ON THE BRST COHOMOLOGY OF N=2 STRINGS *

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ABSTRACT

We analyze the BRST cohomology of the critical N=2 NSR string using chiral bosonization. Picture–changing and spectral flow is made explicit in a holomorphic field basis. The integration of fermionic and $U(1)$ moduli is performed and yields picture– and $U(1)$ ghost number–changing insertions into the string measure for n-point amplitudes at arbitrary genus and $U(1)$ instanton number.

Strings with two world-sheet supersymmetries have been around for almost 20 years, [1] with recently renewed interest. [2] The gauge-invariant N=2 string world-sheet action is given by coupling N=2 supergravity to two complex N=2 scalar matter multiplets (X^μ, ψ^μ), μ=0, 1. Superconformal gauge fixing produces conformal ghosts (b, c), complex N=2 superconformal ghosts (β, γ), and real $U(1)$ ghosts $(\tilde{b}, \tilde{c})$. The N=2 superconformal algebra is generated by the currents $T_{\rm tot}$, $G^{\pm}_{\rm tot}$, and $J_{\rm tot}$.

In contrast to the N=1 string, chiral bosonization of ψ^μ and β, γ depends on the field basis. In the 'real basis', one bosonizes real and imaginary parts of the complex NSR fermions and their ghosts, whereas holomorphic and antiholomorphic combinations are taken in the 'holomorphic basis'. If not indicated otherwise, we shall work in the holomorphic basis which has the advantage of diagonalizing the $U(1)$ symmetry. The $U(1)$ charges appear as $\pm$ superscripts. In this way, $\psi^{\pm\mu}$ and $\beta^\pm, \gamma^\pm$ are replaced by two pairs of bosons, $\phi^\pm$ and $\varphi^\pm$, plus two auxiliary fermion systems $(\eta^\pm, \xi^\pm)$, spanning an extended Fock space containing $\mathbf{Z}\times\mathbf{Z}$ copies of the original fermionic one. It is easy to see that BRST non-trivial operators must have vanishing conformal dimension and $U(1)$ charge. Further grading of the cohomology is effected by the mass level, the total ghost number $u \in \mathbf{Z}$, and two picture numbers $\pi^\pm \in \frac{1}{2}\mathbf{Z}$, with $\pi^+ + \pi^- \in \mathbf{Z}$. Integral and half-integral picture numbers correspond to NS and R states, respectively. For generic momenta, we find[3] that the BRST cohomology on the massless level consists of *four* classes of states for each pair (π^+, π^-), labelled by $v \equiv u - \pi^+ - \pi^- \in \{1, 2, 2, 3\}$ and created by vertex operators of type c, $\tilde{c}$, $c\partial c$ and $\tilde{c}c\partial c$.

Physical states correspond to *classes* of BRST cohomology classes, formed under the following four equivalence relations. First, c– and $c\partial c$–type vertices are to be identified just as in the bosonic string. Second, $\tilde{c}$–type vertex operators get converted to others by applying the $U(1)$ ghost number–changing operator Z^0. Third, two picture–changing operators $Z^\pm$ raise the picture numbers of vertex operators by unit amounts. Fourth, NS and R states are connected by the action of the spectral-flow

*Supported in part by the 'Deutsche Forschungsgemeinschaft'

operators $SFO^\pm$ which move $(\pi^+,\pi^-)\to(\pi^+\pm\frac12,\pi^-\mp\frac12)$. These maps are given by

$$Z^0=\oint\tilde b\,\delta\Big(\oint J_{\rm tot}\Big)\quad,\quad Z^\pm(z)=\delta(\beta^\pm)\,G^\pm_{\rm tot}\quad,\quad SFO^\pm(z)=\exp\{\pm\tfrac12\int^z J_{\rm tot}\}\quad, \tag{1}$$

and commute with $Q_{\rm BRST}$ but are non-trivial. In this fashion, each physical state has a representative $V^{\rm can}$ at $v{=}2$ ($\tilde c c$–type) in the canonical picture $(\pi^+,\pi^-)=(-1,-1)$. On the massless level, only a single scalar excitation survives. For the computation of string amplitudes, however, vertex operators in various other ghost and picture sectors are useful and have been constructed.[3]

Any n–point amplitude involves a sum over genera $h\in\mathbf{Z}^+$ and $U(1)$ instanton number $c\in\mathbf{Z}$. To compute the contribution for fixed h and c, one must integrate out $2h{-}2{\pm}c{+}n$ complex fermionic moduli of $U(1)$ charge ±1, respectively, and $h{-}1{+}n$ complex $U(1)$ moduli, to obtain an integration measure for the remaining $3h{-}3{+}n$ complex metric moduli.[a] The result vanishes for $|c|{>}2h{-}2{+}n$ and symbolically reads[4]

$$\Big\langle\Big|(\oint b)^{3h-3+n}\,(Z^+)^{2h-2+c+n}\,(Z^-)^{2h-2-c+n}\,(Z^0)^{n-1}\Big|^2\prod_{i=1}^h\Big[Z^0(a_i)Z^0(b_i)\Big]\;V_1^{\rm can}\ldots V_n^{\rm can}\Big\rangle \tag{2}$$

where a_i and b_i denote the homology cycles. The picture–changers $Z^\pm$ and Z^0 may be used partially to convert vertex operators to other pictures and/or ghost numbers.

Invariance of correlation functions under spectral flow follows from the fact that a change in monodromies for the world-sheet fermions is equivalent to a shift in the integration over $U(1)$ moduli, which are nothing but the flat $U(1)$ connections on the n–punctured genus–h Riemann surface. It is realized on the vertex operators by[4]

$$V(z)\;\longrightarrow\;V^\theta(z)\;=\;\exp\Big\{\theta\int^z J_{\rm tot}(z')\,dz'\Big\}\;V(z)\quad, \tag{3}$$

with $\theta{=}\pm\frac12$ leading to $SFO^\pm$ mapping from NS to R$^\pm$ sectors.[b] Stated differently,

$$\Big\langle V_1\,V_2\ldots V_n\Big\rangle\;=\;\Big\langle V_1^{\theta_1}\,V_2^{\theta_2}\ldots V_n^{\theta_n}\Big\rangle\qquad\text{for}\qquad\sum_\ell\theta_\ell=0\quad, \tag{4}$$

equating all n–point amplitudes with the same values for h and c.

This work was done in collaboration with S. Ketov. We acknowledge fruitful discussions with N. Berkovits, H. Lü, H. Ooguri, and C. Pope.

1. see, e.g., N. Markus, hep-th/9211059, and references therein.
2. N. Berkovits and C. Vafa, *Nucl. Phys.* **B433** (1995) 123; H. Lü and C.N. Pope, hep-th/9411101; N. Berkovits, hep-th/9412179 and hep-th/9503099.
3. J. Bischoff, S. Ketov and O. Lechtenfeld, *Nucl. Phys.* **B438** (1995) 373; see also: A.Giveon and M. Roček, *Nucl. Phys.* **B400** (1993) 145.
4. S. Ketov and O. Lechtenfeld, hep-th/9503232.

[a] As always, the cases of the sphere and the torus require some modifications.
[b] The two sectors R$^+$ and R$^-$ differ by a unit change in instanton number c.