August 1995 UMDEPP 96-19

Why Auxiliary Fields Matter: The Strange Case of the 4D, N = 1 Supersymmetric QCD Effective Action[1]

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ABSTRACT

Within a four dimensional manifestly N = 1 supersymmetric action, we show that Wess-Zumino-Novikov-Witten (WZNW) terms can be embedded in an extraordinarily simple manner into a purely chiral superaction. In order to achieve this result it is necessary to assign spin-0 and spin-1/2 degrees of freedom both to chiral superfields and as well to non-minimal scalar multiplets. We propose a new formulation for the effective low-energy action of 4D, N = 1 supersymmetric QCD that is consistent with holomorphy through fourth order in the pion superfield. After reduction to a 2D, N = 2 theory we find a new class of manifestly supersymmetric non-linear σ-models with torsion.

[1]Supported in part by National Science Foundation Grant PHY-91-19746 and by NATO Grant CRG-93-0789

1 Introduction

Over a decade ago [1], there began efforts to utilize supersymmetric models to construct the successor to the standard model. These efforts received a further boost with the realization [2] that such theories seem naturally to occur as the low-energy limit of four dimensional superstring and heterotic string theories. A brief survey of the literature would lead one to believe that there are no unresolved issues in how 4D, N = 1 superfields occur in this limit. In fact, there are a number of *assumptions* that are most often not even stated in presentations of the low-energy action (purportedly *derived* from superstrings) upon which most model building is based and phenomenology elucidated. One of these assumptions is that the spin-0 and spin-1/2 fields that are derived from the spectrum of string theory necessarily are described by 4D, N = 1 chiral superfields. It is not generally recognized that this is just an assumption. The reason why this is an assumption lies in the fact that there exist little recognized alternative 4D, N = 1 superfields, the non-minimal scalar multiplet [3] being one, that contain exactly the same on-shell spectrum as the usual chiral multiplet. We named such off-shell representations of 4D, N = 1 supersymmetry "variant representations." Although the non-minimal multiplet has exactly the same on-shell spectrum as the chiral multiplet, it contains a very different set of auxiliary fields. As we pointed out previously, the non-minimal scalar multiplet can appear as an alternate to the usual N = 1 Kähler non-linear σ-models and as well interact with the usual chiral multiplets [4]. Among these latter interactions there is a curious result that if a non-minimal scalar multiplet gains a mass, it can only do so in tandem with a chiral multiplet! In other words, this mechanism provides a natural explanation for the occurrence of Dirac spinors within the context of 4D, N = 1 supersymmetric models.

In most discussions of supersymmetric theories, the issue of auxiliary fields is treated in a cavalier fashion. One would think that there is no essentially important role played by auxiliary fields. Nothing could be further from the truth. One reason this attitude prevails is that there have been few demonstrations of just what dynamical consequences exist when the off-shell spectrum of two multiplets with the same on-shell spectrum are compared. A place where such differences can be shown to have demonstrable consequences is non-linear σ-models. Similarly differences can also be observed in higher derivative theories. Typically, what occurs is that the fields that are usually considered "auxiliary" can become propagating. Under this circumstance, clearly the structure of the auxiliary fields is important. Higher derivative theories are typically characteristic of effective field theories. Among these, perhaps the most important is the low-energy effective Lagrangian $\mathcal{L}_{eff}$ of QCD. It is known that leading terms of this theory are described by a chiral SU(3) $\otimes$ SU(3) non-linear σ-model. Another term of $\mathcal{L}_{eff}$ is the Wess-Zumino-Novikov-Witten term (WZNW) [5].

Along these lines there has been some discussion of what is the structure of the 4D, N = 1 supersymmetric extension of the WZNW term [6]. It is the purpose of this note to show that the introduction of non-minimal scalar multiplets, to describe some of the spin-0 and spin-1/2 fields in the effective action, opens an alternate formulation of the 4D, N = 1 supersymmetric WZNW term. This result highlights the importance of auxiliary fields. Our result also provides the most striking evidence to date that the assumption that only chiral superfields describe the matter seen in Nature is incorrect.

2 Chiral and Non-minimal Multiplet WZNW Theory

Almost every researcher who has investigated four dimensional N = 1 supersymmetry is aware of the chiral scalar or Wess-Zumino multiplet [7]. The multiplet is described by a chiral superfield Φ ($\overline{D}_{\dot\alpha}\Phi = 0$). The component fields are defined by (we use *Superspace* conventions [4])

$$A ~\equiv~ \Phi | \quad , \quad \psi_\alpha ~\equiv~ D_\alpha \Phi | \quad , \quad F ~\equiv~ D^2 \Phi | \quad , \tag{2.1}$$

and appear in the usual linear action as

$$\mathcal{S}_{WZ} ~=~ \int d^4x d^2\theta d^2\overline{\theta} ~\overline{\Phi}\,\Phi ~=~ \int d^4x \Big[-(\partial^{\underline{a}}\overline{A})(\partial_{\underline{a}} A) ~-~ i\overline{\psi}^{\dot\alpha}\partial_{\underline{a}}\psi^\alpha ~+~ \overline{F}F \Big] \quad . \tag{2.2}$$

The non-minimal scalar multiplet is described by a complex linear superfield Σ (subject to the constraint $\overline{D}^2\Sigma = 0$). The component fields are defined by

$$\begin{aligned} B ~&\equiv~ \Sigma | \quad , \quad \overline{\zeta}_{\dot\alpha} ~\equiv~ \overline{D}_{\dot\alpha}\Sigma | \quad , \\ \rho_\alpha ~&\equiv~ D_\alpha \Sigma | \quad , \quad H ~\equiv~ D^2 \Sigma | \quad , \\ p_{\underline{a}} ~&\equiv~ \overline{D}_{\dot\alpha} D_\alpha \Sigma | \quad , \quad \overline{\beta}_{\dot\alpha} ~\equiv~ \tfrac{1}{2} D^\alpha \overline{D}_{\dot\alpha} D_\alpha \Sigma | \quad , \end{aligned} \tag{2.3}$$

and appear in the usual linear action as

$$\begin{aligned} \mathcal{S}_{NM} &= ~-~ \int d^4x d^2\theta d^2\overline{\theta} ~\overline{\Sigma}\,\Sigma \\ &= \int d^4x \Big[-(\partial^{\underline{a}}\overline{B})(\partial_{\underline{a}} B) ~-~ i\overline{\zeta}^{\dot\alpha}\partial_{\underline{a}}\zeta^\alpha ~-~ \overline{H}H \\ &\qquad ~+~ 2\overline{p}^{\underline{a}}p_{\underline{a}} ~+~ \beta^\alpha\rho_\alpha ~+~ \overline{\beta}^{\dot\alpha}\overline{\rho}_{\dot\alpha} \Big] \quad . \end{aligned} \tag{2.4}$$

As is apparent from the last result above, only B and ζ^α are the propagating fields among the off-shell 12 + 12 (bosons + fermions) degrees of freedom of the non-minimal scalar multiplet.

At this point we recall for the reader results in 2D, N = 2 superfield theory [8]. Within this class of theories, it is known that there are two *distinct* minimal scalar multiplets, chiral multiplets and twisted chiral multiplets [9]. The superfield form of the linear kinetic term for the twisted chiral multiplet has a minus sign in comparison to that of the chiral multiplet. We see exactly the same behavior above for the 4D chiral and non-minimal superfield actions. In a 2D, N = 2 non-linear σ-model theory with manifest supersymmetry, the only known way to introduce torsion requires the simultaneous presence of both chiral and twisted chiral superfields. In 4D, the analog of the 2D torsion term is provided by the WZNW term. Thus, it is natural to suggest that we should be able to introduce a 4D, N = 1 supersymmetric WZNW term by utilizing both chiral and non-minimal multiplets.

The starting point in the implementation of this proposal is to note that the condition that Σ is a complex linear superfield (i.e. $\overline{D}^2\Sigma = 0$) necessarily implies that the quantity $\overline{D}_{\dot\alpha}\Sigma$ is a chiral superfield and can therefore lead to a supersymmetric invariant in an F-term! So we introduce a number of chiral superfields $\Phi^{\rm I}$ along with an equal number of non-minimal scalar superfields $\Sigma^{\rm I}$ where ${\rm I} = 1, ..., m$. We also require the existence of a fourth order tensor that is a function of the chiral superfields. We denote this tensor by $\mathcal{J}_{\rm IJKL}(\Phi)$. It follows that the term below is a supersymmetric invariant

$$\mathcal{S}_{WZNW} ~=~ \tfrac{1}{4} \int d^4x \, d^2\theta \; \mathcal{J}_{\rm IJKL}(\Phi)(\overline{D}^{\dot\alpha}\Sigma^{\rm I}\,)\,(\overline{D}^{\dot\beta}\Sigma^{\rm J}\,)\,(\partial^\gamma{}_{\dot\alpha}\Phi^{\rm K}\,)\,(\partial_{\gamma\dot\beta}\Phi^{\rm L}\,) ~+~ {\rm h.c.} \quad . \qquad (2.5)$$

Let us note that the most general non-linear σ-model term involving these superfields takes the form,

$$\mathcal{S}_\sigma ~=~ \int d^4x d^2\theta d^2\overline{\theta} \; \widehat{\Omega}(\,\Phi, \overline{\Phi}; \Sigma, \overline{\Sigma}\,) \quad . \qquad (2.6)$$

The function $\widehat{\Omega}$ is similar to a Kähler potential. However, as shown in the the latter work of [4], the metric for the space for which $\Sigma^{\rm I}$ provides coordinates is not of the form of a Kähler metric. In fact, the metric for the $\Sigma^{\rm I}$-space is <u>not</u> even of hermitian form in general. We thus have a counter-example to the well known folklore that 4D, N = 1 supersymmetric non-linear σ-models necessarily describe Kähler manifolds. (The global description of the complex space described solely by Σ-coordinates has never been given.) Note that one special choice[2] of the function $\widehat{\Omega}$ is given by a

[2]The choice of this function is ultimately done to produce the best phenomenological fit.

fibration in which the Σ-coordinates are fibers over a space with the Φ-coordinates. Such a space is described by

$$\widehat{\Omega} \;=\; \tfrac{1}{2}\,[\, g_{\rm I\,J}(\Phi) \;+\; \overline{g}_{\rm J\,I}(\overline{\Phi})\,]\,[\; \overline{\Phi}^{\rm I}\Phi^{\rm J} \;-\; \overline{\Sigma}^{\rm I}\Sigma^{\rm J}\;] \quad , \tag{2.7}$$

in terms of a holomorphic function $g_{\rm I\,J}(\Phi)$ (for one choice of this function see appendix A). In the limit where we set the non-minimal multiplets to zero, we see that the chiral superfields have a special Kähler geometry[3]. The limiting Kähler potential $K(\Phi,\overline{\Phi})$ can be written in the form $K(\Phi,\overline{\Phi}) = \frac{1}{2}[\overline{\Phi}^{\rm I} g_{\rm I\,J}(\Phi)\Phi^{\rm J} + \Phi^{\rm J}\overline{g}_{\rm I\,J}(\overline{\Phi})\overline{\Phi}^{\rm I}]$ in order to make the special Kähler geometry for the chiral superfields manifest. Defining $\widetilde{\Phi}_{\rm I} \equiv g_{\rm I\,J}(\Phi)\Phi^{\rm J}$, we can re-write $K(\Phi,\overline{\Phi})$ in the form $K(\Phi,\overline{\Phi}) \;=\; \frac{1}{2}[\overline{\Phi}^{\rm I}\widetilde{\Phi}_{\rm I} + \Phi^{\rm I}\overline{\widetilde{\Phi}}_{\rm I}]$ in order to make contact with the recent work on exact results for N = 2 supersymmetric Yang-Mills effective actions [10]. This form also makes obvious the presence of the duality pairs $(\Phi^{\rm I},\, \widetilde{\Phi}_{\rm I})$ that are related by elliptic curves.

Let us offer another interpretation of (2.5), (2.7) and (2.8) below. Within the confines of 2D, N = 2 superconformal field theory, there have been found to exist (c,c) rings and (a,c) rings. The former correspond to functions of chiral multiplets while the latter correspond to twisted chiral multiplets. The interesting point is that (a,c) rings were discovered much later than (c,c) rings. This discovery of these distinct supersymmetry representations in the spectrum of the theory occurred even though they were not put in as elementary representations on the 2D world sheet. This example shows us that non-perturbatively supersymmetric systems are able to generate states that are distinct supersymmetry representations from the elementary states. On the other hand, if (a,c) rings are included at the elementary level, then 2D, N = 2 superconformal field theories can possess an additional symmetry, i.e. mirror symmetry. This suggests that the non-minimal scalar multiplet may be generated non-perturbatively in 4D and if they are included in the underlying supersymmetric renormalizable QCD theory, it may possess a larger symmetry group.

Thus, we should be able to embed the QCD low-energy effective action into a supersymmetric action of the form

$$\mathcal{S}_{eff} \;=\; \mathcal{S}_{\sigma} \;+\; \mathcal{S}_{WZNW} \quad . \tag{2.8}$$

In the next section we will look at the component formulation that follows from the proposal above. However, in closing this section, we note that our proposed description of the 4D, N = 1 supersymmetric QCD low-energy effective action with WZNW term is the <u>first</u> that is consistent with holomorphy [10], the concept that holomorphic functions determine the effective action. In fact, we gave the <u>first</u> demonstration

[3] The first appearance of this type of geometry is in [8].

[8] that the 4D, N = 2 supersymmetric Yang-Mills action is classically determined by holomorphic functions. Recently, major advances have occurred in understanding the 4D, N = 2 supersymmetric Yang-Mills effective action due to the presence of holomorphy and proposals have been made that it should play a role in increasing our understanding of the 4D, N = 1 supersymmetric Yang-Mills effective action.

3 Embedding $\mathcal{L}_{eff}(QCD)$ in a 4D, N = 1 Supersymmetric Theory

The calculation of the component results follows using the by now well established projection technique. We find $\mathcal{S}_{WZNW}$ leads to

$$
\begin{aligned}
&\tfrac{1}{4}\int d^4x d^2\theta\ \mathcal{J}_{\rm IJKL}(\Phi)(\overline{D}^{\dot\alpha}\Sigma^{\rm I})(\overline{D}^{\dot\beta}\Sigma^{\rm J})(\partial^{\gamma}{}_{\dot\alpha}\Phi^{\rm K})(\partial_{\gamma\dot\beta}\Phi^{\rm L})\\
&= \tfrac{1}{4}\int d^4x\Big[-\mathcal{J}_{\rm IJKL}(A)\,(i2\partial^{\alpha\dot\alpha}B^{\rm I} - p^{\alpha\dot\alpha\,{\rm I}})(i2\partial_{\alpha}{}^{\dot\beta}B^{\rm J} - p_{\alpha}{}^{\dot\beta\,{\rm J}})(\partial^{\gamma}{}_{\dot\alpha}A^{\rm K})(\partial_{\gamma\dot\beta}A^{\rm L})\\
&\qquad + 2\mathcal{J}_{\rm IJKL}(A)\,(i\partial_{\alpha}{}^{\dot\alpha}\rho^{\alpha\,{\rm I}} - \beta^{\dot\alpha{\rm I}})\,\overline{\zeta}^{\dot\beta\,{\rm J}}(\partial^{\gamma}{}_{\dot\alpha}A^{\rm K})(\partial_{\gamma\dot\beta}A^{\rm L})\\
&\qquad + \mathcal{J}_{\rm IJKL}(A)\,\overline{\zeta}^{\dot\alpha\,{\rm I}}\overline{\zeta}^{\dot\beta\,{\rm J}}[(\partial^{\gamma}{}_{\dot\alpha}\psi^{\alpha{\rm K}})(\partial_{\gamma\dot\beta}\psi_{\alpha}{}^{\rm L}) + 2(\partial^{\gamma}{}_{\dot\alpha}A^{\rm K})(\partial_{\gamma\dot\beta}F^{\rm L})]\\
&\qquad + 4\mathcal{J}_{\rm IJKL}(A)\,(i2\partial^{\alpha\dot\alpha}B^{\rm I} - p^{\alpha\dot\alpha\,{\rm I}})\,\overline{\zeta}^{\dot\beta\,{\rm J}}(\partial^{\gamma}{}_{\dot\alpha}\psi^{\rm K}_{\alpha})(\partial_{\gamma\dot\beta}A^{\rm L})\\
&\qquad + 2\mathcal{J}_{\rm IJKL,M}(A)\,\psi^{\alpha{\rm M}}\overline{\zeta}^{\dot\alpha\,{\rm I}}\overline{\zeta}^{\dot\beta\,{\rm J}}(\partial^{\gamma}{}_{\dot\alpha}\psi^{\rm K}_{\alpha})(\partial_{\gamma\dot\beta}A^{\rm L})\\
&\qquad - 2\mathcal{J}_{\rm IJKL,M}(A)\,\psi^{\alpha{\rm M}}(i2\partial^{\alpha\dot\alpha}B^{\rm I} - p^{\alpha\dot\alpha\,{\rm I}})\,\overline{\zeta}^{\dot\beta\,{\rm J}}(\partial^{\gamma}{}_{\dot\alpha}A^{\rm K})(\partial_{\gamma\dot\beta}A^{\rm L})\\
&\qquad + \mathcal{J}_{\rm IJKL,M}(A)\,F^{\rm M}\overline{\zeta}^{\dot\alpha\,{\rm I}}\overline{\zeta}^{\dot\beta\,{\rm J}}(\partial^{\gamma}{}_{\dot\alpha}A^{\rm K})(\partial_{\gamma\dot\beta}A^{\rm L})\Big] \quad .
\end{aligned}
\tag{3.1}
$$

As can be seen, only the first line of the rhs consists of purely bosonic terms. Let us focus our analysis by only considering these terms.

It is our first observation that if we set the auxiliary field $p_{\underline{a}}$ to zero, then the purely bosonic terms collapse to

$$
\begin{aligned}
&\tfrac{1}{4}\int d^4x\, d^2\theta\ \mathcal{J}_{\rm IJKL}(\Phi)\,(\overline{D}^{\dot\alpha}\Sigma^{\rm I})\,(\overline{D}^{\dot\beta}\Sigma^{\rm J})\,(\partial^{\gamma}{}_{\dot\alpha}\Phi^{\rm K})\,(\partial_{\gamma\dot\beta}\Phi^{\rm L})|_{phys.\ fields}\\
&\qquad = \int d^4x\Big[\mathcal{J}_{\rm IJKL}(A)\,(\partial^{\alpha\dot\alpha}B^{\rm I})\,(\partial_{\alpha}{}^{\dot\beta}B^{\rm J})\,(\partial^{\gamma}{}_{\dot\alpha}A^{\rm K})\,(\partial_{\gamma\dot\beta}A^{\rm L})\Big] \qquad (3.2)\\
&\qquad = \int d^4x\Big[\mathcal{J}_{\rm IJKL}(A)\,{\rm P}^{\underline{abcd}}\,(\partial_{\underline{a}}B^{\rm I})\,(\partial_{\underline{b}}B^{\rm J})\,(\partial_{\underline{c}}A^{\rm K})\,(\partial_{\underline{d}}A^{\rm L})\Big] \quad .
\end{aligned}
$$

where ${\rm P}^{\underline{abcd}} \equiv [\eta^{\underline{a}[\underline{c}}\eta^{\underline{d}]\underline{b}} + i\epsilon^{\underline{abcd}}]$. Up until this point, we have not made any assumption regarding the explicit form of $\mathcal{J}_{\rm IJKL}(A)$. We could easily choose it to be the (4,0) form that is defined in the non-supersymmetric component WZNW action (see appendix A). However, (3.2) has the consequence that it can describe both the WZNW term as well as the Skyrme term. In the following we we simply concentrate on the WZNW term and thus we choose $\mathcal{J}_{\rm IJKL}(A)$ to be define by (A.6)[4]. Since $p_{\underline{a}}$ actually has a more complicated equation of motion that depends on the leading term of the effective action, its elimination will produce other higher order interactions. However, their presence does not disturb our present results. These and a number of other details will be discussed in a future work.

Now we want the component pion fields that are contained in our QCD superfield WZNW term of (2.8) to agree precisely the non-supersymmetric QCD effective action (see (A.7)). This will be the case if the following identifications are made,

$$\Phi| ~=~ \mathcal{A}(x) ~+~ i\,[\,\Pi(x) ~+~ \Theta(x)\,] ~~,~~ \Sigma| ~=~ \mathcal{B}(x) ~+~ i\,[\,\Pi(x) ~-~ \Theta(x)\,] ~~. \tag{3.3}$$

where $\Pi(x)$ is the pion octet. Thus, we see that the pion superfield is a linear mixture of chiral and complex linear superfields. This is analogous to the fact that a Dirac field in a supersymmetric theory can only occur as a linear combination of basic superfields. We are thus motivated to define the super-pion superfield by

$$\Pi ~\equiv~ -i\tfrac{1}{4}\Big[\,\Phi ~+~ \Sigma ~-~ \overline{\Phi} ~-~ \overline{\Sigma}\,\Big] ~~. \tag{3.4}$$

By the same token we see that in a manifestly supersymmetric world, in addition to the super-pion, there are mirror super-pions defined by

$$\Theta ~\equiv~ -i\tfrac{1}{4}\Big[\,\Phi ~-~ \Sigma ~-~ \overline{\Phi} ~+~ \overline{\Sigma}\,\Big] ~~. \tag{3.5}$$

There are also parity doubles of these fields that are most conveniently defined by

$$\mathcal{A} ~\equiv~ \tfrac{1}{2}\Big[\,\Phi ~+~ \overline{\Phi}\,\Big] ~~,~~ \mathcal{B} ~\equiv~ \tfrac{1}{2}\Big[\,\Sigma ~+~ \overline{\Sigma}\,\Big] ~~. \tag{3.6}$$

Similarly, applying various spinor derivatives to these superfields produce the spin-1/2 pionino SU(3) multiplet and their parity doubles. Here we have some ambiquity. We have enough spinor components to form a Dirac pionino SU(3) multiplet or two Majorana pionino SU(3) multiplets. In the former case, the pionini are isomorphic to the baryon octet that contains the proton!

The leading term in (2.8) will also contain exactly the leading term of (A.7) if we identify the function $g_{\rm IJ}$[5] that appears in (2.7) with that defined in (A.4). Thus, we find that there is a very simple embedding of $\mathcal{L}_{eff}(QCD)$ into our superfield theory.

[4] A few minor modifications are required in the supersymmetric case.

[5] Once again a few minor modifications are required in the supersymmetric case.

4 Conclusion

At this point, it is useful to compare our new suggestion for a 4D, N = 1 supersymmetric extension of the WZNW terms to those that exist in the prior literature. The relevant work occurred in reference [6]. There it was proposed that the 4D, N = 1 supersymmetric extension of the WZNW term is of the form

$$\mathcal{S}_{WZNW} \;=\; \int d^4x d^2\theta d^2\overline{\theta}\, \Big[\beta_{\rm I\, J\bar{K}}(D^{\alpha}\Phi^{\rm I}\,)(\partial_{\alpha\dot{\beta}}\Phi^{\rm J}\,)(\overline{D}^{\dot{\beta}}\overline{\Phi}^{\bar{\rm K}}\,) \;+\; {\rm h.c.} \Big] \quad . \tag{4.1}$$

If we compare our results to the older ones, several features are apparent. Foremost, the previous result utilizes an action that is integrated over the full superspace. (This means for example that all of the chiral superfields contained in (4.1) could be replaced by complex linear superfields and we would then obtain another WZNW-type term.) In particular, the quantity $\beta_{\rm I\,J\bar{K}}$ is <u>not</u> holomorphic. Our choice need only be integrated over a chiral subspace due to its chirality (i.e. holomorphicity). At the level of component fields, the differences are simply tremendous! Our suggestion contains many fewer terms. At most four fermion but not six fermion terms appear in our construction in contrast to (4.1). Finally, there are terms in (4.1) that are quartic in temporal derivatives of bosonic fields. In our proposal no such terms of this high order in temporal derivatives appear. This last point is rather telling. It is certainly true that the non-supersymmetric WZNW terms contains no more than first order temporal derivatives.

Ordinary 4D, N = 1 chiral and non-minimal multiplets possess an uncanny resemblance to 2D, N = 2 chiral and twisted chiral multiplets. This naturally raises the question of whether there might exist some 4D, N = 1 analog to mirror symmetry. We could formally define a 4D mirror operator that sends chiral multiplets into non-minimal multiplets and vice-versa. There are important differences, however. Off-shell chiral and non-minimal multiplets do not possess the same number of degrees of freedom. So there are some issues that require additional study. Finally, we believe that our result regarding the simple embedding of $\mathcal{L}_{eff}(QCD)$ should act as a warning that the sole use of chiral multiplets to describe matter is not always wise. We re-emphasize the cautionary note we made along these lines previously in the second work of [4].

The problem our presentation demonstrates has its ultimate cause in our lack of mastery of string theory. As presently formulated, we simply do not possess a direct (i.e. without making any assumptions) way to derive from string theory the off-shell superfields that presumably emerge in its low-energy limit. For some time, we have believed that it is quite likely that non-minimal scalar multiplets must be involved in this limit. Our reason for this belief is that it appears likely that the 4D, N = 2 low-energy limit of string theory contains at least some non-minimal scalar multiplets! The only known off-shell formulation of 4D, N = 2 hypermultiplets [11] contains 4D, N = 1 non-minimal scalar multiplets. Finally, it is interesting to ponder further WZNW extensions to 4D, N = 2 supersymmetry. The recent advances [10] are silent on the 4D, N = 2 WZNW term. Here we would like to know if the *two* distinct 4D, N = 2 hypermultiplets ([11] and [12]) play roles analogous to that of the 4D, N = 1 chiral and non-minmal multiplets in the 4D, N = 1 WZNW term.

This latest result together with the "natural Dirac mass" associated with a pairing of a chiral superfield together with a complex linear superfield (i.e. $(\Phi^{\rm I}, \Sigma^{\rm I}))$ seems to be hinting that there is something truly fundamental but not understood occurring. As we noted previously, the current generation of supersymmetric phenomenological models totally ignores the possibility that ordinary matter may contain such pairings. We can well imagine scenarios in which one chiral part of a Dirac particle is assigned to chiral superfields and the other chiral part of the same Dirac particle is assigned to complex linear superfields. This might well serve as an intrinsic reason why chiral asymmetry occurs in supersymmetric extensions of the standard model and as well could easily provide the long sought use of supersymmetry to protect the vanishing masses of neutrini. Indeed, if supersymmetry is observed in Nature this could make an attractive explanation for why handedness matters in our universe!

Acknowledgment;
I wish to thank Ms. Lubna Rana for useful discussions.

Appendix A: Brief Review of $\mathcal{L}_{eff}(QCD)$

In this very brief appendix we simply gather together the basic facts concerning the low-energy effective action for QCD. We begin with a definition of the SU(3) pion octet

$$\frac{1}{f_\pi}\Pi \;\equiv\; \frac{1}{f_\pi}\Pi^i\lambda_i \;=\; \frac{1}{f_\pi}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -\eta\sqrt{\frac{2}{3}} \end{pmatrix} \quad . \qquad (A.1)$$

Here λ_1, ..., λ_8 are the Gell-Mann SU(3) matrices. Further f_π is the weak pion coupling constant[6] with the dimensions of mass. Group elements are formed by writing $U(\Pi) = \exp[\, i f_\pi{}^{-1}\Pi\,]$. We define left $(L_m{}^i(\Pi))$ and right $(R_m{}^i(\Pi))$ Maurer-Cartan forms by the equations

$$U^{-1}\partial_{\underline{a}}U \;=\; i f_\pi{}^{-1}(\,\partial_{\underline{a}}\Pi^m\,)\, L_m{}^i(\Pi)\,\lambda_i \quad , \quad (\,\partial_{\underline{a}}U\,)U^{-1} \;=\; i f_\pi{}^{-1}(\,\partial_{\underline{a}}\Pi^m\,)\, R_m{}^i(\Pi)\,\lambda_i \quad . \qquad (A.2)$$

These definitions allow $L_m{}^i(\Pi)$ and $R_m{}^i(\Pi)$ to be calculated as power series in Π^i from [14]

$$\begin{aligned} L_m{}^i(\Pi) &\equiv\; (C_2)^{-1}\mathrm{Tr}\Big[\,T^i\Big(\frac{1-e^{-\Delta}}{\Delta}\Big)T_m\Big] \quad , \\ R_m{}^i(\Pi) &\equiv\; (C_2)^{-1}\mathrm{Tr}\Big[\,T^i\Big(\frac{e^{\Delta}-1}{\Delta}\Big)T_m\Big] \quad , \end{aligned} \qquad (A.3)$$

where $\Delta T_m \equiv i f_\pi{}^{-1}[\Pi\,,\,T_m]$, $\Delta^2 T_m = \Delta\Delta T_m$, etc. and the constant C_2 is determined so that $L_m{}^i(0) = R_m{}^i(0) = \delta_m{}^i$. As a consequence we see

$$\mathcal{S}_\sigma \;=\; \frac{f_\pi^2}{2C_2}\int d^4x\ \mathrm{Tr}[\,(\partial^{\underline{a}}U\,)\,(\partial_{\underline{a}}U^{-1}\,)\,] \;=\; \frac{1}{2}\int d^4x\ \, g_{m\,n}\,(\Pi)\,(\partial^{\underline{a}}\Pi^m\,)\,(\partial_{\underline{a}}\Pi^n\,) \quad , \qquad (A.4)$$

where $g_{m\,n} = \delta_{i\,j}\,L_m{}^i\,L_n{}^j = \delta_{i\,j}\,R_m{}^i\,R_n{}^j$.

The remaining well known term in the QCD effective action is described by the WZNW term. We follow Witten [5] who, using the Vainberg technique [15], showed that with an appropriate normalization this term possesses an integer quantized coefficient, N_C. Using a real parameter y that takes on values between 0 and 1, we define an extended group element $\widehat{U}$ through the relation $\widehat{U} = \exp[\, i y f_\pi^{-1}\Pi\,]$. In terms of the extended group element, the WZNW term is given by

$$\begin{aligned} \mathcal{S}_{WZNW} &=\; -iN_C\,[\,2\cdot 5!\,]^{-1}\int d^4x\int_0^1 dy\ \mathrm{Tr}\Big[\,(\widehat{U}^{-1}\partial_y\widehat{U}\,)\,\widehat{\mathcal{W}}_4\,\Big] \quad , \\ \widehat{\mathcal{W}}_4 &=\; \epsilon^{\underline{abcd}}\,(\partial_{\underline{a}}\widehat{U}^{-1}\,)\,(\partial_{\underline{b}}\widehat{U}\,)\,(\partial_{\underline{c}}\widehat{U}^{-1}\,)\,(\partial_{\underline{d}}\widehat{U}\,) \quad . \end{aligned} \qquad (A.5)$$

[6]It should be noted that we differ from Witten's convention of this parameter by a factor of two.

or more directly using the elements of the pion octet this just becomes

$$
\begin{aligned}
\mathcal{S}_{WZNW} &= \int d^4x \, \epsilon^{\underline{abcd}} \mathcal{J}_{m\,n\,r\,s}(\Pi)(\partial_{\underline{a}}\Pi^m)(\partial_{\underline{b}}\Pi^n)(\partial_{\underline{c}}\Pi^r)(\partial_{\underline{d}}\Pi^s) \quad , \\
\mathcal{J}_{m\,n\,r\,s}(\Pi) &= -N_C[\,8 \cdot 5! f_\pi^5\,]^{-1} f_{a\,b}{}^k \, f_{c\,d}{}^l \mathrm{Tr}\Big[\,\lambda_k\lambda_l\lambda_h\,\Big] \int_0^1 dy \; y^4 \, \Pi^e \widehat{L}_e{}^h \widehat{L}_m{}^a \widehat{L}_n{}^b \widehat{L}_r{}^c \widehat{L}_s{}^d \quad .
\end{aligned} \tag{A.6}
$$

where $\widehat{L}_m{}^i \equiv L_m{}^i(y\Pi)$. Also $f_{a\,b}{}^k$ denotes the structure constants of the group defined by $[\lambda_a, \lambda_b] = i f_{a\,b}{}^k \lambda_k$. The effective QCD Lagrangian is simply given

$$
\mathcal{S}_{eff} \;=\; \mathcal{S}_\sigma \;+\; \mathcal{S}_{WZNW} \tag{A.7}
$$

with $\mathcal{S}_\sigma$ defined in (A.3) and $\mathcal{S}_{WZNW}$ defined in (A.6).

Appendix B: Manifest Supersymmetric Formulation of Kazama-Suzuki Models and New (2,2) Superstrings

In heterotic string theory, one of the well known N = 2 compactification techniques is give by Kazama-Suzuki models [13]. An erstwhile mystery has been, "How does one find a superfield formulation of Kazama-Suzuki models?" Up until now no one has been able to provide an answer. We now wish to suggest that the missing ingredient seems to have been the use of the non-minimal scalar multiplet reduced from 4D, N = 1 superspace down to 2D, N = 2 superspace. The reduction itself is trivial if we introduce the 2D, N = 2 supercovariant derivatives D_α and their conjugates $\overline{D}_\alpha$ which satisfy

$$
\Big[\, D_\alpha, \; D_\beta \,\Big\} \;=\; 0 \quad , \quad \Big[\, \overline{D}_\alpha, \; \overline{D}_\beta \,\Big\} \;=\; 0 \quad , \quad \Big[\, D_\alpha, \; \overline{D}_\beta \,\Big\} \;=\; i(\gamma^c)_{\alpha\beta}\partial_c \quad . \tag{B.1}
$$

The 2D, N = 2 non-minimal multiplet is now defined by $\overline{D}^\alpha \overline{D}_\alpha \Sigma = 0$. The component fields are defined with a few very slight modifications (below we use the chiral components)

$$
\begin{aligned}
& B \equiv \Sigma| \quad , \quad \overline{\zeta}_\pm \;\equiv\; \overline{D}_\pm \Sigma| \quad , \quad \rho_\pm \;\equiv\; D_\pm \Sigma| \quad , \quad H \;\equiv\; -i\, D_+ D_- \Sigma| \quad , \\
& u \equiv \; -i\, \overline{D}_+ D_- \Sigma| \quad , \quad v \;\equiv\; -i\, \overline{D}_- D_+ \Sigma| \quad , \quad p_{\pm\!\!\!\!+} \;\equiv\; -i\, \overline{D}_+ D_+ \Sigma| \quad , \\
& p_{=} \;\equiv\; -i\, \overline{D}_- D_- \Sigma| \quad , \quad \overline{\beta}_\pm \;\equiv\; -i\, D_+ \overline{D}_\pm D_- \Sigma| \quad .
\end{aligned} \tag{B.2}
$$

The complex quantities u and v are the extra components arising from the dimensional reduction of $p_{\underline{a}}$ from 4D. The 2D supersymmetry variations take the forms

$$
\begin{aligned}
\delta_Q B &= \bar{\epsilon}^+ \bar{\zeta}_+ \; + \; \bar{\epsilon}^- \bar{\zeta}_- \; + \; \epsilon^+ \rho_+ \; + \; \epsilon^- \rho_- \quad , \\
\delta_Q \bar{\zeta}_+ &= i\, \epsilon^+ (\partial_{\ddagger} B \; - \; p_{\ddagger}) \; - \; i\, \epsilon^- u \quad , \\
\delta_Q \bar{\zeta}_- &= \; - \, i\, \epsilon^+ v \; + \; i\, \epsilon^- (\partial_{=} B \; - \; p_{=}) \quad , \\
\delta_Q \rho_+ &= -i\, \epsilon^- H \; + \; i\, \bar{\epsilon}^+ p_{\ddagger} \; + \; i\, \bar{\epsilon}^- v \quad , \\
\delta_Q \rho_- &= i\, \epsilon^+ H \; + \; i\, \bar{\epsilon}^+ u \; + \; i\, \bar{\epsilon}^- p_{=} \quad , \\
\delta_Q u &= \epsilon^+ \beta_+ \; - \; \bar{\epsilon}^- \partial_{=} \bar{\zeta}_+ \quad , \\
\delta_Q v &= \epsilon^- (\partial_{=} \rho_+ \; - \; \beta_-) \; - \; \bar{\epsilon}^+ \partial_{\ddagger} \bar{\zeta}_- \quad , \\
\delta_Q H &= - \, i\, \bar{\epsilon}^+ (\partial_{\ddagger} \rho_- \; - \; \beta_+) \; - \; \bar{\epsilon}^- \beta_- \quad , \\
\delta_Q p_{\ddagger} &= \epsilon^+ \partial_{\ddagger} \rho_+ \; + \; \epsilon^- (\partial_{\ddagger} \rho_- \; - \; \beta_+) \; + \; \bar{\epsilon}^- \partial_{\ddagger} \bar{\zeta}_- \quad , \\
\delta_Q p_{=} &= \epsilon^+ \beta_- \; + \; \epsilon^- \partial_{=} \rho_- \; + \; \bar{\epsilon}^+ \partial_{=} \bar{\zeta}_+ \quad , \\
\delta_Q \beta_+ &= i\, \bar{\epsilon}^+ \partial_{\ddagger} u \; + \; i \bar{\epsilon}^- \partial_{=} (\partial_{\ddagger} B \; - \; p_{\ddagger}) \quad , \\
\delta_Q \beta_- &= -i\, \epsilon^- \partial_{=} H \; - \; i \bar{\epsilon}^+ (\partial_{=} \partial_{\ddagger} B \; - \; \partial_{=} p_{\ddagger} \; - \; \partial_{\ddagger} p_{=}) \quad .
\end{aligned}
\qquad (B.3)
$$

Finally for the superfield action that should act as the starting point for the 2D (2,2) Kazama-Suzuki models we propose

$$
\begin{aligned}
\mathcal{S}_{KS} \; = \; & \int d^2\sigma \, d^2\zeta \, d^2\bar{\zeta} \; \widehat{\Omega}(\Phi, \, \bar{\Phi}; \Sigma, \, \bar{\Sigma} \,) \\
& + \; \Big[\int d^2\sigma \, d^2\zeta \; \mathcal{J}_{\rm I \, J}(\Phi) \, (\bar{D}_+ \Sigma^{\rm I} \,) \, (\bar{D}_- \Sigma^{\rm J} \,) \; + \; {\rm h.\, c.} \; + \; ... \Big] \quad .
\end{aligned}
\qquad (B.4)
$$

The terms in the ellipsis represent the introduction of world sheet 2D, N = 1 gauge superfields for the H sub-group in the K-S constructions. In (B.4) the potential $\widehat{\Omega}(\Phi, \, \bar{\Phi}; \Sigma, \, \bar{\Sigma} \,)$ is most likely given by (2.7) with $g_{\rm I \, J}$ constructed from the Maurer-Cartan forms as in (A.4) and $\mathcal{J}_{\rm I \, J}(\Phi)$ is given by

$$
\mathcal{J}_{\rm I \, J}(\Phi) \;\; = \;\; - \, c_0 f_{\rm K \, L \, M} \int_0^1 dy \; y^2 \, \Phi^{\rm N} \widehat{L}_{\rm N}{}^{\rm K} \widehat{L}_{\rm I}{}^{\rm L} \widehat{L}_{\rm J}{}^{\rm M} \quad . \qquad (B.5)
$$

Here the Maurer-Cartan forms are defined in terms of the chiral superfields and the group is arbitrary. However, the final arbiter that determines these functions is 2D, N = 2 superconformal invariance. This is a topic to be studied in the future. Thus, we see for every compact group, there exist a way to construct a 2D, N = 2 action that possesses manifest supersymmetry. Let us emphasize that (2.7) is an explicit construction that associates with every group manifold with metric $g_{\rm I \, J}$ (constructed from the group Maurer-Cartan forms) a special Kähler geometry with a metric whose potential is given by (2.7). To our knowledge this is the first observation relating group manifolds to special Kahler geometry in this manner. It will be of interest to see if the condition of quantum superconformal invariance acts as a restriction to the choices considered by Kazama and Suzuki. Finally, we note that there must exists twisted versions of the action of (B.5). That is the chiral superfields in (B.5) can be replaced by twisted chiral superfields, if simultaneously we replace the complex linear superfields by twisted complex linear superfields, Ξ, (i.e. $\Sigma \to \Xi$ where Ξ satisfies $\overline{D}_+ D_- \Xi = 0$).

Let us be explicit, we expect a subclass of the actions of (B.4) to describe a fundamentally new class of 2D, N = 2 superstrings. As long ago as 1989, we reported that at the level of superfields[7] there were at least three different N = 2 superstring actions. One of these, which actually has an N = 4 rigid supersymmetry (one chiral plus one twisted chiral multiplet) is known to permit a non-trivial axion background unlike the other two version. However, the axion occurs as the second derivative of a potential. Our new theories are not subject to this constraint. So we believe with (B.4) we have yet again increased the number of known 2D, N = 2 superstrings. These new N = 2 superstrings are associated with different choices of auxiliary fields. So even for string theory we have evidence that auxiliary fields matter...a point totally absent in superconformal field theory.

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