arXiv:hep-th/9508144 v2   5 Sep 1995

TYPE IIA-HETEROTIC DUALS WITH MAXIMAL SUPERSYMMETRY

SHYAMOLI CHAUDHURI[†]

Institute for Theoretical Physics
University of California
Santa Barbara, CA 93106-4030

AND

DAVID A. LOWE[*]

Department of Physics
University of California
Santa Barbara, CA 93106-9530

Using finite abelian automorphism groups of $K3$ we construct orbifold candidates for Type IIA-heterotic dual pairs with maximal supersymmetry in six and lower dimensions. On the heterotic side, these results extend the series of known reduced rank theories with maximal supersymmetry. The corresponding Type IIA theories generalize the Schwarz and Sen proposal for the dual of the simplest of the reduced rank theories constructed as a novel Type IIA $\mathbb{Z}_2$ orbifold.

August, 1995

[†] sc@itp.ucsb.edu

[*] lowe@tpau.physics.ucsb.edu

1. Introduction

There is growing evidence for an exact strong-weak coupling duality relating the Type IIA string compactified on a K3 surface and the heterotic string compactified on T^4 [1–7]. The resulting N=2 theory in six dimensions has 24 massless vector multiplets at generic points in the moduli space.

Recently, it has been observed that there are additional heterotic 6d $N = 2$ and 4d $N = 4$ theories on the heterotic side with fewer massless vector multiplets at generic points [8,9]. It is natural therefore to ask whether string duality extends to other maximally supersymmetric theories.

This is the question we will address in this paper. Using orbifold techniques [10], we will construct dual Type II-heterotic theories applying the duality dictionary provided by the soliton string construction of [5,6]. Since we are interested in compactifications with maximal supersymmetry on the heterotic side, we must consider automorphisms of $K3$ which preserve the holomorphic two-form. These are known as symplectic automorphisms. The finite abelian symplectic automorphism groups of Kahler $K3$ surfaces have been classified by Nikulin [11]. Aspinwall [12] has already pointed out the close correspondence between the dimensions of the moduli spaces of the distinguished $K3$ surfaces with automorphism group G, studied in [11], and the rank reduction of the 4d maximally supersymmetric heterotic string models constructed in [8]. In this paper, we will construct a Type II-heterotic dual pair for each abelian group in Nikulin's classification. For the case of the cyclic groups $\mathbb{Z}_n$ ($n = 2, \cdots, 8$) we obtain dual pairs in six dimensions with $N = 2$ supersymmetry. In the other cases G is a product of cyclic groups, and we obtain dual pairs in three, four and five dimensions.

2. Nikulin's Classification and the Dual Pair Construction

In the following, we will use the conjectured strong-weak coupling duality between the heterotic string compactified on a three-torus, and $d = 11$ supergravity compactified on $K3$ [3,4], to construct dual theories in lower dimensions. The strategy will be to compactify further on a circle (or more generally a torus), and then apply the orbifold construction. The symmetries we will utilize combine one of Nikulin's abelian automorphisms with a translation on the circle. Provided these symmetries have no fixed points on the supergravity side, we may apply the duality dictionary of [5,6] to map the action of the symmetry onto the heterotic side.

It is important to bear in mind that the compactification of $d = 11$ supergravity may alternatively be described in an appropriate region of the moduli space as a compactification of the Type IIA string, which involves non-trivial RR background fields [10,13]. The shift in the S^1 of $d = 11$ supergravity corresponds to giving a Wilson line expectation value to the RR $U(1)$ field of the ten-dimensional Type IIA string. In the examples discussed below, the perturbative spectrum in the untwisted sector is insensitive to the presence of the Wilson line, however it does affect the spectrum in the twisted sector. Further discussion of this point may be found in [13].

As Nikulin has shown [11], the action of the automorphism group on the $K3$ generates a corresponding action on the cohomology. These automorphisms leave the self-dual forms

invariant (we choose an anti-self-dual Riemann curvature such that the $(0,2)$, the $(2,0)$ and the Kahler forms are self-dual [14]). For a $K3$ surface, $H^2(K3, \mathbb{Z})$ with its intersection form is isomorphic to an even self-dual lattice with signature $(19,3)$. The action of the group on the cohomology of $K3$ is therefore isomorphic to an automorphism of this even self-dual lattice. According to [5,6], this lattice is to be identified with the Narain lattice [15] of the heterotic string compactified on a three-torus. The three right-moving heterotic string coordinates correspond to the three self-dual elements of $H^2(K3)$, while nineteen of the left-moving heterotic string coordinates correspond to the nineteen anti-self-dual elements of $H^2(K3)$. Invariance of the right-moving coordinates (up to shifts) under this automorphism means that supersymmetry is preserved. As mentioned above, these automorphisms of $K3$ are to be combined with shifts on the additional torus T^m in order that the symmetry be freely acting on the supergravity side. On the heterotic side, we will start with a lattice

$$\Gamma^{(19+m,3+m)} = \Gamma^{(19,3)} \oplus \Gamma^{(m,m)} \ , \tag{2.1}$$

where the $\Gamma^{(m,m)}$ factor corresponds to the torus. The shifts on the supergravity side may be identified with corresponding shifts on $\Gamma^{(m,m)}$.

At this point we recall some useful facts about self-dual lattices. The euclidean even self-dual lattices up to dimension 24 (Niemeier lattices) have been classified [16]. They include 23 Lie algebra lattices and the famous Leech lattice which is unique in having no vectors of norm 2. These lattices and some useful tricks for lattice manipulation are described in appendix A of [17]. The Niemeier classification is a nice starting point for constructing symplectic asymmetric orbifolds.

In the following subsections we consider in turn each of the abelian groups appearing in Nikulin's classification, and construct dual pairs of maximally supersymmetric heterotic and Type II theories. In each subsection we will describe the action of the symmetry on $K3$ and the cohomology, and then discuss the action on a corresponding Narain lattice of the heterotic string compactification, in some cases explicitly constructing such lattices at enhanced symmetry points. First we consider the cyclic groups $\mathbb{Z}_n$ ($n = 2, \cdots, 8$) that appear in Nikulin's list. The $d = 11$ supergravity theory is compactified on

$$X = \frac{K3 \times S^1}{\mathbb{Z}_n} \ , \tag{2.2}$$

where the symmetry acts on the circle as a $\mathbb{Z}_n$ shift. This yields a Type II supergravity theory in six dimensions with $N = 2$ supersymmetry. Later we will consider the noncyclic cases. In these cases we will consider orbifold compactifications down to lower dimensions, with shifts in the additional torus to ensure the symmetry is freely acting on $K3 \times T^m$, where $m = 2, 3, 4$.

The classical moduli spaces of $K3$ surfaces admitting abelian automorphisms are studied in [11]. For each of the orbifold theories we study, a nontrivial moduli space will therefore exist. Since the theories are maximally supersymmetric, the local structure of the moduli space is determined by the number of vector multiplets and takes the form

$$\mathcal{M} = \frac{SO(20 + m - r, 4 + m; \mathbb{R})}{SO(20 + m - r; \mathbb{R}) \times SO(4 + m; \mathbb{R})} \ , \tag{2.3}$$

where r is the reduction in rank relative to the toroidal compactification.

2.1. $\mathbb{Z}_2$

This is the same automorphism used to construct the Type II-heterotic dual pair in [13]. An example of a $K3$ surface which admits this involution is the quartic polynomial in $\mathbb{CP}^3$,

$$\sum_{i=1}^{4}(z^i)^4 = 0 \ . \tag{2.4}$$

The $\mathbb{Z}_2$ involution then acts as $z^1 \to -z^1$, $z^2 \to -z^2$, $z^3 \to z^3$ and $z^4 \to z^4$. This has eight fixed points on $K3$. The involution interchanges eight pairs of anti-self-dual $(1,1)$-forms, leaving the self-dual two-forms and the other anti-self-dual $(1,1)$-forms invariant. To ensure this symmetry is freely acting on $K3 \times S^1$ we accompany it by a π rotation in the S^1. Compactifying $d = 11$ supergravity on $(K3 \times S^1)/\mathbb{Z}_2$ yields a theory with $N = 2$ supersymmetry in six dimensions, with gauge group $U(1)^{16}$ at a generic point.

To describe the action of the symmetry on the heterotic side, first consider a point with enhanced $E_8 \times E_8$ gauge symmetry. It should be understood here and in the following subsections, that when we work at a point of enhanced symmetry on the heterotic side the dual will be a Type II theory compactified on some degenerate $K3$ surface [4,7,18]. The $\mathbb{Z}_2$ acts by interchanging the two E_8 components of the Narain lattice, together with a half period shift δ in the $\Gamma^{(1,1)}$ component of the lattice corresponding to the S^1 of $d = 11$ supergravity. The shift must satisfy $\delta^2 = 0$ for level-matching. Generic points in the moduli space are reached by $SO(20,4)$ rotations that are compatible with the $\mathbb{Z}_2$ action. This is precisely the heterotic asymmetric orbifold construction presented in [9]. The massless spectrum is the same as that found on the $d = 11$ supergravity side at a generic point in the moduli space.

2.2. $\mathbb{Z}_3$

This symmetry has six fixed points on the $K3$. It acts on $H^2(K3)$ by cyclic interchange of three groups of six anti-self-dual $(1,1)$-forms, leaving the other two-forms invariant. We will accompany this action on K3 by a $2\pi/3$ rotation in S^1. The compactification of $d = 11$ supergravity on $(K3 \times S^1)/\mathbb{Z}_3$ has $N = 2$ supersymmetry in six dimensions and gauge group $U(1)^{12}$ at generic points.

To describe the automorphism on the heterotic side it is convenient to start at a point of enhanced gauge symmetry. Consider, for example, the heterotic string compactified on a Narain lattice $\Gamma^{(20,4)}$ with $(D_1^6)_L^3 \times (D_1)_L^2 \times (D_1)_R^4$ symmetry. Such a lattice is obtained from the Niemeier lattice D_6^4 with conjugacy classes given by even permutations of $(0, s, v, c)$, by introducing a shift vector which decomposes one of the D_6 lattices into $D_4 \times D_1 \times D_1$, followed by a flip of the signature of the D_4 lattice, and shifts to give $(D_1)^4$ [17]. In the discussion here and below, (s), (v), and (c), denote the spinor, vector, and conjugate spinor, conjugacy classes of a D_n lattice. The $\mathbb{Z}_3$ symmetry will act as a cyclic interchange of the three $(D_1)^6$ components of the lattice, together with a shift δ by one-third of a period in the $(D_1)_L \times (D_1)_R$ component corresponding to the S^1 of the $d = 11$ supergravity theory. This orbifold action will satisfy level-matching provided $\delta^2 = 0$. A generic point in the moduli space is then reached by a $SO(20,4)$ rotation compatible with the $\mathbb{Z}_3$ symmetry.

3

The massless spectrum of the resulting orbifold agrees with that found on the supergravity side at generic points.

2.3. $\mathbb{Z}_4$

Now we have four fixed points on $K3$ of order four, and four fixed points of order two. Acting on $H^2(K3)$, the symmetry cyclically interchanges four groups of four two-forms, and changes the sign of two additional anti-self-dual $(1,1)$-forms. The self-dual two-forms are invariant. We accompany the action on K3 by a $\pi/2$ rotation in the S^1, to ensure the symmetry is freely acting on $K3 \times S^1$. Compactifying $d = 11$ supergravity on this manifold will yield at generic points a six-dimensional theory with $N = 2$ supersymmetry and $U(1)^{10}$ gauge symmetry.

To describe the action on the heterotic side, we begin with an appropriate $\Gamma^{(20,4)}$ Narain lattice with $((D_1)^4)^4 \times (D_1)_L^2 \times (D_1)_L^3 \times (D_1)_R^3$ symmetry. A symmetric combination of four D_1 lattices, each of which is embedded in a different $(D_1)^4$ factor, is purely right-moving. The $\mathbb{Z}_4$ symmetry then acts as cyclic interchange of the four $(D_1)^4$ factors, as an involution, i.e., -1 on the first two $(D_1)_L$ factors, and as a shift δ by one-quarter of a period in a $(D_1)_L \times (D_1)_R$ factor corresponding to the S^1 of $d = 11$ supergravity. These isometries are easily identified as subgroups of the isometry group of an embedding $D_{16} \times D_2$ lattice, which can be obtained from the D_{24} Niemeier lattice, with conjugacy classes (s) (v), by flipping the signature of a D_4 factor together with necessary shifts. Level-matching is satisfied for $\delta^2 = 0$.

Note that the symmetric embedding of a right-moving D_1 lattice in four $(D_1)^4$ lattices ensures that it is left invariant under the $\mathbb{Z}_4$ interchange. A generic point in the moduli space is reached by rotating the lattice by an element of $SO(20,4)$ consistent with the $\mathbb{Z}_4$ symmetry. The massless spectrum of the resulting orbifold then agrees with that found on the supergravity side.

2.4. $\mathbb{Z}_5$

This automorphism has four fixed points on $K3$. Acting on $H^2(K3)$, it cyclically permutes five groups of four two-forms. The self-dual two-forms are invariant. The action on K3 is combined with a $2\pi/5$ rotation in the S^1. Compactifying $d = 11$ supergravity on $(K3 \times S^1)/\mathbb{Z}_5$ yields a six-dimensional theory with $N = 2$ supersymmetry and $U(1)^8$ gauge symmetry at a generic point.

On the heterotic side, we describe the action of the symmetry by starting with a Narain lattice $\Gamma^{(20,4)}$ with $(D_1^4)_L^5 \times (D_1)_R^4$ symmetry. Such a lattice is obtained from the Niemeier lattice D_4^6 with conjugacy classes (s,s,s,s,s,s), $(0,[0,v,c,c,v])$ by flipping the signature of the first D_4 factor and shifting to give $(D_1)_R^4$. Here $[\cdots]$ denotes a sum over cyclic permutations. The symmetry then simply acts as a cyclic interchange of the five left-moving $(D_1)^4$ factors. One may go to a complex basis where the symmetry acts as a phase rotation. Two of the left-moving complex bosonic coordinates are invariant under this symmetry. The left-moving component of the compact direction corresponding to the S^1 of $d = 11$ supergravity is embedded in these two invariant complex coordinates. The right-moving component corresponds to one of the invariant $(D_1)_R$ factors. We accompany

4

the action of the symmetry by a shift δ by $1/5$ of a lattice vector in this direction. Level-matching is satisfied when $\delta^2 = 0$. A generic point in the moduli space may be reached by acting with a $SO(20,4)$ rotation compatible with the $\mathbb{Z}_5$ symmetry. The massless spectrum of the orbifold then coincides with the supergravity side.

2.5. $\mathbb{Z}_6$

This automorphism has a $\mathbb{Z}_2$ and a $\mathbb{Z}_3$ subgroup. The $\mathbb{Z}_2$ subgroup has six fixed points on the $K3$, the $\mathbb{Z}_3$ subgroup has four fixed points and the $\mathbb{Z}_6$ has two fixed points. The action on $H^2(K3)$ is as follows. Six groups of two (1,1)-forms are cyclically interchanged by the $\mathbb{Z}_6$, three groups of two $(1,1)$-forms are cyclically interchanged by the $\mathbb{Z}_3$, and the two anti-self-dual $(1,1)$-forms pick up minus signs under the $\mathbb{Z}_2$ subgroup. The self-dual two-forms are invariant. This group action is combined with a $\pi/3$ rotation of the S^1. The compactification of $d = 11$ supergravity on this manifold yields a six-dimensional theory with $N = 2$ supersymmetry and $U(1)^8$ gauge symmetry at a generic point.

On the heterotic side, we describe the action of the symmetry by starting with an appropriate Narain lattice $\Gamma^{(20,4)}$ with $(D_1^2)_L^6 \times (D_1^2)_L^3 \times (D_1)_L^2 \times (D_1)_R^4$ symmetry. The symmetry acts as a cyclic interchange of the first six $(D_1)^2$ factors, cyclic interchange of the next three $(D_1)^2$ factors, and -1 on the remaining two left-moving D_1 factors, together with a shift δ by $1/6$ of a lattice vector in the direction corresponding to the S^1 of $d = 11$ supergravity, which is embedded in one of the invariant left-moving coordinates and one of the $(D_1)_R$ factors. The shift satisfies $\delta^2 = 0$. These isometries can be identified in the isometry group of an embedding $D_{12} \times D_6 \times D_2$ lattice, which embeds in D_{24} as in the $\mathbb{Z}_4$ example discussed above. A generic point in the moduli space may be reached by acting with a $SO(20,4)$ rotation preserving the $\mathbb{Z}_6$ symmetry and the massless spectrum of the orbifold then agrees with the supergravity side.

2.6. $\mathbb{Z}_7$

Now we have three fixed points on $K3$. Acting on $H^2(K3)$, the symmetry cyclically permutes seven groups of three two-forms, and leaves one two-form invariant. A basis may be chosen in which four two-forms are invariant under the action of this symmetry. Three of these will correspond to linear combinations of the self-dual $(2,0)$, $(0,2)$ and Kahler forms. The action of this symmetry is to be combined with a $2\pi/7$ rotation in the S^1. Compactifying $d = 11$ supergravity on $(K3 \times S^1)/\mathbb{Z}_7$ yields at a generic point a six-dimensional theory with $N = 2$ supersymmetry and $U(1)^6$ gauge symmetry.

On the heterotic side, we describe the action of the symmetry by starting with a Narain lattice $\Gamma^{(20,4)}$ with $((D_1)^3)^7 \times (D_1)_R^3$ symmetry. We obtain this lattice by beginning with the Niemeier lattice A_3^8 with conjugacy classes $(3, [2,0,0,1,0,1,1])$, and flipping the sign of an $A_3 \times A_1$ sub-lattice followed by the necessary shifts. The necessary sub-lattice is given by the first A_3 factor together with an A_1 embedded symmetrically in the remaining seven A_3 factors. (Note that $SU(4)$ is isomorphic to $SO(6)$ so we can also think of this as a D_3^8 lattice with a Z_7 isometry.) The lattice is chosen such that a symmetric combination of seven D_1 factors, with each D_1 lying in a different $(D_1)^3$ factor, is purely right-moving. The symmetry acts as cyclic interchange of the seven $(D_1)^3$ factors, together with a shift δ by $1/7$ of a lattice vector in the direction corresponding to the S^1 of the $d = 11$ supergravity.

The translation corresponding to the S^1 is identified by pairing a left-moving shift that acts symmetrically in all seven $(D_1)^3$ factors with a right-moving shift in the $(D_1)_R^3$ lattice. Level-matching is satisfied when $\delta^2 = 0$. A generic point in the moduli space may be reached by acting with a $SO(20, 4)$ rotation compatible with the $\mathbb{Z}_7$ symmetry and the massless spectrum of the orbifold is then in agreement with the supergravity side.

2.7. $\mathbb{Z}_8$

This automorphism has a $\mathbb{Z}_2$ and a $\mathbb{Z}_4$ subgroup. The $\mathbb{Z}_2$ subgroup has four fixed points on the $K3$, the $\mathbb{Z}_4$ subgroup has two fixed points and the $\mathbb{Z}_8$ has two fixed points. The action on $H^2(K3)$ is as follows. Eight groups of two two-forms are cyclically interchanged by the $\mathbb{Z}_8$, four groups of two-forms are cyclically interchanged by the $\mathbb{Z}_4$, and one anti-self-dual $(1, 1)$-form picks up a minus sign under the $\mathbb{Z}_2$ subgroup. A basis may therefore be chosen in which four two-forms are invariant. Three of these will correspond to linear combinations of the self-dual $(0, 2)$, $(2, 0)$ and Kahler forms. This group action is combined with a $\pi/3$ rotation of the S^1. The compactification of $d = 11$ supergravity on $(K3 \times S^1)/\mathbb{Z}_8$ gives a six-dimensional theory with $N = 2$ supersymmetry and $U(1)^6$ gauge symmetry at a generic point in the moduli space.

On the heterotic side, we describe the action of the symmetry by starting with a certain Narain lattice with signature $(20, 4)$ and $((D_1)^2)^8 \times (D_1)^4 \times (D_1)_L \times (D_1)_R^3$ symmetry. A symmetric combination of eight D_1 factors, with each D_1 coming from the first component of each $(D_1)^2$ factor, and four other D_1's, each coming from a factor in the $(D_1)^4$ term, is purely right-moving. The symmetry then acts as a cyclic interchange of the eight $(D_1)^2$ factors, cyclic interchange of the four D_1 factors coming from the $(D_1)^4$ term, and -1 on the last left-moving D_1 factor. This isometry group is easily identified as a sub-group of the isometries of $D_{16} \times D_4$, which can be obtained from D_{24} as before. The left-moving component of the direction corresponding to the S^1 of the $d = 11$ supergravity arises from the symmetric combination of eight D_1 factors, with each D_1 coming from the second component of each $(D_1)^2$ factor, and four other D_1's, each coming from a factor in the $(D_1)^4$ term. The generator of the $\mathbb{Z}_8$ is accompanied with a shift δ by $1/8$ of a lattice vector in this direction. The shift satisfies $\delta^2 = 0$. A generic point in the moduli space may be reached by acting with a $SO(20, 4)$ rotation preserving the $\mathbb{Z}_8$ symmetry and the massless spectrum of the orbifold agrees with the supergravity side.

2.8. $(\mathbb{Z}_2)^k$, $k = 2, 3, 4$

In this case we have $2^k - 1$ $\mathbb{Z}_2$ subgroups, each of which have eight fixed points as in the $\mathbb{Z}_2$ example above. Therefore this symmetry group acts with a total of $8(2^k - 1)$ fixed points. The action of the symmetry may be represented by k $\mathbb{Z}_2$ generators, which we denote g_i, $i = 1, \cdots, k$. These act on $H^2(K3)$ as follows. The self-dual two-forms and three anti-self-dual two-forms are invariant. On the remaining sixteen anti-self-dual $(1, 1)$-forms, g_1 acts as

$$((-1)^8, 1^8) \, , \tag{2.5}$$

g_2 acts as

$$((-1)^4, 1^4, (-1)^4, 1^4) \, , \tag{2.6}$$

6

g_3 acts as

$$((-1)^2, 1^2, (-1)^2, 1^2, (-1)^2, 1^2, (-1)^2, 1^2) , \qquad (2.7)$$

and g_4 acts as

$$(-1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1) . \qquad (2.8)$$

Here the superscripts denote repeated entries.

For $k \geq 2$, a shift in the additional S^1 of the $d = 11$ supergravity theory is no longer sufficient to ensure the symmetry is freely acting on $K3 \times S^1$. However, by compactifying down to four dimensions on an additional torus with dimension $k - 1$, i.e. on $K3 \times T^k$, freely acting symmetries may be constructed as follows. The momentum lattice describing the T^k may be described by k integers (m_i), with inner product $\sum m_i^2$. We accompany the action of g_i, $i = 1, \cdots, 4$ by a shift with $m_i = \frac{1}{2}$, and the other components zero. The symmetry will then act freely on $K3 \times T^k$. Compactifying $d = 11$ supergravity on this manifold yields a theory with $N = 2$ supersymmetry in five dimensions and gauge group $U(1)^{14}$ for $k = 2$, $N = 4$ supersymmetry in four dimensions and gauge group $U(1)^{14}$ for $k = 3$ and $N = 8$ supersymmetry in three dimensions and gauge group $U(1)^{15}$ for $k = 4$.

To describe the action of this symmetry on the heterotic side, we start with an even self-dual $\Gamma^{19+k,3+k}$ Narain lattice with $(D_1)_L^{16} \times (D_1)_L^{3+k} \times (D_1)_R^{3+k}$ symmetry. We can identify each of the D_1 factors in the $(D_1)_L^{16}$ with one of the sixteen $(1, 1)$-forms of the $K3$ on which the symmetry acts in a nontrivial way as described above. The action of the g_i on these factors then may be identified with the action on the $(1, 1)$-forms. Likewise we identify the accompanying shifts on the T^k of the $d = 11$ supergravity theory with the same left-right symmetric shifts on the $(D_1)_L \times (D_1)_R$ factors. The massless spectrum of the resulting orbifolds then coincides with that on the supergravity side.

At this point we note that the $(\mathbb{Z}_2)^2$ and $(\mathbb{Z}_2)^3$ asymmetric orbifolds described here recover all of the rank -12 and rank -14 solutions found in the fermionic construction [8,9]. The clearest evidence for this is the existence of $\mathbb{Z}_2$, $(\mathbb{Z}_2)^2$, and $(\mathbb{Z}_2)^3$ orbifolds containing enhanced symmetry points with, respectively, Kac-Moody level 2, level 4, and level 8 realizations of the gauge symmetry. Each $\mathbb{Z}_2$ cyclic automorphism of repeated factor groups doubles the Kac-Moody level. Thus, for example, beginning with the Narain lattice $E_8 \times E_8 \times (D_2)^3$ we obtain reduced rank models with, respectively, $(E_8)_2 \times (SO(4))^3$, $SO(8)_4 \times (SO(4))^3$, and $SO(4)_8 \times (SO(4))^3$ gauge symmetry. Here the subscripts denote the Kac-Moody level. These solutions have already been obtained in the fermionic construction.

2.9. $(\mathbb{Z}_3)^2$

In this case, we have four $\mathbb{Z}_3$ subgroups, each of which is the stationary subgroup for six fixed points. The symmetry is generated by two $\mathbb{Z}_3$ generators which we label g_1 and g_2. The symmetry leaves the self-dual two-forms invariant. The generators act on eighteen of the anti-self-dual $(1, 1)$-forms as follows. g_1 cyclically interchanges three groups of six $(1, 1)$-forms. Within each of these three groups, g_2 acts by cyclically interchanging three sets of two $(1, 1)$-forms. In order that the symmetry be freely acting on the $d = 11$ supergravity side, we consider compactification on $(K3 \times S^1 \times S^1)/(\mathbb{Z}_3)^2$, where each generator g_1, g_2

is accompanied by a $2\pi/3$ rotation in the first and second circle respectively. This yields a theory with $N = 2$ supersymmetry in five dimensions and gauge group $U(1)^{10}$.

On the heterotic side we start by describing the action of this symmetry on a Narain lattice $\Gamma^{(21,5)}$ with symmetry $((D_1^2)^3)_L^3 \times (D_1)_L^3 \times (D_1)_R^5$. The action on the $((D_1^2)^3)_L^3$ components of the lattice are completely analogous to the action of the symmetry on the cohomology. Likewise the generators of the symmetry are accompanied by the shifts described above, acting on the corresponding $(D_1)_L \times (D_1)_R$ factors. Performing a $SO(21,5)$ rotation of the lattice compatible with this symmetry yields an orbifold theory with the gauge group broken down to $U(1)^{10}$ and $N = 2$ supersymmetry in five dimensions, as on the supergravity side.

2.10. $\mathbb{Z}_2 \times \mathbb{Z}_4$

This case is very similar to the $\mathbb{Z}_4$ and $\mathbb{Z}_2$ cases. The symmetry acts with two groups of eight fixed points of order two, and two groups of four fixed points of order four. Let g_1 denote the generator of the $\mathbb{Z}_2$ and g_2 denote the generator of the $\mathbb{Z}_4$. The action of g_2 on $H^2(K3)$ is the same as in the $\mathbb{Z}_4$ case. Recall, this involved cyclically interchanging four groups of four two-forms which we will denote by ω_{ijk} with $i = 1, 2$, $j = 1, 2$ and $k = 1, \cdots, 4$. g_2 also acts by changing the sign of two anti-self-dual $(1, 1)$-forms ω_1 and ω_2. g_1 acts by changing the sign of two of ω_1, ω_2 and two other anti-self-dual $(1, 1)$-forms ω_3 and ω_4. In addition, g_1 interchanges ω_{2jk} with $k = 1, 2$ with $\omega_{2jk'}$ with $k' = 3, 4$ respectively. In order that the symmetry be freely acting on the $d = 11$ supergravity side, we consider compactification on $(K3 \times S^1 \times S^1)/(\mathbb{Z}_2 \times \mathbb{Z}_4)$, where g_1 is accompanied by a π rotation in one circle, and g_2 is accompanied by a $\pi/2$ rotation in the other. This yields a theory with $N = 2$ supersymmetry in five dimensions and gauge group $U(1)^{10}$. The action of the symmetry on the heterotic string compactified on a $\Gamma^{(21,5)}$ lattice follows in an obvious way given the action on the cohomology and the previous description of the $\mathbb{Z}_2$ and $\mathbb{Z}_4$ cases.

2.11. $(\mathbb{Z}_4)^2$

The symmetry acts on the $K3$ surface with six groups of four fixed points of order four. Let us denote the generator of each $\mathbb{Z}_4$ by g_1 and g_2 respectively. To describe the action of this symmetry it is convenient to introduce two two-forms in addition to the 22 two-forms arising from the $K3$. These additional two-forms may be thought of as arising from $H^2(K3 \times T^2)$. The reason we introduce these extra two-forms is that the symmetry may then be written as a combination of cyclic interchanges of linear combinations of the 24 two-forms, which we will denote by ω_{ij}, with $i = 1, \cdots, 6$ and $j = 1, \cdots, 4$. The linear combinations are of course chosen in such a way that the additional two-forms are invariant under the symmetry. g_1 acts by cyclically interchanging on the j index, the ω_{ij} with $i = 1, \cdots, 4$ and interchanges ω_{6j} with $j = 1, 2$ with $\omega_{6j'}$ with $j' = 3, 4$. g_2 acts by cyclically interchanging on the j index, the ω_{ij} with $i = 3, \cdots, 6$ (with the interchange of ω_{4j} being in the reverse order relative to the g_1 action) and interchanges ω_{2j} with $j = 1, 2$ with $\omega_{2j'}$ with $j' = 3, 4$. g_1 and g_2 leave invariant the self-dual two-forms and one anti-self-dual two-form of $K3$. If we accompany g_1 with a $\pi/2$ rotation of one circle, and g_2 with a $\pi/2$ rotation of another circle, then the symmetry acts freely on

$K3 \times S^1 \times S^1$. Compactifying $d = 11$ supergravity on $(K3 \times S^1 \times S^1)/(\mathbb{Z}_4)^2$ then yields a theory with $N = 2$ supersymmetry in five dimensions and gauge group $U(1)^8$. The action of the symmetry on the heterotic side follows straightforwardly from the action on the cohomology given above, and the single $\mathbb{Z}_4$ case worked out above.

2.12. $\mathbb{Z}_2 \times \mathbb{Z}_6$

Let us denote the generator of $\mathbb{Z}_2$ by g_1 and the generator of $\mathbb{Z}_6$ by g_2. The symmetry acts on $K3$ with three groups of six fixed points of order two, and three groups of two fixed points of order six. To describe the action of the symmetry in a convenient way, consider the basis of two-forms ω_{ij}, with $i = 1, 2, 3$ and $j = 1, \cdots, 6$, ω_k with $k = 1, 2, 3$ and one self-dual two-form ω'. g_1 acts by changing the sign of ω_2 and ω_3 and interchanges ω_{2j} with $j = 1, 2, 3$ with $\omega_{2j'}$ for $j' = 4, 5, 6$, respectively. g_1 also acts by interchanging ω_{3j} with $j = 1, 2, 3$ with $\omega_{3j'}$ for $j' = 4, 5, 6$, respectively. On $H^2(K3)$, g_2 changes the sign of ω_1 and ω_2, cyclically interchanges ω_{1j} and ω_{2j} on the j index, and acts with a $\mathbb{Z}_3$ cyclic permutation of the pairs ω_{3j} $(j = 1, 2)$, $\omega_{3j'}$ $(j' = 3, 4)$ and $\omega_{3j''}$ $(j'' = 5, 6)$. Both g_1 and g_2 leave the self-dual two-forms invariant, and one linear combination of the anti-self-dual two-forms. Accompanying g_1 with a π rotation of one circle and g_2 with a $\pi/3$ rotation of another circle, the symmetry is freely on $K3 \times S^1 \times S^1$. The compactification of $d = 11$ supergravity on $(K3 \times S^1 \times S^1)/(\mathbb{Z}_2 \times \mathbb{Z}_6)$ gives a theory in five dimensions with $N = 2$ and gauge group $U(1)^8$. The action of the symmetry on the heterotic side follows straightforwardly from the previous examples.

3. Conclusions

The theories we have constructed should have interesting implications for S-duality in four dimensions. As has been pointed out [19,4] string-string duality in six dimensions between the Type IIA theory compactified on $K3$ and the heterotic string on a four-torus implies S-duality for the theories further compactified to four dimensions on a torus. This follows since string-string duality in six dimensions maps T-duality of one theory to S-duality of the dual. Provided T-duality is not modified quantum mechanically, S-duality of the dual theory follows. A similar story should be true for the orbifold theories described in this paper, although now, in general, only a subgroup of $SL(2, \mathbb{Z})_T$ and likewise $SL(2, \mathbb{Z})_S$ will commute with the orbifolding procedure. This has already been observed in the context of a five-dimensional maximally supersymmetric dual pair, further compactified to four dimensions in [20]. Related phenomena in the context of Type II-Type II dual pairs have been considered in [21].

As has been pointed out [9], the elements of the original $SL(2, \mathbb{Z})_S$ of the toroidally compactified theory which do not commute with the orbifolding, are expected not to leave individual points in the moduli space invariant. In particular, in the $\mathbb{Z}_2$ orbifold theory considered in [9], such elements are expected to exchange points in the moduli space corresponding to dual electric-magnetic pairs. The appearance of dual gauge groups in the same moduli space provides further evidence for S-duality. This phenomenon is expected to generalize in the theories described in this paper, where the non-commuting duality elements act non-trivially within the moduli space. It would be interesting to classify

the enhanced symmetry points in our new heterotic compactifications and to study these duality transformations more carefully.

Finally, we note that the maximally supersymmetric models we have constructed provide a new starting point for obtaining dual pairs with reduced supersymmetry. Type II-heterotic dual pairs with partially broken supersymmetry have recently been studied in [10,20,22].

References

[1] N. Seiberg, "Observations on the Moduli Space of Superconformal Field Theories", *Nucl. Phys.* **B303** (1988) 286.

[2] P. Aspinwall and D. Morrison, "String Theory on K3 surfaces", to appear in "Essays on Mirror Manifolds 2", hep-th/9404151.

[3] C. Hull and P. Townsend, "Unity of Superstring Dualities," hep-th/9410167, *Nucl. Phys.* **B438** (1995) 109.

[4] E. Witten,"String Theory Dynamics in Various Dimensions," hep-th/9503124, *Nucl. Phys.* **B443** (1995) 85.

[5] A. Sen, "String-String Duality Conjecture in Six Dimensions and Charged Solitonic Strings", hep-th/9504027.

[6] J. A. Harvey and A. Strominger, "The Heterotic String is a Soliton," to appear in *Nucl. Phys.* B, hep-th/9504047.

[7] P. Aspinwall, "Enhanced Gauge Symmetries and K3 Surfaces", hep-th/9507012.

[8] S. Chaudhuri, G. Hockney and J. Lykken, "Maximally Supersymmetric String Theories in $D < 10$", to appear in *Phys. Rev. Lett.*, hep-th/9505054. S. Chaudhuri, Talk given at Strings'95 conference, March 1995, USC.

[9] S. Chaudhuri and J. Polchinski, "Moduli Space of CHL Strings", hep-th/9506048.

[10] S. Ferrarra, J. Harvey, A. Strominger and C. Vafa, "Second Quantized Mirror Symmetry", hep-th/9505162.

[11] V.V. Nikulin, "Finite Automorphism Groups of Kahler K3 Surfaces", *Trans. Moscow Math. Soc.* **2** (1980) 71.

[12] P. Aspinwall, Lecture given at Conference on S-duality and Mirror Symmetry, Trieste, June 5-9, 1995; "Some Relationships Between Dualities in String Theory", Cornell preprint CLNS-95/1359, to appear.

[13] J. Schwarz and A. Sen, "The Type IIA Dual of the Six-Dimensional CHL Compactification," hep-th/9507027.

[14] A nice pedagogical discussion of string theory on K3 surfaces can be found in M. Walton, "Heterotic String on the simplest Calabi-Yau manifold and its orbifold limits", *Phys. Rev.* **D37** (1988) 377.

[15] K.S. Narain, *Phys. Lett.* **169B** (1986) 41; K.S. Narain, M.H. Sarmadi and E. Witten, *Nucl. Phys.* **B279** (1987) 369.

[16] H. Niemeier, *Jour. Number Theory* **5** (1973) 142. J. Conway and M. Sloane, *Jour. Number Theory* **15** (1982) 83.

[17] W. Lerche, A. N. Schellekens, and N. P. Warner, "Lattices and Strings", *Phys. Rep.* **177** (1989) 1.

[18] E. Witten, "Some Comments on String Dynamics", hep-th/9507121.

[19] M. Duff, "Strong/Weak Coupling Duality from the Dual String", hep-th/9501030, *Nucl. Phys.* **B442** (1995) 47.

[20] C Vafa and E. Witten, "Dual String Pairs with N=1 and N=2 Supersymmetry in four dimensions", hep-th/9507050.

[21] A. Sen and C. Vafa, "Dual Pairs of Type II String Compactification", hep-th/9508064.

[22] J.A. Harvey, D.A. Lowe and A. Strominger, "N=1 String Duality", hep-th/9507168.