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Geometrobremsstrahlung in the Early Universe

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Abstract

We discuss photon emission from particles decelerlated by the cosmic expansion. Its transition probability does not depend on detailed behavior of the expansion and possesses a new-type conformal anomaly. Moreover the process forces gravitinos with helicity 1/2 to decay during the epoch when they are relativistic.

1 Introduction

Particles in the expanding Robertson-Walker Universe are decelerated in the comoving frame. Ignoring backreaction, the particles obey the geodesic equation and lose their physical momentum P_{phys} like

$$P_{phys} = \frac{P_{conf}}{a} \to 0, \tag{1}$$

where P_{conf} is a conserved conformal momentum and a is a scale factor growing in time. In general the particles in deceleration can emanate radiation, or some massless particles. We call this process geometrobremsstrahlung[1] due to the cosmic expansion.

Many analyses on phenomena in the early Universe have been performed so far using results of high energy particle physics and proposed a lot of interesting features of the Universe[1]. However they are based on calculations of transition matrices in the flat spacetime, emphasizing the fact that rates of interactions often argued are much larger than the Hubble parameter, and no attention seems to be paid to the geometrobremsstrahlung process that really gives no contribution in the flat spacetime.

We shall argue in this paper that the geometrobremsstrahlung may play impotant roles in the early Universe. In section 2, emission of electromagnetic wave from a charged particle in the expanding Universe is discussed classical mechanically, taking account of backreaction. It is shown that classical emission rate may not be simply ignored and the damping time nearly equals to expansion time. In section 3, the photon emission from charged particles is treated quantum mechanically. High momentum limit of the transition probability can be obtained analytically and we also point out that there is a new type of conformal anomaly, where massless limit should be treated carefully and is nontrivial. In section 4, we apply the geometrobremsstrahlung to gravitino decay processes in the early Universe. A prominent enhancement appears in the decay probablility of gravitino especially with helicity 1/2 into a photon and a photino. On the other hand, the inverse geometrobremsstrahlung processes are highly suppressed by the thermal nature of the photons and photinos. By virtue of this mechanism, the helicity 1/2 compo-

[1]This process is also called by DeWitt and Brehme electro-gravitic bremsstrahlung in ref [3].

nent of gravitino produced at the temperature higher than $10^{10} - 10^{10.5}$ GeV can be washed out during the epoch when the gravitino is relativistic.

In this paper, we adopt the natural units, the light velocity $c = 1$ and the Planck constant $\hbar = 1$. Signature of metric is taken as $(+, -, -, -)$.

2 Classical Geometrobremsstrahlung in the early Universe

The radiation reaction has been neglected in the study of the early Universe. However particles are deaccelerated due to the cosmic expansion and thus they will emit the radiation. If the damping time due to the radiation reaction is comparable to the expansion time, the effect of the radiation reaction may not be simply neglected. We shall study in detail this phenomenon in the case of classical charged particles.

The study of the radiation reaction for a charged particle has a long history. The first relativistic calculation was done by Dirac[2]. His calculation has been generalized by DeWitt-Brehme[3] for the motion in gravitational field. They have shown that bremsstrahung induced by the spacetime curvature which we call geometrobremsstrahlung occured in addition to the usual radiation damping. The effect is nonlocal in general which is caused by the so-called tail term in the Green function. It was Hobbs[4] who corrected the result of De Witt-Brehme and pointed out that the tail term vanishes identically in the case of the conformally flat spacetimes. His equation of motion for a particle with 4 velocity u^μ, mass m, charge e without external electromagnetic field may be written in the following form in conformally flat spacetime.

$$m\frac{Du^\mu}{D\tau} = \frac{2e^2}{3}\left(\frac{D^2u^\mu}{D\tau^2} + u^\mu\left(\frac{Du}{D\tau}\right)^2\right) + \frac{2e^2}{3}(\Omega_{,\alpha\beta} - \Omega_{,\alpha}\Omega_{,\beta})\left(g^{\mu\alpha}u^\beta - u^\mu u^\alpha u^\beta\right) \quad (2)$$

where $D/D\tau$ is the absolute derivative along the worldline of the particle with τ the proper time and the $\exp(2\Omega)$ is the conformal factor, $g_{\mu\nu} = e^{2\Omega}\eta_{\mu\nu}$ with $\eta_{\mu\nu}$ the flat Minkowskii metric.

Here we are interested in the radiation reaction induced by the cosmic expansion in the early Universe and thus we will restrict ourselves to the case where the conformal factor depends only on the time variable. We shall take a different approach from Hobbs and evaluate explicitly the damping time scale due to the geometrobremsstrahlung in the early Universe.

We shall take the standard form for the action of a charged particle with mass m and charge e in the gravitational field,

$$S = -m\int\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}d\tau - e\int A_\mu\dot{x}^\mu d\tau - \frac{1}{4}\int d^4x\sqrt{-g}g^{\alpha\beta}g^{\mu\nu}F_{\alpha\mu}F_{\beta\nu}.$$

The dot denotes the derivative with respect to the proper time τ. The equation of motion derived from the above action may be written as follows.

$$m\frac{Du^\mu}{D\tau} = m\left(\frac{du^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta}u^\alpha u^\beta\right) = eF^{\mu\nu}u_\nu.$$

Since we are interested in the early Universe, we may neglect the spatial curvature and thus take the spatially flat Robertson-Walker model as our background geometry,

$$ds^2 = a^2(\eta)(d\eta^2 - d\vec{x}^{\ 2}) = dt^2 - a^2 d\vec{x}^{\ 2}.$$

Since the metric is conformally flat, it is convenient to work in the conformally related flat spacetime. Defining the conformally related proper time $d\tau_f = a^{-1}d\tau$, we shall define the conformally related 4 velocity

$$\tilde{u}^\mu = \frac{dx^\mu}{d\tau_f}.$$

Then the equation of motion may be written as follows.

$$m\frac{Du^\mu}{D\tau} = ma^{-2}\left(\frac{d\tilde{u}^\mu}{d\tau_f} + \frac{a'}{a}\frac{d\eta}{d\tau_f}\tilde{u}^\mu - \frac{a'}{a}\delta^\mu_0\right) = ea^{-3}\eta^{\mu\alpha}F_{\alpha\beta}\tilde{u}^\beta.$$

where the prime denotes the derivative with respect to the conformal time η. By using the self field of the particle in the right hand side of the above equation, we shall obtain the radiation reaction force.

Before calculating the reaction force explicitely, let us compare the timescale between the radiation damping and the cosmic expansion to see the importance of the radiation reaction in the early Universe. The damping time may be roughly evaluated as follows.

$$\frac{1}{t_r} \sim \left|\frac{1}{E_{conf}}\frac{dE_{conf}}{ad\eta}\right| \sim \frac{1}{p_{conf}}\frac{2}{3a}e^2\left(\frac{d\tilde{u}}{d\eta}\right)^2 \sim \frac{2}{3}e^2\frac{p_{phys}H^2}{m^2}$$

where

$$H = \frac{1}{a}\frac{da}{dt}$$

is the Hubble parameter, p_{conf} is the conformal momentum and we have used the fact that the physical momentum $p_{phys} = m\tilde{u} = p_{conf}/a$ decays as a^{-1} if the radiation reaction is neglected. Thus the ratio between the Hubble time $t_{exp} = H^{-1}$ and the damping time is

$$\frac{t_{exp}}{t_r} \sim \frac{2e^2}{3m^2}p_{phys}H.$$

This ratio is much larger than the unity for a relativistic particle at sufficiently early times in the Universe. Thus the radiation reaction may not be simply ignored and might play an important role in the early Universe.

For the calculation of the reaction force, we shall need the field equation derived from the above action,

$$\eta^{\mu\nu}\eta^{\alpha\beta}F_{\alpha\nu,\beta} = e\int d\tau_f\delta^4(x - x(\tau_f))\tilde{u}^\mu.$$

Taking the following non-covariant gauge

$$\eta^{\mu\nu}A_{\mu,\nu} = 0,$$

we arrive at the field equation which has the same form with that in the flat spacetime,

$$\eta^{\alpha\beta}A^\mu_{,\alpha\beta} = e\int d\tau_f\tilde{u}^\mu\delta^4(x - x(\tau_f)).$$

Then the calculation by Dirac [2] applies here and we obtain the standard expression for the reaction force in the flat spacetime,

$$F^\mu_{react} = e\ \eta^{\mu\alpha}F_{\alpha\beta}\tilde{u}^\beta = \frac{2}{3}e^2\left[\frac{d^2\tilde{u}^\mu}{d{\tau_f}^2} + \left(\frac{d\tilde{u}}{d\tau_f}\right)^2\tilde{u}^\mu\right].$$

It can be shown by a direct calculation that our expression of the equation of motion with radiation reaction in the conformally related flat spacetime coincides with eqn(2) when transformed back to the original physical frame.

In order to see the effect of the radiation reaction explicitly, we shall forcus our attention to an 1-dimensional motion. Then the above equation is simplified as

$$\frac{d}{d\tau_f}(am\tilde{u}) = \frac{2}{3}e^2\left[\frac{d^2\tilde{u}}{d\tau_f{}^2} - \frac{\tilde{u}}{1+\tilde{u}^2}\left(\frac{d\tilde{u}}{d\tau_f}\right)^2\right].$$

Without the radiation reaction, the conformal momentum $p_{conf} = am\tilde{u}$ is conserved as expected.

Now we shall rewrite the above equation using the conformal momentum p_{conf} and the background time $dt = ad\eta$,

$$\frac{d^2 p_{conf}}{dt^2} = \left(H + \frac{3m}{2e^2\sqrt{1+(p_{phys}/m)^2}}\right)\frac{dp_{conf}}{dt} + \frac{dH}{dt}p_{conf}. \qquad (3)$$

Notice that there will be no geomotrobremsstrahlung in the case of de Sitter expansion, namely $H = const$. We shall be interested in the relativistic case in the early universe, namely

$$p_{phys} \gg \frac{3m^2}{2e^2H}, \quad m.$$

Then the second term in the coefficient of dp_{conf}/dt in eqn(3) is negligible. Thus when the particle is relativistic, its evolution is governed by the reaction force only and the Hubble time will be the only available time scale in this situation. In fact, the solution in this case may be written as follows.

$$p_{conf}(t) = p_0\left(1 - H(t_0)\int_{t_0}^{t} dt' \exp\left(-\int_{t_0}^{t'} H(x)dx\right)\right)\exp\left(\int_{t_0}^{t} H(t')dt'\right)$$

where we have taken the following initial conditions;

$$p_{conf}(t = t_0) = p_0, \quad \frac{dp_{conf}}{dt}(t = t_0) = 0$$

The second condition expresses the fact that the reaction force is absent at the initial time. The solution shows that the momentum decays in the Hubble time. Thus this process should not be simply neglected. However the above conclusion is obtained as a classical effect and it is not clear if the geometrobremsstrahlung is still effective when the quantum effect is taken into account. We shall discuss quantum geometrobremsstrahlung in the next section.

3 Universality and Conformal Anomaly in Photon Emission

As argued in the section 2, the geometrobremsstrahlung may work classical mechanically in the expanding Universe, interacting with the spatially uniform curvature. Meanwhile whether this notable process survives or not after taking quantum effects into account is nontrivial and this question shall be discussed next.

To define well-behaving quantum amplitudes in the expanding Universe, we consider spacetimes with Minkowskian in and out regions. The way of expansion is chosen arbitrary. The scale factor is described as

$$a(\eta) = C(\lambda\eta) \tag{4}$$

where η is the conformal time, λ^{-1} is a constant exhibiting typical time scale of expansion. The function $C(x)$ in eqn(4) is arbitray except the following constraints,

$$C(-\infty) = b, \tag{5}$$
$$C(\infty) = 1, \tag{6}$$
$$C(x) > 0, \tag{7}$$

where b is some positive constant smaller than 1.

Consider first photon emission in the massive scalar QED with conformal coupling to the background curvature. The action reads

$$S = \int d^4x \quad \sqrt{-g} \quad \Big((\nabla_\mu + ieA_\mu)\Phi^*(\nabla^\mu - ieA^\mu)\Phi + (\frac{1}{6}R - m^2)\Phi^*\Phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\Big) .$$

The photon emission process is prohibited in the flat spacetime by energy-momentum conservation. However in the expanding spacetimes the energy conservation law gets broken and the transition can take place. The transition amplitude is given in the lowest order pertubation such that

$$Amp = -ie \int d^4x \sqrt{-g} i \left(\Phi_f^* \nabla^\mu \Phi_i - \nabla^\mu \Phi_f^* \Phi_i \right) A_\mu^* \tag{8}$$

where $\Phi_i(\Phi_f)$ is initial(final) mode function of massive charged scalar field and A_μ^* is the final mode function of electromagnetic field. The scalar mode functions satisfy

$$\left(\nabla^2 + m^2 - \frac{1}{6} R \right) \Phi = 0. \tag{9}$$

Redefining the field $\tilde{\Phi} = \Phi \cdot a(\eta)$, the wave equation becomes the Klein-Gordon equation with a time-dependent mass,

$$\left(\partial^2 + m^2 a(\eta)^2 \right) \tilde{\Phi} = 0. \tag{10}$$

Here we introduce $g_{\vec{p}_i}^{in}(\eta)$ and $g_{\vec{p}_f}^{out}(\eta)$ satisfying a Schrödinger-type equation,

$$\left[-\frac{d^2}{d\eta^2} - m^2 a(\eta)^2 \right] g_{\vec{p}} = \vec{p}^{\,2} g_{\vec{p}}, \tag{11}$$

with the boundary conditions in the asymptotic in and out regions as

$$\begin{aligned} g_{\vec{p}_i}^{in}(\eta) &\rightarrow \frac{\exp\left(-i\eta \sqrt{\vec{p}_i^{\,2} + m^2 b^2} \right)}{\sqrt{(2\pi)^3 2 \sqrt{\vec{p}_i^{\,2} + m^2 b^2}}} \quad (\eta \sim -\infty), \\ g_{\vec{p}_f}^{out}(\eta) &\rightarrow \frac{\exp\left(-i\eta \sqrt{\vec{p}_f^{\,2} + m^2} \right)}{\sqrt{(2\pi)^3 2 \sqrt{\vec{p}_f^{\,2} + m^2}}} \quad (\eta \sim \infty). \end{aligned}$$

They also satisfy the following normalization condition.

$$i \left(g_{\vec{p}}^* g'_{\vec{p}} - g'^*_{\vec{p}} g_{\vec{p}} \right) = \frac{1}{(2\pi)^3}, \tag{12}$$

where the prime denotes the derivative with respect to η. Then the mode functions can be expressed as

$$\begin{aligned} \Phi_f^* &= \frac{1}{a} \tilde{\Phi}_f^* = \frac{1}{a} e^{-i\vec{p}_f \cdot \vec{x}} g_{\vec{p}_f}^{out*}, \\ \Phi_i &= \frac{1}{a} \tilde{\Phi}_i = \frac{1}{a} e^{i\vec{p}_i \cdot \vec{x}} g_{\vec{p}_I}^{in}. \end{aligned}$$

The electromagnetic final mode function satisfies the Maxwell equation in curved spacetime,

$$\nabla^\mu(\nabla_\mu A^*_\nu - \nabla_\nu A^*_\mu) = 0. \tag{13}$$

Notice that in 4-dimensional conformally flat spacetimes this eqn(13) can be reduced into the same form of that in the flat spacetime,

$$\partial^\mu(\partial_\mu A^*_\nu - \partial_\nu A^*_\mu) = 0. \tag{14}$$

Therefore we get easily the final mode functions

$$A^*_\mu = \epsilon^*_\mu(\vec{k})\frac{\exp\left(i|\vec{k}|\eta - i\vec{k}\cdot\vec{x}\right)}{\sqrt{(2\pi)^3 2|\vec{k}|}}, \tag{15}$$

where ϵ^*_μ is a helicity factor. Using the rescaled field, the amplitude, eqn(8), is rewritten as

$$Amp = -ie\int d^4xi\left(\tilde{\Phi}^*_f\partial^\mu\tilde{\Phi}_i - \partial^\mu\tilde{\Phi}^*_f\tilde{\Phi}_i\right)A^*_\mu. \tag{16}$$

Because the photon emission lasts only during the epoch of expansion, the concept of the probability per unit time is ambiguious. So we shall use the transition probability itself. The transition probability W can be obtained from the amplitude such that

$$W = \sum_{h=L,R}\frac{(2\pi)^3}{V}\int d^3p_f d^3k|Amp|^2, \tag{17}$$

where the summation is performed on the photon helicity and V is the conformal volume of the space which is cancelled by the factor $(2\pi)^3\delta(\vec{0})$ coming from the conformal momentum conservation in $|Amp|^2$. After the helicity summation, the explicit form of W is obtained as follows.

$$\begin{aligned} W &= (2\pi)^6e^2\int\frac{d^3k}{(2\pi)^3 2|\vec{k}|}\int d^3p_f\delta(\vec{k}+\vec{p}_f-\vec{p}_i) \\ &\quad\times\left[(\vec{p}_f+\vec{p}_i)^2\left|\int d\eta\ e^{i|\vec{k}|\eta}g^{out*}_{\vec{p}_f}g^{in}_{\vec{p}_i}\right|^2 - \left|\int d\eta\ e^{i|\vec{k}|\eta}(g^{out*}_{\vec{p}_f}g^{in\prime}_{\vec{p}_i} - g^{out*\prime}_{\vec{p}_f}g^{in}_{\vec{p}_i})\right|^2\right]. \end{aligned}$$

By virtue of the Wronskian relation;

$$\frac{d}{d\eta}\left(g^{out*}_{\vec{p}_f}g^{in\prime}_{\vec{p}_i} - g^{out*\prime}_{\vec{p}_f}g^{in}_{\vec{p}_i}\right) = (\vec{p}_f^{\,2} - \vec{p}_i^{\,2})g^{out*}_{\vec{p}_f}g^{in}_{\vec{p}_i}, \tag{18}$$

the form of W is more simplified such that

$$W = (2\pi)^3 e^2 \int \frac{d^3k}{2|\vec{k}|} 4 \left(\vec{p_i}^{\,2} - \frac{(\vec{k}\vec{p_i})^2}{\vec{k}^2} \right) \left| \int_{-\infty}^{\infty} d\eta \; e^{i|\vec{k}|\eta} g^{out*}_{\vec{p_i}-\vec{k}} g^{in}_{\vec{p_i}} \right|^2 . \tag{19}$$

To grasp the behavior of W in the $|\vec{p_i}| \to \infty$ limit, we first argue a case with scale factor

$$a(\eta) = \Theta(\eta) + b\Theta(-\eta). \tag{20}$$

Then the exact mode functions are derived in this case as

$$\begin{aligned} g^{in}_{\vec{p_i}} &= \Theta(-\eta) \frac{\exp\left(-i\eta\sqrt{\vec{p_i}^{\,2} + m^2 b^2}\right)}{\sqrt{(2\pi)^3 2\sqrt{\vec{p_i}^{\,2} + m^2 b^2}}} \\ &\quad + \Theta(\eta) \frac{A(\vec{p_i}) \exp\left(-i\eta\sqrt{\vec{p_i}^{\,2} + m^2}\right) + B(\vec{p_i}) \exp\left(i\eta\sqrt{\vec{p_i}^{\,2} + m^2}\right)}{\sqrt{(2\pi)^3 2\sqrt{\vec{p_i}^{\,2} + m^2 b^2}}}, \end{aligned} \tag{21}$$

$$\begin{aligned} g^{out}_{\vec{p_f}} &= \Theta(\eta) \frac{\exp\left(-i\eta\sqrt{\vec{p_f}^{\,2} + m^2}\right)}{\sqrt{(2\pi)^3 2\sqrt{\vec{p_f}^{\,2} + m^2}}} \\ &\quad + \Theta(-\eta) \frac{A(\vec{p_f}) \exp\left(-i\eta\sqrt{\vec{p_f}^{\,2} + m^2 b^2}\right) + B(\vec{p_f}) \exp\left(i\eta\sqrt{\vec{p_f}^{\,2} + m^2 b^2}\right)}{\sqrt{(2\pi)^3 2\sqrt{\vec{p_f}^{\,2} + m^2}}}, \end{aligned} \tag{22}$$

where

$$\begin{aligned} A(\vec{p}) &= \frac{1}{2}\left(1 + \sqrt{\frac{\vec{p}^{\,2} + m^2 b^2}{\vec{p}^{\,2} + m^2}}\right), \\ B(\vec{p}) &= \frac{1}{2}\left(1 - \sqrt{\frac{\vec{p}^{\,2} + m^2 b^2}{\vec{p}^{\,2} + m^2}}\right). \end{aligned}$$

Substituting eqn(21) and eqn(22) into eqn(19) and taking the high momentum limit, $|\vec{p_i}| \to \infty$, it is shown that the terms proportinal to B, reflection wave terms in the mode functions, do not contribute to W because of the damping behavior of B. Notice that taking the high momentum limit, the energy conservation law almost restores in the following sense.

$$|\vec{p_i}| \sim |\vec{k}| + |\vec{p_i} - \vec{k}|. \tag{23}$$

Contribution from the region where eqn(23) does not hold is severely suppressed by the energy conservation factor. Using the polar coordinate decomposition $\vec{p_i} \cdot \vec{k} = |\vec{p_i}| k \cos\theta$ with $k = |\vec{k}|$ and taking $|\vec{p_i}|$ much larger than m, it is easily derived that the contribution from the k integral region lying between $|\vec{p_i}|$ and ∞ vanishes. Hence we get

$$\begin{aligned} & W(|\vec{p_i}| \sim \infty) \\ & = \frac{e^2 |\vec{p_i}|}{2(2\pi)^2} \int_{|\vec{p_i}|\delta}^{|\vec{p_i}|} dk \frac{k}{|\vec{p_i}| - k} \int_0^\pi d\theta \sin^3\theta \\ & \times \left| \left[k - \sqrt{\vec{p_i}^{\,2} + m^2} + \sqrt{(|\vec{p_i}| - k)^2 + 2|\vec{p_i}| k (1 - \cos\theta) + m^2} \right]^{-1} \right. \\ & \left. - \left[k - \sqrt{\vec{p_i}^{\,2} + m^2 b^2} + \sqrt{(|\vec{p_i}| - k)^2 + 2|\vec{p_i}| k (1 - \cos\theta) + m^2 b^2} \right]^{-1} \right|^2 . \end{aligned} \tag{24}$$

We need infra-red cutoff $|\vec{p_i}|\delta$ in eqn(24) due to the existance of massless photon. This infra-red divergence is well known one in flat spacetime quantum field theories with massless particles and it should be cancelled by an infra-red divergence of the self energy term [5]. The cutoff δ is physically determined by resolving power of soft photon observation. The θ integration in eqn(24) can be straightforwardly calculated. After performing this integration and taking the high momentum limit $|\vec{p_i}| \to \infty$, the k integration is simplified and we finally obtain

$$W(|\vec{p_i}| \to \infty) = \frac{e^2}{4\pi^2} \left(\ln \frac{1}{\delta} + \delta - 1 \right) \left(\frac{1 + b^2}{1 - b^2} \ln \frac{1}{b^2} - 2 \right). \tag{25}$$

We have taken the helicity sum in eqn(25). It is also possible to evaluate $W(b)$ independently with a fixed photon helicity. For left and right handed helicity, each probability is the same, a half of W in eqn(25).

Furthermore we can also obtain the analytic forms of $W(b)$ in the spinor QED for the case of eqn(20). Because both of the charged fermion and photon have degree of helicity freedom, 4 helicity contributions must be considered separately. The probability in the high momentum limit for 1/2 helicity fermion decaying into fermion with helicitity $h_{fermion}$ and photon with helicity h_{photon} is denoted by $W(1/2; h_{fermion}, h_{photon})$ and is given for each case as follows.

$$W(1/2;1/2,1) = \frac{e^2}{8\pi^2}\left(\ln\frac{1}{\delta}\right)\left(\frac{1+b^2}{1-b^2}\ln\frac{1}{b^2}-2\right). \tag{26}$$

$$W(1/2;1/2,-1) = \frac{e^2}{8\pi^2}\left(\ln\frac{1}{\delta}-\frac{\delta^2}{2}+2\delta-\frac{3}{2}\right)\left(\frac{1+b^2}{1-b^2}\ln\frac{1}{b^2}-2\right) \tag{27}$$

$$W(1/2;-1/2,1) = \frac{e^2}{8\pi^2}\left(1-\frac{b}{1-b^2}\ln\frac{1}{b^2}\right). \tag{28}$$

$$W(1/2;-1/2,-1) = 0. \tag{29}$$

No infra-red cutoff δ appears in eqn(28) and eqn(29) because spinflip of the fermion enables observers to distinguish the bremsstrahlung from the self energy process.

There exists a very useful aspect of W in the high momentum limit. It is supposed that the results eqn(25)$\sim$eqn(29) are exact not only for the special way of the expansion given by eqn(20) but also arbitrary way satisfying eqn(4) $\sim$ eqn(7). This implies that $W(|\vec{p_i}| \to \infty)$ possesses a remarkable universality with respect to the ways of the cosmic expansion. This property may be explained as the Lorentz contraction effect from the view point of the high energy particle. Imagine a particle running in the comoving frame. Suppose that the Universe begins to expand when the particle passes through a point A and the Universe ceases to expand when the particle reaches point B. The particle catches energy from the expansion only while running from A to B. Taking the high momentum limit, the length between A and B contracts to zero in the rest frame of the particle. Therefore the particle cannot see the details of the way how the Universe expands and thus the universality of W crops up.

To see more quantitatively the universality, we shall discuss the scalar QED with an adiabatically slow evolution of the scale factor $a(\eta)$ satisfying eqn(4) $\sim$ eqn(7). In the zeroth order adiabatic approximation(WKB approximation) the mode functions satisfying eqn(11) is written as

$$g_{\vec{p}}^{in} \sim g_{\vec{p}}^{out} \sim \frac{\exp\left[i\vec{p}\cdot\vec{x} - i\int_0^\eta d\eta'\sqrt{\vec{p}^{\ 2} + m^2 a(\eta')^2}\right]}{\sqrt{(2\pi)^3 2\sqrt{\vec{p}^{\ 2} + m^2 a(\eta)^2}}}. \tag{30}$$

Substituting eqn(30) into eqn(19) and introducing the polar coordinate decomposition; $\vec{p}_i \cdot \vec{k} = |\vec{p}_i| k \cos\theta$, we get

$$W \sim \frac{e^2|\vec{p}_i|^2}{2(2\pi)^2}\int_{|\vec{p}_i|\delta}^{\infty} dkk \int_0^{\pi} d\theta \sin^3\theta$$
$$\times \left| \int_{-\infty}^{\infty} d\eta \ \frac{\exp\left[ik\eta - i\int^\eta d\eta'\sqrt{\vec{p}_i^{\ 2} + m^2 a(\eta')^2} + i\int^\eta d\eta' \omega(\vec{p}_i - \vec{k}, \eta')\right]}{\sqrt{\omega(\vec{p}_i - \vec{k}, \eta)\sqrt{\vec{p}_i^{\ 2} + m^2 a(\eta)^2}}} \right|^2 \tag{31}$$

where $\omega(\vec{p}_i - \vec{k}, \eta) = \sqrt{(|\vec{p}_i| - k)^2 + 2|\vec{p}_i| k(1 - \cos\theta) + m^2 a^2}$ and $|\vec{p}_i|\delta$ is the inra-red cutoff. Consider the high momentum limit in eqn(31). Since nonvanishing contribution to W comes from the integral region where the momentum holds the relation eqn(23) as mentioned before, it is enough to restrict the momentum region between $|\vec{p}_i|$ and $|\vec{p}_i|\delta$. Here it should be searched which integral region of θ contributes, accompanied by influence from $a(\eta)$, to the nonvanishing value of W in eqn(31). Due to eqn(23), only emittion to nearly forward direction ($\theta \sim 0$) is permitted and especially the integral region of θ safisfying

$$0 \le \theta \le O\left(m/|\vec{p}_i|\right)$$

gives the scale factor dependence to the W. Several expansions like

$$\sqrt{\vec{p}_i^{\ 2} + m^2 a(\eta')^2} \sim |\vec{p}_i| + \frac{m^2 a(\eta')^2}{2|\vec{p}_i|}$$

yield finally

$$W \sim \frac{e^2|\vec{p}_i|}{2(2\pi)^2}\int_{|\vec{p}_i|\delta}^{|\vec{p}_i|} dk \frac{k}{|\vec{p}_i| - k}\int_0^{O\left(\frac{m}{|\vec{p}_i|}\right)} d\theta \ \theta^3$$
$$\times \left| \int_{-\infty}^{\infty} d\eta \ \exp\left[-i\int_0^\eta d\eta' \left(\frac{m^2 a(\eta')^2}{2|\vec{p}_i|} - \frac{m^2 a(\eta')^2}{2(|\vec{p}_i| - k)} - \frac{|\vec{p}_i| k\theta^2)}{2(|\vec{p}_i| - k)}\right)\right]\right|^2$$

$$= \frac{e^2}{(2\pi)^2} \int_\delta^1 dy \frac{y}{1-y} \int_0^{O(1)} dz z^3$$
$$\times \left| \int_{-\infty}^{\infty} d\tilde{\eta} \ \exp \left[-i \int_0^{\tilde{\eta}} d\tilde{\eta}' \left(C \left(\frac{2\lambda |\vec{p}_i|}{m^2} \tilde{\eta}' \right)^2 - \frac{C(\frac{2\lambda|\vec{p}_i|}{m^2}\tilde{\eta}')^2}{1-y} - \frac{yz^2}{1-y} \right) \right] \right|^2, \tag{32}$$

where we change the integral variables in the following way,

$$\begin{aligned} k &= |\vec{p}_i| y, \\ \theta &= \frac{m}{|\vec{p}_i|} z, \\ \eta &= \frac{2|\vec{p}_i|}{m^2} \tilde{\eta}. \end{aligned}$$

Note that the function $C(\frac{2\lambda|\vec{p}_i|}{m^2}\tilde{\eta})$ in eqn(32) approaches in the high momentum limit to a step function,

$$C = \Theta(\tilde{\eta}) + b \ \Theta(-\tilde{\eta}).$$

Therefore the value of W for arbitrary adiabatical cosmic expansion satisfying eqn(4) $\sim$ eqn(7) must equal to the specified value for eqn(20), and the universality is surely realized. If we dismiss the adiabatic approximation, the mode functions have reflection wave terms like in eqn(21) and eqn(22). However amplitude of the reflection waves vanishes in the high momentum limit and the universality are thought to survive.

Here we have a comment on the rate of the geometrobremsstrahlung. Comparing with the classical results in the section 2, it is noticed from eqn(25)$\sim$ eqn(29) that the gravitobremsstrahlung process does not frequently take place quantum mechanically. This is due to the fact that quantum effect smears position of the classical point particle and dilutes the charge density.

Now it is worth considering the results; eqn(25)$\sim$eqn(29) from the conformal symmetric point of view. Since the massless limit $m \to 0$ forces the speed of the particle to reach the light velocity, the universality with respect to the way of the cosmic expansion is maintained. In the lowest pertubation of the QED, the massless limit is shown to be equivalent with the $|\vec{p}_i| \to \infty$

limit. Therefore again the same results, eqn(25)$\sim$ eqn(29), come up in the $m \to 0$ limit and W really possesses the non-vanishing value. One might naively expect for the massless case that the amplitude in the conformally flat spacetime vanishes as in the Minkowskian spacetime, by virtue of the conformal symmetry, guaranteed at least classical mechanically. However this is *not* true unless $b = 1$ as argued above. This is a sort of conformal anomaly but it emerges in the tree level unlike the usual appearance of the anomaly due to loop effects.

Though the geometrobremsstrahlung in the QED is an interesting phenomenon, unfortunately it is not expected to work efficiently in the real early Universe. The reason comes from its circumstance. Charged particles at the temperature lying between 10^{16} GeV and 1 eV are strongly interacting with each other so that they are soaked in a thermal bath. Their quantum coherence gets broken in a quite short freestreaming time of the thermal bath before collision, and the Universe expands only with $\Delta a/a \ll 1$ during the time. The contribution from the span to the probability is

$$\Delta Prob \propto e^2 \Delta b = e^2 \frac{\Delta t}{2t},$$

where t is the proper time in the comoving frame and we have used the explicit form of the scale factor in the radiation dominant Universe,

$$a \propto t^{\frac{1}{2}}.$$

Transition probability after a long time T in the thermal bath can be estimated summing up the short span contributions between collisions as

$$Prob_{thermal} \propto e^2 \sum_{collisions} \frac{\Delta t}{t} = e^2 \ln \left(\frac{t_o + T}{t_o} \right),$$

which does not reach unity until today due to its logarithm behavior. Hence the geometrobremsstrahlung does not occur in the bath. It should be strongly emphasized that this suppression in the bath appears not only for the particle in the QED but also any thermal particles of general theories.

4 Gravitino Decay by Geometrobremsstrahlung

As mentioned in the section 3, the geometrobremsstrahlung survives in quantum field thoeries. This result tempts us to imagine that the deceleration induced by the cosmic expasion enhances also rates of some high energy particle decay. In this section we shall discuss a possibility of the geometrobremsstrahlung decay of gravitino.

The gravitino is a superpartner of graviton and posseses 3/2 spin. It interacts with other particles weakly through its coupling suppressed by the Planck mass. Because the rate of the interaction is smaller than the Hubble parameter after the Planck epoch, the gravitino in the early Universe is decoupled from other particles (photon, photino, $\cdots$). The gravitino with mass of $1-0.1$ TeV will have lifetime of 10^5-10^8 sec. This longevity of the gravitino would upset the standard big-bang nucleosynthesis in various ways [6, 7]. To avoid the disaster, one should require the reheat temperature after inflation is sufficiently low.

Notice that caluculations of the gravitino decay rate so far is based on the flat spacetime approximation, and taking no account of the geometrobremsstrahlung decay process.

We shall show that the gravitino decay via geometrobremsstrahlung is very effective, especially for 1/2 helicity components. Before the gravitino becomes nonrelativisitic, the momentum of the gravitino in the comoving frame is much larger than its mass m and the Hubble parameter. Therefore in the region from the time the supersymmetry gets broken till the time gravitinos are going to be dust matter, it is valid to adopt the high momentum limit $|\vec{p_i}| \to \infty$ in the section 3. We assume for simplicity that masses of gravitino and photino degenerate in this section.

The action for free gravitino in the Robertson-Walker spacetime reads

$$S_{gravitino} = \int d^4x\sqrt{-g}\left(-\frac{1}{2}\epsilon^{\alpha\beta\mu\nu}\bar{\Psi}_\alpha\gamma_5\gamma_\beta\nabla_\mu\Psi_\nu - \frac{1}{4}m\bar{\Psi}_\alpha[\gamma^\alpha,\ \gamma^\beta]\Psi_\beta\right), \quad (33)$$

where Ψ_μ is the gravitino field, majorana fermion field with spin 3/2, and

$$\gamma^\mu = e^\mu_a\gamma^a, \quad (34)$$

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \quad (35)$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \quad (36)$$

$$e^\mu_a = a(\eta)^{-1}\delta^\mu_a \quad , \quad e^a_\mu = a(\eta)\delta^a_\mu, \quad (37)$$

$$g_{\mu\nu} = e_{\mu a}e^a_\nu. \quad (38)$$

Introducing the rescaled gravitino field as

$$\tilde{\Psi}_\mu = a(\eta)^{\frac{1}{2}}\Psi_\mu, \quad (39)$$

the action can be rewritten into that in the flat spacetime with time-dependent mass $ma(\eta)$,

$$S_{gravitino} = \int d^4x \left(-\frac{1}{2}\epsilon^{abmn}\bar{\tilde{\Psi}}_a\gamma_5\gamma_b\partial_m\tilde{\Psi}_n - \frac{1}{4}ma(\eta)\bar{\tilde{\Psi}}_a[\gamma^a,\ \gamma^b]\Psi_b \right). \quad (40)$$

The action of free photino is given as

$$S_{photino} = \int d^4x\sqrt{-g}\ \frac{1}{2}\ \left(\bar{\lambda}i\gamma^\mu\nabla_\mu\lambda - m\bar{\lambda}\lambda \right), \quad (41)$$

where λ is photino field, majorana fermion field with spin 1/2. Here using rescaled photino field by

$$\tilde{\lambda} = a(\eta)^{\frac{3}{2}}\lambda, \quad (42)$$

the action is transformed into that with mass term $ma(\eta)$ in the Minkowski spacetime,

$$S_{photino} = \int d^4x\ \frac{1}{2}\ \left(\bar{\tilde{\lambda}}i\gamma^a\partial_a\tilde{\lambda} - ma(\eta)\bar{\tilde{\lambda}}\tilde{\lambda} \right). \quad (43)$$

We shall analyze for example a decaying mode to a photon and a photino. Interaction term describing the gravitino decay is given as

$$\begin{aligned} S_{int} &= \int d^4x\sqrt{-g}\frac{1}{8iM_G}\bar{\Psi}_\mu[\gamma^\alpha,\gamma^\beta]\gamma^\mu\lambda F_{\alpha\beta} \\ &= \int d^4x\frac{1}{8iM_G a(\eta)}\bar{\tilde{\Psi}}_m[\gamma^a,\gamma^b]\gamma^m\tilde{\lambda}F_{ab}, \end{aligned} \quad (44)$$

where M_G is the Planck mass. The same method in the section 3 yields the transition probability $W(b)$ of the gravitino decay into a photon and a photino in the high momentum limit. The following tables show the leading behaviors, up to factors, of $W(b)\frac{M_G^2}{m^2}$ near $b = 0$.

TABLE 1 :Decay of Gravitino with helicity 3/2

	photon helicity +1	photon helicity -1
photino helicity +1/2	$O(1) \neq 0$	$1/b^2$
photino helicity -1/2	$\ln b$	0

TABLE 2 :Decay of Gravitino with helicity 1/2

	photon helicity +1	photon helicity -1
photino helicity +1/2	$\ln b$	$1/b^4$
photino helicity -1/2	$1/b^2$	$1/b^2$

A remarkable enhancement appears in the mode in which a gravitino with helicity 1/2 decays into a photon with helicity -1 and a photino with helicity 1/2 and the probability $W(b)$ behaves as

$$W_{\frac{1}{2}}(b) \sim O\left(\left[\frac{m}{M_G}\right]^2\right)\frac{1}{b^4} \qquad (b \sim 0). \tag{45}$$

The decay occurs when $W_{\frac{1}{2}}(b) \sim 1.$ and the radiation dominant Universe expands like

$$\frac{a_f}{a_i} = \frac{1}{b} = \left(\frac{t_f}{t_i}\right)^{\frac{1}{2}}, \tag{46}$$

where $t_i(t_f)$ is initial(final) proper time in the comoving frame. Therefore we get

$$\Gamma_{gb} = \frac{1}{t_f} \sim \frac{m}{M_G}H_i, \tag{47}$$

where $H_i = 1/t_i$ is the Hubble parameter at the production time of the gravitino. On the other hand ordinary nongravitational decay process gives us its rate,

$$\Gamma_o \sim \left(\frac{m}{M_G}\right)^2 m. \tag{48}$$

Taking the mass $m \sim 1 - 0.1$ TeV, the lifetime $1/\Gamma_o$ is estimated as $10^5 - 10^8$ sec, while the geometrobremsstrahlung process gives much shorter lifetime $1/\Gamma_{gb} \leq 10^{-13} - 10^{-11}$ sec for the production temperature of the gravitino higher than $10^{11} - 10^{10.5}$GeV. Thus the gravitinos with 1/2 helicity decay during the epoch when they are relativistic, as opposed to arguments so far which insist that they decay after becoming dust matter.

The inverse geometrobremsstrahlung processes (photino $\rightarrow$ gravitino + photon, photon $\rightarrow$ gravitino + photino) are highly suppressed due to the quantum decoherence by the thermal bath. The photons and photinos in the thermal bath collide frequently with other particles until the Universe expands enough and the quantum coherence of the particles is lost. By the similar argument in the section 3, the decay probability producing gravitino behaves like

$$W \sim O\left(\frac{m^2}{M_G^2}\right) \ln\left(\frac{t_f}{t_i}\right), \tag{49}$$

and the photon and photino in the bath does not decay until today by this mechanism. Therefore gravitino is produced just by the two-body scatterings in the bath as usually argued [7]. Thus if an amount of gravitinos from the vacuum produced by pair creation where energy is typically order of the Hubble parameter is negligibly small, the 1/2 helicity components of gravitino produced from the thermal bath at the temperature beyond $10^{10} - 10^{11}$ GeV are washed out during the epoch when the gravitino is relativistic. Hence they do not affect to the nucleosysthesis scenario and also not contribute to the mass density in the Universe today.

The 3/2 helicity components of gravitino have, however, smaller decay probability than that of 1/2 components. Maximum value of $W(b \sim 0)$ appears at a decaying mode into a photino with helicity 1/2 and a photon with helicity -1, and it behaves like

$$W_{\frac{3}{2}}(b \sim 0) \sim O\left(\left[\frac{m}{M_G}\right]^2\right) \frac{1}{b^2}. \tag{50}$$

Analyzing its lifetime using eqn(50), it is derived assuming no inflation that the 3/2 helicity components of gravitino produced at the Planck epoch decays until the gravitino becomes nonrelativistic matter. However,unfortunately, the particles produced later are not washed out by this mechanism. Thus we conclude that the gravitino problem itself is not solved by the geometro-bremsstrahlung.

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