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CERN-TH/95-229**“FASTER THAN LIGHT” PHOTONS AND ROTATING BLACK HOLES****R.D. Daniels* and G.M. Shore*†**

** Department of Physics
University of Wales Swansea
Singleton Park
Swansea, SA2 8PP, U.K.*

*† TH Division, CERN
CH1211 Genève 23, Switzerland*

Abstract

The effective action for QED in curved spacetime includes equivalence principle violating interactions between the electromagnetic field and the spacetime curvature. These interactions admit the possibility of superluminal yet causal photon propagation in gravitational fields. In this paper, we extend our analysis of photon propagation in gravitational backgrounds to the Kerr spacetime describing a rotating black hole. The results support two general theorems – a polarisation sum rule and a ‘horizon theorem’. The implications for the stationary limit surface bounding the ergosphere are also discussed.

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1. Introduction

The possibility, originally discovered by Drummond and Hathrell in 1980 [1], that photons propagating in curved spacetime may travel with speeds exceeding the usual speed of light has been the subject of renewed interest in the last two years[2-5]*. The phenomenon arises because vacuum polarisation in QED induces interactions between the electromagnetic field and spacetime curvature. Such interactions violate the strong principle of equivalence and allow the possibility of spacelike photon propagation without necessarily implying any violation of causality.

In their original paper[1], Drummond and Hathrell studied photon propagation in the Schwarzschild spacetime, together with other examples including gravitational wave and Robertson-Walker backgrounds. In our previous paper[2], we extended this analysis to the Reissner-Nordström geometry describing a charged black hole. Here, we complete our survey of photon propagation in black hole spacetimes by considering the Kerr geometry describing a rotating black hole.

Our study is motivated by the hope that by examining special cases with a particularly rich structure it may be possible to uncover general properties of photon propagation in gravitational fields. That hope is borne out by the results presented here. In particular, it had been observed previously that the light cone for radially directed photons in both the Schwarzschild and Reissner-Nordström geometries remains unperturbed, i.e. at $k^2 = 0$. For the Kerr spacetime, we find that this is no longer true and photons travelling on radial trajectories may indeed have velocities differing from 1, either greater or smaller than the usual velocity of light depending on the polarisation. However, we also find that these corrections always vanish at the event horizon itself. The light cone always remains $k^2 = 0$ at the horizon.

This result, together with a further observation that in all cases the corrections to the photon velocity are equal and opposite for the two transverse polarisations, was the motivation for ref.[12] where they are formalised into two theorems – a polarisation sum rule and a ‘horizon theorem’ (see section 5). This latter paper also contains some related observations about electromagnetic birefringence and the rôle of the conformal anomaly in photon propagation.

2. Photon Propagation in Curved Spacetime

In this paper, we consider the nature of photon propagation implied by the action

$$\Gamma = \int dx \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{m^2} \left(a R F_{\mu\nu} F^{\mu\nu} + b R_{\mu\nu} F^{\mu\lambda} F^{\nu\lambda} + c R_{\mu\nu\lambda\rho} F^{\mu\nu} F^{\lambda\rho} \right) \right) \quad (2.1)$$

where the mass scale m and constants a , b and c are regarded as free parameters. This is the simplest action which incorporates explicit equivalence principle violating interactions

* For earlier papers related to ref.[1] see refs.[6,7]. Similar phenomena in non-gravitational backgrounds are discussed in refs.[8-11,3].

and leads to possible modifications of the light cone. (Other more complicated actions are considered in ref.[5].)

In fact, the expression (2.1) is generated as the effective action in QED in curved spacetime including vacuum polarisation effects[1]. It is valid in the approximation of weak curvature and low frequency photons, so that the neglect of higher powers of the curvature tensor and extra covariant derivatives in the interaction terms is justified. Roughly, (see ref.[13] for a careful discussion), this effective action is a good approximation in the parameter range $\lambda_c < \lambda < L$, where λ is the photon wavelength, L is a typical curvature scale, and $\lambda_c = 1/m$ is the electron Compton wavelength, m being its mass. The electron provides the appropriate quantum scale as the effect is generated by vacuum polarisation diagrams involving an internal electron loop. In this case, the coupling constants are given to one loop in terms of the fine structure constant by $a = -\frac{1}{144}\frac{\alpha}{\pi}$, $b = \frac{13}{360}\frac{\alpha}{\pi}$ and $c = -\frac{1}{360}\frac{\alpha}{\pi}$. Notice, however, that if this is the origin of the action (2.1), then the validity of the results we deduce here concerning photon propagation is only established for relatively low frequency photons, whereas to consider the speed of real signal propagation and the implications for causality we need to consider high frequencies. The question of dispersion, the generalisation of the effective action to include higher derivative terms and the physical interpretation of the results will be discussed in ref.[13].

The Bianchi identity and equation of motion following from the action (2.1) are*

$$D_\mu F_{\nu\lambda} + D_\nu F_{\lambda\mu} + D_\lambda F_{\mu\nu} = 0 \quad (2.2)$$

and

$$D_\mu F^{\mu\nu} - \frac{1}{m^2} \left(2b R^\mu{}_\lambda D_\mu F^{\lambda\nu} + 4c R^{\mu\nu}{}_{\lambda\rho} D_\mu F^{\lambda\rho} \right) = 0 \quad (2.3)$$

where we have assumed that both the photon wavelength and m^{-1} are small compared to the curvature scale.

The simplest way to determine the characteristics of photon propagation from these equations is to use geometric optics. In the leading geometric optics approximation, we write the electromagnetic field strength as the product of a slowly varying amplitude and a rapidly varying phase, i.e. $F_{\mu\nu} = f_{\mu\nu} \exp i\theta$, where the wave vector is $k_\mu = \partial_\mu \theta$. In the quantum particle interpretation, we identify k_μ as the photon momentum. The amplitude is constrained by the Bianchi identity to be of the form $f_{\mu\nu} = k_\mu a_\nu - k_\nu a_\mu$, where the direction of a_μ specifies the polarisation. Light rays (photon trajectories) are defined as the integral curves of the wave vector (photon momentum). Without the additional term in eq.(2.3), these may easily be shown to be null geodesics. This is no longer true when the equivalence principle violating interactions are included, nor is the light cone condition $k^2 = 0$ necessarily satisfied.

Our main concern is with the local modifications to the light cone induced by these direct curvature interactions. Introducing an orthonormal frame using vierbeins defined by $g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$, where η_{ab} is the Minkowski metric, the photon equation of motion (2.3) gives

$$k^2 a_b - \frac{2b}{m^2} \left(R_{ac} \left(k^a k^c a_b - k^a k_b a^c \right) \right) - \frac{8c}{m^2} \left(R_{abcd} k^a k^c a^d \right) = 0 \quad (2.4)$$

* See ref.[12] for an extended version of this section.

The polarisation vector is spacelike normalised, $a^b a_b = -1$, and can be taken orthogonal to the momentum, $k^b a_b = 0$. Of the three remaining degrees of freedom, in the quantum theory only the two transverse polarisation states are physical. Given a polarisation vector a^b satisfying this equation, the corresponding light cone condition is simply

$$k^2 - \frac{2b}{m^2} R_{ac} k^a k^c + \frac{8c}{m^2} R_{abcd} k^a k^c a^b a^d = 0 \quad (2.5)$$

We shall now explore the consequences of these equations for the Kerr geometry.

3. The Kerr Spacetime

The Kerr spacetime (see, e.g., refs.[14-16]) is described by the metric

$$ds^2 = -\rho^2 \frac{\Delta}{\Sigma^2} dt^2 + \rho^2 \frac{1}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{1}{\rho^2} \Sigma^2 \sin^2 \theta (d\phi - \omega dt)^2 \quad (3.1)$$

where

$$\begin{aligned} \omega &= \frac{2aMr}{\Sigma^2} & \rho^2 &= r^2 + a^2 \cos^2 \theta \\ \Delta &= r^2 - 2Mr + a^2 & \Sigma^2 &= (r^2 + a^2)^2 - a^2 \sin^2 \theta \end{aligned} \quad (3.2)$$

This metric represents the exterior spacetime of a rotating black hole. It is axially symmetric about the rotation axis $\theta = 0$. It is specified by two parameters, M and a , where M is the mass and Ma the angular momentum as measured from infinity. For $a = 0$, it reduces to the Schwarzschild spacetime. In the description below, we assume $a \leq M$.

The condition $\Delta(r) = 0$, for which there is a coordinate singularity similar to that in the Schwarzschild metric, has two solutions, $r = r_{\pm} = M \pm \sqrt{M^2 - a^2}$. The larger, $r = r_+$, is in fact the event horizon, from within which no particle on a timelike or null trajectory can escape to infinity. The region $r < r_-$ contains a ring singularity. Just as in the Schwarzschild geometry, there is a Killing vector which is timelike in the asymptotic region (large r) but which is spacelike within the event horizon. However, for the Kerr spacetime, there is a further region outside the horizon, $r_+ \leq r < r_E(\theta)$, for which the Killing vector remains spacelike. This is the ergosphere. $r_E(\theta) = M + \sqrt{M^2 - a^2 \cos^2 \theta}$ is the value of r for which the metric component $g_{tt} = -\left(1 - \frac{2Mr}{\rho^2}\right)$ vanishes and the Killing vector is null. The outer limit of the ergosphere, the stationary limit surface, is the inner boundary of the region where particles travelling on timelike curves can remain at rest relative to infinity. Within the ergosphere, even null curves are pulled round in the direction of the rotation. Particles may, however, escape to infinity from this region. The situation is summarised pictorially in Fig.1, taken from ref.[15]. The stationary limit surface and event horizon coincide at the poles, where the stationary limit surface becomes null. At the equator, r_E is equal to $2M$, the Schwarzschild radius.

We now introduce a local orthonormal frame. The appropriate basis 1-forms are e^a ($a = 0, 1, 2, 3$) with

$$e^0 = e^0_t dt \quad e^1 = e^1_r dr \quad e^2 = e^2_\theta d\theta \quad e^3 = e^3_\phi (d\phi - \omega dt) \quad (3.3)$$

where the vierbeins are

$$e^0_t = -\rho \frac{\sqrt{\Delta}}{\Sigma} \quad e^1_r = \frac{\rho}{\sqrt{\Delta}} \quad e^2_\theta = \rho \quad e^3_t = -\omega e^3_\phi = -\frac{\omega}{\rho} \Sigma \sin \theta \quad (3.4)$$

The Kerr metric is Ricci flat, so $R_{ab} = 0$. There are six independent non-vanishing components of the Riemann curvature which we can choose in this frame to be [16]

$$\begin{aligned} R_{0101} &= A & R_{0202} &= B & R_{0123} &= C \\ R_{0231} &= D & R_{0102} &= E & R_{0113} &= F \end{aligned} \quad (3.5)$$

The complete set of non-vanishing components is

$$\begin{aligned} R_{2323} &= -R_{0101} & R_{1313} &= -R_{0202} & R_{1212} &= -R_{0303} = R_{0101} + R_{0202} \\ R_{0312} &= -R_{0123} - R_{0231} & R_{3132} &= R_{0102} & R_{0223} &= -R_{0113} \end{aligned} \quad (3.6)$$

together with those related by the usual symmetries of R_{abcd} . The quantities A, \dots, F are given by

$$\begin{aligned} A &= \frac{Mr}{\rho^6} (r^2 - 3a^2 \cos^2 \theta) \frac{1}{\Sigma^2} \left(2(r^2 + a^2)^2 + a^2 \Delta \sin^2 \theta \right) \\ B &= -\frac{Mr}{\rho^6} (r^2 - 3a^2 \cos^2 \theta) \frac{1}{\Sigma^2} \left((r^2 + a^2)^2 + 2a^2 \Delta \sin^2 \theta \right) \\ \Rightarrow A + B &= \frac{Mr}{\rho^6} (r^2 - 3a^2 \cos^2 \theta) \\ C &= -\frac{aM \cos \theta}{\rho^6} (3r^2 - a^2 \cos^2 \theta) \frac{1}{\Sigma^2} \left(2(r^2 + a^2)^2 + a^2 \Delta \sin^2 \theta \right) \\ D &= \frac{aM \cos \theta}{\rho^6} (3r^2 - a^2 \cos^2 \theta) \frac{1}{\Sigma^2} \left((r^2 + a^2)^2 + 2a^2 \Delta \sin^2 \theta \right) \\ \Rightarrow C + D &= -\frac{aM \cos \theta}{\rho^6} (3r^2 - a^2 \cos^2 \theta) \\ E &= -\frac{aM \cos \theta}{\rho^6} (3r^2 - a^2 \cos^2 \theta) \frac{3a\sqrt{\Delta}}{\Sigma^2} (r^2 + a^2) \sin \theta \\ F &= \frac{Mr}{\rho^6} (3r^2 - a^2 \cos^2 \theta) \frac{3a\sqrt{\Delta}}{\Sigma^2} (r^2 + a^2) \sin \theta \end{aligned} \quad (3.7)$$

Introducing the notation $U_{ab}^{01} = \delta_a^0 \delta_b^1 - \delta_b^0 \delta_a^1$ etc., we can rewrite the complete Riemann tensor compactly in the following form:

$$\begin{aligned} R_{abcd} &= 2A \left(U_{ab}^{01} U_{cd}^{01} - U_{ab}^{23} U_{cd}^{23} - U_{ab}^{03} U_{cd}^{03} + U_{ab}^{12} U_{cd}^{12} \right) \\ &\quad + 2B \left(U^{02} U^{02} - U^{13} U^{13} - U^{03} U^{03} + U^{12} U^{12} \right) \\ &\quad + C \left(U^{01} U^{23} + U^{23} U^{01} - U^{03} U^{12} - U^{12} U^{03} \right) \end{aligned}$$

$$\begin{aligned}
& + D \left(-U^{02}U^{13} - U^{13}U^{02} - U^{03}U^{12} - U^{12}U^{03} \right) \\
& + E \left(U^{01}U^{02} + U^{02}U^{01} + U^{13}U^{23} + U^{23}U^{13} \right) \\
& + F \left(U^{01}U^{13} + U^{13}U^{01} - U^{02}U^{23} - U^{23}U^{02} \right)
\end{aligned} \tag{3.8}$$

where we have suppressed the a, b, c, d indices after the first line for clarity. To obtain the curvature components $R_{\mu\nu\lambda\rho}$ in the coordinate frame (t, r, θ, ϕ) , simply replace U_{ab}^{01} by $U_{\mu\nu}^{01} = e^0{}_\mu e^1{}_\nu - e^1{}_\nu e^0{}_\mu$ etc. in this expression.

Notice the following simplifications for special cases. On the event horizon, $\Delta(r_+) = 0$, so $E = F = 0$, $A = -2B$ and $C = -2D$. In the equatorial plane $\theta = \pi/2$, $\cos \theta$ vanishes and so $C = D = E = 0$.

4. Photon Propagation in Kerr Spacetime

We now return to eq.(2.4) describing photon propagation in curved spacetime. For Ricci flat spacetimes such as Kerr, this reduces to

$$k^2 a_b + \epsilon R_{abcd} k^a k^c a^d = 0 \tag{4.1}$$

where we have written $\epsilon = -8c/m^2$. This is a set of three simultaneous linear equations for the independent components of the polarisation a_b . To solve these[1], it is convenient to introduce the following linear combinations of momentum components:

$$\ell_b = k^a U_{ab}^{01} \quad m_b = k^a U_{ab}^{02} \quad n_b = k^a U_{ab}^{03} \tag{4.2}$$

together with the dependent combinations

$$\begin{aligned}
p_b &= k^a U_{ab}^{12} = \frac{1}{k^0} (k^1 m_b - k^2 \ell_b) \\
q_b &= k^a U_{ab}^{13} = \frac{1}{k^0} (k^1 n_b - k^3 \ell_b) \\
r_b &= k^a U_{ab}^{23} = \frac{1}{k^0} (k^2 n_b - k^3 m_b)
\end{aligned} \tag{4.3}$$

The vectors ℓ, m, n are independent and orthogonal to k^a .

We can therefore rewrite (4.1) as a set of equations for the independent polarisation components $a.\ell$, $a.m$ and $a.n$ by contracting appropriately. Substituting eq.(3.8) for the Riemann tensor, we therefore arrive at the following set of equations:

$$\begin{aligned}
0 &= k^2 a.\ell + 2A\epsilon \left(\ell^2 a.\ell - \ell.r a.r - \ell.n a.n + \ell.p a.p \right) \\
&+ 2B\epsilon \left(\ell.m a.m - \ell.q a.q - \ell.n a.n + \ell.p a.p \right) \\
&+ C\epsilon \left(\ell^2 a.r + \ell.r a.\ell - \ell.n a.p - \ell.p a.n \right)
\end{aligned}$$

$$\begin{aligned}
& + D\epsilon\left(-\ell.m\ a.q - \ell.q\ a.m - \ell.n\ a.p - \ell.p\ a.n\right) \\
& + E\epsilon\left(\ell^2\ a.m + \ell.m\ a.l + \ell.q\ a.r + \ell.r\ a.q\right) \\
& + F\epsilon\left(\ell^2\ a.q + \ell.q\ a.l - \ell.m\ a.r - \ell.r\ a.m\right) \\
0 & = k^2\ a.m + \dots \\
0 & = k^2\ a.n + \dots
\end{aligned} \tag{4.4}$$

To save space, we have only written the first in full. The second two have a similar form and are easily reconstructed.

In principle, this set of equations could now be solved in general. However, it is much more illuminating to look at a selection of special cases which illustrate the most important features. First, consider photon propagation in the equatorial plane.

(i) *Equatorial plane, radial motion*

To illustrate the method of solution, consider first radial motion confined to the equatorial plane, where $C = D = E = 0$. The photon momentum components satisfy $k^2 = k^3 = 0$. The various momentum-dependent terms appearing in eqs.(4.4) therefore simplify considerably. In particular, we have $\ell^2 = k^0 k^0 - k^1 k^1$, $m^2 = n^2 = k^0 k^0$ and $\ell.m = \ell.n = m.n = 0$, while for the others we find $p^2 = q^2 = k^1 k^1$, $m.p = n.q = k^0 k^1$, with all other contractions vanishing. For the polarisation projections, $a.p = k^1/k^0 a.m$, $a.q = k^1/k^0 a.n$ and $a.r = 0$. Substituting these special results into eqs.(4.4) and rewriting the system in matrix form, we find

$$\begin{pmatrix}
k^2 + 2A\epsilon\ell^2 & 0 & F\epsilon\ell^2\frac{k^1}{k^0} \\
0 & k^2 + 2A\epsilon k^1 k^1 + 2B\epsilon(k^0 k^0 + k^1 k^1) & 0 \\
F\epsilon k^0 k^1 & 0 & k^2 - 2A\epsilon k^0 k^0 - 2B\epsilon(k^0 k^0 + k^1 k^1)
\end{pmatrix}
\times \begin{pmatrix}
a.l \\
a.m \\
a.n
\end{pmatrix} = 0 \tag{4.5}$$

In general we would have to diagonalise to find the polarisation eigenvectors, the corresponding values of k^2 being given as solutions of the vanishing of the determinant. In this case, however, there is a further simplification. We should regard ϵ as a small parameter (in fact, it is the dimensionless combinations $A\epsilon$, $B\epsilon$ etc. which are small) since the original action involving single powers of the curvature will be valid only for $\lambda_c < L$, and work to consistent order in small ϵ . Since with $\epsilon = 0$ the light cone condition is just $k^2 = 0$, in general we have $k^2 = O(\epsilon)$. Now, for radial motion $\ell^2 = -k^2$ so is of $O(\epsilon)$, and therefore the off-diagonal entry proportional to F is actually of $O(\epsilon^2)$ and should be neglected at lowest order.

We therefore find the solutions:

$$k^2 = 0, \quad \text{corresponding to the polarisation } a_a \text{ proportional to } \ell_a,$$

$k^2 + 2A\epsilon k^0 k^0 + 2B\epsilon(k^0 k^0 + k^1 k^1)$, corresponding to a_a proportional to m_a ,

$k^2 - 2A\epsilon k^0 k^0 - 2B\epsilon(k^0 k^0 + k^1 k^1)$, corresponding to a_a proportional to n_a .

The solution with $k^2 = 0$ corresponds to an unphysical polarisation, $\ell_a = k^0 \delta_a^1 - k^1 \delta_a^0$, while the two physical transverse polarisations are proportional to $m_a = k^0 \delta_a^2$, i.e. polarisation in the θ direction, and $n_a = k^0 \delta_a^3$, i.e. polarisation in the ϕ direction. At this level the effect is non-dispersive and the (phase or group) velocity shift is simply

$$\delta v = \left| \frac{k^0}{k^1} \right| - 1 = \pm(A + 2B)\epsilon = \mp\epsilon \left(\frac{3Ma^2 r^3}{\rho^6 \Sigma^2} \Delta \right) \quad (4.6)$$

for θ (ϕ) polarisation respectively.

Two features of this result are immediately apparent. First, the shift in k^2 , or equivalently the velocity shifts away from the conventional speed of light, are equal and opposite for the two transverse polarisations. Second, the shift vanishes on the event horizon, since $\Delta(r_+) = 0$. Both these observations turn out to be examples of general theorems and we discuss them further in the next section.

(ii) *Equatorial plane, orbital motion*

Now consider photons travelling in the orbital (ϕ) direction in the equatorial plane. In this case, $k^1 = k^2 = 0$. The analysis goes through in the same way as described above and gives the solutions:

$k^2 + 2A\epsilon k^0 k^0 - 2B\epsilon k^3 k^3 - 2F\epsilon k^0 k^3 = 0$, corresponding to a_a proportional to ℓ_a ,

$k^2 - 2A\epsilon k^3 k^3 + 2B\epsilon k^0 k^0 + 2F\epsilon k^0 k^3 = 0$, corresponding to a_a proportional to m_a ,

$k^2 - (2A + 2B)\epsilon k^0 k^0 = 0$, corresponding to a_a proportional to n_a .

The two physical transverse polarisations, in the radial and θ directions, are given by the first two of these solutions respectively. Notice, however, that now the light cone condition is not quadratic in each of the momentum components separately due to the presence of the F term. This introduces a splitting in the velocities for propagation with $k^3 > 0$ and $k^3 < 0$. For the radial polarisation we find

$$\delta v = (A - B \mp F)\epsilon = \frac{3Mr^3}{\rho^6 \Sigma^2} \left((r^2 + a^2)^2 + a^2 \Delta \mp 3a(r^2 + a^2)\sqrt{\Delta} \right) \quad (4.7)$$

depending on whether the motion is with or against the direction of spin. For the θ polarisation, δv has the opposite sign. This expression simplifies on the horizon, where we have

$$\delta v|_{\text{horizon}} = \frac{3M}{r_+^3} \epsilon \quad (4.8)$$

and on the stationary limit surface $r_E = 2M$, where (for $k^3 \leq 0$)

$$\delta v|_{\text{stat. lim.}} = \frac{3}{2} \frac{1}{r_E^4 (r_E^2 + 2a^2)} \left(r_E^4 + 5a^2 (r_E^2 + a^2) \right) \epsilon \quad (4.9)$$

(iii) *Radial motion, arbitrary direction*

In the Schwarzschild and Reissner-Nordström spacetimes, the light cone for radial photons remains $k^2 = 0$ for any direction (the solutions are of course spherically symmetric) and for any value of the radial coordinate r . In the Kerr spacetime, however, we have just seen that $k^2 \neq 0$ for radial photons in the equatorial plane except at the horizon $r = r_+$. We now want to check whether this result remains true independent of the angle θ to the polar axis at which the photons are directed.

The calculation follows the lines of (i) except that now the curvature components C , D and E are non-vanishing. Eq.(4.5) generalises to:

$$\begin{pmatrix} k^2 + 2A\epsilon\ell^2 & E\epsilon\ell^2 & F\epsilon\ell^2\frac{k^1}{k^0} \\ E\epsilon k^0 k^0 & k^2 + 2A\epsilon k^1 k^1 + 2B\epsilon(k^0 k^0 + k^1 k^1) & -(C + 2D)\epsilon k^0 k^1 \\ F\epsilon k^0 k^1 & -(C + 2D)\epsilon k^0 k^1 & k^2 - 2A\epsilon k^0 k^0 - 2B\epsilon(k^0 k^0 + k^1 k^1) \end{pmatrix} \times \begin{pmatrix} a.\ell \\ a.m \\ a.n \end{pmatrix} = 0 \quad (4.10)$$

Again there is a solution $k^2 = 0$ corresponding to the unphysical polarisation a_a proportional to ℓ_a . The transverse polarisations diagonalising (4.10) are complicated r and θ dependent linear combinations of the unit vectors in the θ and ϕ directions, the corresponding velocity shifts being

$$\begin{aligned} \delta v &= \pm \left((A + 2B)^2 + \frac{1}{4}(C + 2D)^2 \right)^{\frac{1}{2}} \\ &= \pm \epsilon \Delta \frac{3Ma^2 \sin^2 \theta}{\rho^6 \Sigma^2} \left(r^2 (r^2 - 3a^2 \cos^2 \theta)^2 + \frac{1}{4} a^2 \cos^2 \theta (3r^2 - a^2 \cos^2 \theta)^2 \right)^{\frac{1}{2}} \end{aligned} \quad (4.11)$$

Notice that $\delta v = 0$ along the polar axis, while of course the previous result is recovered in the equatorial plane. Most important, however, we again see that the velocity shifts are equal and opposite for the transverse polarisations and that, independently of θ , $\delta v = 0$ on the event horizon where $\Delta(r_+)$ vanishes.

5. The Polarisation Sum Rule, the Horizon and the Stationary Limit Surface

These results on modifications of the light cone in special cases, together with those previously obtained in refs[1,2], motivated the formulation of two general theorems which were stated precisely and proved in ref.[12] (see also ref.[17]). These are:

Polarisation Sum Rule

This relates the sum over the transverse polarisation states to an appropriate projection of the Ricci tensor, viz.

$$\sum_{\text{pol}} k^2 = -\frac{1}{m^2} (4b + 8c) R_{ac} k^a k^c \quad (5.1)$$

Horizon Theorem

This states that at the event horizon, the light cone for photons with momentum directed normal to the horizon remains $k^2 = 0$, independent of the polarisation.

Clearly, the specialisation of the polarisation sum rule to Ricci flat spacetimes, i.e. $\sum_{\text{pol}} k^2 = 0$, or equivalently $\sum_{\text{pol}} \delta v = 0$, is satisfied by all the examples in Kerr spacetime discussed in section 4.

Similarly, for radial photons, eq.(4.12) ensures that $k^2 = 0$, or equivalently $v = 1$, at the event horizon even though the velocity differs from 1 for all other values of the radial coordinate. The horizon theorem therefore ensures that the geometric event horizon remains a true horizon for photons propagating according to the action (2.1), even in a spacetime with as rich a structure as Kerr.

In the light of this, it is interesting to ask whether the geometric stationary limit surface $r = r_E(\theta)$ specified by $g_{tt} = 0$ retains its defining property for real photon propagation. Recall that this is the surface within which even light signals emitted against the direction of rotation are pulled round, through the phenomenon of dragging of inertial frames[14,15], so that as measured by an asymptotic observer they propagate in the direction of rotation of the black hole.

The results of section 4(ii) show that this is not true. The fact that the light cone is modified for photons emitted in the negative ϕ direction (see eqs.(4.7)(4.9)) even on the stationary limit surface shows that the effective stationary limit surface is shifted from $r_E(\theta)$ to a larger or smaller value of r depending on the photon polarisation.

The shift is most readily found using a trick used in ref.[1] to calculate the modification to the bending of light in a Schwarzschild spacetime. As we have seen, the light cone condition is modified to $\tilde{\eta}_{ab}k^ak^b = 0$ where $\tilde{\eta}_{ab}$ differs from the orthonormal (Minkowski) metric by the terms of $O(\epsilon)$ calculated in section 4. Propagation with this modified light cone in a spacetime with metric $g_{\mu\nu}$ is therefore equivalent to conventional propagation with light cone $\eta_{ab}k^ak^b = 0$ in a modified geometry with metric $\tilde{g}_{\mu\nu} = \tilde{\eta}_{ab}e^a{}_{\mu}e^b{}_{\nu}$.

In this case, we have determined $\tilde{\eta}_{ab}$ for propagation in the counter-orbital direction in the equatorial plane (section 4(ii)). Since $g_{tt} = -e^0{}_te^0{}_t + e^3{}_te^3{}_t$, we see that evaluated on the (unperturbed) stationary limit surface $e^0{}_t = e^3{}_t$. We therefore have immediately that

$$\tilde{g}_{tt} = g_{tt} + 2\epsilon(A - B + F)e^0{}_te^0{}_t \Big|_{\theta=\frac{\pi}{2}, r=2M} \quad (5.2)$$

Setting $\tilde{g}_{tt} = 0$ gives the effective stationary limit surface. In the equatorial plane, we therefore find

$$r_E^{\text{eff}} = 2M \pm \epsilon(A - B + F)\frac{a^2}{a^2 + 2M^2} \quad (5.3)$$

depending on the polarisation, the magnitude of the velocity shift term being given in eq.(4.9).

Finally, we should emphasise again that these results follow from taking the action (2.1) literally. If instead we regard it as the lowest-order effective action for QED, we should

ask to what extent the inherent approximations can be relaxed. The weak curvature ($\lambda_c < L$) approximation is difficult to improve on, so the magnitude of the results will necessarily be extremely small in the domain where the derivation is reliable. This limits the interest of these results for astrophysical black holes. This is, however, just the usual situation for quantum field phenomena in curved spacetime such as, e.g., the Hawking effect[18]. The predicted effects are of the order of a quantum scale divided by a curvature scale, so are expected to become large only for microscopic black holes or in the very early universe. A more serious limitation of the action (2.1) is the implicit restriction to relatively low-frequency photons ($\lambda_c < \lambda$), which poses serious questions as to the observability and relevance for signal propagation and causality of the shifts in the light cone considered here. These issues will be addressed in a forthcoming paper[13]. Despite the need for caution, however, the general theorems inferred from our study of photon propagation in black hole spacetimes may well prove to be valid outside these limitations. In particular, the horizon theorem, whose proof relied on the physical result that classically no gravitational radiation (or matter) crosses the horizon[19], looks sufficiently robust to conjecture that it may be true in general.

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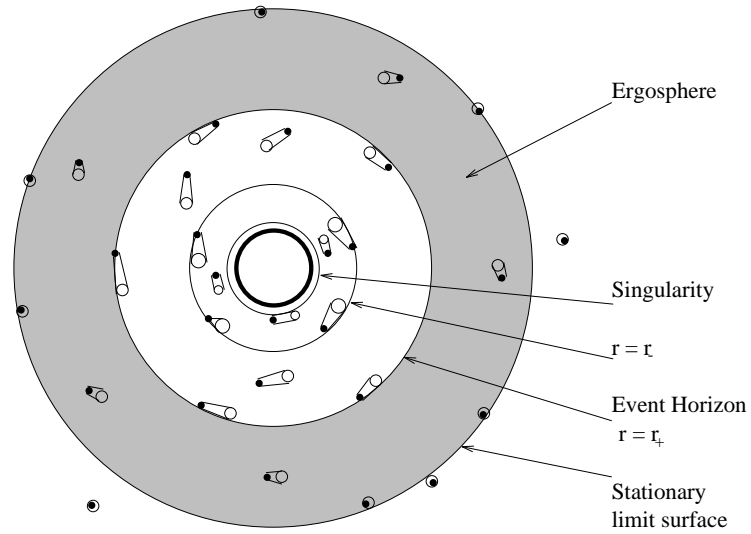


Fig. 1 A picture of the equatorial plane of the Kerr geometry with $a^2 < M^2$. The circles represent the position after a short time interval of photons emitted in the r, ϕ plane from the points represented by the heavy dots. The distinctive ‘frame-dragging’ effect giving rise to the ergosphere is readily seen, as is the property of the horizon that it cannot be crossed even by null trajectories.