^2c^4} > 0\ ,\ N = \sqrt{\frac{E+mc^2}{2mc^2}}\ ,$$

$$u^{(r)\dagger}(\vec{p})u^{(r)}(\vec{p}) = \frac{E}{mc^2},\ v^{(r)}(\vec{p}) = -\gamma_5 u^{(r)}(\vec{p}),\ (r = 1,\ 2).$$

It is well known that the negative-energy solution become appreciable during the compression of a electron packet and is responsible for the phenomena of zitterbewegung [10, 13, 29]. The interpretation will be inconsistent if the negative states are all filled as in the hole theory.

Either in free case or in an external field, the positive and negative solutions together constitute a complete set. Starting from this point, Ma and Ni derived the Levinson theorem for Dirac particles [30]. In Eq. (20) of Ref. [30], it was shown that when the particle is moving in a short range attractive central potential $V(r)$ the phase shift is positive for the positive-energy states and negative for the negative-energy states. This implies that for the latter case the particle is repulsed by the potential and so behaves as an antiparticle.

In this paper, we show that it is the coherent excitation of antiparticle ingredient inside a particle state which is responsible for the generation of "kinetic mass" of the particle. Where the rest mass comes from?

Let us look at the mass-energy relation again:

$$E^2 = m^2c^4 = m_0^2c^4 + p^2c^2 \ . \qquad (C.1)$$

Notice that in a composite particle , the "total mass" m of a constituent particle becomes part of the "rest mass" m_0 of composite particle. This fact implies that the rest mass m_0 and the "kinetic mass" (p/c) must be stemming from the same origin. However, the right triangle relation between m_0 and (p/c), Eq. (C.1), strongly hints that they must be generated from different (orthogonal) mechanism.

In Ref. [31], basing on NJL model [14c], we redrive the formula (C.1) with m_0 being an energy gap in the new vacuum, which is formed after the condensation of massless particle-antiparticle pairs in original (naive) vacuum. So we see that the rest mass and kinetic mass are generated via different orthogonal mechanism (many body effect versus single particle effect), while both of them are stemming from the common origin—the coexistence of particle-antiparticle and the $\mathcal{PC} = \mathcal{T}$ symmetry.

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Figure Caption

FIG.1. The special relativity and quantum mechanics are two pillars of modern physics as viewed by the achievement of particle physics. However, it is just the development of particle physics which reveals the microscopic essence of special relativity. For more detail, see text.

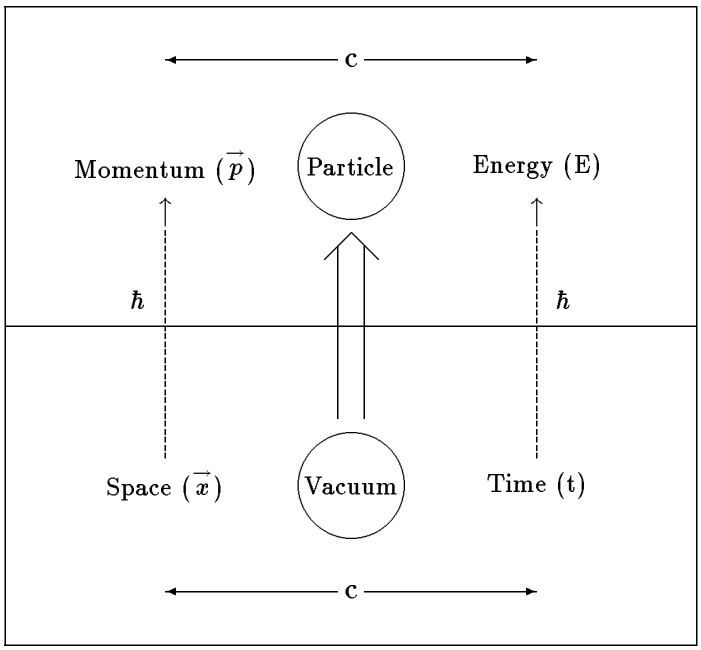
c
Momentum $(\vec{p})$
Particle
Energy (E)
$\hbar$
$\hbar$
Space $(\vec{x})$
Vacuum
Time (t)
c

FIG. 1.