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On the mass splittinq between axial and vector heavy-light mesons

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Abstract

Mass splitting between axial and vector $\bar{Q}q$ mesons is considered within the standard QCD sum rules. In agreement with the first experimental data on the B_1 meson ($J^P = 1^+$) we find that the splitting for B is about the same as for D and show that $1/m_Q$ corrections to the meson masses are small.

Several groups[1] recently announced that they had observed candidates for the axial open beauty state ($J_j^P = 1^+_{3/2}$) about 500 MeV above the corresponding vector meson $B^*(5325)$. The splitting appears to be of the same magnitude as in the case of D mesons, as predicted e.g. in the instanton liquid model[2]. This fact however looks somewhat strange from the viewpoint of expansion of the meson mass with respect to $1/m_Q$, where m_Q is the heavy quark mass,

$$m = m_Q + E_0 + \frac{E_1}{m_Q} + ..., \tag{1}$$

and indicates that the $1/m_Q$ corrections are either very close in both the vector and axial cases, or small. On one hand, there appears to be no reason for these corrections to be close (e.g., the resonance energy, E_0, is different in both cases[3]). On the other hand, it is known that for the couplings, such as f_D and f_B, these corrections are very important, and $1/m_Q$-expansion breaks down for f_D[4, 5]. In the present paper we calculate the mass of the axial B_1 meson and demonstrate, using a non-relativistic version[3, 5] of the standard QCD sum rules[6], that $1/m_Q$ corrections to the $\bar{Q}q$ vector and axial meson masses are indeed rather small.

The sum rules for vector and axial $\bar{Q}q$ mesons are obtained by considering correlators of vector ($j_\mu = \bar{Q}\gamma_\mu q$) and axial ($j_\mu = \bar{Q}\gamma_\mu\gamma_5 q$) currents

$$C_{\mu\nu}(q) = i\int d^4x e^{iqx}\langle 0|T\{j_\mu(x), j^+_\nu(0)|0\rangle \tag{2}$$

at Euclidean momentum $-q^2 > 1$ GeV^2. We choose the tensor structure proportional to $g_{\mu\nu}$, since for $q_\mu q_\nu$ the lowest state in the axial case is the pseudoscalar meson, and take into account operators with dimension $d \leq 5$ omitting the gluon condensate whose contribution is negligible. After the standard Borel transformation the sum rule takes the form[7, 8]

$$\begin{aligned}
&\frac{m^4_\pm}{g^2_\pm}\exp(-m^2_\pm/M^2) \\
&\quad = \frac{M^4}{8\pi^2}\int_{m_Q^2}^{s_0} ds s e^{-s/M^2}\left(1-\frac{m_Q^2}{s}\right)^2\left(2+\frac{m_Q^2}{s}\right)\left(1+\frac{4\alpha_s(M^2)}{3\pi}F(\frac{m_Q^2}{s})\right) \\
&\quad \pm m_Q\langle\bar{q}q\rangle L^{4/9}\exp(-m_Q^2/M^2) \mp \frac{1}{4M^4}m_Q^3 m_0^2\langle\bar{q}q\rangle\exp(-m_Q^2/M^2)\ .
\end{aligned} \tag{3}$$

The couplings $g_\pm$ are defined according to $\langle 0|j_\mu|1^\pm\rangle = (m^2_\pm/g_\pm)e_\mu$, where e_μ is the meson polarization vector. The function $F(x)$ decribes the α_s-corrections[9],

$$F(x) \quad = \quad \frac{13}{4} + 2Li_2(x) + \ln x\ln(1-x) + \frac{3}{2}\frac{x}{2+x}\ln\left(\frac{x}{1-x}\right)$$

$$- \quad \ln(1-x) - \frac{4-x-x^2}{(1-x)^2(2+x)} x \ln x - \frac{5-x-2x^2}{(1-x)(2+x)} \, . \tag{4}$$

The other quantities in Eq.(3) are: $m_0^2 = \langle \bar{q}\sigma_{\alpha\beta}G_{\alpha\beta}q\rangle / \langle \bar{q}q\rangle = 0.8 \ \text{GeV}^2$, $\langle \bar{q}q\rangle = -(0.24)^3$ GeV^3, $L = \ln(M/\Lambda)/\ln(\bar{\mu}/\Lambda)$, $\Lambda = 0.15$ GeV, $\bar{\mu} = 0.5$ GeV. The meson mass is obtained from Eq.(3) by taking the logarithmic derivative with respect to M^2. The only difference between the vector and axial sum rule (apart from the value of the continuum threshold, s_0) is the sign of the terms with $\langle \bar{q}q\rangle$. Neglecting the continuum and the anomalous dimension factor, it is easy to obtain the following estimate

$$\frac{m_+^2 - m_-^2}{m_+^2 + m_-^2} \approx -\frac{4\pi^2 m_Q \langle \bar{q}q\rangle}{M^4} \left(1 - \frac{5m_0^2 m_Q^2}{12M^4}\right) \, . \tag{5}$$

In the limit $m_Q \to \infty$ the Borel parameter scales as $M^2 \sim 2m_Q\mu$, where $\mu = 1/\tau$ is the non-relativistic Borel parameter and τ is the typical euclidean time over which the correlators change significantly[5]. Thus, one expects for the splitting between axial and vector $\bar{Q}q$ states at $m_Q \to \infty$

$$m_+ - m_- \approx -\frac{\pi^2 \langle \bar{q}q\rangle}{\mu^2} \left(1 - \frac{5m_0^2}{48\mu^2}\right) \, . \tag{6}$$

The r.h.s. of the above equation has a flat maximum at $\mu \approx 0.4$ GeV which corresponds to $m_+ - m_- \approx 0.4$ GeV. The question is whether b and c quarks are heavy enough for B and D mesons to satisfy Eq.(6), i.e. whether $1/m_Q$-corrections to masses are really small. Our experience with the couplings[5] indicates that these corrections may be important.

In Figs. 1 and 2 we show the results of numerical analysis of the sum rules of Eq.(3) in the corresponding working windows in M^2 for $m_c = 1.35$ GeV, $m_b = 4.7$ GeV and the optimal values of continuum thresholds $s_0 = 6$, 8, 35 and 40 GeV^2 for D^*, D_1, B^* and B_1, respectively[7, 8, 10]. We see that the mass splitting is indeed the same for D and B:

$$m_+ - m_- = (500 \pm 50)\, MeV \, . \tag{7}$$

For the masses, the sum rules give the values $m_{B_1} = (5.9 \pm 0.15)$ GeV, $m_{B^*} = (5.4 \pm 0.15)$ GeV, $m_{D_1} = (2.5 \pm 0.15)$ GeV and $m_{D^*} = (2.0 \pm 0.15)$ GeV. The errors correspond to allowed variations of continuum threshold s_0. A better accuracy for the splitting is due to partial cancelation of continuum contributions in this case. For the couplings we get $g_{B_1} = 22 \pm 3$, $g_{B^*} = 27 \pm 3$, $g_{D_1} = 10.5 \pm 1.5$ and $g_{D^*} = 9 \pm 1.5$ (the last two couplings were obtained before[7, 8]). It is worth mentioning that α_s-corrections are rather important: without them the splittings would be about 30% bigger.

Now, we will explicitly demonstrate that the $1/m_Q$-correction to the meson mass in Eq.(1) is small by expanding the sum rule of Eq.(3) around the limit $m_Q \to \infty$. Following the same procedure as in[5], we introduce the non-relativistic continuum threshold E_c and Borel parameter μ, $s_0 = (m_Q + E_c)^2$, $M^2 = 2m_Q\mu$, and obtain

$$E_0 = 3\mu \frac{F_2(E_c/\mu)a_2^{rad} \pm \pi^2 m_0^2\langle\bar{q}q\rangle/144\mu^5}{F_1(E_c/\mu)a_1^{rad} \pm \pi^2\langle\bar{q}q\rangle L^{4/9}/6\mu^3 \mp \pi^2 m_0^2\langle\bar{q}q\rangle/96\mu^5} , \tag{8}$$

$$\begin{aligned} E_1 = & - \frac{1}{2}E_0^2 - \frac{48}{C_0\pi^2}\exp(E_0/\mu)\mu^4(4\mu - E_0)F_2(E_c/\mu)a_2^{rad} \\ & - \frac{3}{2C_0\pi^2}\exp[(E_0 - E_c)/\mu]E_c^4[(E_0 - E_c)a_1^{rad} - (11\mu/2)a_2^{rad}] . \end{aligned} \tag{9}$$

where C_0 determines the asymptotic behavior of the residue in the limit $m_Q \to \infty$,

$$\frac{1}{g^2} = \frac{C_0}{m_Q^3}\left(1 + \frac{C_1}{m_Q} + ...\right) . \tag{10}$$

The coefficient C_0 itself is determined from the following sum rule

$$C_0 = \exp(E_0/\mu)\left(\frac{6\mu^3}{\pi^2}F_1(E_c/\mu)a_1^{rad} \pm \langle\bar{q}q\rangle L^{4/9} \mp m_0^2\langle\bar{q}q\rangle/16\mu^3\right) . \tag{11}$$

Upper and lower signs in Eqs.(8) and (11) correspond to axial and vector cases, respectively. Once E_1 is known, $1/m_Q$-correction to the coupling can be obtained from the sum rule

$$\begin{aligned} C_1 = & - 4E_0 + \frac{E_0^2 + 2E_1}{2\mu} - \frac{48}{C_0\pi^2}\exp(E_0/\mu)\mu^4 F_2(E_c/\mu)a_2^{rad} \\ & + \frac{3}{2C_0\pi^2}\exp[(E_0 - E_c)/\mu]E_c^4 a_1^{rad} . \end{aligned} \tag{12}$$

The functions F_1 and F_2 in Eqs.(8), (9), (11) and (12) are the standard functions describing quark loop and continuum contributions,

$$\begin{aligned} F_1(x) &= 1 - (1 + x + x^2/2)e^{-x} , \\ F_2(x) &= 1 - (1 + x + x^2/2 + x^3/6)e^{-x} . \end{aligned} \tag{13}$$

The factors a_1^{rad} and a_2^{rad} contain radiative corrections [1],

[1] As in ref.[5], the arguments of the logs in $a_{1,2}^{rad}$ correspond to the maxima of the integrands in the dispersion integrals. In the numerical analysis we put $m_Q = m_b$. Thus, we do not actually go to the limit $m_Q \to \infty$ in the radiative corrections. For a rigorous treatment of radiative corrections in this limit within the heavy quark effective theory, see e.g.[11].

$$
\begin{aligned}
a_1^{rad} &= 1+\alpha_s\left(2.35+\frac{2}{\pi}\log(m_Q/4\mu)\right) \ , \\
a_2^{rad} &= 1+\alpha_s\left(2.35+\frac{2}{\pi}\log(m_Q/6\mu)\right) \ .
\end{aligned} \tag{14}
$$

Note, that in the leading order in $1/m_Q$ the sum rules for $J=1$ and $J=0$ meson masses[3] are the same. The hyperfine splitting is contained in the $1/m_Q$-corrections.

The asymptotic coefficients E_0 and C_0 were calculated in ref.[3], and the splitting $E_0^+ - E_0^- = 800 \pm 200$ MeV was obtained. Our results for E_0^+ and E_0^- are presented in Fig.3 for the non-relativistic continuum thresholds $E_c = 1.8$ GeV and $E_c = 1$ GeV in the axial and vector cases respectively. We obtained a smaller value, $E_0^+ - E_0^- = 600 \pm 100$ MeV. We were not able to trace the source of this difference, since the axial case was discussed very briefly in ref.[3]. For $C_0^\pm$ we then get $C_0^+ \approx 0.71$ and $C_0^- \approx 0.16$ (Fig.4) which agrees with ref.[3] [2]. Using these values in Eq.(9) we obtain the results for $1/m_Q$ mass corrections presented in Fig.5. We see that the stability of the sum rule for E_1 and the accuracy is rather poor in the axial case. However, it is clear that the $1/m_Q$-corrections turn out small both on the scale of m_b and m_c [3]. Finally, using the values $E_1^+ = 0$ and $E_1^- = -0.12$ GeV^2 we get from Eq.(12) $C_1^+ \approx -2.6$ GeV and $C_1^- \approx -5.5$ GeV (Fig.6).

Thus, we have shown that the splittings between axial and vector mesons with open charm and beauty calculated from QCD sum rules are rather close and agree with experiment. By a non-relativistic expansion of the sum rules we checked that $1/m_Q$-corrections to meson masses are very small. This is in contrast with similar corrections to the couplings which are very important (in the vector case $1/m_Q$ expansion does not work for g_{B^*}!).

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[2]This value of C_0^- is in agreement with the estimate obtained in an earlier paper[12].

[3]$1/m_Q$ corrections to vector and pseudoscalar meson masses were calculated within the sum rules approach in Refs.[13] and [14]. Our value for E_1 in the vector case is of the same sign as in[13], but four times smaller in the absolute value. On the other hand, it is two times smaler in the absolute value and of the opposite sign than in[14]. The disagreement is disturbing and should be resolved. But we do not discuss it here, since the corrections are small in any case.

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Figure Captions:

- Fig. 1: Mass splitting between D_1 and D^* from (3) in GeV.
- Fig. 2: Same as Fig. 1 for B_1 and B^*.
- Fig. 3: E_0 in GeV.
- Fig. 4: C_0 in GeV^3.
- Fig. 5: E_1 in GeV^2.
- Fig. 6: C_1 in GeV.