CP violation in multi-Higgs supersymmetric models

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Abstract

We consider supersymmetric extensions of the standard model with two pairs of Higgs doublets. We study the possibility of spontaneous CP violation in these scenarios and present a model where the origin of CP violation is soft, with all the complex phases in the Lagrangian derived from complex masses and vacuum expectation values (VEVs) of the Higgs fields. The main ingredient of the model is an approximate global symmetry, which determines the order of magnitude of Yukawa couplings and scalar VEVs. We assume that the terms violating this symmetry are suppressed by powers of the small parameter $\epsilon_{PQ} = O(m_b/m_t)$. The tree-level flavor changing interactions are small due to a combination of this global symmetry and a flavor symmetry, but they can be the dominant source of CP violation. All CP-violating effects occur at order ϵ_{PQ}^2 as the result of exchange of *almost*-decoupled extra Higgs bosons and/or through the usual mechanisms with an *almost*-real CKM matrix. On dimensional grounds, the model gives $\epsilon_K \approx \epsilon_{PQ}^2$ and predicts for the neutron electric dipole moment (and possibly also for ϵ'_K) a suppression of order ϵ_{PQ}^2 with respect to the values obtained in standard and minimal supersymmetric scenarios. The predicted CP asymmetries in B decays are generically too small to be seen in the near future. The mass of the lightest neutral scalar, the strong CP problem, and possible contributions to the Z decay into b quarks (the R_b puzzle) are also briefly addressed in the framework of this model.

1   Introduction

Although the standard model is today in impressive agreement with all particle physics data, its scalar sector has not been proven yet. A scalar sector defined by elementary fields seems to contradict the possibility of two very different mass scales (namely, the electroweak and the grand unification or Planck scales). Supersymmetry (SUSY) [1] would offer an explanation for the stability at the quantum level of the different scales of the theory, provided that it is broken only by soft terms below the TeV region. A lot of attention has been paid to the minimal SUSY extension of the standard model (MSSM), which presents appealing features such as a consistent grand unification of the gauge couplings or a candidate for the cold dark matter of the universe. Experimentally, the MSSM has been so far *flexible* enough to avoid conflict with any measurement, but its most compelling prediction, the presence of a light neutral Higgs (lighter than the Z boson at the tree level), is still missing.

The MSSM would also offer distinctive predictions for CP violating processes. Arbitrary complex phases ψ in soft gaugino masses and scalar trilinears would give fermion electric dipole moments (EDMs) well above their present experimental limits. This implies generically $\psi \leq 10^{-2}$ [2], a somewhat unnaturally small number. The MSSM would also predict unsupressed flavor changing neutral currents (FCNCs) unless there is some degree of degeneracy between squark masses (something which occurs for supergravity Lagrangians with canonical kinetic terms) and correlation between the Cabibbo-Kobayashi-Maskawa (CKM) matrix and its equivalent in the squark sector. In the usual SUSY scenario [3] CP violation in K and B physics depends essentially on only one phase (the CKM phase ϕ), whereas the set of small phases in soft terms (uncorrelated to the family structure) may have experimental relevance only in fermion EDMs. This scenario, however, holds only for highly degenerated squark masses. In general, taking the experimental limit $\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2} \leq \frac{1}{30}$ [4] from $K - \bar{K}$ mixing one obtains that acceptable complex phases in gaugino masses may have an impact on the K system. For complex gluino masses this was shown in [5], and a model with small phases has been recently proposed in [6]. It was also shown [7] that due to large top-quark effects acceptable complex phases in chargino mass terms may also contribute to CP asymmetries in the K system.

A different approach to the origin of CP violation which is specially appealing in

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SUSY models is the idea of spontaneous CP violation (SCPV) [8]. All the phases in the Lagrangian (initially CP-conserving) would have their origin in a small number of complex vacuum expectation values (VEVs) of scalar fields. Moreover, the sizes of these phases could be correlated by approximate symmetries suppressing some couplings in the Lagrangian [9]. Unfortunately, in the minimal Higgs sector of the MSSM there is no room for SCPV [10, 11], and *hard CP* violation is required. The possibility of SCPV has been also studied in SUSY extensions with singlet fields [7, 12], where it can be obtained but seems to require certain ammount of fine tuning.

The SUSY models with more than two Higgs doublets are an obvious extension of the MSSM [13]. They are minimal in the sense that no new species are introduced, but just repeated. From the model building point of view there is no compelling reason to disregard them, and they could appear naturally in models with fermion-Higgs unification (like E_6) [14] or left-right symmetric scenarios, where two bidoublets are required in order to obtain realistic fermion masses and mixings. Four Higgs doublet (4HD) models require an intermediate scale to be consistent with grand unification, but even this could be more in line with recent data on $\alpha_s(M_Z)$ than the desert scenario [15]. Since more than one Higgs doublet couples to quarks of a given charge, a possible concern in this type of models is the presence of FCNC at the tree level. The experimental limits, however, can be easily avoided just by invoking the action of an approximate flavor symmetry (see next section). On the other hand, a nonminimal scalar sector opens the possibility of SCPV and, in general, widens the parameter space relevant in low-energy precision measurements (this could be convenient, for example, if the anomalous value of R_b persists).

In this paper we present a 4HD SUSY model which seems to contain satisfactory answers to many phenomenological questions. CP violation appears softly, in complex Higgs masses and VEVs. The main ingredient of the model is a Peccei-Quinn like approximate symmetry which determines the order of magnitude of Yukawa couplings and scalar VEVs. We define this symmetry in such a way that the additional pair of doublets has small VEVs with order one complex phases and is weakly coupled to all matter fields. As a consequence, the ratio m_b/m_t, CP-violating effects in K physics, and the neutron EDM will appear suppressed by powers of the small parameter ϵ_{PQ} that parametrizes the violation of this symmetry. The CP asymmetries in B decays are predicted to be typically two or three orders of magnitude smaller than in CKM scenarios (the CKM

matrix in the model is essentially real), a signal that can be used to discriminate this 4HD model with respect to the MSSM or the standard model.

The plan of the paper is as follows. In Section 2 we write the generic Lagrangian for 4HD SUSY models and review previous results on spontaneous CP violation. We show that a realistic scenario for soft CP violation requires complex Higgs masses in the initial effective model. In Section 3 we define our model and minimize the Higgs potential. We show that the order of magnitude of Yukawa couplings, complex scalar VEVs, and the CKM complex phase are correlated by the approximate global symmetry. In Section 4 we explore the implications of the model on K and B physics as well as on the neutron EDM. In Section 5 we discuss the resulting spectrum of scalar fields (in particular, the mass of the lightest neutral mode) and other possible phenomenological impacts of the 4HD model. Section 6 is devoted to conclusions. Details about the minimization of the scalar potential can be found in the Appendix.

2   Complex VEVs in four Higgs doublet models

The most general superpotential with four higgs doublets is given by

$$
\begin{aligned}
W \;=\; & Q(\mathbf{h}_1 H_1 + \mathbf{h}_3 H_3)D^c + Q(\mathbf{h}_2 H_2 + \mathbf{h}_4 H_4)U^c + L(\mathbf{h}_1^e H_1 + \mathbf{h}_3^e H_3)E^c \\
+\; & \mu_{12}H_1 H_2 + \mu_{32}H_3 H_2 + \mu_{14}H_1 H_4 + \mu_{34}H_3 H_4,
\end{aligned}
\tag{1}
$$

where Q stands for quark doublets, D^c for down quark singlets, U^c for up quark singlets, L for lepton doublets, E^c for charged lepton singlets, and $\mathbf{h}_i$ are the Yukawa matrices (family indices are omitted). The Higgs doublets H_1, H_3 and H_2, H_4 have hypercharges -1 and $+1$, respectively.

Including soft SUSY breaking terms the effective potential for the Higgs fields is

$$
\begin{aligned}
V \;=\; & m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 H_3^\dagger H_3 + m_4^2 H_4^\dagger H_4 + \\
+\; & (m_{12}^2 H_1 H_2 + h.c.) + (m_{32}^2 H_3 H_2 + h.c.) + \\
+\; & (m_{14}^2 H_1 H_4 + h.c.) + (m_{34}^2 H_3 H_4 + h.c.) + \\
+\; & (m_{13}^2 H_1^\dagger H_3 + h.c.) + (m_{24}^2 H_2^\dagger H_4 + h.c.) + V_D^{4HD} + \Delta V,
\end{aligned}
\tag{2}
$$

where V_D^{4HD} contains the D-terms and ΔV the radiative corrections. For the neutral

components ϕ_i of the doublets one has

$$V_D^{4HD} = \frac{1}{8}(g^2 + g'^2) \, [\phi_1^\dagger \phi_1 + \phi_3^\dagger \phi_3 - \phi_2^\dagger \phi_2 - \phi_4^\dagger \phi_4]^2. \tag{3}$$

The radiative contributions ΔV are generated by SUSY breaking effects. In our arguments it will suffice to consider the terms derived from large top (and possibly bottom) quark Yukawa interactions [16]:

$$\Delta V = \sum_q \frac{3}{16\pi^2} \{ m_{\tilde{q}}^4 [ln(\frac{m_{\tilde{q}}^2}{Q^2}) - \frac{3}{2}] - m_q^4 [ln(\frac{m_q^2}{Q^2}) - \frac{3}{2}] \}, \tag{4}$$

where $q = t, b$, $m_t^2 = |h_{2t}\phi_2 + h_{4t}\phi_4|^2$, $m_b^2 = |h_{1b}\phi_1 + h_{3b}\phi_3|^2$, and $m_{\tilde{q}}^2 = m_s^2 + m_q^2$.

Our first comment about the viability of 4HD models should make reference to the size of FCNCs via Yukawa interactions. If the Yukawa matrices in (1) are uncorrelated, there is no reason to expect that the unitary transformations defining mass eigenstates also diagonalize (in flavor space) the couplings to the extra Higgs doublets. This would introduce unsuppressed FCNCs at tree level. The observed pattern of quark masses and mixings, however, strongly suggests the possibility of an approximate flavor symmetry as the origin of the hierarchies required in the Yukawa matrices. In the simplest scenarios [17, 18] the effect of such a symmetry would be to generate fermion matrices with off-diagonal elements of order $O(\sqrt{m_i m_j}/v)$, where m_i is the mass of the ith quark and v is the weak scale. If the extra Higgs doublets are a replica (with respect to this flavor symmetry) of the first doublet, they will introduce Yukawa matrices with the same approximate structure. In that case the smallness of Yukawa couplings is enough to keep all FCNC within the experimental limits. In particular, for extra Higgs masses around 1 TeV the tree-level contributions to $K - \bar{K}$ and $B - \bar{B}$ mixings would be of the same order as the standard contributions [18, 19]. In our 4HD model we will assume this type of approximate flavor symmetry at work.

Our main motivation to study 4HD models concerns the origin of CP violation. In these models explicit CP violation seems even more inconvenient than in the MSSM, due to new processes mediated by the Yukawa interactions described above (the approximate flavor symmetry would not explain, for example, the small value of ϵ_K [19]). The possibility of SCPV in 4HD models has been addressed in a recent paper [20]. There we assume that all the parameters in the Lagrangian are real and the CP-violating phases appear via VEVs $v_i e^{i\delta_i}$ of the Higgs fields. We showed that at tree level ($i.e.$, $\Delta V = 0$)

the minimum equations for the phases can be solved in terms of a simple geometrical object but the remaining conditions are then uncompatible. Namely, after a redefinition of masses and fields that cancels the terms m_{13}^2 and m_{24}^2, the four tree-level minimum conditions for the VEVs v_i with nonzero phases read [20]

$$v_1 \frac{\partial V}{\partial v_1} = v_1^2 \left[m_1^2 - \frac{m_{12}^2 m_{34}^2 - m_{14}^2 m_{32}^2}{m_{32}^2 m_{34}^2} \frac{1}{h(\mathbf{v})} + g(\mathbf{v}) \right] = 0$$

$$v_2 \frac{\partial V}{\partial v_2} = v_2^2 \left[m_2^2 - m_{12}^2 m_{32}^2 h(\mathbf{v}) - g(\mathbf{v}) \right] = 0$$

$$v_3 \frac{\partial V}{\partial v_3} = v_3^2 \left[m_3^2 + \frac{m_{12}^2 m_{34}^2 - m_{14}^2 m_{32}^2}{m_{12}^2 m_{14}^2} \frac{1}{h(\mathbf{v})} + g(\mathbf{v}) \right] = 0$$

$$v_4 \frac{\partial V}{\partial v_4} = v_4^2 \left[m_4^2 + m_{14}^2 m_{34}^2 h(\mathbf{v}) - g(\mathbf{v}) \right] = 0 \,, \tag{5}$$

where $g(\mathbf{v}) = \frac{1}{8}(g^2 + g'^2)[v_1^2 + v_3^2 - v_2^2 - v_4^2]$, and

$$h(\mathbf{v}) = \sqrt{m_{12}^2 m_{34}^2 - m_{14}^2 m_{32}^2} \sqrt{\frac{\frac{1}{m_{32}^2 m_{34}^2} v_1^2 - \frac{1}{m_{12}^2 m_{14}^2} v_3^2}{m_{12}^2 m_{32}^2 v_2^2 - m_{14}^2 m_{34}^2 v_4^2}} \,. \tag{6}$$

Since the four equations above depend on only two combinations of VEVs, $g(\mathbf{v})$ and $h(\mathbf{v})$, there will be no solution (i.e., phases different from 0 or π) unless a fine tuned value of the mass parameters is imposed. Moreover, if this fine tuning were used it would imply the presence of two massless scalar fields.

The effect of the radiative corrections is twofold: they relax the ammount of fine tuning required in the equations above, and they generate masses for the two massless modes. For these two effects to be sizeable we need (see ΔV in Eq. 4) large squark masses and at least two large Yukawa couplings. These could be the two top quark couplings h_{2t} and h_{4t} or one top (h_{2t}) plus one bottom (h_{1b}) coupling. However, for $m_s \leq 5$ TeV and Yukawas smaller than ≈ 1.2 (as required to avoid Landau poles before the Plank scale) we find that the two light scalar fields have masses smaller than ≈ 30 GeV. In consequence we conclude that in 4HD models with all the parameters real the presence of nontrivial complex phases in the Higgs VEVs implies two scalar fields apparently too light. (A more detailed examination of the parameter space might show, however, that this possibility is not entirely excluded by current limits on the masses of the scalar fields.) The situation here is then similar to the MSSM (where the allowed mass of the light scalar field is already excluded [11]) or the singlet model (which relay on radiative effects to give mass to a mode with negative tree-level mass [12]).

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In 4HD scenarios there is, however, still another possibility which seems consistent with the idea of SCPV. It requires that the four Higgs mass parameters μ in the super-potential (or, equivalently, the six parameters m_{ij}^2 in Eq. 2) are allowed to be complex. This could be justified since the Higgses are the only superfields which are not protected of mass contributions by the gauge symmetry. They could acquire their masses in an intermediate scale, via complex VEVs of singlet fields with no sizeable effect on the rest of the low-energy effective Lagrangian. We will not assume complex phases on all soft SUSY-breaking terms, since in principle these singlets do not couple to gauginos or squarks (we will neglect the possibility of further phases or new contributions to SUSY-breaking parameters due to the presence of nonsinglet heavy fields [21]). The hypothesis of complex Higgs masses, consistent with a *soft* origin of CP violation, is not possible in the MSSM or the singlet model, since there (unlike here) all the Higgs masses can be made real by field redefinitions.

In the next sections we study the implications of a 4HD model where all the parameters in the initial Lagrangian are real except for the Higgs mass parameters.

3 Definition of the model

The approximate flavor symmetry described in the previous section suppresses all FCNC amplitudes to acceptable limits. However, multi-Higgs models face a potential problem also with CP violation: if the Yukawa couplings are complex with phases of order one, CP-violating signals in K physics would be too large. In particular, ϵ_K would be typically two or three orders of magnitude larger than observed [19]. Thus it seems that a general model of many Higgs doublets requires another ingredient in addition to the flavor symmetry. Its effect should be either a suppression of the Yukawa couplings of the new doublets, or to make the complex phases small. The first approach is typical in models with natural flavor conservation (NFC) [22], whereas a natural suppression of the phases has been obtained in the superweak model with SCPV proposed in [9]. In our scenario these two effects will be achieved by the action of a global symmetry.

We will assume that the effective Lagrangian of the model obeys an approximate Peccei-Quinn like symmetry with the following assignment of charges [23]:

$$Q(H_3) = +1 \quad Q(H_4) = -1 \quad Q(D^c) = +1 \; . \tag{7}$$

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All other superfields have zero charge[1]. The symmetry is approximate in the sense that couplings of operators violating the symmetry are suppressed by powers of a small parameter ϵ_{PQ}. In this section we discuss the impact of this global symmetry first on the Higgs scalar part of the potential and then on the Yukawa sector.

The assignment of charges tells us that in the scalar potential m_{14}^2, m_{32}^2, m_{13}^2, m_{24}^4 are suppressed by ϵ_{PQ}. m_{12}^2 and m_{34}^2 remain unsuppressed (of the order of the SUSY-breaking scale $m_s \leq 1$ TeV). For easy reading we will write the suppression factors explicitly; for example m_{32}^2 becomes $\epsilon_{PQ} m_{32}^2$, where $m_{32}^2 = O(m_s^2)$. The tree-level scalar potential (we neglect radiative corrections in the following) involving only neutral Higgs fields is then given by

$$
\begin{aligned}
V &= \left(\begin{array}{cc} \phi_1^\dagger & \phi_3^\dagger \end{array} \right) \left(\begin{array}{cc} m_1^2 & \epsilon_{PQ} m_{13}^2 \\ \epsilon_{PQ} m_{13}^{2*} & m_3^2 \end{array} \right) \left(\begin{array}{c} \phi_1 \\ \phi_3 \end{array} \right) \\
&+ \left(\begin{array}{cc} \phi_2^\dagger & \phi_4^\dagger \end{array} \right) \left(\begin{array}{cc} m_2^2 & \epsilon_{PQ} m_{24}^2 \\ \epsilon_{PQ} m_{24}^{2*} & m_4^2 \end{array} \right) \left(\begin{array}{c} \phi_2 \\ \phi_4 \end{array} \right) \\
&+ [\left(\begin{array}{cc} \phi_1 & \phi_3 \end{array} \right) \left(\begin{array}{cc} m_{12}^2 & \epsilon_{PQ} m_{14}^2 \\ \epsilon_{PQ} m_{32}^2 & m_{34}^2 \end{array} \right) \left(\begin{array}{c} \phi_2 \\ \phi_4 \end{array} \right) + h.c.] \\
&+ \frac{1}{8}(g^2 + g'^2)[\left(\begin{array}{cc} \phi_1^\dagger & \phi_3^\dagger \end{array} \right) \left(\begin{array}{c} \phi_1 \\ \phi_3 \end{array} \right) - \left(\begin{array}{cc} \phi_2^\dagger & \phi_4^\dagger \end{array} \right) \left(\begin{array}{c} \phi_2 \\ \phi_4 \end{array} \right)]^2 .
\end{aligned}
\tag{8}
$$

As explained in the previous section, we assume that the mass parameters m_{ij}^2 are complex. The first two mass matrices above can be diagonalized through two unitary transformations of order ϵ_{PQ} of the scalar fields:

$$
\left(\begin{array}{c} \phi_1' \\ \phi_3' \end{array} \right) = \mathbf{U}_1 \left(\begin{array}{c} \phi_1 \\ \phi_3 \end{array} \right) ; \quad \left(\begin{array}{c} \phi_2' \\ \phi_4' \end{array} \right) = \mathbf{U}_2 \left(\begin{array}{c} \phi_2 \\ \phi_4 \end{array} \right) .
\tag{9}
$$

The quartic term in the potential will not change its form and can be obtained just by replacing unprimed by primed fields. The relative size of the four complex masses in the third mass matrix above will stay the same (i.e., the off-diagonal elements are still suppressed by ϵ_{PQ}). A phase transformation of the fields ϕ_i can be used to remove three of the four phases, leaving only one phase α in the scalar potential. Dropping the prime to specify transformed quantities, the mass terms read

$$
V_m = \left(\begin{array}{cc} \phi_1^\dagger & \phi_3^\dagger \end{array} \right) \left(\begin{array}{cc} m_1^2 & 0 \\ 0 & m_3^2 \end{array} \right) \left(\begin{array}{c} \phi_1 \\ \phi_3 \end{array} \right) + \left(\begin{array}{cc} \phi_2^\dagger & \phi_4^\dagger \end{array} \right) \left(\begin{array}{cc} m_2^2 & 0 \\ 0 & m_4^2 \end{array} \right) \left(\begin{array}{c} \phi_2 \\ \phi_4 \end{array} \right)
$$

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[1]There should also be charge assignments in the lepton sector, e.g. $Q(E^c) = +1$. We will comment on this possibility when discussing the electron EDM.

$$+ \; \left[\begin{pmatrix} \phi_1 & \phi_3 \end{pmatrix} \begin{pmatrix} m_{12}^2 & \epsilon_{PQ} e^{i\alpha} m_{14}^2 \\ \epsilon_{PQ} m_{32}^2 & m_{34}^2 \end{pmatrix} \begin{pmatrix} \phi_2 \\ \phi_4 \end{pmatrix} + h.c. \right] . \tag{10}$$

where the masses are now real and the phase α of m_{14}^2 has been written explicitly. We assume α to be of order one, since there is no symmetry reason for it to be suppressed. It is easy to see how the Higgs field redefinitions above change the mass parameters μ_{ij} in the superpotential W (these parameters are relevant since they will appear in scalar trilinears). We obtain μ_{12} and μ_{34} real up to order ϵ_{PQ}^2 whereas μ_{14} and μ_{23} will be mass coefficients with arbitrary complex phases but suppreessed by a power of ϵ_{PQ}.

We now go to the minimization of the Higgs potential. In particular, we want to find what are the relative size and the phases of the scalar VEVs suggested by the approximate symmetry. We write

$$< \phi_1 >= \frac{1}{\sqrt{2}} v_1 \; ; \quad < \phi_3 >= \frac{1}{\sqrt{2}} v_3 e^{i\delta_3} , \tag{11}$$

and

$$< \phi_2 >= \frac{1}{\sqrt{2}} v_2 e^{i\delta_2} \; ; \quad < \phi_4 >= \frac{1}{\sqrt{2}} v_4 e^{i\delta_4} , \tag{12}$$

where a global hypercharge transformation has been used to rotate away the phase of $< \phi_1 >$. A detailed discussion of the minimum equations can be found in the Appendix. The results are the following. For nonzero values of the phase α the minimum is allways complex. The suppression in terms of ϵ_{PQ} of the mass parameters determines the order of magnitude of the VEVs and phases:

$$
\begin{aligned}
v_1 \, , \; v_2 &= O(v) \\
v_3 \, , \; v_4 &= O(\epsilon_{PQ} v) \\
\delta_2 &= O(\epsilon_{PQ}^2) \\
\delta_3 \, , \; \delta_4 &= O(1)
\end{aligned}
\tag{13}
$$

where v denotes the weak scale. A remarkable feature of the model is that one can understand its structure in terms of an expansion in ϵ_{PQ} from the model with just two doublets. In the limit $\epsilon_{PQ} = 0$ the sectors (H_1, H_2) and (H_3, H_4) decouple; the minimum gives then equations for v_1 and v_2 identical to the VEVs in the MSSM, whereas $v_3 = v_4 = 0$. The phase δ_2 is then zero, while the phases δ_3 and δ_4 are irrelevant. Turning on a small value of ϵ_{PQ} gives (proportionally) VEVs to the extra pair of scalars. Simultaneously it allows a nonzero value of α, which translates into unsuppressed complex phases in the

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(H_3, H_4) sector and a phase of order ϵ_{PQ}^2 in the (H_1, H_2) *standard* sector. The mixings between the two sectors are small: either in a basis of scalar mass eigenstates or in a basis where $v_3' = v_4' = 0$ (useful when discussing FCNCs via Yukawas), both are obtained from the original basis just by unitary transformations of order ϵ_{PQ}.

We now turn to the Yukawa sector of the theory. The charge assignments dictates that the matrix $\mathbf{h}_2$ is unsuppressed, $\mathbf{h}_1$ and $\mathbf{h}_4$ are suppressed by a factor of ϵ_{PQ}, while $\mathbf{h}_3$ is suppressed by ϵ_{PQ}^2. Making this suppression explicit the Yukawa sector for the quark fields reads

$$\mathcal{L}_Y = Q(\epsilon_{PQ}\mathbf{h}_1 H_1 + \epsilon_{PQ}^2 \mathbf{h}_3 H_3)D^c + Q(\mathbf{h}_2 H_2 + \epsilon_{PQ}\mathbf{h}_4 H_4)U^c \,, \qquad (14)$$

where now $\mathbf{h}_i$ ($i = 1, ..., 4$) just carry the suppression from the flavor symmetry. In our model the Yukawa couplings of the initial Lagrangian are real. However, the field redefinitions performed to leave only one phase α in the Higgs potential will also redefine the Yukawa couplings and introduce complex phases. As we show, these phases translate into a CKM phase and complex FCNC couplings which are naturally suppressed by the approximate global symmetry.

We first performed the unitary transformations in Eq. (9), which redefine the fields H_i by (complex) factors of order ϵ_{PQ}. They imply a redefinition of the Yukawa matrices which introduces phases of order ϵ_{PQ}^2 in $\mathbf{h}_1$ and $\mathbf{h}_2$ and of order one in $\mathbf{h}_3$ and $\mathbf{h}_4$. Then we performed the (order one) phase redefinitions of the Higgs doublets that make all mass parameters real except for m_{14}^2. This translates into overall phases of order one multiplying the Yukawa matrices $\mathbf{h}_i$. However, we can still redefine the quark fields and absorb the phases which multiply $\mathbf{h}_1$ and $\mathbf{h}_2$ (the leading Yukawa couplings). The net result is that the Lagrangian in (14) expressed in terms of the Higgs fields used to minimize the potential has real (up to order ϵ_{PQ}^2) couplings in $\mathbf{h}_1$ and $\mathbf{h}_2$ and arbitrary complex phases in $\mathbf{h}_3$ and $\mathbf{h}_4$.

After spontaneous symmetry breaking, the structure of VEVs in (13) suggests (note that v_1 amd v_2 are not suppressed by powers of ϵ_{PQ}) $\tan\beta \equiv \sqrt{\frac{v_2^2 + v_4^2}{v_1^2 + v_3^2}} = O(1)$ and $\epsilon_{PQ} = O(m_b/m_t)$. Thus, the approximate global symmetry is used to accommodate the small ratio m_b/m_t, while the hierarchy between generations of the same charge is left to the flavor symmetry (the flavor symmetry would be exact in the limit with only the third generation being massive). The complex Yukawas $\mathbf{h}_3$ and $\mathbf{h}_4$ and the corresponding VEVs (v_3 and v_4) are both suppressed by a power of ϵ_{PQ}, whereas the leading Yukawas

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and VEVs are real up to order ϵ_{PQ}^2. We then obtain quark mass matrices where all the entries have complex phases of order ϵ_{PQ}^2. In consequence, the complex phase in the CKM matrix is also of order ϵ_{PQ}^2.

It is also easy to see what is the pattern of FCNC and CP violation via scalar exchange predicted by the model. It will be convenient to define a basis where only two of the four Higgs fields develop VEV and then only the second pair of scalars mediates FCNC processes. Again, this involves a unitary transformation of order ϵ_{PQ}, which leaves almost decoupled the extra pair of Higgses (essentially (H_3, H_4)). In addition, the mixings in the scalar mass matrices between the two sectors are also suppressed. The overall suppression by a power of ϵ_{PQ}, when added to the one with origin in the flavor symmetry, renders these tree-level FCNCs smaller than CKM (box) diagrams typically by a factor of ϵ_{PQ}^2. In particular, the $K - \bar{K}$ and $B - \bar{B}$ mixings are dominated here by the standard contributions, like the W-exchange box diagram in Fig. 1. This fact will distinguish our scenario from typical multi-Higgs models with soft CP violation where the tree-level superweak interactions are the main source of flavor changing processes [9]. On the other hand, the sector (H_3, H_4) involves arbitrary phases in Yukawa couplings and scalar VEVs. Although suppressed by a factor of ϵ_{PQ}^2, these couplings can be the dominant source of CP violation through diagrames like the one shown in Fig. 2. In particular, as shown in the next section, they are the main source of complex phases in K physics and compete with box contributions in B physics.

We need as well that FCNC contributions via SUSY particles (wino and gluino box diagrams) are within the experimental limits, which in general requires certain degree of squark-quark alignment and squark degeneracy. In fact, the squark-quark alignment could appear here as a natural consequence of the flavor symmetry[24]. Since we assumed no complex phases in soft-SUSY parameters others than Higgs masses, their CP violating effects will follow the same pattern described above. We explore these and other phenomenological implications of the model in the next sections.

4 CP violation in K and B physics and the neutron EDM

K physics. As our first example we look at the K system. In this scenario the FCNC

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processes via Yukawa interactions (see previous section) are highly suppressed, and the dominant contribution to Re ΔM_{12} comes from the box diagram in Fig. 1. The leading imaginary contribution to ΔM_{12}, however, will come from the neutral Higgs exchange in Fig. 2. The flavor-changing Yukawa couplings are complex (with phases of order 1) and generically suppressed by the Peccei-Quinn and the flavor symmetries (for example, in Fig. 2 the couplings are of order $\epsilon_{PQ}\sqrt{m_d m_s}/v$). When the mass of the exchanged scalar is around 1 TeV, this (complex) diagram is roughly suppressed by ϵ_{PQ}^2 with respect to the box diagram in Fig. 1. Since the CKM matrix and then the box contributions are approximately real, the leading contribution to Im (ΔM_{12}) comes from the Higgs exchange in Fig. 2. On dimensional grounds, the CP violating parameter ϵ_K (see [25] for definitions and notation) in the K system and ϵ_{PQ} are related:

$$|\epsilon_K| \approx \frac{1}{2\sqrt{2}} \frac{\text{Im }\Delta M_{12}}{\text{Re }\Delta M_{12}} \approx \epsilon_{PQ}^2. \tag{15}$$

The parameter ϵ_{PQ} sets the overall strength of Yukawa couplings of the Higgs doublets and suggests the order of magnitude of all the scalar VEVs. In particular (see section 3), one expects $\tan\beta = O(1)$ and $\epsilon_{PQ} = O(m_b/m_t)$. Then the relation above establishes $\epsilon_K \approx 10^{-3}$, as experimentally required.

Other sizeable contributions to ϵ_K may come from SUSY box diagrams with chargino or gluino exchange. Both of them require large SUSY contributions to ΔM_{12} (of the same order as the standard box diagram). Chargino box diagrams would then give [7] contributions of order $\epsilon_K \approx 10^{-1}\epsilon_{PQ}^2$, whereas gluino boxes [6] could be as large as $\epsilon_K \approx \epsilon_{PQ}^2$ (i.e., of the same order as the dominant tree-level scalar exchange). The factors ϵ_{PQ}^2 above derive from the suppresion in Yukawa couplings or extra scalar VEVs. Large gluino box contributions, however, also require large left-right squark mixing $\delta_{LR} \equiv m_{LR}^2/m_s^2 \approx 10^{-3}$ (a naive estimate would give $\delta_{LR} \approx \frac{A\sqrt{m_d m_s}}{m_s^2} \approx 10^{-4}$). In addition, the tree-level contributions to ϵ_K can be easily enhanced [23] assuming Higgs masses ligheter than 1 TeV, so the clear tendency in our model is that this type of nonstandard Higgs exchange provides the dominant contribution to ϵ_K.

In contrast, the expected value for ϵ_K' differs in principle from the standard model prediction. An estimate of ϵ_K' can be obtained from the phase $t_0 \equiv \text{Im}A_0/\text{Re}A_0$, where A_i is the decay amplitude of a K^0 into two pions of isospin i (see [25] for notation). In

particular one has the experimental constraint

$$t_0 \approx \sqrt{2} \left| \frac{A_0}{A_2} \right| |\epsilon_K'| \leq 10^{-4} \ . \tag{16}$$

The dominant contribution to $\mathrm{Re}A_0$ arises from the standard penguin diagram:

$$\mathcal{L}_p \approx \frac{\alpha_s \alpha_W}{3m_W^2} \sin\theta_c \ln \frac{m_c^2}{m_K^2} \mathcal{O}_{\mathcal{LR}} + H.c. \ , \tag{17}$$

where

$$\mathcal{O}_{\mathcal{LR}} = (\bar{s}_L \gamma_\mu T^a d_L)(\bar{q}_R \gamma_\mu T^a q_R) \tag{18}$$

and T^a are the 3-dimensional generators of $SU(3)$. In our scenario, however, the imaginary part of this penguin diagram is suppressed by the smallness of the phase (of order $\epsilon_{PQ}^2 \approx 10^{-3}$) in the CKM matrix. Since the standard model prediction for t_0 is of order $s_{13}s_{23}/s_{12} \approx 10^{-3}$, we obtain a first contribution of order 10^{-6}. Other contributions to t_0 may come from penguin diagrams with chargino and stop (Figure 3) and tree-level diagrams with charged scalars. The first contributions have been studied in [7] in the context of SUSY models with SCPV. It is found that they are typically of order $t_0 \approx 10^{-3} \sin\delta$, being δ the complex phase in the VEV of H_2 (the scalar field giving mass to the top quark). In our model the Higgs fields H and $\tilde{H}$ in Fig. 3 can correspond to H_2 or H_4. In the first case the phase δ is of order ϵ_{PQ}^2, and in the second case the same degree of suppression comes from the small VEV and the small Yukawa coupling of H_4. The contributions via exchange of charged Higgses have been analyzed in [9] in the context of two-Higgs doublets models. In our model either the Yukawas are *almost* real (for the doublets with dominant couplings, as it happens in [9]), or the Yukawas themselves are suppressed. Hence, from the three types of diagrams we obtain contributions (taking $\epsilon_{PQ}^2 \approx 10^{-3}$) of order $t_0 \approx 10^{-6}$ or $\frac{\epsilon_K'}{\epsilon_K} \approx 10^{-5}$ (for $|A_2/A_0| \approx 1/22$).

Potentially larger contributions to ϵ_K' are expected from gluino-mediated penguin diagrams (Figure 4). Although gluino masses in our model are real, there will be complex phases of order ϵ_{PQ}^2 (see discussion of chargino penguin above) in left-right squark mixings. This type of contributions to ϵ_K' have been studied in detail in [6]. There it is found that for complex phases in gluino masses of order 10^{-3} they could result in values as large as $\frac{\epsilon_K'}{\epsilon_K} \approx 10^{-3}$. An analogous result has been recently obtained in [26], with contributions from squark mixings of CKM type and small CP-violating phases (which are natural in our scenario). These values are only obtained, however, when gluino box diagrams

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saturate the value of ϵ_K. Since we have assumed that the FCNCs are here dominated by standard box diagrams, we expect a value for ϵ'_K typically smaller. Modulo hadronic matrix element uncertainties, we estimate

$$\frac{\epsilon'_K}{\epsilon_K} \approx 10^{-4} - 10^{-5} \ , \tag{19}$$

with the possibility to consistently increase this value via gluino penguin contributions.

B physics. We consider now CP violation in B physics [27]. Although today the only observed CP violation is in the K system, the standard model predicts clear signals in B decays that should be observed in the near future. These CP asymmetries are generally parametrized in terms of the complex phases λ_{iq}, which in turn depend on the product of phases in three amplitudes: the direct b decay, the $B - \bar{B}$ mixing, and (possibly) the $K - \bar{K}$ mixing. In CKM scenarios the phases λ_{iq} are constrained by unitarity and have a simple geometrical interpretation (these predictions are not expected to change much in minimal SUSY models). However, in our 4HD scenario the three amplitudes above have complex phases of order ϵ^2_{PQ} (see below), and the predictions change to the extent that no CP asymmetries will be observed at the projected B factory at SLAC.

To see why this is so, we will first consider the decay amplitude of a b quark into lighter flavors. The main contribution corresponds to a tree-level diagram with W exchange. Since it will be proportional to elements of the CKM matrix, its imaginary component will be suppressed by a factor of ϵ^2_{PQ}. The decay via charged Higgs are suppressed by the same factor due to the relative smallness of their Yukawas and the smallness of the mixing (in the scalar mass matrix) between the standard and the extra Higgs sectors. In non-SUSY models with NFC the second effect can give significant contributions (proportional to m_t) in B decays and in flavor-changing processes [9].

The main contribution in this model to $B - \bar{B}$ mixing $\Delta M_{B\overline{B}}$ comes from the standard box diagrams, and is proportional to CKM elements. The tree-level diagrams with exchange of neutral scalar give contributions which are suppressed by the flavor and the Peccei-Quinn symmetries, with a relative factor of ϵ^2_{PQ} with respect to the box diagrams. Both types of diagrams introduce imaginary components of order ϵ^2_{PQ} with respect to the main real component, and the contribution to the complex phase λ_{iq} from $B - \bar{B}$ mixing is negligible (of that order). As discussed above, the same conclusion applies to the contribution from $K - \bar{K}$ mixing.

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In consequence, in this scenario one expects that all CP asymmetries in B decays negligibly small (of order $\epsilon_{PQ}^2 \approx 10^{-3}$). This type of prediction is shared, for example, by non-SUSY multi-Higgs doublet models [9, 19] or SUSY models with real Yukawa matrices [7, 6]. In CKM scenarios the situation is essentially different. There the smallness of CP violation in the K system is atributed to the smallness of the CKM elements involving the third family of quarks, whereas CP-violating asymmetries in the B system are large: the $B - \bar{B}$ mixing and the b decays are proportional to elements of the CKM matrix with arbitrary complex phases. The absence of CP asymmetries at the SLAC B factory would point to a non-CKM origin of CP violation, and many-Higgs doublet model (SUSY or non-SUSY) would appear as a natural candidate.

Neutron EDM: As in usual SUSY scenarios, the prediction of our model for the neutron EDM d_n is much larger than in the non-SUSY standard model. In the MSSM the explicit phases ψ in SUSY-breaking gaugino mases and scalar trilinears introduce contributions which roughly require a suppression of 2 or 3 orders of magnitude: $d_n \approx 10^{-25}(\frac{\psi}{7 \times 10^{-3}})$ e cm [2]. These diagrams are also present in our scenario, but their contribution has a natural suppression of order ϵ_{PQ}^2 respect to the MSSM. The origin of this factor is (again!) either the smallnes of the complex phase δ_2 (of order ϵ_{PQ}^2) in the two standard Higgs doublets, or the combined relative smallness of VEVs and Yukawa couplings (both suppressed by a factor of ϵ_{PQ}) of the two extra doublets.

To illustrate this fact, let us consider the contribution from the the one-loop chargino-squark diagram (Figure 5). When H corresponds to H_1, then the complex phase in the Yukawa coupling is suppressed. When H is H_3, then the VEV and the Yukawa coupling are of order ϵ_{PQ}. In consequence, in this 4HD one expects

$$d_n \approx 10^{-23}\epsilon_{PQ}^2 \ e \ \text{cm} \approx 10^{-26} \ e \ \text{cm} , \tag{20}$$

a value which is close to the present experimental limit $|d_n| < 1.2 \times 10^{-25} e$ cm [28].

Here we also comment on the lepton sector. We still have the freedom to assign a global symmetry charge to E^c (or even L). For simplicity let us consider $E^c = +1$, which would be consistent with $m_\tau = O(m_b)$. In this sector all FCNC processes via nonstandard scalars will be completely negligible (the size of Yukawas suggested by the flavor symmetry would be enough to control all these processes). The pattern of CP violation will be analogous to the one discussed in the quark sector, with the relevant

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complex phases suppressed by a factor of ϵ_{PQ}^2. The leading contribution to the electron EDM comes from a diagram similar to the one shown in Fig. 5. If all the superparticles have comparable masses it is expected that $d_e \approx 10^{-2}d_n$ [29], so that in our model the electron EDM is not far from the present experimental limits.

5   Other phenomenological implications

As shown by Flores and Sher in [13], the presence of a light neutral scalar field (with a tree-level mass smaller than M_Z) is a prediction shared by all SUSY models with Higgs doublets only, regardless of the number of doublets. Since in the limit $\epsilon_{PQ} \rightarrow 0$ the scalar sector of our 4HD model essentially coincides with the MSSM, we expect small corrections to the standard predictions.

To see how these corrections arise we will first consider the model with $\alpha = 0$ and, consequently, with all the VEVs real. The approximate symmetry dictates that $v \approx v_1 \approx v_2$ and $\epsilon_{PQ}v \approx v_3 \approx v_4$. We can perform two rotations of order ϵ_{PQ} of the Higgs fields (one in the space $\phi_1 - \phi_3$ and another in $\phi_2 - \phi_4$) in such a way that $v_3 = v_4 = 0$. It is then straightforward to find the mass 4×4 matrix M_h^2

$$M_h^2 = \left(\begin{array}{cc} M_0^2 & M_1^2 \\ M_1^{2\,T} & M_2^2 \end{array} \right) \tag{21}$$

for the CP-even scalar fields h_i. The 2×2 matrix M_0^2 corresponding to $h_1 - h_2$ is identical to the one obtained in the MSSM, with an eigenvalue m_h^2 smaller than M_Z^2 and another $m_H^2 \approx m_s^2$. The submatrix M_2^2 in the $h_3 - h_4$ sector has two eigenvalues of order m_s^2. The Peccei-Quinn symmetry forces all the elements in $M_1^2 = O(\epsilon_{PQ}m_s^2)$ and, through mixing, tends to lower the lightest eigenvalue in M_h^2 by terms of order $\epsilon_{PQ}^2 m_s^2$. For nonzero values of α the scalar VEVs will be allways complex (see Section 3), introducing mixing betweeen CP-odd and CP-even states. Due to the approximate symmetry, however, the mixings of the lightest scalar field with CP-odd states are small and introduce corrections of the same order. In consequence, we conclude that these corrections do not change significantly the tree-level bound $m_h < M_Z$ (for $\epsilon_{PQ}^2 = 10^{-3}$ and $m_s = 500$ GeV this bound is lowered by less than 2 GeV). However, we expect radiative top quark effects to be much more important.

We note that the spontaneous breaking of the (approximate) global symmetry do not introduce light fields. The reason for this is that in the limit of exact symmetry

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($\epsilon_{PQ} = 0$) the only VEVs breaking the symmetry (v_3 and v_4) go to zero too: there are no light pseudo-goldstone states because the size of the spontaneous and the explicit symmetry breaking terms is of the same order.

It is also easy to see that this model accommodates the small ratio m_b/m_t without need of fine tuning to avoid too light charginos [30] (in the MSSM, a small mass ratio m_b/m_t based on a large value of $\tan\beta$ implies such fine-tuning problem). The chargino mass matrix is here

$$\begin{pmatrix} \mu_{12} & \epsilon_{PQ}\mu_{14} & \frac{gv}{\sqrt{2}} \\ \epsilon_{PQ}\mu_{32} & \mu_{34} & \frac{g\epsilon_{PQ}v}{\sqrt{2}} \\ \frac{gv}{\sqrt{2}} & \frac{g\epsilon_{PQ}v}{\sqrt{2}} & M \end{pmatrix} , \qquad (22)$$

where we used the VEVs in (13) and denoted the gaugino mass by M. This structure has no light eigenvalues.

Another possible implication of 4HD models concerns the value of R_b. Within the standard model, the partial width of the Z boson to $b\bar{b}$ seems to be very sensitive to top-quark radiative correction. For the top observed in CDF the predicted value is well below (a three-σ deviation) the present experimental limits [31]. In minimal SUSY scenarios the main correction results from the balance between Z vertices with Higgs-top and their SUSY partners, and the anomalous value of R_b can be alleviated for light charginos and/or light stop scalars [32]. In 4HD SUSY models the situation is similar (especially in our scenario, due to the global approximate symmetry assumed in the Yukawa sector), with more freedom than in the MSSM to adjust the corrections. Note, for example, that large bottom Yukawa couplings do not imply necessarily a large value of $\tan\beta$ ($\equiv \sqrt{\frac{v_2^2+v_4^2}{v_1^2+v_3^2}}$).

Our last comment concerns the strong CP problem. In the model under consideration there are (tree-level) contributions to θ of order $\epsilon_{PQ}^2 \approx 10^{-3}$, a value much bigger than the present experimental limit ($\theta < 10^{-9}$). It seems possible, however, that the intermediate scale used to break CP would also define a realistic axion scenario. Some of the ingredients (a Peccei-Quinn symmetry or singlet VEVs breaking the global symmetries) are already present in the model. Of course, for this scenario to work other requirements (on the dimension of the operators breaking the anomalous Peccei-Quinn symmetry, on the ratio of the scales involved,...) are also needed.

6   Conclusions

The origin of CP nonconservation in SUSY models provides a good reason to explore nonminimal extensions. In the usual MSSM scenario CP-violating phases occur in two different sectors: in Yukawa couplings, where they would be responsible for CP violation in K and B physics, and in SUSY-breaking terms (gaugino masses and scalar trilinears), where they would induce too large fermion EDMs unless suppressed by two or three orders of magnitude.

We have presented here an extension of the MSSM with four Higgs doublets where the complex phases appear spontaneously, induced by explicit phases in Higgs masses. An approximate Peccei-Quinn symmetry *almost* decouples the pair of extra Higgs fields, but their small couplings (also suppressed by the flavor symmetry) turn out to be responsible for all CP-violating phenomena. In particular, tree-level FCNC diagrams are irrelevant in $\mathrm{Re}\Delta M_{12}$ but responsible for ϵ_K. The resulting CKM matrix of the model has a negligible complex phase of order $\epsilon_{PQ}^2 \approx 10^{-3}$. This suppression appears in all CP signals either from small phases in the dominant scalar sector or from small ratios of VEVs and Yukawa couplings in the extra sector.

On dimensional grounds, the parameter ϵ_{PQ} specifying the violation of the Peccei-Quinn symmetry sets:

- the ratio $m_b/m_t \approx \epsilon_{PQ}$, and the relative suppression of the Yukawa couplings of the extra Higgses ($\mathbf{h}_3/\mathbf{h}_1 \approx \mathbf{h}_4/\mathbf{h}_2 \approx \epsilon_{PQ}$);

- the parameter $\epsilon_K \approx \epsilon_{PQ}^2$ and the ratio $\epsilon_K'/\epsilon_K \leq \epsilon_{PQ}^2$ (with a preferred value $(10^{-1} - 10^{-2})\epsilon_{PQ}^2$);

- the neutron EDM $\approx 10^{-23}\epsilon_{PQ}^2 e$ cm, being 10^{-23} an estimate for typical SUSY-breaking parameters;

- and the CP violating asymmetries $\lambda_{iq} \approx \epsilon_{PQ}^2$ involved in B physics.

In consequence, a neutron EDM close to its present experimental limit, negligible CP-violating effects on B physics, and a small value of the ϵ_K' parameter could be regarded as typical predictions of the model. In addition, we have estimated the effects of the extra sector on the mass of the lightest neutral scalar and commented on other aspects of the model (R_b and the strong θ parameter).

We think that 4HD models constitute an interesting possibility in SUSY extensions which, however, seems almost absent in the literature. We have defined a scenario where

CP violation is brought under control in a consistent way (due to the action of an approx- imate symmetry), in contrast to SUSY models where the complex phases are assumed small without explanation. Although we have analyzed a particular model, we think that it contains essential ingredients which may be shared by any satisfactory multi-Higgs SUSY model. In a generic multi-Higgs model hard (CKM-like) CP violation seems to imply too large imaginary FCNCs mediated by the extra Higgs fields. This fact strongly suggests a soft origin of CP violation. Then the problem of containing simultaneously FCNC and too large CP violation forces these models to have, for example, unobservable CP asymmetries in B decays, a prediction that will be tested in the near future.

Acknowledgments

Appendix: Solutions to minimum equations

The vacuum expectation value of the scalar potential is

$$
\begin{aligned}
< V > \;=\; & \frac{1}{2}m_1^2 v_1^2 + \frac{1}{2}m_2^2 v_2^2 + \frac{1}{2}m_3^2 v_3^2 + \frac{1}{2}m_4^2 v_4^2 + m_{12}^2 v_1 v_2 \cos\delta_2 + \\
& + \; \epsilon_{PQ} m_{14}^2 v_1 v_4 \cos(\delta_4 + \alpha) + \epsilon_{PQ} m_{32}^2 v_3 v_2 \cos(\delta_3 + \delta_2) + m_{34}^2 v_3 v_4 \cos(\delta_3 + \delta_4) + \\
& + \; \frac{1}{32}(g^2 + g'^2)[v_1^2 + v_3^2 - v_2^2 - v_4^2]^2.
\end{aligned}
\tag{23}
$$

The conditions at the minimum are

$$
\begin{aligned}
v_1 \frac{\partial V}{\partial v_1} \;=\;& m_1^2 v_1^2 + m_{12}^2 v_1 v_2 \cos\delta_2 + \epsilon_{PQ} m_{14}^2 v_1 v_4 \cos(\delta_4 + \alpha) + v_1^2 g(\mathbf{v}) = 0 \,, \\
v_2 \frac{\partial V}{\partial v_2} \;=\;& m_2^2 v_2^2 + m_{12}^2 v_1 v_2 \cos\delta_2 + \epsilon_{PQ} m_{32}^2 v_3 v_2 \cos(\delta_3 + \delta_2) - v_2^2 g(\mathbf{v}) = 0 \,, \\
v_3 \frac{\partial V}{\partial v_3} \;=\;& m_3^2 v_3^2 + \epsilon_{PQ} m_{32}^2 v_3 v_2 \cos(\delta_3 + \delta_2) + m_{34}^2 v_3 v_4 \cos(\delta_3 + \delta_4) + v_3^2 g(\mathbf{v}) = 0 \,, \\
v_4 \frac{\partial V}{\partial v_4} \;=\;& m_4^2 v_4^2 + m_{34}^2 v_3 v_4 \cos(\delta_3 + \delta_4) + \epsilon_{PQ} m_{14}^2 v_1 v_4 \cos(\delta_4 + \alpha) - v_4^2 g(\mathbf{v}) = 0 \quad (24)
\end{aligned}
$$

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where $g(\mathbf{v}) = \frac{1}{8}(g^2 + g'^2)[v_1^2 + v_3^2 - v_2^2 - v_4^2]$, and

$$-\frac{\partial V}{\partial \delta_2} = m_{12}^2 v_1 v_2 \sin \delta_2 + \epsilon_{PQ} m_{32}^2 v_3 v_2 \sin(\delta_3 + \delta_2) = 0 \, ,$$

$$-\frac{\partial V}{\partial \delta_3} = \epsilon_{PQ} m_{32}^2 v_3 v_2 \sin(\delta_3 + \delta_2) + m_{34}^2 v_3 v_4 \sin(\delta_3 + \delta_4) = 0 \, ,$$

$$-\frac{\partial V}{\partial \delta_4} = m_{34}^2 v_3 v_4 \sin(\delta_3 + \delta_4) + \epsilon_{PQ} m_{14}^2 v_1 v_4 \sin(\delta_4 + \alpha) = 0. \tag{25}$$

First, we can estimate the relative sizes of VEVs. The solutions to (24) are consistent with either one of the pairs of VEVs (v_1, v_2) or (v_3, v_4) being suppressed by order ϵ_{PQ} with respect to the weak scale. Depending on the sizes of the unsupressed parameters m_1^2, m_3^2, m_{13}^2 and m_2^2, m_4^2, m_{24}^2 the absolute minimum will prefer one of the above choices. This can easily be seen from the following. Imagine for a moment that $\epsilon_{PQ} = 0$. The equations in (24) reduce to two pairs of equations, with first pair depending on the ratio $\frac{v_1}{v_2}$ and $g(\mathbf{v})$, and the second on $\frac{v_3}{v_4}$ and the same function $g(\mathbf{v})$. Thus, one pair of VEVs is forced to be zero. Turning back on a small ϵ_{PQ}, the terms in the Lagrangian suppressed by ϵ_{PQ} can make the previously trivial pair nonzero, but suppressed. The only question left is which pair of the VEVs is small, and this will depend on the choice of the unsuppressed mass parameters in the potential. We will assume that the parameters are such that (v_1, v_2) are unsuppressed (and of the order weak scale), while (v_3, v_4) are of the order ϵ_{PQ} times the weak scale. This assumption does not involve fine tuning but only "halves" the available parameter space. For the phases δ_i, it was shown in [20] that in the limit $\alpha = 0$ there is no nontrivial solution (*i.e.*, δ_i different from 0 or π). It is easy to see, however, that for $\alpha \neq 0$ the equations (24) do *not* have this trivial solution, and complex phases are guaranteed. In particular, for $\alpha = O(1)$ from the two first equations in (25) it follows that δ_2 is of order ϵ_{PQ}^2 (modulo π, depending on the sign of m_{12}^2) and δ_3 and δ_4 are unsuppressed.

In summary, the structure of values of VEVs and their phases is

$$\begin{aligned}
v_1 \, , v_2 &= O(v) \\
v_3 \, , v_4 &= O(\epsilon_{PQ} v) \\
\delta_2 &= O(\epsilon_{PQ}^2) \\
\delta_3 \, , \delta_4 &= O(1)
\end{aligned} \tag{26}$$

where v denotes the weak scale.

We now explore this structure in more detail (at first order in ϵ_{PQ}). The first equation in (25) gives δ_2 to be of order ϵ_{PQ}^2 up to[2] a factor of π. Such δ_2 does not contribute to leading order (its contributions are $O(\epsilon_{PQ}^2)$) to the minimum of the scalar potential (23), and can be neglected in the rest of equations. The minimum equations (24) to leading order are

$$v_1 \frac{\partial V}{\partial v_1} = m_1^2 v_1^2 + m_{12}^2 v_1 v_2 + v_1^2 g_o(\mathbf{v}) = 0,$$

$$v_2 \frac{\partial V}{\partial v_2} = m_2^2 v_2^2 + m_{12}^2 v_1 v_2 - v_2^2 g_o(\mathbf{v}) = 0,$$

$$v_3 \frac{\partial V}{\partial v_3} = m_3^2 v_3^2 - \epsilon_{PQ} m_{32}^2 v_3 v_2 \cos \delta_3 + m_{34}^2 v_3 v_4 \cos(\delta_3 + \delta_4) + v_3^2 g_o(\mathbf{v}) = 0,$$

$$v_4 \frac{\partial V}{\partial v_4} = m_4^2 v_4^2 + m_{34}^2 v_3 v_4 \cos(\delta_3 + \delta_4) + \epsilon_{PQ} m_{14}^2 v_1 v_4 \cos(\delta_4 + \alpha) - v_4^2 g_o(\mathbf{v}) = 0, \quad (27)$$

where $g_o(\mathbf{v}) = \frac{1}{8}(g^2 + g'^2)[v_1^2 - v_2^2]$, and

$$-\frac{\partial V}{\partial \delta_3} = -\epsilon_{PQ} m_{32}^2 v_3 v_2 \sin \delta_3 + m_{34}^2 v_3 v_4 \sin(\delta_3 + \delta_4) = 0,$$

$$-\frac{\partial V}{\partial \delta_4} = m_{34}^2 v_3 v_4 \sin(\delta_3 + \delta_4) + \epsilon_{PQ} m_{14}^2 v_1 v_4 \sin(\delta_4 + \alpha) = 0. \quad (28)$$

The first two equations in (27) are just the equations of the MSSM, and they fix v_1 and v_2 in the usual way. Thus, we expect both v_1 and v_2 to be of the order of weak scale and $\tan \beta = O(1)$ (*i.e.* no supression by ϵ_{PQ}, and no fine tuning producing large $\tan \beta$).

The goal now is to find the phases δ_3 and δ_4 in terms of quantities $c = \epsilon_{PQ} m_{32}^2 v_3 v_2$, $f = m_{34}^2 v_3 v_4$ and $y = -\epsilon_{PQ} m^2 v_1 v_4$ and the angle α in order to eliminate them in the third and fourth equation of VEVs in (27). We note that the three quantities c, f, and y are of order $O(\epsilon_{PQ}^2)$, and so we expect δ_3 and δ_4 unsupressed. In order to find δ_3 and δ_4 we use a geometrical interpretation similar to the one devised in [20]. It is possible to see that the two equations (28) define one of the objects shown in Figure 6 (which one it is will depend whether $1/c$, $1/f$ and $1/y$ can form a triangle or not). The quantities p and q there are not independent, and can be expressed in terms of c, f, y and α. The difference between the two types of solutions can be understood in the limit $\alpha \rightarrow 0$, where only the trivial solutions $\delta_3 = 0$ and $\delta_4 = \pi$ exist. For $\alpha = 0$ the object in Fig. 6(b) implies nonzero δ_3 and δ_4, a type of solution which requires fine tuning between mass parameters

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[2] In the following we choose m_{12}^2 positive without loss of generality and thus $\delta_2 \sim \pi + O(\epsilon_{PQ}^2)$.

once it is substituted in the equations for v_3 and v_4. In consequence, for small values of α only the solutions of the type in Fig. 6(a) appear. When α is nonzero the fine tuning is lifted, and both types of objects define possible solutions to the minimum equations.

We performed numerical solutions to the above equations when $\alpha \neq 0$ and large (order 1) and we found that the minima satisfy the structure given in (26). For simplicity and to illustrate the discussion above we will give the equations for v_3 and v_4 at first order in α.

Fig. 6(a): When $\alpha \to 0$ we see that $\delta_3 \to 0$ and $\delta_4 \to \pi$, while $1/p \to 1/c + 1/f$ and $1/q \to 1/y - 1/f$. From the figure we first find cosines of the relevant angles ($\delta_3 + \delta_4$, δ_3 and $\delta_4 + \alpha$) to leading order in α. Then we are in the position to find v_3 and v_4 by substituting these expressions into the two last equations of (27)

$$
v_3 \frac{\partial V}{\partial v_3} = m_3^2 v_3^2 - c[1 - \frac{\alpha^2}{2}(\frac{\frac{1}{c}}{\frac{1}{f} + \frac{1}{c} - \frac{1}{y}})^2] - f[1 - \frac{\alpha^2}{2}(\frac{\frac{1}{f}}{\frac{1}{f} + \frac{1}{c} - \frac{1}{y}})^2] + v_3^2 g(\mathbf{v}) = 0\,,
$$

$$
v_4 \frac{\partial V}{\partial v_4} = m_4^2 v_4^2 - f[1 - \frac{\alpha^2}{2}(\frac{\frac{1}{f}}{\frac{1}{f} + \frac{1}{c} - \frac{1}{y}})^2] + y[1 - \frac{\alpha^2}{2}(\frac{\frac{1}{y}}{\frac{1}{f} + \frac{1}{c} - \frac{1}{y}})^2] - v_4^2 g(\mathbf{v}) = 0 \,(29)
$$

where, again, $c = \epsilon_{PQ} m_{32}^2 v_3 v_2$, $f = m_{34}^2 v_3 v_4$ and $y = -\epsilon_{PQ} m^2 v_1 v_4$. These equations, although still complicated, can be solved in v_3 and v_4, (remember that v_1 and v_2 are already fixed). Then we can go back and find δ_3 and δ_4, thus completing the search for the first case.

Fig. 6(b): In this case $1/p \to 1/y$ and $1/q \to 1/c$, while δ_3 and δ_4 tend to go to angles in the triangle with sides $1/c$, $1/f$ and $1/y$ (we denote this (order $O(1)$) asymptotic angles as δ_3^o and δ_4^o). We can again find the relevant angles to leading order in α and then find v_3 and v_4 by substituting these expressions into the two last equations of (27)

$$
v_3 \frac{\partial V}{\partial v_3} = v_3^2[m_3^2 + \frac{m_{32}^2 m_{34}^2}{m_{14}^2}\frac{v_2}{v_1} + g^0(\mathbf{v})] + 2\alpha f \frac{\sin(\delta_3^o + \delta_4^o)}{\tan \delta_3^o \tan \delta_4^o} = 0\,,
$$

$$
v_4 \frac{\partial V}{\partial v_4} = v_4^2[m_4^2 + \frac{m_{34}^2 m_{14}^2}{m_{32}^2}\frac{v_1}{v_2} - g^0(\mathbf{v})] + 2\alpha f \frac{\sin(\delta_3^o + \delta_4^o)}{\tan \delta_3^o \tan \delta_4^o} = 0\,,
$$

$$
(30)
$$

Remembering that v_1 and v_2 are already fixed in terms of m_1^2, m_2^2 and m_{12}^2, we see clearly the fine tuning for vanishing α[20]: the terms in square brackets would be forced to vanish, implying two relations between mass parameters. However, once we include $\alpha \neq 0$ these degeneracies get lifted.

References

[1] For a review on SUSY see H.P. Nilles, Phys. Rep. **110** (1981) 1.

[2] A. de Rújula, M.B. Gavela, P. Pène, and F.J. Vegas, Phys. Lett. **B245** (1990) 640.

[3] See for example I.I. Bigi and F. Gabbiani, in *Proceedings of the BNL Summer Study on CP violation* Brookhaven National Laboratory, 1990, edited by S. Dawson and A. Soni.

[4] J. Ellis, D.V. Nanopoulos, Phys. Lett. **B110** (1982) 44.

[5] P. Langacker and B. Sathiapalan, Phys. Lett. **B144** (1984) 401.

[6] K.S. Babu and S.M. Barr, Phys. Rev. Lett. **72** (1994) 2831.

[7] A. Pomarol, Phys. Rev. **D47** (1993) 273.

[8] T.D. Lee, Phys. Rev. **D8** (1973) 1226.

[9] J. Liu and L. Wolfenstein, Nucl. Phys. **B289** (1987) 1.

[10] N. Maekawa, Phys. Lett. **B282** (1992) 387.

[11] A. Pomarol, Phys. Lett. **B287** (1992) 331.

[12] K.S. Babu and S.M. Barr, Phys. Rev. **D49** (1994) 2156.

[13] For previous work on four Higgs doublet SUSY models see R.A. Flores and M. Sher, Ann. Phys. **148** (1983) 95; K. Griest and M. Sher, Phys. Rev. **D42** (1990) 3834; H. Haber and Y. Nir, Nucl. Phys. **B335** (1990) 363.

[14] F. Gursey, P. Sikivie, and P. Ramond, Phys. Lett. **B60** (1976) 177; A string model with four Higgs doublets has been proposed in F. del Aguila, M. Masip, and L. da Mota, Nucl. Phys. **B440** (1995) 3.

[15] B. Brachmachari and R.N. Mohapatra, University of Maryland preprint No. UMD-PP-95-138 (hep-ph/9505347).

[16] J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. **B257** (1991) 83.

[17] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. **B147** (1979) 277; T.P. Cheng and M. Sher, Phys. Rev. **D35** (1987) 3484.

[18] A. Antaramian, L.J. Hall and A. Rašin, Phys. Rev. Lett. **69** (1992) 1871.

[19] L. Hall and S. Weinberg, Phys. Rev. **D48** (1993) 979.

[20] M. Masip and A. Rašin, University of Maryland preprint No. UMD-PP-95-143 (hep-ph/9506471), to appear in Phys. Rev. **D**.

[21] A. Dannenberg, L. Hall and L. Randall, Nucl. Phys. **B271** (1985) 575.

[22] S. Glashow and S. Weinberg, Phys. Rev. **D15** (1977) 1958.

[23] A. Rašin, talk given at the XXXth Moriond conference on Electroweak Interactions and Unified Theories, Les Arcs, 1995, University of Maryland preprint No. UMD-PP-95-142 (hep-ph/9507239).

[24] Y. Nir and N. Seiberg, Phys. Lett. **B309** (1993) 337.

[25] L.L. Chau, Phys. Rep. **95** (1983) 1.

[26] A. Pomarol and D. Tommasini, CERN preprint No. CERN-TH/95-207 (hep-ph/9507462).

[27] For a review and further references see Y. Nir and H. Quinn, Ann. Rev. Nucl. Part. Sci. **42** (1992) 211.

[28] I.S. Altarev *et al.*, Phys. Lett. **B276** (1992) 242; K.F. Smith *et al.*, Phys. Lett. **B234** (1990) 191.

[29] W. Bernreuther and M. Suzuki, Rev. Mod. Phys. **63** (1991) 313.

[30] A.E. Nelson and L. Randall, Phys. Lett. **B316** (1993) 516.

[31] *Review of Particle Properties*, Phys. Rev. **D50** (1994) 1173.

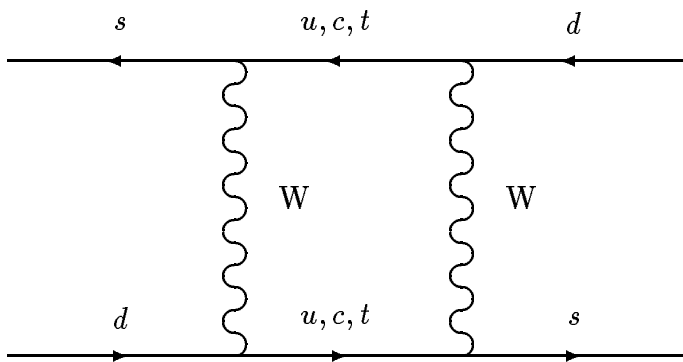[32] See for example D. García, R.A. Jiménez, and J. Solà, Phys. Lett. **B347** (1995) 2737.

Figure 1: Leading contribution to Re ΔM_{12}.

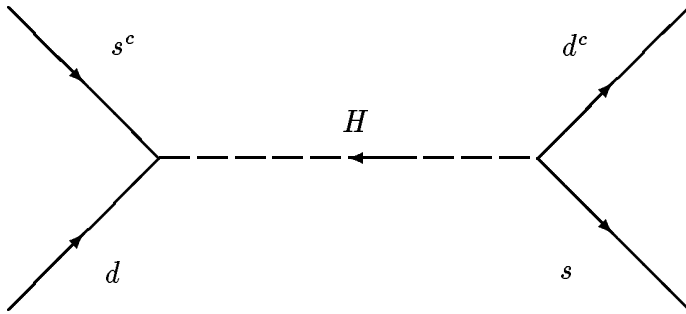Figure 2: Leading contribution to Im ΔM_{12}.

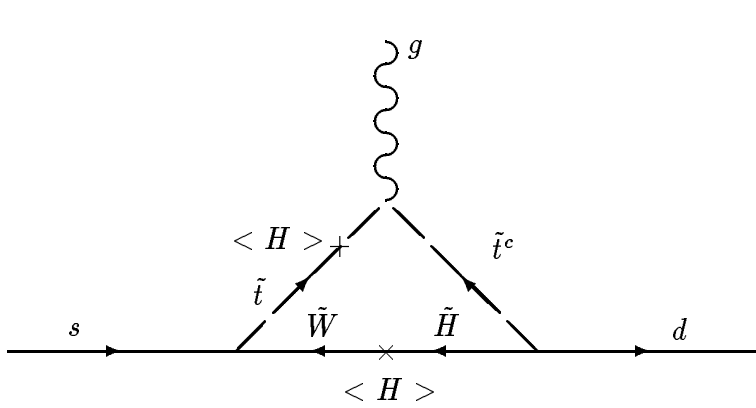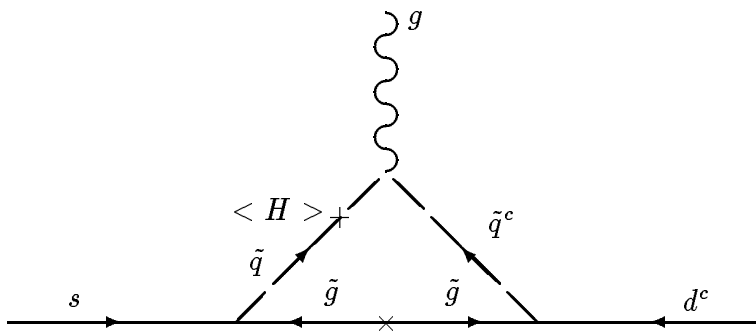Figure 3: Chargino contribution to ϵ'_K.

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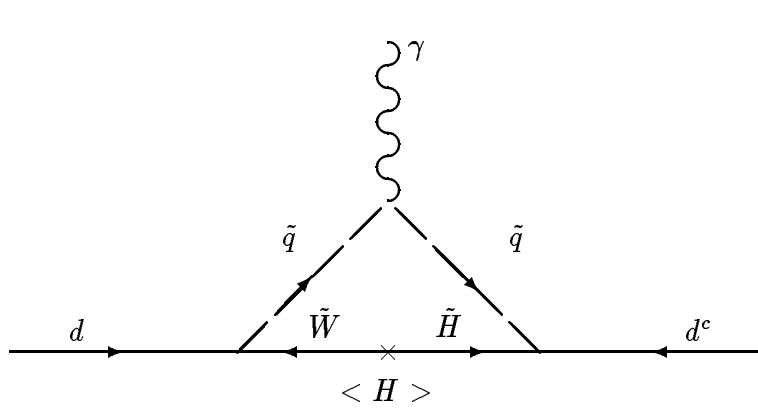Figure 4: Gluino contribution to ϵ'_K.
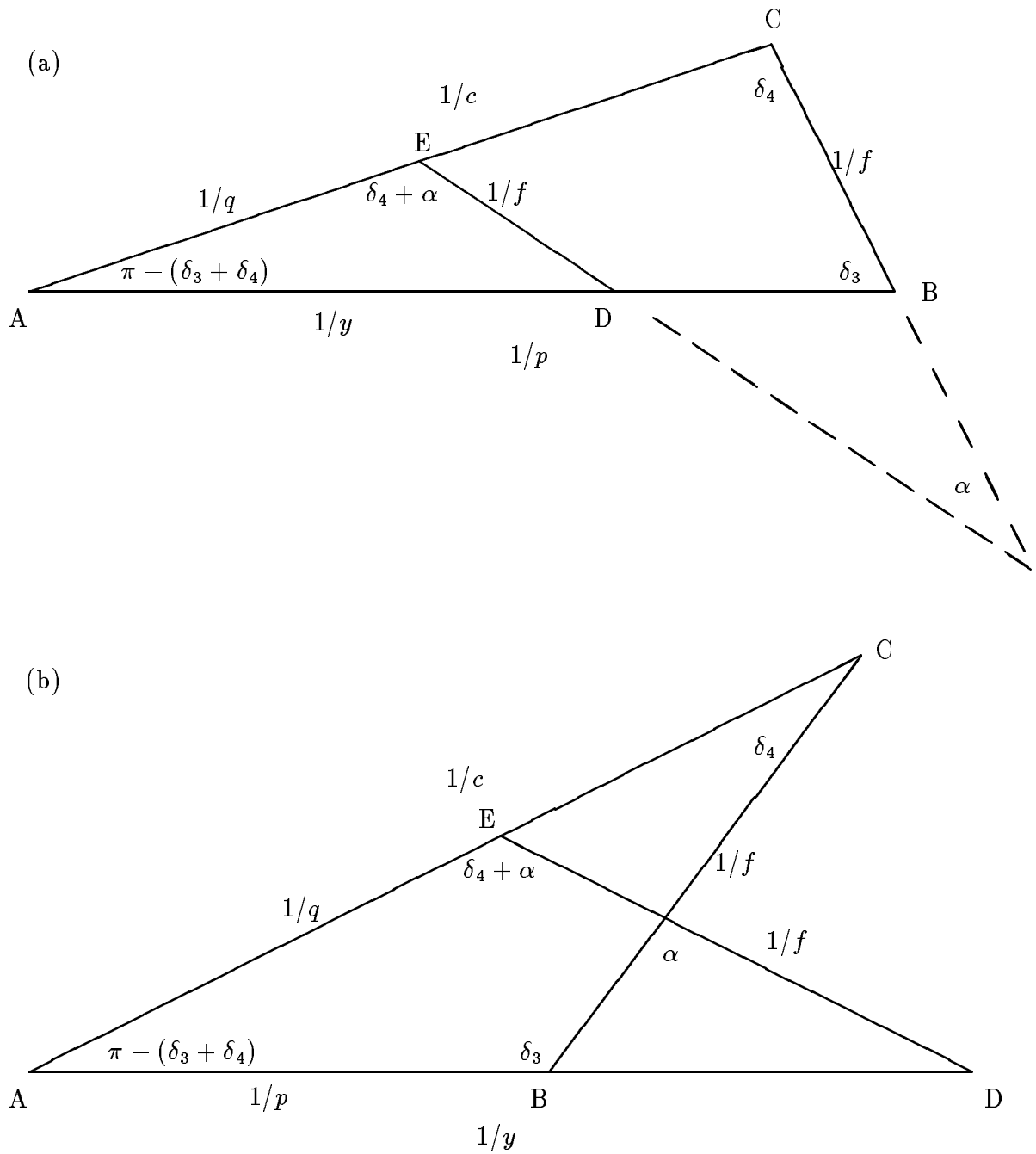
Figure 5: Chargino contribution to the neutron EDM.

Figure 6: The two possible geometrical objects which represent the CP nontrivial solution of equations (29) generated by a nonzero soft phase α. Each object consists of two triangles, ABC and ADE. The sides of the triangles are AB $= 1/p$, BC $= 1/f$, AC $= 1/c$, AD $= 1/y$, DE $= 1/f$, AE $= 1/q$. (a) The object is such that the sides $1/c$, $1/f$ and $1/y$ cannot form a triangle. (b) The object is such that the sides $1/c$, $1/f$ and $1/y$ can form a triangle.

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