

# Flavor-Changing Magnetic Dipole Moment and Oscillation of a Neutrino in a Degenerate Electron Plasma

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## Abstract

The standard model prediction for a magnetic dipole moment of a neutrino is proportional to the neutrino mass and extremely small. It also generates a flavor-changing process, but the GIM mechanism reduces the corresponding amplitude. These properties of a neutrino magnetic moment change drastically in a degenerate electron plasma. We have shown that an electron-hole excitation gives a contribution proportional to the electrons' Fermi momentum. Since this effect is absent in  $\mu$  and  $\tau$  sector, the GIM cancellation does not work. The magnetic moment induces a neutrino oscillation if a strong enough magnetic field exists in the plasma. The required magnitude of the field strength that affects the  $\nu_e$  burst from a supernova is estimated to be the order of  $10^8$  Gauss.

A magnetic dipole moment of a neutrino induces interesting phenomena, such as a spin rotation when its travelling in a static magnetic field [1, 2] or a transition radiation when passing an interface between two different media [3]. It may also affect the stellar cooling by the decay of plasmons [4], which is known as a dominant cooling process in a dense star [5, 6]. The standard model predicts a nonzero value for it through the processes depicted in Fig. 1, but the magnitude is far below the one that current experiments can detect. To leading order in  $m_l^2/m_W^2$ , the result is independent of the mass  $m_l$  of the charged leptons in the internal lines; it is instead proportional to the neutrino mass  $m_\nu$  [7],

$$\mu_\nu = \frac{3eG_F m_\nu}{8\sqrt{2}\pi^2} = 3 \times 10^{-19} \left( \frac{m_\nu}{1\text{eV}} \right) \mu_B, \quad (1)$$

where  $\mu_B$  is the Bohr magneton. There are many orders of magnitude between this prediction and the present empirical limits,  $\mu_\nu < 10^{-6} - 10^{-11} \mu_B$  [8].

The leptonic charged current coupling to the W boson generates a generation mixing for massive neutrinos as is the case for the quark sector. The processes in Fig. 1 can then generate, so to say, a flavor-changing electromagnetic current. The independence of the leading contribution from  $m_l$ , however, subjects the corresponding amplitude to the GIM suppression [7]: The sum of the leading contributions from all three generations cancels each other because of the unitarity of the leptonic CKM mixing matrix. Thus the amplitude, such as the one for the decay  $\nu_\mu \rightarrow \nu_e \gamma$ , gets a nonzero contribution from the next-to-leading effect and is further suppressed by a factor  $(m_l^2/m_W^2)$  than one estimates naively with  $\mu_\nu$ .

The purpose of this letter is to show that these properties of a neutrino magnetic moment change drastically in a degenerate electron plasma. In the gas of high density and relatively low temperature, i.e., where the Fermi momentum  $p_F$  is much larger than the temperature  $T$ , most of the electrons degenerates into the Fermi sphere. Electromagnetic potential  $A_\mu$  excites one of the electrons out of the Fermi sphere and leaves a hole in it. Subsequently the excited electron comes back into the sphere emitting a pair of neutrinos by exchanging a W boson with the hole. This polarization of a electron-hole pair turns out to add a contribution of the order of  $eG_F p_F$  to the magnetic moment (See Eqs. (11) and (14) below). Since this effect is intrinsic to degenerate electrons and absent in the  $\mu$  and  $\tau$  sectors, the GIM mechanism no longer washes out this contribution.

The Fermi momentum of the electron gas at the core of massive stars becomes as large as or larger than the electron mass  $m_e$  at the later stage of their evolution. This gives a possibility that the resulting magnetic moment becomes so large that some observations can detect it. We will consider a neutrino oscillation induced by the flavor-changing magnetic moment, which may take place in the stars. Our calculation uses a zero temperature approximation, and the result should be applied to the cases of  $p_F \gg T$ . In the following, we will first describe the result for the magnetic moment briefly and then consider the induced neutrino oscillation.

We assume the masses that the neutrinos,  $\nu_e$ ,  $\nu_\mu$ , and  $\nu_\tau$ , get at the electroweak symmetry breaking are of Majorana type.<sup>1</sup> The corresponding fields, which we collectively denote by  $\nu(x)$  in the left-handed two-component notation, have an expansion

$$\nu(x) = \frac{1}{\sqrt{V}} \sum_{\vec{p}, s} \left[ e^{-iE(\vec{p})x^0 + i\vec{p}\cdot\vec{x}} u(\vec{p}, s) a_{\vec{p}, s}(\nu) + e^{iE(\vec{p})x^0 - i\vec{p}\cdot\vec{x}} v(\vec{p}, s) a_{\vec{p}, s}^\dagger(\nu) \right] \quad (2)$$

in the annihilation,  $a_{\vec{p}, s}(\nu)$ , and the creation,  $a_{\vec{p}, s}^\dagger(\nu)$ , operators of the state with the momentum  $\vec{p}$  and the helicity  $s$ ; the spinors  $u(\vec{p}, s)$  and  $v(\vec{p}, s)$  are defined by

$$u(\vec{p}, s) = \sqrt{\frac{E(\vec{p}) - s|\vec{p}|}{2E(\vec{p})}} \chi(\hat{p}, s) \quad (3)$$

$$v(\vec{p}, s) = -\sqrt{\frac{E(\vec{p}) + s|\vec{p}|}{2E(\vec{p})}} \epsilon \chi(\hat{p}, s)^*, \quad (4)$$

where  $E(\vec{p}) = (\vec{p}^2 + m_\nu^2)^{1/2}$ ,  $\chi(\hat{p}, s)$  is the normalized eigenspinor for the helicity,

$$(\hat{p} \cdot \vec{\sigma}) \chi(\hat{p}, s) = s \chi(\hat{p}, s), \quad \chi^\dagger(\hat{p}, s) \chi(\hat{p}, t) = \delta_{st}, \quad (5)$$

defined with the Pauli matrices  $\vec{\sigma}$ , and  $\epsilon \equiv i\sigma^2$ . The quantisation volume  $V$  in (2) should be taken as the size of the plasma. Note that for states with relativistic momentum,  $\nu(x)$  is dominated by annihilation operators of  $s = -1$  and creation operators of  $+1$ .

The electrons and positrons are described by  $e(x)$  and  $e^c(x)$  in the two-component notation.  $e(x)$  has the charged current coupling to a W boson, while  $e^c(x)$  does not. They have the same expansion as Eq. (2) if one does proper replacements; in  $e(x)$  the annihilation operators stand for electrons,  $a_{\vec{p}, s}(e)$ , and the creation operators for positrons,  $b_{\vec{p}, s}^\dagger(e)$ , while in  $e^c(x)$  the annihilation operators stand for positrons,  $b_{\vec{p}, s}(e)$ , and the creation operators for electrons,

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<sup>1</sup>For the case of Dirac neutrinos, one readily gets the corrections by simply replacing  $a_{\vec{p}, s}^\dagger(\nu)$  in (2) with  $b_{\vec{p}, s}^\dagger(\nu)$ , the creation operator for the anti-neutrino state. It does not affect our discussion in this letter.

$a_{\vec{p},s}^\dagger(e)$ ; the energy in the expressions of  $u(\vec{p},s)$  and  $v(\vec{p},s)$  is understood to be calculated with  $m_e$ .

In terms of these fields, the one-loop induced electromagnetic vertex for the neutrinos has the form

$$i\mathcal{L}_{\text{eff}} = \left( \frac{-ieg^2}{2} \right) V_{\nu_a e} V_{\nu_b e}^* \nu_a^\dagger(p_1) F^\mu(p_1, p_2) \nu_b(p_2) A_\mu(q), \quad (6)$$

where,  $e$  and  $g$  are the electromagnetic and charged current coupling constants,  $V_{\nu_a e}$  is the element of the CKM matrix between  $\nu_a$  ( $a = e, \mu, \tau$ ) and  $e$ , and the fields are written in their Fourier components of momenta  $q$ ,  $p_1$ , and  $p_2$ , that satisfy  $q + p_1 + p_2 = 0$ . The structure function  $F^\mu$  in Eq. (6) is given from the expectation value of a T-product,

$$F^\mu(p_1, p_2) \equiv \int \int dx dy e^{-ip_1 x - ip_2 y} \times \langle | \text{T} W_\nu^+(x) \bar{\sigma}^\nu e(x) [e^\dagger(0) \bar{\sigma}^\mu e(0) - e^{c\dagger}(0) \bar{\sigma}^\mu e^c(0) + \dots] e^\dagger(y) \bar{\sigma}^\lambda W_\lambda^-(y) | \rangle, \quad (7)$$

where the state

$$| \rangle = \prod_{|\vec{p}| < p_F, s = \pm 1} a_{\vec{p},s}^\dagger(e) | 0 \rangle \quad (8)$$

represents the degenerate electron plasma;  $\bar{\sigma}^\mu \equiv (1, -\vec{\sigma})$ . We have abbreviated the contribution from  $W$ 's electromagnetic vertex in Eq. (7). The calculation is carried out by modifying the one for a non-relativistic plasma [9]. The usual Feynman rule applies if one uses the propagators that take Pauli exclusion principle into account. They are obtained from Eq. (1) in Ref. [5]. Since we are interested in the effect proper to a degenerate plasma, we subtract the vacuum contribution  $F_{\text{vc}}^\mu$  from  $F^\mu$  and concentrate on the remaining term  $F_{\text{dg}}^\mu = F^\mu - F_{\text{vc}}^\mu$  ( $F_{\text{vc}}^\mu$  is defined by  $F^\mu$  with zero  $p_F$ ).

We first specify the form of  $F_{\text{dg}}^\mu$  taking various conditions into account. Since we have assumed the plasma is isotropic and homogeneous, the temporal component  $F_{\text{dg}}^0$  is a scalar while the spacial components  $\vec{F}_{\text{dg}}$  are vectors. To leading order in  $1/m_W$ , only the contribution of Fig. 1 (a) remains and the  $W$  boson propagator can be safely contracted to a point form,  $G_{\mu\nu} \sim g_{\mu\nu}/m_W^2$ . This is because the loop momentum is restricted by  $p_F$  in  $F_{\text{dg}}^\mu$ . Thus  $F_{\text{dg}}^\mu$  depends only on  $q$ . There are two structures, 1 (unit matrix) and  $\vec{\sigma} \cdot \vec{q}$ , for  $F_{\text{dg}}^0$  and four structures,  $\vec{q}$ ,  $(\vec{\sigma} \cdot \vec{q})\vec{q}$ ,  $\vec{\sigma}$ , and  $\vec{\sigma} \times \vec{q}$ , for  $\vec{F}_{\text{dg}}$ . They are also constrained by the gauge invariance,  $q^0 F_{\text{dg}}^0 - \vec{q} \cdot \vec{F}_{\text{dg}} = 0$ . Finally, we recall the study on  $F_{\text{dg}}^\mu$  in Ref. [5] for the plasmon decay.

Although it was done with the four Fermi interaction, the result still applies because the W boson propagator is safely contracted. The four Fermi interaction is Fierz-transformed into the product of the neutral current of neutrinos and the V-A current of electrons.  $F_{\text{dg}}^\mu$  is then divided into two components, one from electrons' vector current and the other from their axial vector current [5]. The vector component is related by  $F_{\text{dg}}^\mu \sim \Pi_\nu^\mu \bar{\sigma}^\nu / 2m_W^2$  to the polarization tensor  $\Pi_{\mu\nu}$  and, thus, turns out to consist of two independent structures. For the axial component, we are left with only one available structure  $\vec{\sigma} \times \vec{q}$ . We realize that  $F_{\text{dg}}^\mu$  has the form

$$F_{\text{dg}}^0 = f_l \left[ |\vec{q}|^2 + q^0(\vec{\sigma} \cdot \vec{q}) \right], \quad (9)$$

$$\vec{F}_{\text{dg}} = f_l q^0 \vec{q} \left[ 1 + \frac{q^0}{|\vec{q}|^2} (\vec{\sigma} \cdot \vec{q}) \right] + f_t \left[ |\vec{q}|^2 \vec{\sigma} - (\vec{\sigma} \cdot \vec{q}) \vec{q} \right] + f_m i(\vec{\sigma} \times \vec{q}) \quad (10)$$

with three form factors  $f_l$ ,  $f_t$ , and  $f_m$ , which depend on the rotational invariants  $|\vec{q}|$  and  $q^0$ .<sup>2</sup> Applying the time reversal invariance of  $|\rangle$ , we see that all of the form factors are an even function of  $q^0$ .

The flavor-changing magnetic dipole moment  $\mu_{ab}$  is related to  $f_m$  by

$$\mu_{ab} = \frac{eg^2}{2} V_{\nu_a e} V_{\nu_b e}^* f_m, \quad (11)$$

where we have used the definition that the corresponding Lagrangian is written as

$$\mathcal{L}_m = \mu_{ab} (\nu_a^\dagger \vec{\sigma} \nu_b) \cdot \vec{B}, \quad (12)$$

with the magnetic field  $\vec{B}$ . Under the CP transformation,  $f_m$  is odd (while  $f_l$  and  $f_t$  are even). That means  $\mu_{ab}$  changes its sign if the plasma is made of positrons, or positrons and electrons contribute destructively to  $\mu_{ab}$  if they coexist in a plasma.

We evaluated  $I \equiv \text{Tr}(\sigma^j F_{\text{dg}}^i) / 2$  to extract out  $f_m$ . Keeping only the terms that are proportional to the anti-symmetric tensor  $\epsilon_{ijk}$ , we found

$$\begin{aligned} I &= i\epsilon_{ijk} \left( \frac{1}{m_W^2} \right) \int \frac{d\vec{p}}{(2\pi)^3} \\ &\times \left[ \left( \frac{p^k + q^k}{E(\vec{p} + \vec{q})} - \frac{p^k}{E(\vec{p})} \right) \frac{\theta(|\vec{p} + \vec{q}| - p_F) \theta(p_F - |\vec{p}|) (E(\vec{p} + \vec{q}) - E(\vec{p}))}{(q^0)^2 - ((E(\vec{p} + \vec{q}) - E(\vec{p}))^2} \right. \\ &\left. - \left( \frac{p^k + q^k}{E(\vec{p} + \vec{q})} + \frac{p^k}{E(\vec{p})} \right) \frac{\theta(p_F - |\vec{p} + \vec{q}|) (E(\vec{p} + \vec{q}) + E(\vec{p}))}{(q^0)^2 - ((E(\vec{p} + \vec{q}) + E(\vec{p}))^2} \right], \quad (13) \end{aligned}$$

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<sup>2</sup>As far as the decay of a plasmon is concerned,  $f_l$  and  $f_t$  give dominant contribution over  $f_m$  [5] (See also [10]).

where  $E(\vec{p}) = (\vec{p}^2 + m_e^2)^{1/2}$ . The first term comes from the electron-hole excitation; the second represents the vacuum polarization of an electron-positron pair that is now forbidden if the electron has the momentum in the Fermi sphere. We are interested in the static component of  $\mu_{ab}$ ; for  $q^0 = 0$  and  $|\vec{q}| \ll p_F$ , we found

$$f_m = -\frac{1}{4\pi^2} \frac{p_F}{m_W^2}. \quad (14)$$

Details of the calculation will be presented elsewhere [11].

Let us turn to a neutrino oscillation induced by  $\mu_{ab}$ . The oscillation we are considering here is the one where a neutrino in one of the flavors (mass eigenstates) oscillates into another flavor in the presence of an external magnetic field  $\vec{B}$ . Thus, it is conceptually different from the vacuum oscillation [12] in which a neutrino, created in an eigenstate of the weak interaction coupled to an electron, oscillates into another kind. We assume the neutrino energy  $E$  is relativistic,  $m_\nu/E \ll 1$ , in the Lorentz frame where the plasma has zero mean velocity. We also assume the deviation of  $\vec{B}$  from the completely static and homogeneous configuration is small, and the energy and momentum transfer from  $\vec{B}$  to the neutrino is negligible compared with  $E$ .

In a vacuum, where a flavor-changing process is suppressed, a physically interesting process induced by a magnetic moment is an oscillation between two different helicities. This is the spin rotation discussed in Refs. [1, 2]. This helicity-flipping process is, however, suppressed by the factor  $(m_\nu/E)$  as one can immediately see from the explicit form for the coupling, Eq. (12), and the expansion, Eq. (2).<sup>3</sup> An advantage in a degenerate plasma is that an oscillation is possible between two different flavors keeping the helicity intact and thus without the suppression of  $m_\nu/E$ .

For the sake of clearness of the argument, we specifically consider an oscillation between  $\nu_e$  and  $\nu_\mu$  with  $s = -1$ . We adopt a two-state approximation; we restrict the Hilbert space with two states,  $|\nu_e\rangle = a_{\vec{p}_1, -1}^\dagger(\nu_e)|\rangle$  and  $|\nu_\mu\rangle = a_{\vec{p}_2, -1}^\dagger(\nu_\mu)|\rangle$ , and consider an oscillation just between them. Based on the assumptions mentioned above, we assume the energies are the same. Then the momenta (whose magnitudes are  $\sqrt{E^2 - m_{\nu_e}^2}$  and  $\sqrt{E^2 - m_{\nu_\mu}^2}$ ) should be slightly different, which is taken account for by a small momentum transfer  $\vec{q}$  from  $\vec{B}$ .

The Hamiltonian  $H$ , a  $2 \times 2$  matrix in our approximation, get off-diagonal matrix elements

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<sup>3</sup>The proportionality of  $\mu_\nu$  to  $m_\nu$  comes from this factor in our two component notation.

in the presence of  $\vec{B}$ . It reads

$$H = \begin{pmatrix} E & A \\ A^* & E \end{pmatrix}, \quad (15)$$

where

$$A = \int_V d\vec{x} \langle \nu_e | (-\mathcal{L}_m) | \nu_\mu \rangle \simeq \mu_{e\mu} \cos \theta |\vec{B}|, \quad \hat{p} \cdot \hat{B} = \cos \theta. \quad (16)$$

The new eigenstates are a mixture of  $|\nu_e\rangle$  and  $|\nu_\mu\rangle$  with equal weights and a different relative phase; their energy difference is  $\Delta E = 2|A|$ . The half-oscillation length  $L^{(1/2)} = \pi/2\Delta E$  characterizes the oscillation; the relativistic neutrino that is initially in  $|\nu_e\rangle$  gets a fifty percent probability to be detected as  $|\nu_\mu\rangle$  after a travel of this length.

We mention a few of the characteristics of  $L^{(1/2)}$ , other than the absence of the suppression of  $m_\nu/E$  mentioned above. First, it does not depend on the energy or the mass difference of the neutrinos, while in the vacuum oscillation the oscillation length is given by  $\pi E/(m_{\nu_\mu}^2 - m_{\nu_e}^2)$ . Secondly, it inversely proportional to  $\cos \theta$ ; the oscillation takes place most efficiently for the momenta nearly parallel to  $\vec{B}$ , while the spin rotation is most efficient when the neutrino travels perpendicular to  $\vec{B}$  [1].

The stellar interiors are the candidates where the magnetic-moment-induced neutrino oscillation may have a physical importance. We relate  $p_F$  to the mass density  $\rho$  of the interior and the electron's number fraction  $Y_e$  per baryon, and write the oscillation length as

$$L^{(1/2)} = 6.1 \times 10^{17} \text{ cm} [Y_e \rho (\text{g/cm}^3)]^{-1/3} [|\vec{B}| (\text{G})]^{-1} [|V_{\nu_e e} V_{\nu_\mu e}^* \cos \theta|]^{-1} \quad (17)$$

with values of  $\rho$  in units of gram per cubic centimeter and of  $|\vec{B}|$  in Gauss. Our zero temperature calculation applies for the case  $p_F \gg T$ , which reads  $[Y_e \rho (\text{g/cm}^3)]^{1/3} \gg 1.7 \times 10^{-8} T (\text{K})$  with  $T$  in units of Kelvin. For the solar neutrinos, the oscillation is too slow; even taking optimistic values,  $\rho \sim 10^2 \text{ g/cm}^3$  and  $|\vec{B}| \sim 10^3 \text{ G}$ , we get  $L^{(1/2)} \sim 10^{14} \text{ cm} Y_e^{-1/3} [|V_{\nu_e e} V_{\nu_\mu e}^* \cos \theta|]^{-1}$ , which is much larger than the solar radius  $\sim 7 \times 10^{10} \text{ cm}$ .

An interesting possibility is that the oscillation may convert  $\nu_e$  from a supernova to  $\nu_\mu$  or  $\nu_\tau$ . A massive star, whose mass is bigger than  $8M_\odot$ , has its core consist mainly of the Fe elements at the final stage of its evolution. The degenerate electron plasma plays an important role to keep the core from collapsing. It, however, fails when the mass of the core exceeds the Chandrasekhar mass. The core begins to collapse and  $\nu_e$  is copiously produced by the electron capture by the Fe nuclei. The collapse eventually stops when the central density reaches to

the nuclear density, and a shock wave is formed at the central region of the core. The shock wave then produces the  $\nu_e$  burst by the neutronization of free protons when it propagates outward in the core [13]. The  $\nu_e$  flux generated by these processes in the very early stage of the explosion is converted to another kind if there penetrates a strong enough magnetic field in the core; the resulting flux becomes a composition of  $\nu_e$  and a substantial amount of  $\nu_\mu$  or  $\nu_\tau$ . We estimate the required field strength for this to happen by the condition that  $L^{(1/2)}$  is shorter than  $R_{\text{core}}$ , the core radius. Using the values [13],  $R_{\text{core}} \sim 10^7 \text{ cm}$  and  $\rho \sim 10^{10} \text{ g/cm}^3$ , we find  $|\vec{B}| > 10^8 \text{ G } Y_e^{-1/3} [|V_{\nu_e} V_{\nu_\mu}^* \cos \theta|]^{-1}$ . This magnitude seems realizable if one compares it with the value  $|\vec{B}| \sim 10^{13} \text{ G}$  which is possible for a supernova or a neutron star as discussed in Ref. [1].

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## Figure Caption

Fig. 1: The Feynman diagrams for the magnetic dipole moment of a neutrino in the standard model.